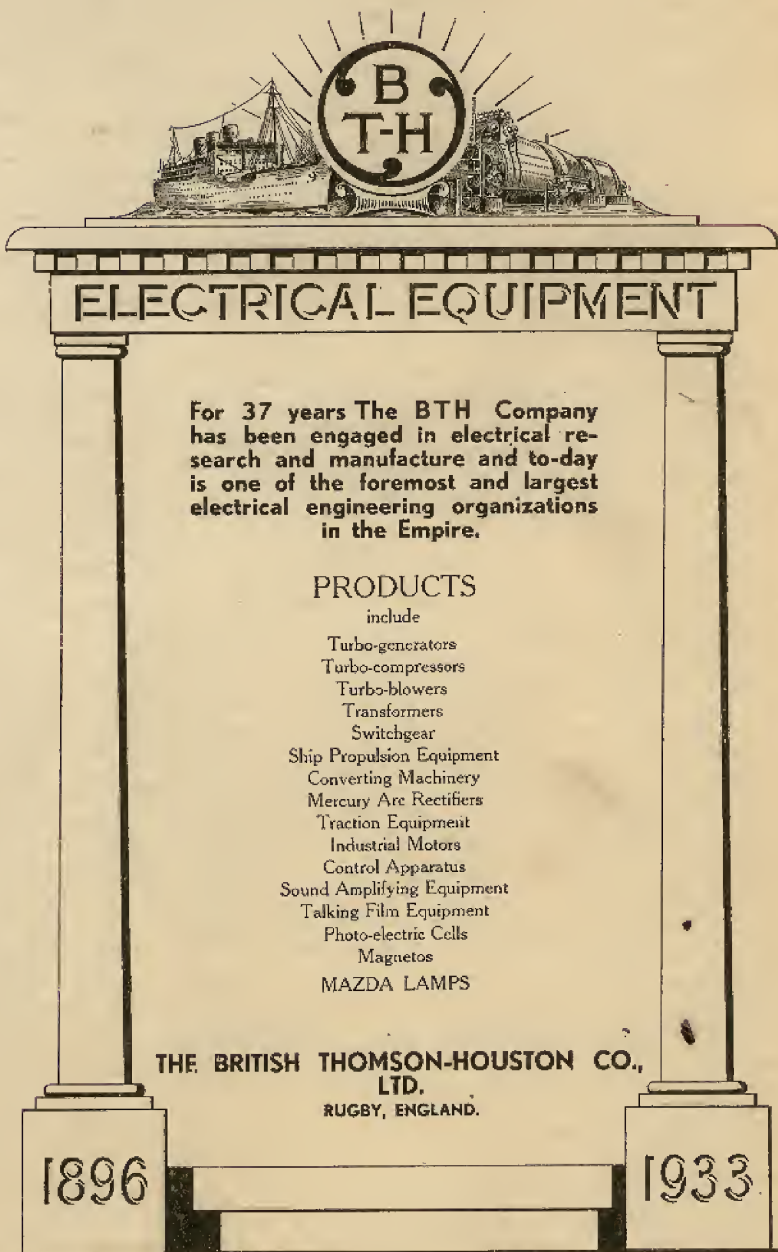


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ADMIRALTY HANDBOOK

OF

WIRELESS TELEGRAPHY,

1931.

December, 1931.

The Lords Commissioners of the Admiralty have decided that a standard work on Wireless Telegraphy is required for the information and guidance of Officers and Men of H.M. Fleet; for this purpose the "Admiralty Handbook of Wireless Telegraphy, 1931" has been prepared at H.M. Signal School.

This book supersedes "The Admiralty Handbook of Wireless Telegraphy, 1925."

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PREFATORY NOTE.

It was considered that the value of this Handbook would be enhanced by the addition of a brief elementary account of the main principles of wireless communication ; and such an account will be found in Appendix A, immediately following the final chapter.

While it is primarily intended for the use of Junior Officers and Ratings, it is hoped that this Appendix may also prove of service to others who are interested in the subject, but are precluded by temporal or other factors from making a more comprehensive study.

It will also be found that certain paragraphs in the main text have been distinguished by an asterisk, to indicate that they are of a more difficult nature than the rest of the book. Those who are unable to follow the treatment in the asterisked paragraphs may omit them without detriment to the sequence of the argument.

NOMENCLATURE OF WAVES.

THE range of frequencies of the æther waves used in wireless communication is now subdivided as follows :—

Below 100 kc./s.	Low Frequencies (L.F.).
100–1,500 kc./s.	Medium Frequencies (M.F.).
1,500–6,000 kc./s.	Intermediate Frequencies (I.F.).
6,000–30,000 kc./s.	High Frequencies (H.F.).
Above 30,000 kc./s.	Very high Frequencies (V.H.F.).

It has been a common practice in the past to refer to the oscillatory currents produced by these waves in a receiving aerial as H.F. currents, and to differentiate them from the currents of audible frequency produced after detection by using the term L.F. for the latter. It is obvious that this usage conflicts with the nomenclature in the table above ; hence in this Handbook the term **Radio Frequency (R.F.)** is applied to all currents directly produced by an incoming signal, and the currents flowing after detection are called **Audio Frequency (A.F.)** currents.

It also frequently happens that an oscillatory current whose frequency falls within the wireless range is generated by the action of a receiver, *e.g.*, in superheterodyne and quench receivers. When describing the action of such receivers it is desirable to distinguish these currents from the R.F. currents produced by an incoming signal. The designation of **Supersonic Frequency (S.F.)** currents has therefore been adopted for these currents.

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TABLE I.

The Greek Alphabet is here given for reference, as many Greek letters appear in the Text.

Letter.		Name.	English Equivalent.
Small.	Capital.		
α	A	Alpha	a
β	B	Beta	b
γ	Γ	Gamma	g
δ	Δ	Delta	d
ϵ	E	Epsilon	ē (as in "met")
ζ	Z	Zeta	z
η	H	Eta	ēē (as in "meet")
θ	Θ	Theta	th
ι	I	Iota	i
κ	K	Kappa	k
λ	Λ	Lambda	l
μ	M	Mu	m
ν	N	Nu	n
ξ	Ξ	Ksi	x
\omicron	O	Omicron	ō (as in "olive")
π	Π	Pi	p
ρ	P	Rho	r
σ	Σ	Sigma	s
τ	T	Tau	t
υ	Υ	Upsilon	u
ϕ	Φ	Phi	ph
χ	X	Chi	ch (as in "school")
ψ	Ψ	Psi	ps
ω	Ω	Omega	o (as in "broke")

TABLE II.

Symbols for Quantities for Use in Electrical Equations, etc.

Number.	Quantity.	Sign.
1	Length	l
2	Mass	m
3	Time	t
4	Angles	α, θ, ϕ
5	Work or Energy	W
6	Power	P
7	Efficiency	η
8	Period	T
9	Frequency	f
10	$2\pi \times$ frequency	ω
11	Wavelength	λ
12	Phase displacement	ϕ
13	Temperature, centigrade	t or θ
14	Temperature, absolute	T or Θ
15	Quantity or charge of electricity	Q
16	Current	I
17	Voltage (E.M.F. or P.D.)	E or V
18	Resistance	R
19	Specific Resistance or Resistivity	ρ
20	Conductance	G
21	Specific Conductance or Conductivity	γ
22	Specific Inductive Capacity or Dielectric Constant	K
23	Electrostatic Field Strength	X
24	Electrostatic Displacement or Flux Density	D
25	Electrostatic Flux	ψ
26	Capacity	C
27	Magnetic Pole Strength	m
28	Permeability	μ
29	Magnetic Field Strength	H
30	Magnetic Induction or Flux Density	B
31	Magnetic Flux	Φ
32	Magnetic Reluctance	S
33	Magneto Motive Force	G
34	Self Inductance	L
35	Mutual Inductance	M
36	Reactance	X
37	Impedance	Z
38	Susceptance	B
39	Admittance	Y
40	Base of Napierian logs	e
41	Damping Factor	α
42	Logarithmic Decrement	δ
43	Aerial Capacity	σ
44	Valve mutual conductance	g_m
45	Valve A.C. resistance (impedance)	r_a
46	Valve amplification factor	μ

TABLE III.

Distinguishing Symbols for Constant and Virtual Values of Quantities.

Number.	Quantity.	Constant Value.	Maximum Value.	Arithmetic Mean Value.	Virtual Value.	Instantaneous Value.
1	Potential Difference	V	\mathcal{V}	\bar{V}	V	v
2	E.M.F.	E	\mathcal{E}	\bar{E}	E	e
3	Charge	Q	\mathcal{Q}	\bar{Q}	Q	q
4	Current	I	\mathcal{I}	\bar{I}	I	i
5	Flux	Φ	Φ_m	$\bar{\Phi}$	Φ	ϕ
6	Magnetic Field ..	H	\mathcal{H}	\bar{H}	H	h
7	Electric Field ..	X	\mathcal{X}	\bar{X}	X	x

TABLE IV.

Prefixes for Multiples and Submultiples of Quantities.

Number.	Multiple or Submultiple.	Name.	Prefix.
1	10^6	Mega-	M
2	10^3	Kilo-	k
3	10^2	Hekto-	H
4	10^{-2}	Centi-	c
5	10^{-3}	Milli-	m
6	10^{-6}	Micro-	μ
7	10^{-9}	Millimicro-	m μ
8	10^{-12}	Micro-micro	$\mu\mu$

TABLE V.

Signs for Units Employed after Numerical Values.

Number.	Unit.	Abbreviation.
1	Ampere	A
2	Volt	V
3	Ohm	Ω
4	Coulomb	C
5	Joule	J
6	Watt	W
7	Farad	F
8	Henry	H
9	Watt-hour	Wh
10	Volt-Ampere	VA
11	Ampere-hour	Ah
12	Kilowatt	kW
13	Kilo-volt-ampere	kVA
14	Kilowatt-hour	kWh

CHAPTER I.

GENERAL INTRODUCTION.

1. In Wireless Telegraphy or Telephony we deal with the transmission, propagation and reception of Electromagnetic or Æther waves. It is the object of this book to explain this means of communication.

WAVE MOTION.

2. A "**Wave**" is a progressive disturbance in any medium, formed by the propagation of alternating pressures and tensions, without any permanent displacement of the medium itself in the direction in which these stresses are propagated.

The more concrete examples of wave motion, the waves of the sea, sound waves, &c., are very much more familiar than the wave motions associated with light, heat and wireless waves. This is because the media in which they are propagated are material. In such material media it is a movement of matter at the source which gives rise to the wave. A very simple example of this is given by the dropping of a large stone vertically into a pond of water. A wave motion is immediately produced on the surface by the impact, spreading out radially in all directions from the point where the stone strikes the water. Its form is due to alternate states of pressure and tension at the surface of the water, a crest corresponding to a pressure and a trough to a tension. It derives its energy from the loss of kinetic energy by the stone as its velocity is diminished on striking the surface.

Such a wave has certain characteristics, which we shall see later are more or less common to all types of wave motion.

- (1) **The form of the wave** travels forward, although the water itself does not travel forward to any appreciable extent. Actually the particles of water have approximately an up-and-down movement about their positions of equilibrium, and after a complete wave has passed they are in the same position as they were just before its arrival.
- (2) **The energy** possessed by the wave on starting travels forward with it and is expended on the shore of the pond. During the passage of the wave some of this energy is lost through friction, and as a result the size of the wave diminishes the further it is from its source.
- (3) **The speed of the waves** varies very little at different points along its direction of propagation. It depends primarily on the depth of the water in shallow water, and in deep water depends on surface tension, density of the water, and the length and size of the wave.

3. Any type of wave motion has four different quantities associated with it, its **Velocity**, **Frequency**, **Wavelength** and **Amplitude**.

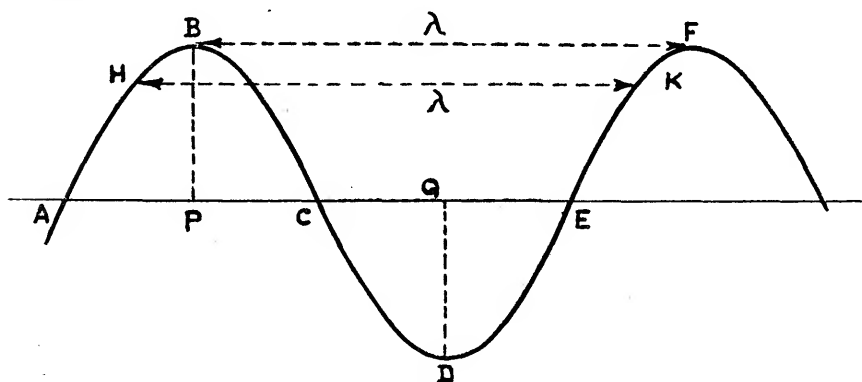


FIG. 1.

The form of the wave and the energy which it is conveying travel outwards with a certain **Velocity**.

The Frequency is the number of waves that pass a fixed point in a given interval of time. For the type of wave motion associated with wireless the unit of time is one second, and the frequency is therefore the number of waves passing a given point per second. It is denoted by the symbol "f."

The Wavelength is the distance between one wave crest and the next, or, more generally, the distance between two consecutive points at which the moving particles of the medium have the same displacement from their mean position and are moving at every instant in the same direction. It is denoted by the symbol "λ."

The Amplitude is the maximum displacement of a moving particle of the medium from its mean position, BP or DQ in Fig. 1.

To these definitions may be added that of the **Period** of the wave, which is the time in which the waveform moves forward through one wavelength, or the interval of time between passage of successive waves past a given point.

A **cycle** of a wave is one complete set of varying conditions, *i.e.*, in Fig. 1, the curve A B C D E. Portions of the wave above the mean line may be termed positive half cycles, below it, negative half cycles.

4. There are certain relationships between these quantities.

(a) The Period being the time between successive waves and the Frequency the number of waves per second,

$$\text{Period} = \frac{1}{f} \text{ seconds.}$$

- (b) The Frequency being the number of waves per second and the wavelength the distance between them, the product of these two quantities gives the speed at which the wave is travelling, i.e., its velocity, written v .

As a formula, $f \times \lambda = v$.

$$f = \frac{v}{\lambda}$$

$$\lambda = \frac{v}{f}$$

5. The first elementary type of wave we took for illustrative purposes, that on the surface of water, is one in which the particles of the transmitting medium move at right angles to the direction of propagation. Such waves are called **Transverse Waves**.

Other examples of the above are the ether waves we shall be concerned with mainly in this book. There is another class of waves called **Longitudinal Waves**, in which the particles of the transmitting medium move to and fro in the same direction as the waves are propagated. The most common example is the sound wave.

SOUND WAVES.

6. When a bell rings, it vibrates at a rate depending on its mass, shape and material, and alternately presses forward and drags backward the particles of air immediately surrounding it. These particles communicate their motion to the particles adjacent to themselves, and the action is carried on, resulting in a wave motion being set up in the atmosphere. At any given instant there are alternate states of compression and rarefaction along any direction outwards from the source of sound, and these, impinging on the ear, make the ear drum vibrate backwards and forwards at the same frequency as the source of sound. The particles of the transmitting medium, the air, which are set into vibration, move backwards and forwards in the same direction as the sound is going, i.e., radially from the transmitter, and so this is a case of longitudinal vibration.

Just as in the case of visible waves on water, the frequency will be the number of waves passing per second—in this case, the number of states of compression passing a fixed point per second, and the wavelength will be the distance between states of maximum compression.

7. **Method of Representation.**—A transverse wave has a form that can be seen, but a longitudinal wave has not. If, however, we draw lines at right angles to the direction of propagation and of lengths proportional to the amount of compression (drawn positively) or rarefaction (drawn negatively), and join up the ends of these lines, we shall get what looks like a transverse wave, and the

definitions of the quantities associated with the wave may seem more obvious.

Such a waveform may be drawn as above with the horizontal axis an axis of distance, and the waveform representing an instantaneous set of conditions, or it might represent the variations in compression and rarefaction passing a fixed point, in which case the horizontal axis would be an axis of time.

8. The physiological sensation produced by a sound wave of a certain frequency is termed its "**Pitch.**" Doubling the frequency of a sound wave raises its pitch one octave. For example, the pitch of middle C on a piano corresponds to a frequency of 256 cycles per second, the pitch of the next higher C to a frequency of 512 cycles per second, and so on.

It does not follow that all sound waves are **audible.** The best known form of sensitive receiver for sound waves is the human ear, but even this will only respond to a very limited range of pitch.

The lower limit of audibility is about 16 cycles per second. Below this frequency the ear separates the sound into its constituent parts, and the impression of a musical note is lost. In the same way the impression of continuity is lost if a cinema film is run at less than about 16 pictures per second. The upper limit of audibility varies from about 15,000 to 30,000 cycles per second, according to the person concerned.

The pipe organ usually ranges in frequency from 16 to 8,000.

The range of frequency for the human voice in singing is from 60 for a low bass voice to about 1,300 for a very high soprano.

The average male speaking voice has a frequency of about 130, and the average female speaking voice is an octave higher, frequency about 260.

9. Applying the formula of paragraph 4 (b) to the case of sound waves, we can work out their wavelength given their velocity. Sound waves in air travel outwards in all directions from their source in three dimensions with a velocity of about 1,130 feet per second, or about 12·8 miles per minute, or about 770 miles per hour.

The wavelength of the note "middle C" would therefore be $\frac{1,130}{256}$ feet = 4·4 feet, while that of the "treble C" would be $\frac{1,130}{512}$ = 2·2 feet.

10. Sound signals produced in air are very erratic in their range and intensity, due to the fact that sound may be carried by the wind or reflected or refracted (bent from its course) by layers of air of different densities, so that a sound may be audible some distance away and yet be quite inaudible at points nearer the transmitting agency in the same straight line.

11. Sound waves may also be propagated through water, as is done by a "submarine bell." They travel at a higher speed in water, their speed in any medium being given by a formula involving the density and elasticity of the medium. For water this formula gives about four times the speed in air; actually, 4,700 feet per second in fresh water, 4,900 feet per second in sea water.

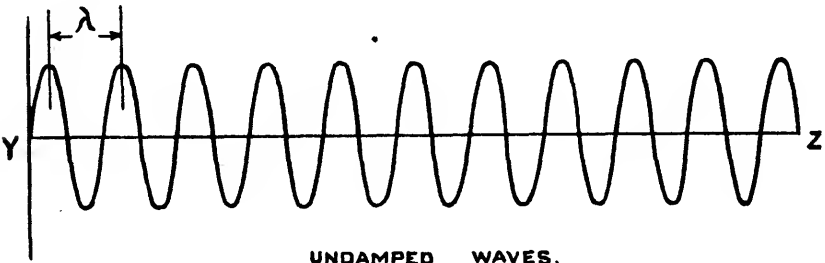
The wavelength of "middle C" in sea water is $\frac{4,900}{256} = 19.14$ feet, and in fresh water, $\frac{4,700}{256} = 18.3$ feet.

Sound waves in water can be received on a suitable receiver, such as a hydrophone, an S/T oscillator, or even through the hull of a ship.

UNDAMPED AND DAMPED WAVES.

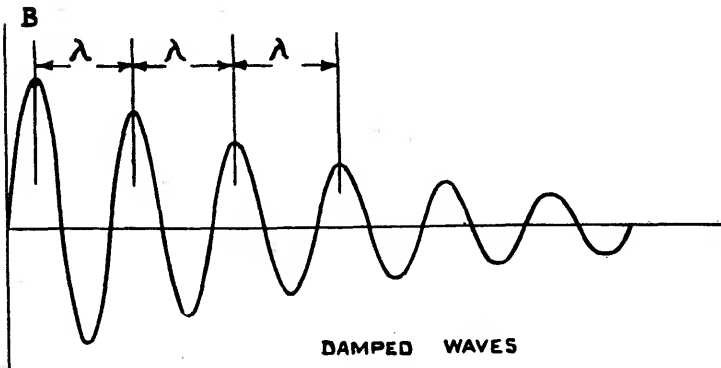
12. There is another method by which waves may be divided into two classes, viz. :—

- (1) Continuous or "**undamped**" waves, represented graphically by Fig. 2 (a) below.
- (2) "**Damped**" waves, represented by Fig. 2 (b).



UNDAMPED WAVES.

(a)



DAMPED WAVES

FIG. 2.

In these figures the horizontal axis is an axis of time, and the graphs represent amplitudes of waves passing a fixed point.

The essential feature of **undamped** or **continuous waves** is that the amplitude remains constant; if we take a common type of wave motion, **sound**, such waves are the kind produced by an organ note, where a continuous force is applied to producing the note all the time the key is pressed down.

The **damped** wave, on the other hand, has a varying amplitude; an example is the piano note, where the vibrating wire which produces the sound is set into agitation when the key is struck, and vibrates to and fro with less and less amplitude as time goes on. After the first impulse there is no steady supply of energy to it to overcome the "damping" effect of air resistance, &c., and so keep its amplitude of vibration constant.

The distinction between these types of wave is very important in wireless, because some types of transmitting apparatus produce undamped or continuous waves and some types damped waves. A group of waves as in Fig. 2 (b) is often called a wave train.

THE ÆTHER.

13. In the foregoing instances of wave motion, viz., surface sea waves and sound waves, the wave motion has depended on some movement of matter (para. 24) at the source.

There are other sorts of wave motion or vibration, called "æther waves," which are generated by the movement of electrons at their source.

Now we have seen that sound is conveyed from transmitter to receiver in the following manner: the transmitter is set in vibration; the intervening medium is set in vibration; the vibration of the medium sets the receiver in vibration.

We are led to believe in the existence of the medium we term the æther for the following reasons:—

The earth continually receives enormous quantities of energy from the sun in the form of light and heat, which travel through a space known to be empty of ordinary matter. Filaments of incandescent lamps give off light and heat, although the bulb contains practically no gas or air.

It is unreasonable to suppose that the energy in the sun or in an electric circuit disappears there and reappears at the earth (or the receiving circuit) without having been conveyed across the intervening space.

It must be conveyed across either as an actual molecular movement, like the flow of a river, or as a wave motion, like the passage of sound through air.

All experience goes to show that light and electromagnetic energy generally are transmitted through space as a wave motion, and we are led to the supposition that all space is occupied by a

medium which conveys the energy, and that this medium has properties different from those possessed by ordinary matter.

We call this medium "æther."

The medium called the æther must necessarily be universally diffused and must inter-penetrate all matter. It cannot be exhausted or removed from any place, because no material is impervious to it.

The presence of what we know as matter in its various forms may, however, modify the properties of æther so far as these æther waves are concerned.

For example, a light wave can pass through a glass window, but cannot pass through a brick wall, while a wireless wave can pass through a brick wall but cannot pass through a sheet of copper.

The æther must possess in great degree some form of elasticity—that is, resistance to any change of state produced in it—and it must also possess inertia, or a quality in virtue of which a change so made in it tends to persist.

It is clear that it has the property of being capable of storing up energy in large quantities and transmitting it from one place to another, as shown by the fact that enormous amounts of energy are hourly being transmitted from the sun to the earth.

Vibrations of the æther are **only** produced by the electric and magnetic fields associated with electrons in motion. (See Chapter II.)

Electric and magnetic stresses are passed through the æther with a definite velocity, which has been found to be* **300,000 kilometres** (i.e., 3×10^8 metres), or **186,000 miles per second**.

All movements of the æther consist of electric and magnetic forces, alternating in direction; they produce a disturbance, spreading outwards, which is called an "electro-magnetic wave," or simply an "æther wave."

ÆTHER WAVES.

14. According to their frequencies, æther waves produce different effects, and require different methods of generation and detection. All these waves convey energy, which can be converted to heat energy by means of a suitable detector.

The range of frequencies so far discovered is shown in Table VI.

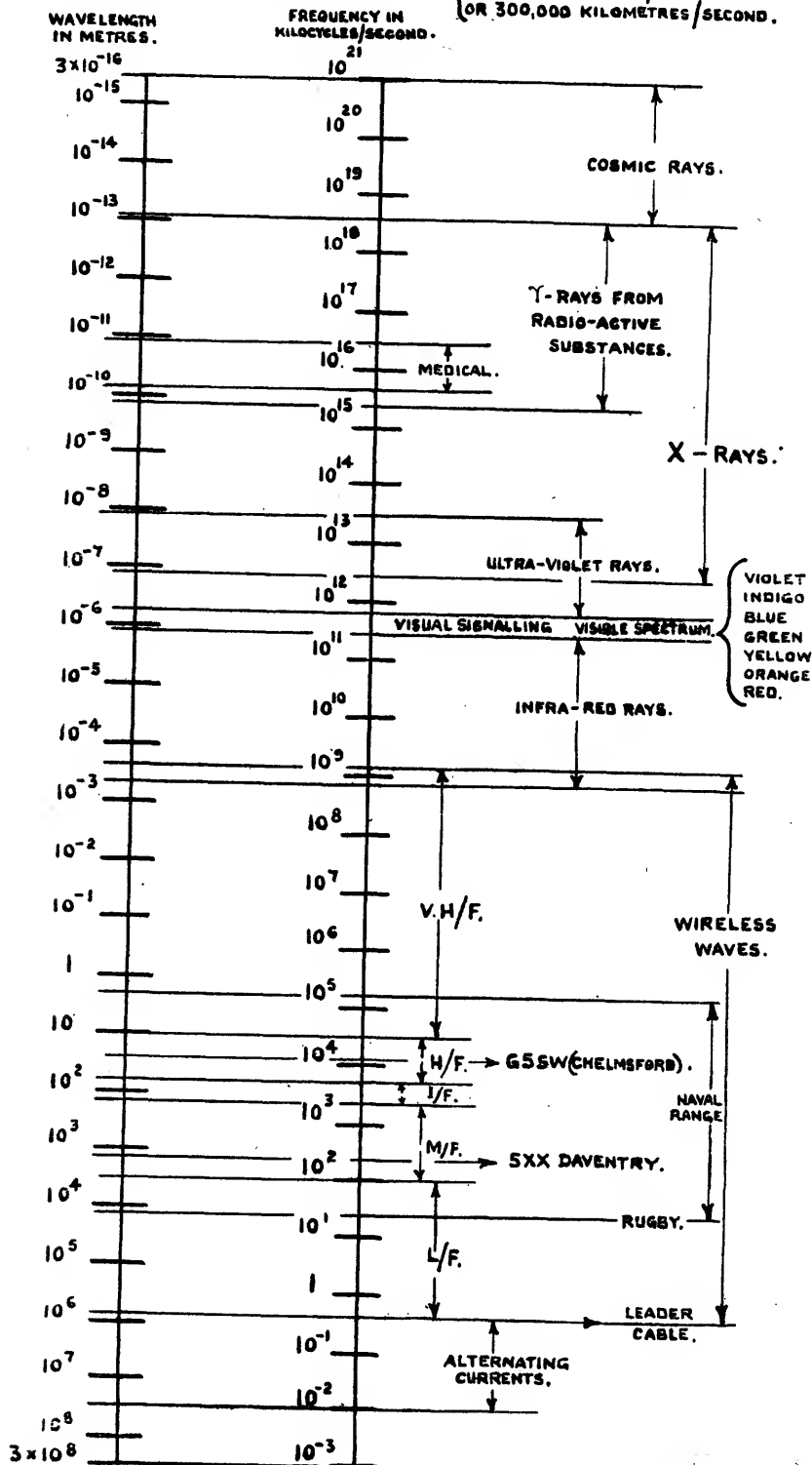
Æther waves are of a totally different nature from sound waves, and even at audible frequencies they do not affect the ear directly and cannot be heard.

They are classified in groups or bands according to the way in which they are generated. It will be seen that certain bands on the chart overlap, e.g., the ultra-violet and X-ray bands, and the infra-red and wireless bands. This merely means that the same

* This figure is an approximation. The figure which is generally accepted as being accurate is 2.9982×10^8 metres per second.

TABLE VI WAVES IN FREE ÆTHER.

VELOCITY OF PROPAGATION = $\begin{cases} 3 \times 10^8 \text{ METRES/SECOND.} \\ \text{OR } 300,000 \text{ KILOMETRES/SECOND.} \end{cases}$



wave can be generated by two different methods. The properties of the wave are unaffected by its mode of generation.

The waves of highest frequency so far discovered are the so-called "penetrating radiation" or cosmic rays, which appear to be produced in the depths of space and reach the earth in all directions.

The next range is that of X-rays, which are produced by the sudden stoppage of very fast-moving electrons, and are of value for medical purposes. The higher frequencies of this range are also covered by the Gamma (γ)-rays emitted in the disintegration of radio-active substances, some of which are used in medicine.

At its lower end, the X-ray band overlaps with the ultra-violet rays, which are radiated from very hot bodies and ionised gases. These are the rays which affect a photographic plate, and are also valuable rays in "sun-bathing."

The next range is the visible spectrum—the band of frequencies which can be directly detected by the eye. When all present, as in the case of the sun's radiation, they give the sensation of white light, but the different frequencies present can be made visible by passing the white light through a glass prism.

The rays below the red end of the visible spectrum, or infra-red rays, are radiated from hot bodies, *e.g.*, a poker not heated to redness. Their lowest frequencies overlap with the waves on wires produced by electrical means, *i.e.*, the highest frequencies of the wireless range.

This brings us to the range of wireless waves from about 10 kc/s to 10^{10} kc/s.

They are of too low a frequency to be perceived directly by the eye, and have to be collected on an aerial, and then made perceptible to the senses in one of a variety of ways—generally by giving rise to sounds in a pair of telephones.

They have the supreme advantages over any other form of signalling that they follow the curvature of the earth, and do not suffer nearly so much from dissipation in the atmosphere, and so are suitable for signalling to the greatest possible distances. It is only a question of the use of suitable power and receiving gear for stations to communicate with half the circumference of the earth between them.

15. Table VI is drawn on a "logarithmic" scale, in which equal distances along the vertical axis represent, not equal differences in magnitudes of numbers, but equal differences in their logarithms.

The frequencies and wavelengths correspond according to the formula of para. 4:—

$$f \times \lambda = v.$$

In the case of electromagnetic waves this velocity is 3×10^8 metres per second, so that, for instance, a frequency of 10^6 cycles/second corresponds to a wavelength of 300 metres.

The above velocity is, strictly speaking, only correct for æther, but the velocity of electromagnetic waves in air is practically the same. In other substances it may be very different, a common example being that the velocity of light in glass is different from what it is in air. Light of the same frequency, however, moving through glass or air sets up the same sensation of colour in the eye, so it is the frequency which distinguishes one electromagnetic wave from another, and not the wavelength.

Hence the common practice hitherto of alluding to wireless waves by their wavelengths is not so correct as if we allude to them by their frequencies. It has been assumed that they are propagated in air. For this reason, and others, it has been decided that wireless waves will be quoted in frequencies in future, and, owing to the large quantities that would be involved, using "cycles per second," the unit "kilocycles per second" is used. A **kilocycle** is 1,000 cycles.

The product of wavelength in metres and frequency in kilocycles per second is equal to the velocity in kilometres per second. Hence, to convert a given wavelength into frequency in kilocycles, it is necessary to divide **300,000** by the wavelength in metres, and *vice versa*.

Thus the expression "a wavelength of 300 metres" is more accurately represented by "a frequency of 1,000 kilocycles per second." The recognised abbreviation for "kilocycles per second" is kc/s.

In the case of V.H.F. waves, the number which gives the frequency in kc/s. becomes very large and such frequencies are usually quoted in "megacycles per second" (Mc/s.).

$$\begin{aligned} 1 \text{ megacycle per second} &= 1,000,000 \text{ cycles per sec.} \\ &= 1,000 \text{ kilocycles per sec.} \end{aligned}$$

To express the frequency in Mc/s., **300** should be divided by the wavelength in metres and *vice versa*, e.g., a wavelength of 20 metres corresponds to a frequency of $\frac{300}{20} = 15 \text{ Mc/s.}$

16. For a proper understanding of W/T, the main essential is a thorough grasp of the principles of electricity and of the laws governing alternating and direct currents. Once these are mastered, W/T in itself will be found to be easy of comprehension, provided the student has some imagination, since one is dealing with a wave motion which is invisible, inaudible and intangible.

The problem in W/T is to maintain an electrical oscillation in an aerial circuit.

The nature and appearance of an aerial may be assumed to be familiar to everyone nowadays.

An aerial circuit is a natural electrical oscillator.

In virtue of certain properties termed "inductance" and "capacity," which are associated with it, it is just as ready to be set in electrical oscillation as is the balance wheel of a watch to be set in mechanical oscillation.

17. The balance wheel of a watch is such a very useful and accurate analogy all through the study of W/T that we may well stop for a moment to consider it.

The wheel is carefully balanced and mounted in perfect bearings. To its centre is attached one end of a fine spring—the “hair spring.”

The tension of this hair spring is adjustable by means of the “regulator.”

The balance wheel oscillates at a rate depending on its weight and the tension on the hair spring.

In its oscillation, it operates the escapement, which controls the rate at which the main spring is allowed to uncoil and move the hands of the watch; conversely, the main spring supplies the energy requisite for maintaining the oscillation of the balance wheel.

Now the weight or inertia of the wheel and the elasticity of the hair spring are two mechanical properties which correspond respectively to the electrical properties of the aerial referred to above—**inductance and capacity.**

The aerial circuit may be thought of as the balance wheel and the various systems of energising such a circuit described in this book as the hair spring and escapement.

18. The four methods employed for the transmission of W/T are :—

- (a) The spark system.
- (b) The arc system.
- (c) The valve system.
- (d) The high frequency alternator system.

Method (a) generates damped waves.

Methods (b) and (d) generate undamped or continuous waves; method (c) generates damped or undamped waves at will.

In the Naval service we are not concerned with the high frequency alternator system.

19. The relative merits of these four systems may be classified as follows. (The full significance of the points dealt with below may not be appreciated until the chapters dealing with the respective systems have been read. They are inserted here for convenience.)

(a) The Spark System.

Advantages.

- (1) Robust and durable.
- (2) Faults easily cleared.
- (3) Emits a wave which forces its way well through interference.

Disadvantages.

- (1) Wasteful of power.
- (2) Short range as compared with continuous wave generators.
- (3) Interferes badly.
- (4) Requires high insulation on account of initial peak voltages.

(b) The Arc System.*Advantages.*

- (1) Robust and durable.
- (2) Faults easily cleared.
- (3) Can be easily constructed to handle large powers.

Disadvantages.

- (1) Slow in starting up.
- (2) Presents certain "keying" difficulties.
- (3) High-power sets radiate harmonics badly.
- (4) Unsuitable for use in a fleet, as it is not possible to "listen through" for Admiral's signals, messages of distress, &c.
- (5) High frequencies cannot be produced, the limiting value being 250 kc/s.

(c) The Valve System.

- (1) Radiates a very pure wave.
- (2) Easy to key.
- (3) Very suitable for radio-telephony.
- (4) Transmits damped or undamped waves at will.
- (5) Quick in starting up.
- (1) Valves are fragile and require frequent replacement.
- (2) If faults develop, they are not so easy to trace as in other sets.

(d) The High Frequency Alternator System.

- (1) Radiates a wave which is very pure and free from harmonics.
- (2) Easy to key.
- (3) Very suitable for radio-telephony.
- (4) Suitable for high power working.
- (1) Requires very expert supervision and maintenance.
- (2) Its frequency cannot be varied so readily as in other systems.
- (3) Its first cost is high.
- (4) Only suitable for low frequencies.

The first three systems are described fully in subsequent chapters.

20. When the aerial circuit is set in oscillation by one of the above methods, a succession of waves is set up in the æther.

These waves spread out in all directions over the surface of the earth, in circles of ever-increasing radius.

Whenever they encounter any other aerial, they endeavour to set it in electrical oscillation, as the wind sets trees shaking.

The currents in the receiving aerial are passed through a "detector," which renders them suitable for energising a device—a pair of telephones, a loudspeaker, a tape machine, &c.—which renders them perceptible to one of the senses.

21. Notice the sequence—the transmitter—the intervening medium—the receiver.

In the same way, in speaking, the transmitter is the human throat and mouth, the medium which carries the sound waves is the air, the receiver is the drum of the ear.

In signalling with a flashing lamp, the transmitter is the lamp, the medium is the æther which carries the light waves, the receiver is the eye.

22. Further, wireless may be used for telephony. The only difference is that the human voice is passed through a microphone in order to vary or modulate the aerial current, instead of using a morse key. The wave is carried to the receiver and detected in the same manner as above.

The direction from which a wireless wave is coming may be determined by the use of "direction-finding" apparatus. This is of very great benefit in navigation and is dealt with in a subsequent chapter.

We must now proceed to discuss electricity in general, direct and alternating currents, and the machinery required for producing them, before passing to the details of the various transmitting and receiving circuits used in wireless telegraphy.

CHAPTER II.

ELECTRICITY AND MAGNETISM.

23. In this chapter the elementary principles of electricity and magnetism are explained, in so far as they affect wireless telegraphy and the circuits used in wireless telegraphy.

CONSTITUTION OF MATTER.

24. Matter.—It is difficult, if not impossible, to define rigorously what is meant by matter. It may be taken loosely to mean anything which occupies space and is attracted to the earth's surface when in the neighbourhood of the earth, *i.e.*, has weight.

Matter may exist in three states—solid, liquid and gaseous.

Solid matter tends to preserve its shape almost indefinitely, *e.g.*, jewels and ornaments found in the excavated remains of ancient civilisations still retain as sharp outlines as on the day they left the hand of the craftsman.

Liquid matter has no shape of its own and takes the shape of any vessel in which it may be contained.

Gaseous matter also has no shape of its own, but in addition, unlike liquids, it will adapt its volume to that of its containing vessel and fill it completely.

Matter may assume all three states according to the temperature and pressure to which it is subjected. Thus, at normal atmospheric pressure, water (liquid between temperatures of 0°C. and 100°C.) can become ice (solid below a temperature of 0°C.) or steam (gaseous above 100°C.).

Air (gaseous at ordinary temperature and pressure) can become liquid at a very low temperature and high pressure, and solid at an even lower temperature.

25. Molecules.—It is not possible to take a piece of matter and go on dividing it indefinitely into smaller pieces. A stage is reached in which any further division alters completely the properties of the original matter.

The smallest portion of any substance which can be subdivided without its properties being altered is called a "molecule" of the substance. It exhibits the same chemical and physical properties as the substance in bulk.

There are as many different kinds of molecules in the universe as there are different kinds of substances—an almost limitless number.

The distinction between the different states of matter may be examined in the light of this molecular theory.

26. In all states of matter molecules are in continuous rapid motion.

In a **solid** the molecules are crowded very closely together so that, although their motion is rapid, it is an oscillation about an average position. It may be likened to that of a man in a crowd where it is almost or quite impossible for him to leave the space he occupies between his neighbours; yet he may turn round and have some motion from side to side. It is the attractive force between molecules, very great because of their proximity to each other, which makes it difficult to alter the shape of solids.

In a **liquid** the molecules are usually less closely packed and there is less cohesive force between them. They are sufficiently free to move from one point to another of the liquid. Their motion may be likened to that of a man moving in a crowded thoroughfare. The case of ice, however, which contracts when it is turned into water, shows that the distance between molecules in a liquid may be less than in a solid. It is the difference in the type of motion, *i.e.*, a change in position of the whole molecule as compared with a vibration about a mean position which gives the essential difference in nature between the liquid and the solid states.

In a **gas** the movement of the molecules is still freer. They are relatively far apart compared with their dimensions. Because of their great speeds, however, they are still continually colliding with other molecules. The cohesive force is practically absent—in other words, gases expand or contract easily.

27. The effect of heat is normally to increase both the amplitude and the speed of the molecular agitation so that the number of collisions increases. The spaces between the neighbouring molecules increase in size and this is observable as an increase in size of the body which is being heated. In the case of a gas heated in an enclosed vessel, the volume cannot increase and so only the speed of the molecules increases. This is observable as an increase in pressure on the walls of the vessel.

As was mentioned above, such changes of temperature may be sufficient to alter the matter from one state to another. The molecules under such conditions are themselves unaltered, it is merely their organisation which has been changed.

28. The Atom.—Molecules are capable of further sub-division but the resulting particles are no longer molecules. They are called “atoms” and have different properties from the molecules of which they formed a part. They are, for example, incapable of independent existence for any length of time except in the special case when the molecule contains only one atom, *i.e.*, when the molecule and the atom are the same.

An atom is the smallest portion of matter that can enter into chemical combination, or the smallest portion of matter obtainable by chemical separation.

A molecule may consist of one, two or more atoms of the same kind or it may consist of two or more atoms of different kinds. In chemistry the term "element" is applied to a substance which is composed entirely of atoms of the same kind: thus, two atoms of hydrogen (H) will combine to form a molecule of hydrogen (H_2). Hydrogen is therefore an element. Two atoms of hydrogen and one atom of oxygen will combine to form a molecule of water (H_2O). Water is not an element but a "chemical compound." The number of atoms in the molecule depends on the substance. In a molecule of salt there are two atoms; in a molecule of alum about 100 atoms. Examples of molecules which consist of only one atom are furnished by the rare gases of the atmosphere such as helium and neon.

The number of different atoms is limited; it is believed to be 92 and the enormous number of different substances, and therefore different molecules, in the world is given by varying combinations of this limited number.

29. The extremely small dimensions of these divisions of matter are shown by the following figures.

The mass of the hydrogen atom is 1.63×10^{-24} gram.

The diameter of the atom, regarding it as a sphere, is from about 2.4×10^{-8} cms. to 5.0×10^{-8} cms.

There are about 3×10^{23} molecules of hydrogen in a gram of the substance.

The average velocity of the hydrogen molecule at 15° C. is 1,694 metres per second.

30. Atoms vary in mass and size, the hydrogen atom being the lightest.

The **atomic weight** of a substance is the ratio of the weight of an atom of the substance to the weight of an atom of hydrogen.

The atomic weight of oxygen is 16*, of sodium 23, of molybdenum 96, and so on.

ATOMIC STRUCTURE.

31. Atoms may be further sub-divided into their constituents, viz., **Protons** and **Electrons**, but these are quite different in their nature from matter as we normally conceive it.

An **electron** is a minute particle of negative electricity which, when dissociated from the atom of which it is a part, shows none of the properties of ordinary matter. All electrons are similar, no matter what type of atom they are associated with. It is important

* In practice, atomic weights are referred to that of oxygen, taken as 16. On this basis the atomic weight of hydrogen is 1.008, and not exactly unity as would appear from the text.

to realise that the electron is **nothing but electricity** and is the smallest possible quantity of negative electricity.

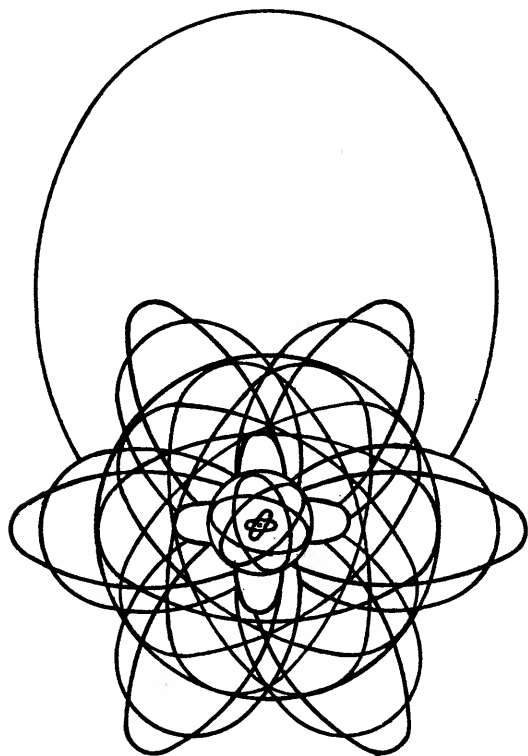
The charge of electricity we have called an electron is equal to 1.59×10^{-19} coulombs, so that there are 6.29×10^{18} electrons in a coulomb (paragraph 46).

The radius of an electron is of the order of 10^{-18} cm.

A **proton** is electrically the exact opposite of an electron. It also is supposed to be purely electrical in nature, but it consists of positive electricity. The proton and the electron are thus equal electrical charges, but of opposite sign. Due to some difference as yet unexplained, this has the effect of making the mass of the proton very much greater than that of the electron, so that the mass of an atom is, for all practical purposes, the mass of the protons it contains.

The mass of the proton is 1.63×10^{-24} grams.

32. The structural arrangement of these constituents in an atom appears to take the form of a central positive nucleus around which circulate a number of electrons in various orbits like the



Copper Atom. (29 Planetary Electrons.)

FIG. 3.

planets round the sun, except that these orbits are described in different planes.

Fig. 3 is a conventional representation of this idea for the case of the copper atom, the orbits of the various planetary electrons being projected on to the plane of the paper. For clearness in the figure, only the outer parts of the outer electron orbits have been drawn. They are continued, of course, round the nucleus.

33. The hydrogen atom will be considered first, as it is the simplest.

The nucleus of the hydrogen atom consists of a single proton, round which rotates a single electron in a planetary orbit. The atom is electrically neutral, the charge of negative electricity which is the electron being neutralised by the charge of positive electricity which is the proton. The mass of the atom is almost entirely concentrated in the proton.

The distance of the planetary electron from the proton is about 100,000 times the dimensions of the latter, and in more complex atoms the distances are of a like order, so that Fig. 3 must be understood to be a purely conventional drawing and not in any way drawn to scale.

34. The more complex atoms are made up in the same way of central nuclei and surrounding electrons. The nucleus itself, in these cases, is not composed simply of protons, but of a combination of protons and electrons. For instance, in the case of helium, the central nucleus is made up of four protons and two electrons, while two other electrons revolve round it. In every case the atom is electrically neutral, *e.g.*, in the case of helium the charges on the four protons neutralise the four electrons.

All atoms, and hence all molecules, all elements, all chemical compounds, and in fact the whole of matter, are merely different combinations of positive protons and negative electrons.

The **atomic weight** is practically equal to the number of protons in the atom.

The **atomic number** of an element is the number of surplus positive charges in the inner structure, or nucleus, and it is this quantity which determines the nature of the atom: it is also the number of planetary electrons.

The largest known atomic number is that of uranium, which has 92 revolving electrons. Consequently, as was mentioned before, there are at least 92 different kinds of atoms.

We shall take the more complicated example shown in Fig. 3 to illustrate this theory further.

Copper has an atomic weight of about 64 and its atomic number is 29.

That means that the nucleus of the copper atom contains 64 protons, while there are 29 outer electrons associated with it. As the atom is electrically neutral, there must be $64 - 29 = 35$

electrons in the nucleus, so the total composition of the central nucleus is 64 protons + 35 electrons, and it has an excess positive charge of 29 protons.

The nucleus may conveniently be thought of as a charge of positive electricity concentrated at a point.

The atomic number is generally about half the atomic weight.

35. The planetary electrons revolve at inconceivably great speeds round the positive nucleus. Thus it is apparent that energy is associated with and locked up in every atom.

Electrons from the outer revolving structures of some atoms can, as will be shown later, be caused to move from an orbit in one atom to an orbit in another atom, thus producing electrical phenomena.

The statement that electricity is "generated" by a battery simply means that electricity, already in existence, is given a motion in a particular direction.

A battery or dynamo does not generate electricity in the wires connected to it any more than a pump which is impelling a stream of water in a pipe generates the water.

THE ELECTRON THEORY OF ELECTRICITY.

36. Ions and Ionisation.

Under ordinary conditions the electrons of any atom are firmly held in the atom and the positive and negative charges neutralise each other as far as effects external to the atom are concerned.

In certain substances some of the constituent electrons describing outer orbits can be removed from the atom if sufficient energy is applied. The removal of such an electron **does not alter the nature of the atom**, but it alters its state of electrification and generally its chemical properties.

An atom which has either a deficit or a surplus of electrons beyond its natural complement is called an "**ion**" and is said to be **ionised**.

If a neutral atom loses an electron, the atom becomes a **positive ion** because it is left with more positive than negative charges.

If an electron is added to a neutral atom, the atom becomes a **negative ion** because it now contains an excess of negative over positive charges.

A positive ion will attract electrons and so will have a strong tendency to become a neutral atom again, while a negative ion will repel electrons and will readily part with the excess electron it has acquired.

The process by which a neutral atom becomes ionised is called "**ionisation**" and can be achieved in various ways.

37. Positive and Negative Electrification.

A **positively charged body** is one whose atoms have a deficit of electrons.

A **negatively charged** body is one whose atoms have an excess of electrons.

The amount of electrification depends upon the number of ionised atoms. Normally no single atom gains or loses more than one mobile electron.

It will be seen from the above that, when the poles of a battery are joined by a metallic conductor, the "electric current" which flows "from **positive** to **negative**," in accordance with the ideas of the earlier experimenters, is really an electron current flowing from negative to positive, *i.e.*, a surplus of electrons flowing to where there is a deficit so as to equalise the distribution.

When a "negative charge" is spoken of, a surplus of electrons is meant; when a "positive charge" is spoken of, a deficit of electrons is meant.

It is convenient to use the old method of treatment because of its greater familiarity, but in the explanation of certain facts, especially in the theory of the thermionic valve, the electron theory and the resultant direction of flow of the electron current is important and will be specially pointed out.

Charged bodies exercise forces of attraction and repulsion upon charges and upon each other. If a positively charged body is placed in a gas in an ionised condition, in which both positive and negative ions and electrons may be present, it will attract the electrons and negative ions and repel the positive ions.

38. Conductors and Insulators.—One of the earliest discoveries in the study of electricity was that certain substances, such as amber and glass, when rubbed with silk or flannel, acquired the property of attracting small light objects such as bits of paper or fluff. The experiment may be readily tried, for instance, with a fountain pen rubbed on the hair. Other substances, mostly metals, did not exhibit this property. The name "electricity" is derived from the Greek word for amber.

In terms of the electron theory, this is due to the transfer of electrons from the glass to the silk because of the friction created between them. The glass thus acquires a positive charge and the silk a negative charge. This transfer of electrons takes place in nearly every case where two substances are rubbed together. The distinguishing feature between silk and metals is that the silk possesses the ability to retain the extra electrons it receives, but if the metal is held in the hand, for instance, while performing the experiment, the electrons readily escape from it *via* the hand to earth, *i.e.*, an electron current flows to earth. This gives a general division of all substances into two kinds when their electrical properties are under consideration. **Conductors** are substances which readily permit a flow of electrons to take place under the influence of electrical forces, and **insulators** are substances in which, under the same circumstances, there is no flow of electrons.

No rigid line can be drawn between conducting and insulating substances. In practice the term "insulator" is applied to substances in which the flow of electrons is so minute compared with that in a good conductor that it may be considered as negligible.

Examples of good insulators are dry air, ebonite, sulphur, mica, indiarubber, shellac, silk and oil; on the other hand, most metals are good conductors.

39. Types of Electric Current.—The explanation on the electron theory of the processes by which electric currents can flow in different substances makes it possible to classify such currents into three well-defined types :—

- (a) Conduction currents.
- (b) Displacement currents.
- (c) Convection currents.

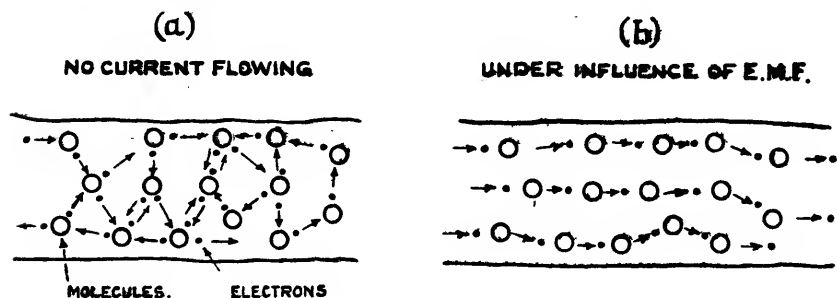
40. Conduction Current.—This is the type of current which flows in a metallic conductor such as a copper wire when it is connected to the terminals of a battery.

If the representation of the copper atom in Fig. 3 is studied, it will be seen that one of the planetary electrons describes a much larger orbit than any of the others. In a piece of copper, the molecules are closely crowded and each preserves its average position. Its motion consists of a rapid vibration about this position. Under these conditions the outer electron of the copper atom is likely often to come under as great an attraction from the nuclei of neighbouring atoms as that which keeps it attached to its own nucleus. The result is that the outer electrons of various neighbouring copper atoms possess, as it were, a divided allegiance, and readily interchange the nuclei to which they are normally attached. While these changes are occurring, such electrons may be looked upon as free from the sway of any particular nucleus. They are therefore referred to as **free** or **mobile** electrons.

If there are no external electric forces, this transference of electrons will not lead to anything which we can recognise as an electric current. The electrons are just as liable to transfer themselves from atom to atom in one direction as in another. This state of affairs is shown diagrammatically in Fig. 4 (a). The net transference of electricity in any particular direction is nil, and no electric current would be observed.

Suppose now that by some means the free electrons, when transferring from atom to atom, were given a tendency to make their transfer in one particular direction. This is essentially what happens when a copper wire is joined between the terminals of a battery. On the average, more electrons will transfer in this direction (towards the positive terminal) than in any other. This is very roughly illustrated by the arrows in Fig. 4 (b). The tendencies that can be given in this way produce only small effects compared with the normal haphazard motion of the free electrons.

It is unlikely that any one electron ever moves more than the distance between two neighbouring atoms, but the consequence on the large scale is to produce a slow drift of electrons in the direction of the positive terminal. It is this which we recognise as an electric current in the wire.

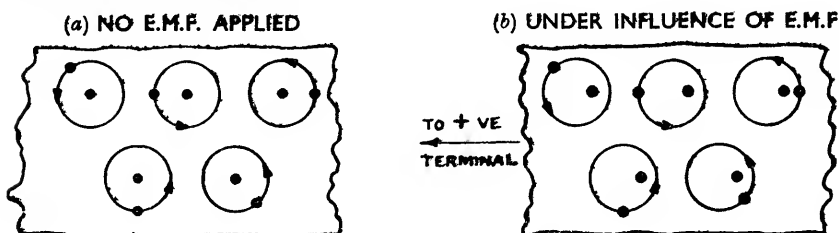


A Conducting Substance.

FIG. 4.

Conduction currents are thus due to the motion of free electrons. The atoms of the conductor do not alter their mean positions, as may easily be recognised. When a current is flowing along a copper conductor, copper is not transferred from one part of the conductor to another. The more free electrons a substance possesses and the "freer" they are, the better are its conducting properties.

41. Displacement Current.—In insulators the number of free electrons is negligibly small compared with the number present in a good conductor. The planetary electrons in each atom at every point in their orbits come under a greater attraction from their own nucleus than from the nucleus of any neighbouring atom, and there is no interchange of planetary electrons between adjacent atoms. Thus, under the action of steady electrical forces no current flows in an insulating substance.



An Insulating Substance.

FIG. 5.

The electrons in their orbits, however, must experience the same tendency as the free electrons discussed above. If the insulating substance is connected to the terminals of a battery, the electrons tend to move in the direction of the positive terminal. This

"external" attractive force, together with the "internal" attraction of the nucleus and the repulsion of the other planetary electrons, determines the resultant orbit which any particular electron describes. Compared with the orbit described by the electron under the atomic forces alone, this new orbit is displaced to some slight extent in the direction of the positive terminal. This is illustrated roughly in Fig. 5. Fig. 5 (a) shows the orbits of some planetary electrons, represented as circles whose centres are the nuclei of their respective atoms, when no E.M.F. is applied to the insulator. (The meaning of E.M.F. is explained in paragraph 48; for the moment it may be understood as a method of imposing motion in a particular direction on free electrons, such as is supplied by a battery.) Fig. 5 (b) shows the orbits displaced as the result of an applied E.M.F. The amount by which the orbits are displaced from their normal positions obviously depends on the E.M.F. applied. The greater the E.M.F., the greater the displacement.

As long as the E.M.F. is steady, the electrons will continue to revolve in the orbits to which they have adjusted themselves and there will be no continuous movement of electricity in any particular direction, *i.e.*, no current. When the E.M.F. is altered, however, the electrons have to adjust themselves to a new orbit before settling down under the different conditions. This adjustment will be in the same direction for all the electrons and so will be observed as a momentary current during the time the E.M.F. is changing. Whenever, for instance, an E.M.F. is applied to an insulator, *i.e.*, when the E.M.F. changes from zero to a certain value, a momentary current of this kind will flow. Such currents are called "displacement currents."

If an alternating E.M.F. (*i.e.*, an E.M.F. whose value is always changing) is applied across an insulator, the electrons are never able to settle down, but are continually readjusting their orbits to accord with the variation in E.M.F. In other words, there is always a displacement current in the insulator. It is in this sense that we are able to say that a condenser allows alternating currents to pass through it, although it acts as a barrier to direct currents. (paragraph 292.)

If the applied E.M.F. increases to such an extent that the nucleus cannot hold all its electrons against the powerful external attraction, an electron is pulled from the atom and a current of the type of paragraph 40 is established. By cumulative action a large current may be set up, and this effect is called **rupture** of the dielectric, or its insulation is said to "break down."

42. Convection Current.—This kind of current is due to the movement of electricity—electrons or positive and negative ions—through a liquid or a gas, or in a perfect vacuum. Common examples are the currents that flow through the chemical solution in a cell, across a spark gap and a Poulsen arc, and between the

filament and grid or anode of a thermionic valve. In each case the action is accompanied by ionisation, except in the case of the perfectly exhausted valve, where the convection current is nothing but electrons proceeding from filament to grid or anode. Each of these types of convection current is fully dealt with in the chapters on these subjects.

UNITS.

43. The derivation and definition of the units used in studying electrical phenomena will now be given. They are built up to a large extent from ideas developed in the study of mechanics, and the Appendix on Mechanics at the end of the book, and more particularly the remarks on energy and power, should be carefully studied at this stage.

The other primary consideration in developing electrical units is based on the observed effects of electric currents, and these will now be dealt with.

44. Effects of Electric Current.—The more important effects observed when an electric current is flowing are :—

- (a) **The heating effect.** A wire carrying a current becomes heated. This property is made use of in electric radiators and lamps.
- (b) **The magnetic effect.** A wire carrying a current is surrounded by a magnetic field. This property is of very great importance, and its various effects are dealt with in due course.
- (c) **The chemical effect.** This occurs in chemical solutions. Electroplating is a commercial example of its use. It finds a practical application to wireless work in connection with primary and secondary batteries.

The heating effect is not suitable for standardising purposes because of the difficulty of measuring quantities of heat accurately.

The magnetic effect is that on which the absolute electromagnetic unit of current is based.

The chemical effect gives the practical standard for measuring current.

An account of the flow of electric current in chemical solutions is given below (paragraph 108). The passage of such currents is accompanied by the deposition of the substances composing the chemical in solution, and the amounts deposited are proportional to the current and to the time during which the current is flowing. Measurements of weight and time can both be performed with great accuracy, and so the chemical effect is very suitable for defining a standard current. The chemical used is silver nitrate dissolved in water and the unit of current defined from it is called the **International Ampere**.

45. The International Ampere is the unvarying electric current which, when passed through a solution of nitrate of silver in water, deposits silver at the rate of 0.00111800 gram per second.

For measuring very small currents we use two other units :—

The milliampere (symbol mA) = $\frac{1}{1000}$ ampere.

The microampere (symbol μ A) = $\frac{1}{10^6}$ ampere.

Current is denoted by the letter I.

46. The Coulomb.—The idea of a current or rate of flow implies the idea of quantity. Quantity of electricity is therefore defined as the amount passing when a certain rate of flow is maintained for a certain time. The quantity of electricity passing through any cross-section of a wire when current is flowing is, of course, merely the sum of the charges of all the free electrons crossing that section.

Quantity is denoted by the letter Q.

The **coulomb** is the quantity of electricity passing per second when the current strength is one ampere.

Hence if a current of I amperes is flowing for t seconds, the quantity of electricity, Q, is given in coulombs by $Q = It$ coulombs.

Conversely, of course, an ampere is a rate of flow of 1 coulomb per second, and $I = \frac{Q}{t}$.

A coulomb is 6.29×10^{18} electrons.

An ampere is a rate of flow of 6.29×10^{18} electrons past a given point in an electric circuit per second.

Example 1.

If a current of 10 amperes flows for 5 seconds, then the quantity of electricity that passes a point in the circuit will be $Q = I \times t = 10 \times 5 = 50$ coulombs.

A larger unit of **quantity**, used in connection with capacities of accumulators, for example, is the **ampere-hour**. This is equivalent to 1 coulomb per second (*i.e.*, 1 ampere) for 3,600 seconds, *i.e.*, 3,600 coulombs.

47. Energy.—The derivation of the idea of energy is explained in Appendix C, which should be revised at this stage. The meaning of the principle of conservation of energy is there discussed, and the transformation of energy from one kind to another in natural processes is illustrated. Here we are concerned with the transformations that accompany electrical phenomena.

When an electric current flows in a conductor, the conductor becomes heated (paragraph 44). Heat is a form of energy, being due to the vibrations of the molecules of the conductor (paragraph 27). It follows that the passage of current is associated with the transformation into heat energy of some other form of energy.

We may give this form the name of "electrical energy," as it is connected with an electric current. This electrical energy is linked with the directed motion imposed on the free electrons when a current is flowing (paragraph 40). The electrons acquire energy of motion during their journey from molecule to molecule under the action of the electric forces. When recaptured, part of this energy may help to set free another electron; the rest is transferred to the molecule and serves to increase the violence of molecular vibration, *i.e.*, the electron's kinetic energy is converted to heat energy of the conductor.

Some other form of energy must have been converted into this electrical energy or extra kinetic energy of the electrons. The two commonest forms are chemical and mechanical. Chemical energy is converted to electrical energy when a primary cell or accumulator sustains an electric current. The chemical reaction between the substances of the cell sets free a certain amount of energy (paragraph 109). Mechanical energy is the source of electrical energy from a dynamo, the dynamo armature being rotated by some mechanical arrangement.

48. Electromotive Force and Potential Difference.—Suppose we consider a simple closed electric circuit, such as a conductor connected between the terminals of a battery. There are two energy transformations going on concurrently. Chemical energy is being converted to electrical energy by the battery, electrical energy is being converted to heat energy in the conductor. (There will also be a conversion of electrical energy to heat, due to the flow of ions in the battery itself, but we may ignore it for the moment.) These two processes provide the basis of two important ideas in the description of electrical phenomena.

Wherever there is introduced in any part of an electrical circuit another form of energy capable of being converted into electrical energy, we say that an electromotive force (E.M.F.) is acting in the circuit.

If between any two points in a circuit electrical energy may be converted into any other form of energy, we say that a potential difference (P.D.) is established between the two points. Thus in the simple circuit above, the battery is said to supply an E.M.F. as chemical energy is there being converted to electrical energy. Between any two points on the conductor electrical energy is being converted to heat energy; there is therefore a P.D. between any two points on the conductor.

49. Electromotive Force (E.M.F.) is produced in four different ways:—

- (1) **Chemical.** By two dissimilar metals or other substances being immersed in certain chemical solutions known as electrolytes—such as the acids in primary and secondary

cells. Chemical energy is transformed into electrical energy. This subject is treated further in paragraphs 107 to 127.

- (2) **Thermo-electric.** By two dissimilar conductors being placed in contact and their junction heated. Heat energy is transformed to electrical energy. A practical case is the thermal junction of steel and eureka wire used in wavemeters.

This subject is treated further in the chapter on wavemeters (Chapter XX).

- (3) **Electromagnetic.** By interaction in certain circumstances between a conductor and "lines of magnetic flux," *e.g.*, in the dynamo, alternator, transformer. Mechanical and "magnetic" energy are converted to electrical energy.

This is treated further in Chapters III, IV and VI.

- (4) **Electrostatic.** Various frictional machines produce the result required by converting mechanical energy to electrical energy. From our point of view this method is of no importance.

50. Units of E.M.F. and P.D.—The energy transformations discussed above are used to derive units of E.M.F. and P.D. We shall first consider the unit of P.D.

It has been seen that electrical energy is converted to heat energy between two points in a conductor carrying a current. Each free electron has approximately the same extra energy of motion when recaptured by a molecule, and so the more electrons there are travelling round the circuit the greater will be the heat developed. We might thus take the amount of electrical energy converted per electron between any two points as a measure of the P.D. between the two points. Actually, we consider the energy conversion per coulomb, as the coulomb and not the electron is the practical unit of quantity. The C.G.S. unit of energy is the "erg," the work which can be done by a force of one dyne acting through a distance of one centimetre. This unit is inconveniently small and for electrical measurements a practical unit called a "joule" is used. One joule is equal to ten million (10^7) ergs. (Appendix C.)

The symbol used for energy is *W* (Work).

The two units, the coulomb and the joule, are used to derive a unit of P.D., which is called the "volt."

If the amount of electrical energy converted to other forms of energy when one coulomb passes between two points of a conductor is equal to one joule, then the **potential difference** between the two points is said to be one **volt**.

The symbol generally used for P.D. is *V*.

Thus, if two coulombs are carried from one point to another and the conversion of electrical energy is 5 joules, then the P.D. in volts between the two points is

$$V = \frac{5 \text{ joules}}{2 \text{ coulombs}} = 2.5 \text{ volts.}$$

More generally, if Q coulombs are carried from one point to another and the conversion of electrical energy is W joules, then the P.D. in volts between the two points is

$$V = W \div Q.$$

Volts = joules \div coulombs.

It is not necessary for the coulombs actually to pass in order to find the P.D. If the electrical energy conversion which would occur if one coulomb were allowed to pass, can be calculated, then the P.D. can be found from the formula above. This is the method employed in electrostatic calculations of P.D. (paragraph 102).

51. The idea of electromotive force is also derived from the conversion of energy, and so the **volt** may also be used as the **unit of E.M.F.**

If the amount of other forms of energy converted to electrical energy at any point of a circuit is one joule per coulomb of electricity which passes the point, then the **E.M.F.** developed at that point is said to be one **volt**.

Thus, if a 2-volt accumulator is being used as a source of E.M.F. in a circuit, for every coulomb of electricity that flows round the circuit (including the accumulator itself), 2 joules of chemical energy are converted to electrical energy.

It will be seen that the term electromotive "force" for this concept is rather unfortunate. The concept in electricity which corresponds most closely to that of force in mechanics is "electric field strength," which is discussed under Electrostatics (paragraph 97). The concept in mechanics with which E.M.F. and P.D. are best compared is that of potential energy per unit mass (head or level).

As the volt is the unit of both E.M.F. and P.D., they are often both indiscriminately referred to as "voltage," but the identity of the unit must not be allowed to cause confusion between the two ideas.

52. The above discussion brings out the ideas underlying the definition of the volt, but their application does not provide a very convenient method of measurement. The definition of the International Volt is thus derived from a development of these ideas and will be given when that development has been explained (para. 60).

Other units used are :—

The millivolt (symbol mV) = $\frac{1}{1,000}$ volt.

The microvolt (symbol μV) = $\frac{1}{10^6}$ volt.

The kilovolt (symbol kV) = 1,000 volts.

53. Potential.—When the height of a mountain is referred to it is generally assumed to mean the vertical distance between the top of the mountain and mean sea-level. If it were given by different observers as the vertical distance from their own position, like the heights referred to **bench** marks by architects, the numbers quoted by these observers would merely give the **difference** in height between the observer and the mountain top. This may be compared to P.D. in electricity. If we fix on one point in a circuit, then the P.D. between it and any other point depends on the position of the other point.

It is found convenient in electricity to have an idea corresponding to mean sea level. Just as the difference in height between the top of a mountain and mean sea level is called the height of the mountain, so the difference in potential between a point in an electrical circuit and any point on the earth's surface is called the "potential" of the point, the word "difference" being dropped. This is possible because there is no potential difference between any two points on the earth's surface. If such a P.D. developed momentarily a current would flow until the P.D. had disappeared.

In other words, the earth's surface is taken as being at zero potential, just as mean sea level is taken as being at zero height.

Many parts of the earth's surface are below sea level, *e.g.*, the ocean bottom. If heights above sea-level are taken as positive, then depths below it might be considered negative. A similar state of affairs occurs with regard to electrical potential. It may be positive or negative.

A point has a positive potential if in the passage of a quantity of positive electricity **from the point to earth** electrical energy is converted to other forms of energy.

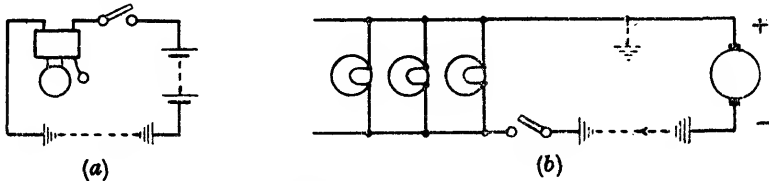
A point has a negative potential if in the passage of a quantity of positive electricity **from earth to the point**, electrical energy is converted to other forms of energy.

These definitions would be reversed if we considered what **actually** happens, *viz.*, the passage of a number of electrons (negative electricity) between the point and earth.

The distinction between potential and P.D. should be carefully observed. Thus if a lamp has 110 volts applied across it from the mains, the correct expression is that there is a P.D. of 110 volts across the lamp. If the negative main is earthed, it is permissible to say that the positive main is at a potential of 110 volts, for 110

volts is then the potential difference between the positive main and earth.

54. Earth Return Circuits.—Advantage is often taken of the fact mentioned above, that the earth's surface is a good conductor, to utilise an "earth return" as part of a circuit. The hull of a ship may also be used for this purpose.



Earth return circuits.

FIG. 6.

For example, in Fig. 6 (a) the battery shown is being used to ring a bell. The positive terminal of the battery is joined to the bell through a switch, but connection from the negative battery terminal is made by using an earth return between it and the bell.

Similarly, in Fig. 6 (b) the dynamo is connected to a number of lamps in parallel through a single-line wire, but the return connection is made by earthing the negative dynamo brush and one side of the switch.

This method of economising wire can only be used in cases when the insulation of the single conductor can be relied upon.

If any earth were to develop on the line wire, as shown dotted, the dynamo would be short-circuited, an excessive current would flow, and the machine would be burnt out.

The earth has such a large cross-section that its resistance can be neglected. If the earth return is found to have an unduly high resistance, the fault probably lies in the method in which connection is made to earth.

Frequent use is made in W/T of earth connections.

55. If a point in a circuit which contains a source of E.M.F. is earthed, electrons may have either to flow out of, or into, the earth. For example, if in Fig. 6 (b) the dynamo is developing a P.D. of 100 volts between its brushes, its positive brush will be at 100 volts potential above earth (or zero) potential.

A positive electrification denotes a deficiency of electrons, so that in this case electrons will flow from earth through the lamps, in at the positive brush, through the dynamo and back to earth at the negative brush.

If the brush connections are reversed, the brush connected to the line wire will be at 100 volts potential negative to earth, and the electron current is reversed in direction.

56. Power.—We have dealt with the total conversion of energy taking place between different points in a circuit without reference to the time involved, *e.g.*, the time taken for a coulomb to pass from one point to another. The consideration of the time taken leads to the idea of power—the rate of working, or rate at which energy is being transformed.

The unit of power derived from the erg, the C.G.S. unit of energy, is a transformation of energy at the rate of one erg per second.

The practical unit of power is the **watt**, the rate of working when one joule of energy is transformed per second.

$$1 \text{ watt} = 1 \text{ joule per second.}$$

The symbol for power is P .

$$P = W/t.$$

Since one joule of energy is transformed when one coulomb passes between two points whose P.D. is one volt, if this passage takes place in one second the rate of working, or power, is one watt. But the passage of one coulomb per second represents a current of one ampere, so that one watt is the power developed when a current of one ampere flows between two points whose P.D. is one volt.

If the P.D. between the points is V volts and the current is I amps., the power developed in watts is $P = IV$.

$$\text{Watts} = \text{volts} \times \text{amps.}$$

57. A power of one watt developed over a period of one hour will correspond to the transformation of

$$\frac{1 \text{ joule}}{\text{sec.}} \times 3,600 \text{ secs.} = 3,600 \text{ joules.}$$

of energy. This amount of **energy** is taken as a unit called the **watt-hour** when a larger unit than the joule is required.

$$1 \text{ watt-hour} = 3,600 \text{ joules.}$$

A still larger unit of energy, the **kilowatt-hour**, is also employed, corresponding to a power of 1,000 watts developed over a period of one hour, *i.e.*, 3,600,000 joules. This unit of energy is known as the Board of Trade Unit (B.O.T.U.).

58. Ohm's Law.—If any given conductor is kept at constant temperature and the P.D. between its ends is compared with the current flowing in it, there is found to be a constant relationship between the two, *e.g.*, if the current is doubled, the P.D. is doubled, and so on.

We might anticipate some such relation on the electron theory of metallic conduction. Increasing the current means that the free electrons have to travel faster from molecule to molecule and so have a greater amount of kinetic energy to transfer to the molecules when recaptured, *i.e.*, the conversion of energy per free electron and therefore the P.D. is increased.

This relationship is called Ohm's Law and may be stated symbolically as

$$\frac{V}{I} = \text{constant, or } V = \text{constant} \times I.$$

The constant is a measure of the difficulty the electrons find in escaping recapture by molecules; the greater the constant, the greater is V for a given I , *i.e.*, the greater is the energy transferred to the molecules by the free electrons. It is a criterion of the opposition the conductor offers to the passage of current. It is called the **resistance** of the conductor and denoted by the letter R .

As we should expect, the constant is much smaller in conductors than in insulators. In other words, conductors have low resistance; insulators have high resistance.

Ohm's Law may therefore be stated as:—

$$\frac{V}{I} = R \text{ or } V = RI.$$

It is true only for conduction currents and the particular type of convection current which flows in chemical solutions. It does not hold for gaseous convection currents.

Further, it only applies in the above form when the current is steady and cannot be used when the current is changing.

59. If $V = 1$ when $I = 1$ in the formula $V = RI$, then R must also be equal to 1 if the formula is to be satisfied.

This enables us to define a **unit** of resistance called the **ohm**.

A conductor has a resistance of one ohm if the P.D. between its ends is one volt, when the current flowing in it is one ampere. The symbol Ω (omega) is used as a contraction for ohm.

Owing to the practical difficulty of measuring the volt from the amount of energy transformed, this definition is not convenient for the standardisation of resistances. Consequently, the logical procedure has been reversed and the ohm has been defined for standardising purposes as the resistance of a certain conductor of dimensions adjusted to give it a value as nearly equal as possible to that of the ohm derived as above.

60. The International Ohm (*i.e.*, the standard value fixed by international agreement) is "the resistance offered to an unvarying electric current by a column of mercury at the temperature of melting ice, 14.4521 grammes in mass, of constant cross-sectional area, 1 sq. mm., and of length 106.300 cms.

The following multiples of the ohm are frequently used:—

1 megohm (symbol $M \Omega$) = 10^6 ohms) = a million ohms.

1 microhm (symbol $\mu \Omega$ (mu—omega)) = $\frac{1}{10^6}$ or 10^{-6} ohm
= a millionth of an ohm.

By using Ohm's Law a practical way of defining the volt for measuring purposes can now be derived.

The **International Volt** is the P.D. between the ends of a conductor whose resistance is one International Ohm when the current flowing in the conductor is one International Ampere.

These definitions allow accurate comparisons to be made of different resistances and potential differences.

61. Resistance Formula.—The resistance of a conductor depends on its dimensions, its material and its temperature.

We should expect this on the electron theory. The material determines the number of free electrons and its temperature gives a measure of the vibratory motion of its molecules, which is bound to affect the chance of recapture by the molecules of the free electrons. The greater the area of cross-section of a conductor, the more electrons will there be crossing any cross-section at a time. The longer the conductor, the greater the number of recaptures by molecules. Hence we find that the resistance of a conductor varies directly as its length and inversely as its cross-section.

At constant temperature the resistance of a conductor may thus be written as

$$R = \rho l/A,$$

where l is its length and A its cross-sectional area. ρ (rho) is the factor which takes account of the material of the conductor. It is called the **specific resistance** or resistivity of the material. If $l = 1$ and $A = 1$ in the formula, then $R = \rho$. The specific resistance may thus be defined as follows :—

62. The specific resistance of a material is the resistance of a piece of the material one centimetre in length and having a cross-section of one square centimetre, usually called a centimetre cube of the material. Since resistance alters with change of temperature, the specific resistance is usually quoted for 20° C.

Some specific resistances are, approximately :—

Copper—1·7 microhms per cm. cube.

Platinum—11 " "

Mercury—94 " "

Insulators have enormously high specific resistances compared with the above. As was pointed out before, the difference between conductors and insulators (in the practical sense of the word) is merely one of degree. There are no perfect insulators in the sense of substances that allow absolutely no conduction current to flow, but the current is very small indeed for substances usually termed insulators.

A column of air, one inch long, offers as much resistance to an electric current as a copper cable thirty thousand million miles long, of the same cross-section, *i.e.*, a cable long enough to reach from the earth to Arcturus and back twenty times.

When testing insulation resistance, for example, by the bridge megger (para. 190), it is the passage of a very minute current through the insulating substance that allows the instrument to operate.

Some specific resistances for insulators are :—

Distilled water— 7×10^{10} ohms per cm. cube.

Silica—over 5×10^{18} ,, ,,

Mica— 5×10^{16} „ „

63. Temperature Coefficients.—In all cases the specific resistance depends to some extent on the temperature.

For most materials it increases uniformly with the temperature and the **temperature coefficient** is defined to be the fractional increase of resistance per degree increase of temperature above a definite temperature, usually taken as 20° C.

Thus if α is the temperature coefficient for 20° C., and ρ is the specific resistance at 20° C., the specific resistance at θ° C. is given by

$$\rho' \text{ (at } \theta^\circ \text{ C.)} = \rho \{1 + \alpha (\theta^\circ - 20^\circ)\}.$$

The temperature coefficient of copper is 0.00393 .

The temperature coefficient of iron and steel is 0.006.

Alloys have in general very much smaller temperature coefficients than pure metals, and so are used in the construction of standard resistances which are required to vary as little as possible.

The temperature coefficient of German silver, for instance, is about 0.0004; of manganin, 0.000006.

The temperature coefficients of carbon, glass and electrolytes are negative, *i.e.*, their resistances decrease as their temperatures increase.

In the formula $R = \rho l/A$, the various quantities must always be expressed in the appropriate units, l in centimetres and A in square centimetres.

If ρ is expressed in microhms per cm. cube, then R will be found in microhms.

If ρ is expressed in ohms per cm. cube, then R will be found in ohms.

64. Conductance is defined as the reciprocal of resistance, provided direct currents only are being considered. Its meaning is generalised when applied to the effects of alternating current.

It is a measure of the ease with which current may flow through a conductor, as opposed to resistance which is a measure of the difficulty experienced by a current in flowing.

The symbol for conductance is G , i.e., $G = \frac{1}{R}$ for any conductor through which direct current is flowing.

The unit of conductance is the "reciprocal ohm" or **mho** and is defined as the reciprocal of the International Ohm.

The reciprocal of the specific resistance of a material is called its **conductivity** and given the symbol γ (gamma).

Example 2.

Find (a) the resistance and (b) the conductance of a copper conductor 5 miles long and 0.08 inch in diameter.

It is first necessary to reduce the various lengths and areas to the units for which the specific resistance is quoted, *i.e.*, centimetres and square centimetres.

$$\begin{aligned} 5 \text{ miles} &= 5 \times 1.61 \text{ kilometres} \\ &= 8.05 \times 10^5 \text{ cms.} \end{aligned}$$

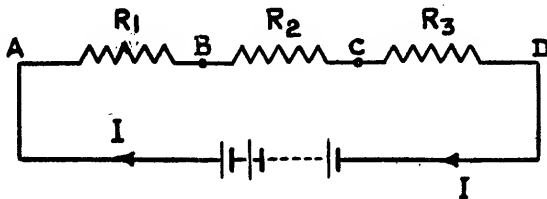
$$\begin{aligned} \text{Cross-sectional area} &= \pi \times 0.04^2 \text{ sq. inches.} \\ &= 0.005026 \text{ sq. inches.} \\ &= 0.005026 \times 6.45 \text{ sq. cms.} \\ &= 0.03242 \text{ sq. cms.} \end{aligned}$$

$$\rho = 1.7 \times 10^{-8} \text{ ohms per cm. cube.}$$

$$\therefore R = \frac{1.7 \times 10^{-8} \times 8.05 \times 10^5}{0.03242} = \frac{1.37}{0.03242} = 42 \text{ } \Omega \text{ approx.}$$

$$\text{Conductance } G = \frac{1}{R} = \frac{1}{42} = 0.024 \text{ mho.}$$

65. Resistances in Series.—Resistances are said to be in series when they are connected end to end as shown in Fig. 7. We wish to derive an expression for the total resistance (R) of a number of resistances in series in terms of the individual resistances (R_1 , R_2 and R_3).



$$E.M.F. = E. \quad \text{Internal resistance} = r.$$

FIG. 7.

In a closed circuit in which current is flowing it is not possible for electricity to pile up at any point. Thus in Fig. 7 the same current must flow through each of the three resistances. Suppose this current is I amps. and that the total P.D. from A to D is V volts; in other words, when one coulomb passes from A to D the work done (heat energy produced) is V joules. The total resistance R is then given by

$$R = \frac{V}{I}$$

according to Ohm's Law.

The work done by one coulomb in passing from A to D can be divided into three parts, viz., from A to B, B to C, and C to D. If these three quantities of work are V_1 , V_2 and V_3 joules respectively, then $V = V_1 + V_2 + V_3$.

But by definition V_1 , V_2 and V_3 are the P.D.s in volts from A to B, B to C, and C to D.

Therefore by Ohm's Law $V_1 = IR_1$, $V_2 = IR_2$, and $V_3 = IR_3$.

$$\therefore V = V_1 + V_2 + V_3 = IR_1 + IR_2 + IR_3 = I(R_1 + R_2 + R_3).$$

$$\therefore R = \frac{V}{I} = R_1 + R_2 + R_3$$

i.e., the equivalent resistance of a number of resistances in series is equal to the sum of their individual resistances.

Example 3.

Fig. 8 indicates a battery, whose internal resistance is 2 ohms and E.M.F. 10 volts, connected to a resistance of 3 ohms. (We may give a definite value to the internal resistance of the battery, *i.e.*, the resistance of the chemical solution between its plates, because Ohm's Law applies to such solutions.) Find the strength of the current.

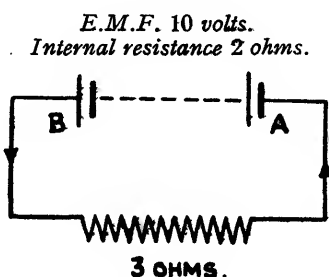


FIG. 8.

The total fall of potential round the circuit (including the battery itself) must be the same as the E.M.F., for all the energy converted to electrical energy from chemical energy finally appears as heat in the battery and external resistance.

The internal resistance of 2 ohms and the 3-ohm resistance are in series and so the total resistance is $2 + 3 = 5$ ohms.

$$\therefore \text{by Ohm's Law, } I = \frac{V}{R} = \frac{10}{5} = 2 \text{ amps.}$$

This current, as mentioned above, must be the same at all points in the circuit.

There is a P.D. (also called a "voltage drop" or "IR drop") between any two points in the circuit. From B to A in the external circuit a current of 2 amps. flows through a resistance of 3 ohms and so from B to A there is a voltage drop of 6 volts.

A voltmeter connected across A and B will read 6 volts.

This is called the terminal P.D. of the battery. It obviously depends on the current flowing. The larger the current, the smaller the terminal P.D. because the IR drop in the battery is greater.

More generally, if we have a battery of internal resistance r ohms and E.M.F. E volts sending current through an external resistance of R ohms, then by the above argument

$$I = \frac{E}{R + r}$$

Similarly, if V in this case is the terminal P.D. of the battery

$$I = \frac{V}{R} = \frac{\text{Terminal P.D.}}{\text{External resistance}}$$

$$\text{and also } I = \frac{E - V}{r} = \frac{\text{Voltage drop in battery}}{\text{Internal resistance}}$$

In other words, V is less than E , or some fraction of the energy supplied to the electrons is lost within the source of E.M.F. because of its internal resistance.

66. Resistances in Parallel.—Resistances are said to be in parallel when arranged as shown in Fig. 9. At each end they are connected to a common point. The equivalent resistance of such an arrangement may be found as follows.

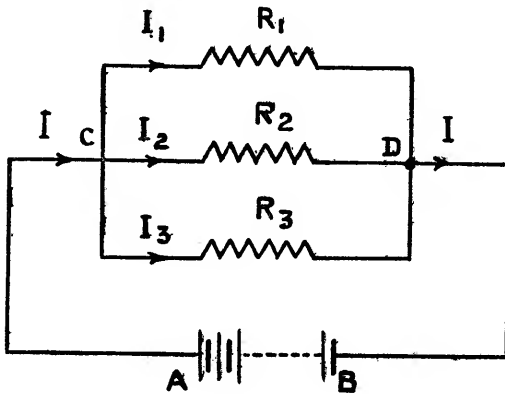


FIG. 9.

Referring to Fig. 9, let the current flowing from the battery be I amps. The P.D. across each resistance must be the same, as it is the P.D. between the points C and D in the circuit. Let it be V volts. As electrons cannot accumulate at the point C, but must move on round the circuit, the sum of the currents I_1 , I_2 and I_3 , flowing in the three resistances R_1 , R_2 and R_3 , must be equal to I , i.e., $I = I_1 + I_2 + I_3$.

But, by Ohm's Law,

$$I_1 = \frac{V}{R_1}, I_2 = \frac{V}{R_2}, I_3 = \frac{V}{R_3}.$$

$$\therefore I = I_1 + I_2 + I_3 = \frac{V}{R_1} + \frac{V}{R_2} + \frac{V}{R_3} = V \left[\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} \right]$$

If R is the equivalent resistance of the whole arrangement, then

$$I = \frac{V}{R}.$$

$$\therefore \frac{V}{R} = V \left[\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} \right].$$

$$\therefore \frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3},$$

from which R may be determined.

This result may also be put in the form

$$G = G_1 + G_2 + G_3$$

where G is the equivalent conductance $\left(= \frac{1}{R} \right)$ and G_1 , G_2 and G_3 are the conductances of R_1 , R_2 and R_3 .

In other words, the equivalent conductance of a number of resistances in parallel is equal to the sum of their individual conductances.

The equivalent conductance is thus greater than any of the individual conductances, and so its reciprocal, the equivalent resistance, is less than any of the individual resistances.

The equivalent resistance of n equal resistances of value R arranged in parallel is given by $\frac{R}{n}$.

With only two resistances R_1 and R_2 in parallel, a simple formula for the equivalent resistance can be obtained.

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} = \frac{R_1 + R_2}{R_1 R_2}$$

$$\therefore R = \frac{R_1 R_2}{R_1 + R_2} = \frac{\text{product of individual resistances.}}{\text{sum of individual resistances.}}$$

67. Use of Shunting Resistance.—When only a small proportion of a given current is required in a particular path an alternative path of much lower resistance is connected in parallel. The total current then splits up between the two paths, and the latter takes the major part. This principle is frequently met with in measuring instruments, *e.g.*, ammeters, where the current required to flow through the instrument is very small.

Let R_m be the resistance of the instrument and R_s that of the shunt, and I_m and I_s the currents through each.

I is the total current to be measured.

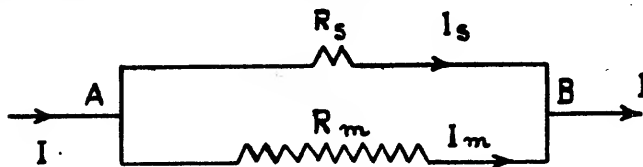


FIG. 10.

The voltage drop between A and B (Fig. 10) is $I R_s = I_m R_m$

$$= I \times \text{equivalent resistance}$$

$$= I \frac{R_m R_s}{R_s + R_m}$$

$$\therefore I_m = I \frac{R_s}{R_s + R_m}$$

If R_s is small compared with R_m , this current I_m is a small fraction of I .

For example, to measure a current up to 10 amperes with an ammeter reading up to 1 ampere, we should want only 1/10th of the total current to pass through the instrument, 9/10ths going through the shunt.

$$I_m = \frac{1}{10} I = I \frac{R_s}{R_s + R_m}$$

$$\therefore \frac{R_s}{R_s + R_m} = \frac{1}{10}, \text{ or } R_s = 1/9\text{th of } R_m$$

With this shunt resistance across the ammeter, any reading on the ammeter scale would indicate that 10 times that amount of current was flowing in the circuit.

68. Examples on Resistances in Series and Parallel.

Example 4.

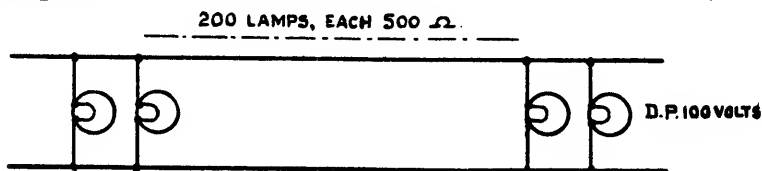


FIG. 11.

Let 200 lamps, each of hot resistance 500Ω , be connected across 100-volt mains, as in Fig. 11.

$$\text{Current in each lamp} = \frac{E}{R} = \frac{100}{500} = 0.2 \text{ ampere.}$$

Total current (200 paths in parallel).

$$= 200 \times 0.2 \text{ amp.} = 40 \text{ amperes.}$$

Alternatively, the effective resistance of 200 resistances in parallel, each $500\ \Omega$, is $\frac{500}{200} = 2.5$ ohms.

$$\therefore \text{Total current} = \frac{100}{2.5} = 40 \text{ amperes.}$$

Example 5.—Five cells in parallel, each with a resistance of 0.25 ohm, are connected to an external resistance made up of 0.55 ohm in series with three resistances of 1 ohm, 4 ohms and 0.8 ohm in parallel. The E.M.F. of each cell is 1.5 volts. Find the current in each cell, the current in the 0.55 ohm resistance, and the currents in the separate external resistances.

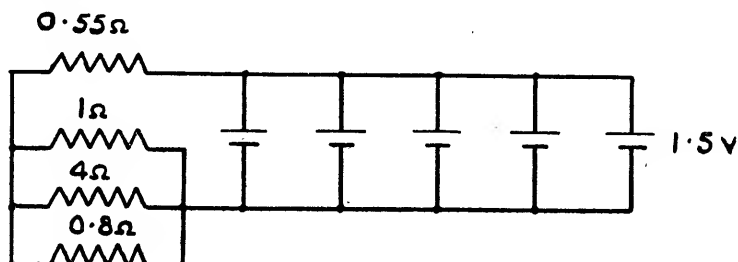


FIG. 12.

$$\text{Total resistance of circuit} = \frac{0.25}{5} + 0.55 + R,$$

where R is given by

$$\frac{1}{R} = \frac{1}{1} + \frac{1}{4} + \frac{1}{0.8} = 2.5.$$

$$\therefore R = 0.4 \text{ ohm.}$$

$$\therefore \text{Total resistance} = 0.05 + 0.55 + 0.4 = 1 \text{ ohm.}$$

E.M.F. is the same as that of one cell = 1.5 volts.

$$\therefore \text{Total current in the circuit is } 1.5 \text{ amperes.}$$

It follows that

(1) Current through the 0.55 ohm resistance is 1.5 amps.

(2) The current through each cell is $\frac{1.5}{5} = 0.3$ amp.

The current through the separate parallel resistances is given by the P.D. across them divided by their values in ohms.

P.D. across them is 1.5×0.4 ohms (their effective resistance)

$$= 0.6 \text{ volt.}$$

$$\therefore \text{Separate currents are } \frac{0.6}{1}, \frac{0.6}{4}, \frac{0.6}{0.8} \text{ amps.}$$

i.e., $0.6, 0.15, 0.75$ amp., and the sum of these is 1.5 amps., the total current.

69. Power.—It was shown in paragraph 56 that the power developed in any part of an electrical circuit is the product of the current flowing in it and the P.D. across it.

$$P = IV,$$

or watts = amps. \times volts.

By the application of Ohm's Law, expressions for the power may be obtained either in terms of current and resistance or of P.D. and resistance.

$$(a) V = IR.$$

$$\therefore P = IV = I \times IR = I^2R.$$

$$(b) I = \frac{V}{R}$$

$$\therefore P = IV = \frac{V}{R} \times V = \frac{V^2}{R}.$$

Thus, referring back to Example 3 (para. 65), the power developed in the 3 Ω resistance may be calculated in any of the following three ways :—

- (1) Current (I) in 3 Ω resistance = 2 amps.

P.D. (V) across 3 Ω resistance = 6 volts.

$$\therefore \text{Power} = IV = 2 \text{ amps.} \times 6 \text{ volts} = 12 \text{ watts.}$$

- (2) Current (I) in 3 Ω resistance (R) = 2 amps.

$$\therefore \text{Power} = I^2R = 4 \times 3 = 12 \text{ watts.}$$

- (3) P.D. (V) across 3 Ω resistance (R) = 6 volts.

$$\therefore \text{Power} = \frac{V^2}{R} = \frac{36}{3} = 12 \text{ watts.}$$

The power loss in the internal resistance of the battery (2 Ω) is

$$2^2 \times 2 = 8 \text{ watts,}$$

so that the total power which must be supplied to the circuit is $12 + 8 = 20$ watts when 2 amps. is flowing

It is easily verified that this is the rate of working of the battery,

$$EI = 10 \times 2 = 20 \text{ watts.}$$

70. The meaning of such expressions as a "10 kW. 100-volt" dynamo or a "40-watt 220-volt" lamp should be clearly understood.

The first expression denotes a dynamo which will supply a current of $\frac{10,000}{100} = 100$ amperes at 100 volts pressure without being overloaded when rotated at its designed speed.

The second denotes a lamp which requires a current of $\frac{40}{220} = 0.18$ amp. at 220 volts to keep it burning at its designed brilliancy.

71. The idea that I^2R represents heat, energy, or even force, is very prevalent. This is quite wrong. I^2R represents the **rate** at which light and heat energy are produced in the lamp, or the **rate** at which mechanical energy is turned out by the motor, or the **rate** at which a load is raised.

A cyclist travels at 10 miles per hour. This does not tell us the distance he covers, but simply the **rate** at which he covers it. To know the distance traversed we must know the length of time for which he rides.

$$\begin{aligned}\text{Distance} &= \text{rate} \times \text{time} \\ &= \frac{10 \text{ miles}}{1 \text{ hour}} \times 2 \text{ hours (say).} \\ &= 20 \text{ miles.}\end{aligned}$$

Similarly, when a certain power is employed for a certain length of time, an amount of energy—in various forms—is available.

$$\begin{aligned}\text{Energy} &= \text{Power} \times \text{Time} \\ &= EI \text{ (or } I^2R) \text{ watts} \times t \text{ seconds} \\ &= EIt \text{ (or } I^2Rt) \text{ joules.}\end{aligned}$$

Example 6.

A 100-volt lamp of 500 ohms hot resistance is connected across 100-volt mains. Find the energy taken from the mains in one minute and the heat developed by the lamp.

$$\begin{aligned}\text{Energy} &= \text{power} \times \text{time.} \\ &= \frac{E^2}{R} \text{ joules per sec.} \times 60 \text{ secs.} \\ &= \frac{100^2}{500} \times 60 = 1,200 \text{ joules.} \\ \text{or } I &= \frac{100}{500} = 0.2 \text{ amp.}\end{aligned}$$

$$\begin{aligned}\text{Energy} &= EIt = 100 \times 0.2 \times 60 = 1,200 \text{ joules,} \\ \text{or } \text{Energy} &= I^2Rt = 0.2^2 \times 500 \times 60 = 1,200 \text{ joules.}\end{aligned}$$

This amount of energy is converted into heat. The unit of heat is the “**calorie**” and

$$1 \text{ calorie} = 4.2 \text{ joules.}$$

$$\text{Thus, } 1,200 \text{ joules} = \frac{1,200}{4.2} = 286 \text{ calories.}$$

Example 7.

Ten accumulators, each having an E.M.F. of 2 volts and a resistance of 0.05 ohm, are to be charged with a current of 8 amps.

Find the amount of energy stored in them in 15 minutes (assuming that their E.M.Fs. remain at 2 volts) and the amount of heat energy wasted.

E.M.F. of battery opposing applied E.M.F. = 20 volts.

Resistance of battery = $0.05 \times 10 = 0.5$ ohm.

Voltage to drive 8 amps. through battery resistance
 $= 8 \times 0.5 = 4$ volts.

Voltage to overcome back E.M.F. of battery = 20 volts.

Hence required charging voltage = 24 volts.

Energy stored up in 15 minutes in the form of chemical energy
 $= E \times I \times t$

$$= 20 \times 8 \times (15 \times 60) = 144,000 \text{ joules,}$$

$$\text{or } 20 \times 8 \text{ watts} \times \frac{1}{4} \text{ hour} = 40 \text{ watt-hours.}$$

Heat energy = $E \times I \times t$

$$= 4 \times 8 \times \frac{1}{4} = 8 \text{ watt-hours,}$$

$$\text{or } 4 \times 8 \times 15 \times 60 \text{ joules} = 28,800 \text{ joules}$$

$$= \frac{28,800}{4.2}$$

$$= 6,860 \text{ calories.}$$

72. Kirchhoff's Laws.—These are generalisations of the ideas developed above which facilitate the calculation of the currents in a complicated arrangement of resistances. They are two in number.

(1) **Kirchhoff's First Law.**—At any junction of resistances, the sum of the currents flowing towards the junction is equal to the sum of the currents flowing away from it.

This merely expresses the fact mentioned above that electricity cannot accumulate at any point of a circuit (paragraph 65).

(2) **Kirchhoff's Second Law** is a generalisation of Ohm's Law. It states that in any closed circuit the sum of the E.M.Fs. reckoned positive in the direction of the current and negative in the opposite direction is equal to the sum of the products of current and resistance in every part of the closed circuit.

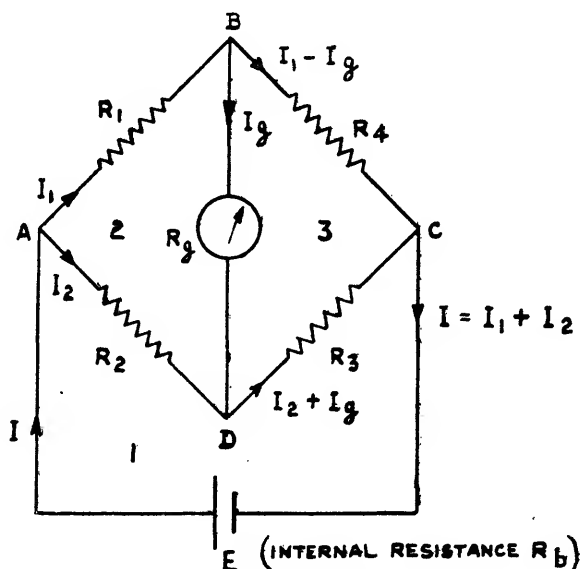
73. Wheatstone's Bridge.—This is a network of resistances which is in common use for the measurement of resistance, *e.g.*, in the Bridge Megger (paragraph 190). Its theory will be considered here as an example of the application of Kirchhoff's Laws.

Fig. 13 shows the arrangement. R_1 , R_2 , R_3 and R_4 are four resistances, one of which is unknown and whose value is to be determined. A current-measuring instrument G , whose resistance is R_g , is connected as indicated and current is sent through the arrangement by the battery E of internal resistance R_b .

We shall first obtain the equations for finding the current in any part of the circuit in terms of the resistances and E.M.F., *i.e.*, the various currents labelled I_1 , I_2 , I_g , &c.

The current flowing through the battery is taken to be I . At A this divides into two parts, I_1 and I_2 . Kirchhoff's First Law states that $I = I_1 + I_2$. According to the values of I_1 and I_2 , current will flow through G either from B to D or D to B . It is assumed to flow from B to D in this case and to be of value I_g . If it actually flows from D to B , I_g will turn out to be negative in the solution, *i.e.*, in the other direction to that assumed for it.

Kirchhoff's First Law then enables us to derive the currents in the other branches as labelled in the figure.



Wheatstone's Bridge.

FIG. 13.

The next step is to apply the Second Law to each closed circuit in the arrangement. These are labelled (1), (2) and (3). (EABC and ABCD are also closed circuits, but it will be found that no information is derived from them which is not contained in that given by the application of the law to (1), (2) and (3).)

Circuit (1). The sum of the products of current and resistance, taking account of direction, is

$$R_b I + R_2 I_2 + R_3 (I_2 + I_g).$$

The E.M.F. is E and acts in the direction of I .

$$\therefore R_b I + R_2 I_2 + R_3 (I_2 + I_g) = E. \quad (1)$$

Circuit (2). There is no E.M.F. acting and so the sum of the IR drops is zero, *i.e.*, $R_1 I_1 + R_g I_g - R_2 I_2 = 0$. (2)

Note that I_2 acts in the opposite direction round circuit (2) to that of I_1 and I_g , and so must be reckoned as negative if they are taken to be positive.

Circuit (3) gives in a similar manner

$$R_g I_g + R_3 (I_2 + I_g) - R_4 (I_1 - I_g) = 0. \quad (3)$$

These three equations, together with $I = I_1 + I_2$, (4) enable the values of I , I_1 , I_2 and I_g to be found in terms of the other quantities. As the solution, although straightforward, is rather heavy, it will not be worked out here.

74. In measuring resistance by the bridge megger, the values of the known variable resistances are adjusted until no current flows through the measuring instrument, *i.e.*, in the above equations $I_g = 0$. The relationship between the bridge resistances in this case may then be found as follows:—

Putting $I_g = 0$ in (2) and (3) gives

$$R_1 I_1 - R_2 I_2 = 0, \text{ or } R_1 I_1 = R_2 I_2 \text{ from (2)}$$

$$\text{and } R_3 I_2 - R_4 I_1 = 0, \text{ or } R_3 I_2 = R_4 I_1 \text{ from (3).}$$

From these relations by division it is easily seen that

$$\frac{R_1}{R_4} = \frac{R_2}{R_3}.$$

If three of these resistances are known, the fourth may thus be calculated.

LAMPS.

75. Incandescent lamps are rated by their voltage, power and candle power. There are three types in common use—carbon filament, metallic filament, and “gas-filled.” The bulb of the ordinary metallic filament lamp is evacuated; the gas-filled lamp has its bulb filled with an inert gas such as argon. The filaments of both are usually made of tungsten. These two kinds are of more modern introduction and, though perhaps more costly, are more efficient (as regards consumption of electrical energy) and do not blacken as do carbon lamps, owing to deposition of carbon.

Each type is referred to as consuming so many “watts per candle power.” Carbon lamps take, roughly, 3.5 to 4 watts per c.p., whereas metallic filament lamps vary, roughly, from 1 to 2 watts per c.p., and gas-filled lamps take $\frac{1}{2}$ watt per c.p.

The two latter consume less power to give the same amount of light, and hence are more efficient.

Example 8.

A 220-volt 32-c.p. metallic filament lamp of efficiency 1.7 watts per c.p. is burned from 220-volt mains; find what current it takes.

$$\text{Watts supplied} = 32 \times 1.7 = 54.4 = EI.$$

$$I = \frac{EI}{E} = \frac{54.4}{220} = 0.247 \text{ amps.}$$

For general lighting purposes metallic filament lamps are preferable, but for accumulator charging on a small scale, where lamps in parallel are used to regulate the charging current, carbon lamps are preferable, as they pass more current, and fewer are required.

MAGNETISM AND MAGNETIC FIELDS.

76. Permanent Magnets.—Pieces of a certain natural iron ore, called lodestone, are found to exercise on each other forces of attraction or repulsion, which vary according to definite laws. These are called permanent magnets. When freely suspended by a piece of silk fibre, any such permanent magnet sets itself in a definite direction with regard to the North and South Magnetic Poles of the Earth. That end which tends to point towards the North Magnetic Pole is termed the "North-seeking" or "North" (N) pole of the magnet, and that end which tends to point towards the South Magnetic Pole of the Earth is termed the "South-seeking" or South (S) pole of the magnet.

The magnetism inherent in the magnet is found to be more concentrated towards the ends, so that for purposes of investigation each end can be considered as a magnetic pole of a certain polarity; the longer and thinner the permanent magnet, the more is this assumption justified.

This concentration of magnetism at the ends or poles gives them the property of attracting small pieces of iron. Either end of a small piece of iron is attracted by either pole of a permanent magnet, and we shall see that this is explained by the fact that the iron, supposed unmagnetised at the start, is made into a temporary magnet under the influence of the permanent magnet and the resultant attraction is simply an example of the general forces of attraction and repulsion which exist between magnetic poles.

Permanent magnets can be artificially produced by bringing a bar of steel under the influence of magnetising forces. A common example of a permanent magnet is a compass needle.

77. Laws Governing Magnetic Action.—The forces of attraction and repulsion can be studied by suspending two magnets by pieces of silk fibre and bringing them close to each other.

It is found that :—

- (1) Like poles repel, unlike poles attract, each other.
- (2) The force exerted between two poles is inversely proportional to the square of the distance between them.

The magnitude of the force exerted gives rise to the idea of pole strength. Two poles are said to have equal pole strengths if the forces they exert on another pole at the same distance in the same medium are equal. By the use of the C.G.S. units of force and distance, the dyne and the centimetre, a definition of **unit pole strength** is then obtained. If two poles of equal strength, when placed 1 cm. apart in a vacuum, exert on each other a force of one dyne, they are said to possess **unit pole strength** or to be **unit poles**. A unit North pole is taken as the positive unit, and so a unit South pole has a pole strength of -1 .

- (3) With this definition of the strength of a unit pole, it is found that the force between two poles is proportional to the product of the strengths of the poles.

Combining these results, it may be stated that if two poles have strengths m_1 and m_2 , i.e., they are m_1 and m_2 times a unit pole, respectively, and they are placed d centimetres apart *in vacuo*, the force between them will be

$$\frac{m_1 m_2}{d^2} \text{ dynes.}$$

- (4) The two poles of the same magnet have the same strength.

The results of (2) and (3) above are modified if the medium between the poles is other than a vacuum, according to a property of the medium called its "permeability."

Permeability is denoted by the letter μ , and will be investigated further in the following paragraphs.

The formula of section (3) becomes, for an intervening medium with permeability, μ ,

$$F = \frac{m_1 m_2}{\mu d^2} \text{ dynes.}$$

It will now be seen that the unit magnetic pole defined above is derived on the basis that the permeability of a vacuum is unity ($\mu = 1$). We can only compare the relative permeabilities of various media, and so this has been adopted as the simplest assumption for investigating magnetic phenomena, but it introduces an arbitrary element into the system of units developed from the unit magnetic pole. This system is called the **electromagnetic system of units**. The unit pole defined above, for instance, is called the **electromagnetic unit (E.M.U.)** of pole strength.

78. The Magnetic Field.—The magnetic field is the space in the neighbourhood of a magnet or magnets in which the forces of attraction and repulsion mentioned above can be observed. If a small magnetised compass needle is introduced into such a field and freely suspended, the various forces of attraction and repulsion acting on its two poles will have some resultant effect, so that it will settle in some position of equilibrium and its N. (or S.) pole will point in some definite direction. The direction in which its **North pole** points is by convention taken as the **direction of the field**.

As the compass needle is moved from one position to another in the field, the direction in which it points varies, so that the magnetic field has a varying direction. If the magnetic field we are considering is produced by one bar magnet, and the small compass needle we are to use as our indicating device is brought very near to the N. pole of the bar magnet, the N. pole of the needle will point directly away from the magnet. If the compass needle is then moved carefully along the direction in which its North pole is pointing,

that direction will continuously vary, and the path traced out by the compass needle will come round to the South pole of the bar magnet. The tangent to this path at any point is the direction of the magnetic field there. By starting at different points, a series of different paths can be obtained, with the result shown in Fig. 14.

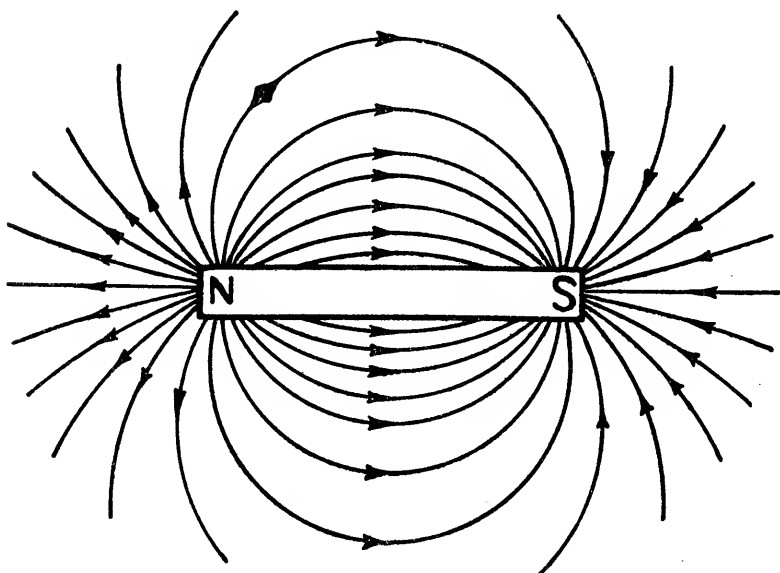


FIG. 14.

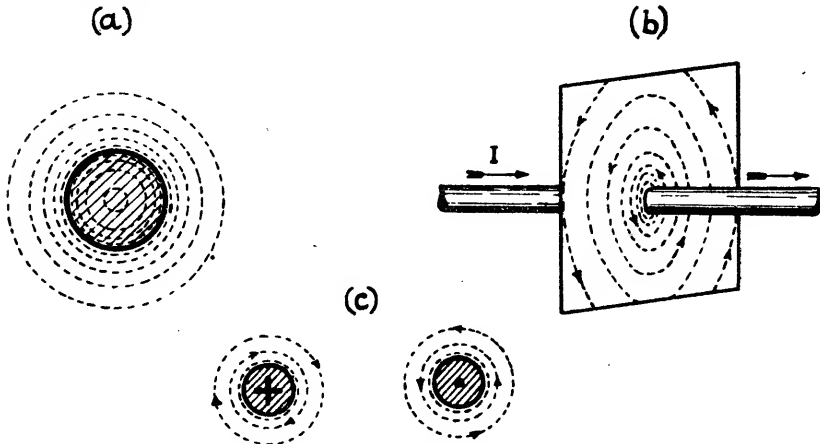
The curved paths above are known as **Lines of Magnetic Force**, and they denote the **direction** of the field.

79. Magnetic Fields Produced by Electric Currents.—So far we have only considered so-called permanent magnets, which are either natural magnetic ores or, for example, bars of steel which have been artificially "magnetised." An explanation of the latter process is bound up with the investigation of the effect of an **electric current in setting up a magnetic field**, which we now go on to consider.

It was mentioned in para. 44, as one of the effects of electric currents, that a wire carrying a current, *i.e.*, electricity in motion, sets up a magnetic field around it. This field is composed of lines of force which are in the form of concentric circles round the wire carrying the current, both inside and outside the conductor. (See Fig. 15.)

In other words, a small compass needle in the vicinity of a current-carrying conductor is acted on by exactly the same type of forces of magnetic attraction and repulsion as if it were brought near to a bar magnet.

The direction of the magnetic field is shown by the two figures 15 (c) below, which are end-on views of the conductor. The left-hand figure illustrates an electric current flowing away from the reader, in which case the positive direction of the lines of force is in a clockwise direction.



Magnetic Field round a Conductor carrying a Current.

FIG. 15.

The right-hand figure illustrates an electric current flowing towards the reader, in which case the positive direction of the lines of force is in a counter-clockwise direction.

The symbol \otimes is meant to illustrate the tail-feathers of an arrow going into the conductor, *i.e.*, a current receding; and the symbol \odot the point of an arrow coming out, *i.e.*, a current advancing.

The magnetic field is strongest at the surface of the conductor, and its diminishing intensity is roughly illustrated in the figures by the distances between successive lines.

The standard rule for determining the relation between direction of current flow and direction of the field is that they bear the same relation to one another as the direction of movement and that of rotation of an ordinary corkscrew.

80. Resultant Magnetic Fields.—If two wires carrying currents are placed parallel to one another the resultant magnetic field will be as indicated in Fig. 16 (a), if the currents are in opposite directions, or as indicated in Fig. 16 (b) if they are in the same direction.

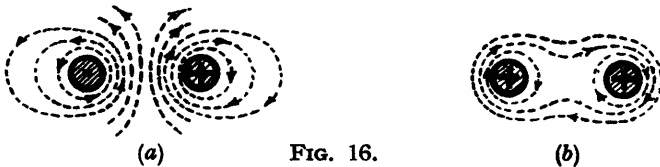


FIG. 16.

In the first case the wires tend to be pushed apart, and in the second case to come together.

If the wire carrying the current is wound in a loop, the lines of force all pass through the loop in the same direction, as shown in Fig. 17.

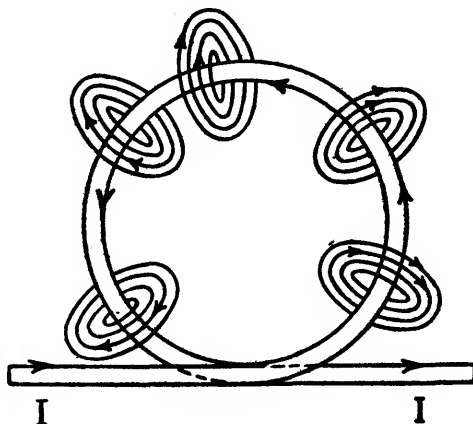


FIG. 17.

A coil of wire is simply a number of continuous loops and a current sent through such a coil produces lines of force in a lengthwise direction through the coil, emerging at the ends, and completing the magnetic circuit through the surrounding medium. The diagram, Fig. 18, shows how the concentric lines of force merge together to give the resulting field.

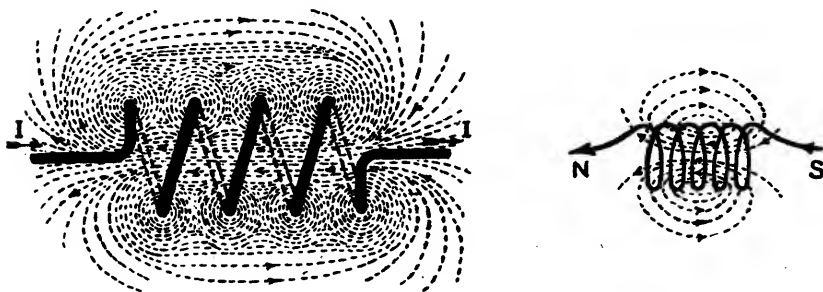
*Field round Solenoid.*

FIG. 18.

Every line of force is closed on itself, and in our original investigation of a bar magnet this statement is justified by assuming the lines of force, which experimentally can only be plotted outside the magnet, to continue from the S. pole of the magnet to the N. pole inside the magnet itself.

81. A coil of wire carrying a current and so having a magnetic field as shown is called a **Solenoid**. The end of the coil from which the lines of force emerge is termed its **North Pole** and the end at which they enter is termed its **South Pole**, just as in the case of the bar magnet.

The "End Rule" or "Clock Rule" for determining the polarity of a coil is:—"Look at one end of the coil; if the current flows in a clockwise direction (either towards or away from you) then that end will have south polarity; if anti-clockwise, north polarity."

This is illustrated in Fig. 19, the arrows on the letters S and N being, as shown, indicative of the current being clockwise or not.

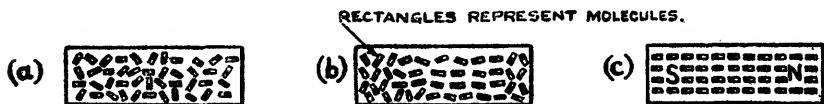


FIG. 19.

It should be obvious that the solenoid produces exactly the same distribution of magnetic field as the bar magnet—in other words, as long as the solenoid is carrying current, it is an electromagnet.

82. Molecular Theory of Magnetism.—This similarity between the magnetism produced by an electric current and that produced by a permanent magnet leads to the conclusion that it is the same fundamental cause that is operative in the two cases. An electric current is nothing but a movement of electrons in some definite direction, and the magnetism of the permanent magnet is due to the fact that the electrons which revolve round the nucleus of the atom do so in a less haphazard way than in a non-magnetic substance. The electrons rotating round the nucleus of any atom set up magnetic effects, because they constitute a current of electricity. Each molecule is thus effectively a small magnet, or equivalent to a small current-carrying coil. In non-magnetised substances the molecules lie "anyhow," so that the minute magnetic field due to any one of them is neutralised by the field of some other one, which is in such a position that their fields oppose and annul each other.

In a magnetised substance, however, it appears that a large proportion of the molecules have their axes pointing in the same direction, so that their magnetic fields are in the same direction and therefore additive.



Illustrating Magnetisation.

FIG. 20.

When a substance is completely magnetised, all the molecules have their axes turned in the same direction and the resultant effect is strongest. These conditions are illustrated in Fig. 20, the three diagrams representing non-magnetised, partially magnetised, and completely magnetised substances. When completely magnetised a substance is said to have attained "saturation."

83. Induced Magnetism.—If a piece of magnetic material, say, soft iron, is introduced into the magnetic field of a solenoid, it becomes magnetised by "**induction.**" Under the influence of the magnetic field, the elementary magnets, the molecules of which it is composed, tend to come into the definite arrangement referred to in the last paragraph, and so the soft iron itself is a **temporary magnet**, as long as the current which produces the effect is kept flowing.

As a result the following effects are noticed :—

- (1) The strength of the original magnetic field is increased, due to the fact that the magnetic effects of the constituent molecules of the soft iron now act in conjunction with the magnetic effect of the current in the coil.
- (2) Forces of attraction and repulsion are operative between the solenoid, which is, as we saw, a magnet, and the soft iron, which is now a temporary magnet.
- (3) The iron tends to move from weaker to stronger parts of the field.

When the piece of soft iron is removed from the magnetic field, or the current is cut off, it loses most of the temporary magnetisation it has acquired.

Similar effects are observed if steel is substituted for soft iron ; but, for the same inducing field, the induction in steel is less than in soft iron, and the residual magnetism immediately after the field is removed is also smaller. It is found, however, that heating to red heat, mechanical disturbance, or the mere passage of time has less effect on magnetised steel than on magnetised soft iron. Steel is, therefore a more suitable material for the construction of permanent magnets.

The term "retentivity" has been used to describe the property of retaining magnetism after the magnetising field has been removed.

All materials do not give the results quoted in this paragraph. The substances, such as iron, steel and nickel, which show magnetic properties strongly when subjected to a magnetising force are called **ferro-magnetic**.

Substances which show magnetic properties to a very slight degree are called **para-magnetic**, and those which act in the opposite way—that is, diminish the strength of a magnetic field when placed in it, are termed **dia-magnetic**.

The latter two characteristics are of little practical importance in our study of electro-magnetism.

84. Field Strength.—So far we have dealt very little with the quantitative aspect of magnetism, and in paragraph 78 only considered the direction of the magnetic field without reference to its intensity. The intervening paragraphs have been concerned with a descriptive account of phenomena which must now be investigated more strictly.

The **Intensity** of the magnetic field at any point is the force in dynes acting on a unit pole placed at that point.

This is also referred to as the **Magnetic Field Strength** at the point, and is denoted by the letter H . It is measured in dynes per unit pole, sometimes called "gauss."

Unit field strength exists at a point where the force on a unit pole equals one dyne, and therefore unit field exists at a distance of one centimetre *in vacuo* from a unit pole. This is the **electromagnetic unit**. The field strength at a distance of one centimetre from a unit pole in a medium of permeability μ is $\frac{1}{\mu}$ gauss or dynes per unit pole.

85. Lines of Magnetic Flux.—It has been seen that the direction of the magnetic field strength at any point in the field is quite definite, being given by the tangent to the "line of magnetic force" passing through the point. In other words, the force of attraction or repulsion on a magnetic pole is fixed both in magnitude and direction at any point of the field.

To provide some picture of how such "action at a distance" may take place, it has been found convenient to look upon the "lines of magnetic force" as if they had a physical existence, and to regard them as if they were in the nature of tentacles which a magnetised body shoots out into the space around it, and which act directly on other magnetised bodies in the neighbourhood and pull and push them about. It has further been found possible to give a quantitative significance to the number of these tentacles in any part of the field. They are then called "lines of magnetic flux."

86. Magnetic Flux.—The two quantities with which we might associate a number of "lines of magnetic flux" for quantitative purposes are:—

- (1) The strength of the magnetic pole producing them.
- (2) The mechanical forces which are exerted on poles at various parts of the field, *i.e.*, the magnetic field intensity.

The latter suffers from the disadvantage that it changes abruptly in value when the medium changes, so that if, for instance, a piece of iron were in a magnetic field mostly consisting of air as medium, and if we identified number of lines with field strength, we should be forced to assume that many of the "tentacles" vanished at one edge of the iron and only reappeared at the other edge. Their

existence in the iron would have to be denied as the field strength in the iron is very much less than in the surrounding air.

It has been found more convenient to assume that these lines of magnetic flux are continuous and their number independent of the medium, and for this reason the number of lines is derived from the strength of the magnetic pole from which they set out. Their direction, of course, at any point is the direction of the magnetic field at that point, *i.e.*, the direction of the "lines of magnetic force."

87. The Unit of Flux.—In the field round a unit pole concentrated at a point, the field strength is the same at all points the same distance away from the pole ($H = \frac{1}{\mu r^2}$ at a distance r). All such points lie on a sphere whose centre is the unit pole, and the direction of the field is radially outwards. The lines of magnetic flux thus are straight and diverge radially from the pole like the spokes of a wheel (in three dimensions). The surface area of a sphere of radius r is $4\pi r^2$ and so the surface area of a sphere of 1 cm. radius is 4π sq. cms.

The number of lines of magnetic flux emanating from a unit pole is taken to be such that one line goes through each sq. cm. of the surface of a sphere of radius 1 cm. described about the pole as centre.

Thus, for a pole strength of three units, three lines would go through every sq. cm. on the surface of such a sphere.

In the case of a unit pole, the total number of lines crossing this sphere is 4π , *i.e.*, 4π lines of magnetic flux emanate from a unit pole.

Lines of magnetic flux used as quantitative units in this way are usually called "**lines**." The name "Maxwell" for this unit has also been suggested. It is an E.M.U.

The total number of lines of magnetic flux passing through any area is called the "**flux**" through that area and denoted by the letter Φ (phi).

The "**flux density**" or "**magnetic induction**" at any point in a magnetic field is the flux passing through an area of 1 sq. cm. at right angles to the direction of the field at that point. It is denoted by B . B is thus measured in lines per sq. cm. or Maxwells per sq. cm. $\Phi = BA$ for an area A , which the flux Φ (of uniform density B) crosses at right angles.

88. Relation between Flux Density and Field Strength.—From the derivation of the "line" it will be seen that the flux density at a distance of 1 cm. from a unit pole is $B = 1$ line per sq. cm. The field strength *in vacuo* at this distance is $H = 1$ gauss. Thus, *in vacuo*, $B = H$ numerically.

In any other medium the field strength at a distance of 1 cm. from a unit pole is $H = \frac{1}{\mu}$ gauss, where μ is the permeability of the medium. B is unaffected by the medium and depends only on the

pole strength, *i.e.*, B is still 1 line per sq. cm., and so $B = \mu H$ in a medium of permeability μ , *i.e.*, the permeability μ now appears as the numerical ratio of the flux density to the magnetic field strength at any point in the field.

$$\mu = \frac{B}{H}$$

This ratio of flux density to field strength, or permeability, μ , differs widely for different substances, and it is variable in value for

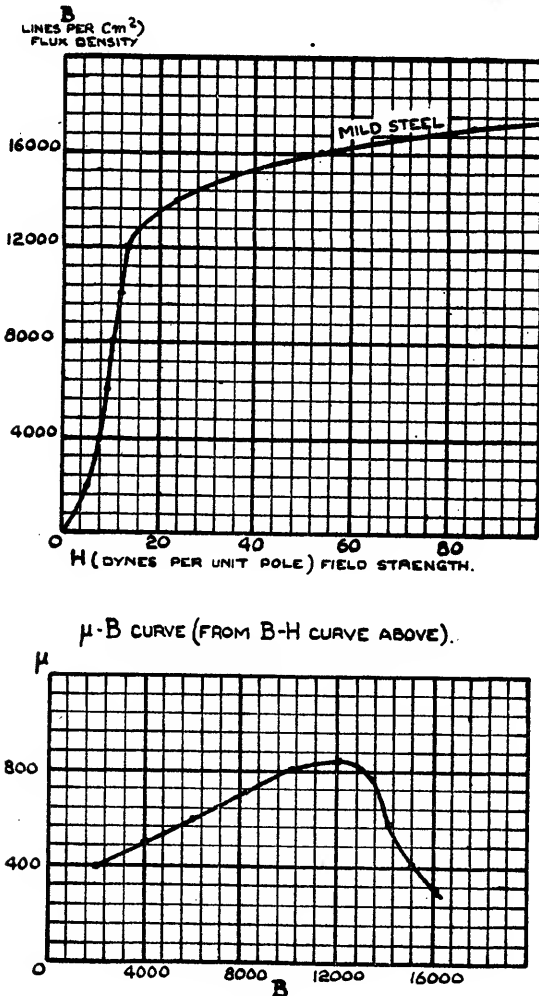


FIG. 21.

the same substance. In the case of iron it may vary with the quality of the iron, the value of B , and the temperature of the iron.

The distinction between materials drawn at the end of para. 83 can now be explained more accurately :—

Ferro-magnetic materials, such as iron, steel, nickel, cobalt and their alloys, have a high and variable value of μ . It may vary between, say, 200 and 1,000, for some specimen, but these numbers are simply indicative of its order of magnitude.

Para-magnetic materials, such as the ordinary non-magnetic metals, copper, aluminium, &c., have values of μ which are constant and very little greater than unity.

Dia-magnetic materials, such as bismuth, have μ constant and very little less than unity.

In practice, we may neglect the deviation of μ from unity in all substances except the ferro-magnetic ones. In other words, for all substances, except ferro-magnetic substances, it may be taken that $B = H$, numerically. The units of B and H are, of course, quite different (the line per sq. cm. and the gauss respectively) as they represent different properties of the field.

A curve of B against H for a ferro-magnetic material is shown in Fig. 21, and also a permeability curve, showing variation of μ with B .

89. Hysteresis.—The property of steel already referred to as retentivity is a special case of a more general property exhibited by ferro-magnetic materials and known as **hysteresis**. This property may be studied by winding a coil of wire round an anchor ring of

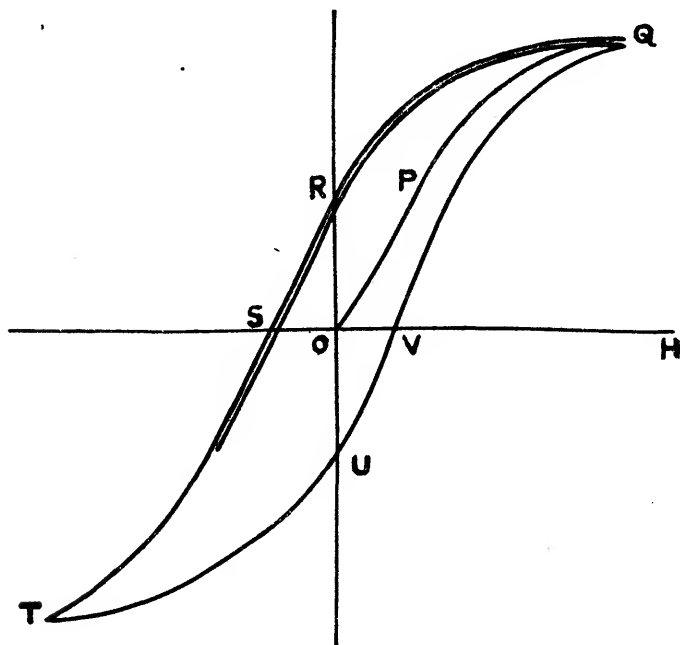


FIG. 22.

some ferro-magnetic metal, say, cast steel, and sending a current through the coil. It will be seen in the next paragraph that the field strength in the ring is proportional to the current. Instruments for measuring flux directly are also available. Thus, corresponding values of flux and magnetising current may be measured, and the equivalent values of B and H obtained from them. The curve OPQ in Fig. 22 is obtained in this way by gradually increasing the current from zero to a certain value and measuring the amount of flux produced by various currents.

If now the current is gradually reduced from its maximum value, the flux density does not diminish along the same curve as it increased, but along QR. When the current is reduced to zero, there is a residual magnetisation, shown in Fig. 22 by OR, and if the current is then increased in the opposite direction and a complete cycle of current changes completed, the flux density curve assumes the shape of the loop QRSTUVQ.

The magnetisation may be said to lag continually behind the magnetising force, and the word **Hysteresis**, meaning "lagging behind," is used to describe this effect. It can be proved that the area of the **Hysteresis Loop** is indicative of energy loss and is a definite factor to be considered in all apparatus where changing currents set up changing states of magnetisation. The energy is expended in heating the iron. This property is common, in varying degrees, to all ferro-magnetic substances.

90. The electromagnetic unit of current is derived from the magnetic field at the centre of a single circular loop carrying a current such as is shown in Fig. 17. It will be seen that the field inside the loop due to the current is in a direction coming out of the plane of the paper. At the centre of the loop the direction of the resultant field is perpendicular to the plane of the paper; its magnitude is everywhere proportional to the current flowing in the loop. The E.M.U. of field strength has already been defined (paragraph 84) and the measurement of field strength in dynes per unit pole explained. The E.M.U. of current is based on such measurements.

The **E.M.U. of current** is such that when flowing in a circular loop of wire of radius 1 cm., the magnetic field it produces at the centre of the loop is 2π dynes per unit pole.

The " 2π " appears because 2π cms. is the length of the circumference of the circular loop specified in the definition. Every small part of the loop contributes equally to the resultant field at the centre so that any one cm. length of the circumference contributes one dyne per unit pole to the total. The E.M.U. of current may therefore be based directly on the E.M.U. of field strength by a rather more artificial definition as follows:—

The E.M.U. of current is such that when it flows in a conductor of length 1 cm. shaped into an arc of a circle whose radius is 1 cm.,

it produces at the centre of the circle a field of one dyne per unit pole.

The E.M.U. of current is rather a large current and the ampere or practical unit was originally taken to be one-tenth of it,

$$\text{i.e., } 1 \text{ E.M.U.} = 10 \text{ amperes.}$$

Later, when the International Ampere was defined from the chemical effect for standardising purposes, its value was arranged to be as nearly as possible 0.1 E.M.U. This was the reason for choosing what seems otherwise such a peculiar amount as 0.001118 gm. silver in the definition of the International Ampere. It follows directly from the above that

$$1 \text{ E.M.U. of quantity of electricity} = 10 \text{ coulombs.}$$

Unit P.D. on the E.M. system is the P.D. between two points when the work done in carrying 1 E.M.U. of quantity from one point to the other is 1 erg,

$$\begin{aligned} \text{i.e., } 1 \text{ E.M.U. of P.D.} &= \frac{1 \text{ erg}}{1 \text{ E.M.U. of quantity}} = \frac{\frac{1}{10^7} \text{ joule}}{10 \text{ coulombs}} \\ &= \frac{1 \text{ joule}}{10^8 \text{ coulomb}} = \frac{1}{10^8} \text{ volt,} \end{aligned}$$

or 1 volt = 10^8 E.M.U. of P.D. (or E.M.F.).

The E.M.U. of resistance may now be compared with the ohm.

$$\begin{aligned} 1 \text{ ohm} &= \frac{1 \text{ volt}}{1 \text{ amp.}} = \frac{10^8 \text{ E.M.U. of P.D.}}{\frac{1}{10} \text{ E.M.U. of current}} \\ &= 10^9 \text{ E.M.U. of resistance.} \end{aligned}$$

91. Field Strength inside a Solenoid.—It can be proved that, if I be the current in amperes, N the number of turns in the solenoid, and l the mean length of the path of the lines of force measured in centimetres, the field strength inside a solenoid is uniform and given by

$$H = \frac{4\pi IN}{10l} = 1.257 \frac{IN}{l}.$$

By definition, this is the force in dynes which would act on a unit magnetic pole placed inside the solenoid.

Theoretically this is only true for an infinitely long, uniformly wound solenoid, or for a uniformly wound annular solenoid.

The latter case is an important one in practice, and such a solenoid, as illustrated in Fig. 23, is known as a **Toroid**, or **Toroidal Coil**. For such a coil, if the cross-section is small compared with the radius, the field strength can be taken as uniform over the cross-section.

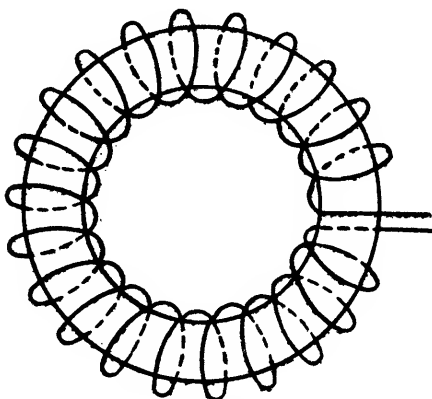


FIG. 23.

92. Magneto-motive Force. Reluctance.—In Fig. 23 suppose the magnetic circuit (the core of the toroid) to be filled with a material of permeability μ .

If H is the field strength, $B = \mu H$ is the flux density in the magnetic circuit.

The total flux Φ is given by the product of flux density and cross-sectional area.

$$\begin{aligned}\Phi &= BA = \mu HA \\ &= \mu A \cdot \frac{4\pi IN}{10l}.\end{aligned}$$

This can be re-written as

$$\frac{4\pi}{10} (IN) = \Phi \times \frac{l}{\mu A}.$$

The product $\frac{4\pi}{10} (IN)$, or $1.257 IN$, is called the **Magneto-Motive Force (M.M.F.)** by analogy with E.M.F.: for, by the above argument, it is equal to HI , *i.e.*, to the work done in carrying a unit magnetic pole round the magnetic circuit. It is denoted by G (not to be confused with the symbol for conductance, which is also G).

The term $\frac{l}{\mu A}$ is called the **Reluctance** of the circuit and the equation can therefore be expressed as

$$\text{Magneto-Motive Force} = \text{Flux} \times \text{Reluctance}.$$

The product IN is known as the **ampere-turns**, *i.e.*, the number of amperes multiplied by the number of turns in the coil.

From examination of the above equation it will be seen to bear a close analogy to the equation $\text{E.M.F.} = \text{current} \times \text{resistance}$.

Just as in an electric circuit there is a resistance to current flow which in conjunction with the E.M.F. applied gives a certain value of current, so in the magnetic circuit we can look upon the reluctance

as a factor to be overcome when a certain M.M.F. tries to establish a flux through the circuit, and which, in conjunction with the M.M.F., establishes a certain value of flux.

The **M.M.F.** depends on the product $I \times N$, because the greater the current the stronger should be the magnetic flux through the solenoid, and similarly the greater the number of turns the stronger should be the flux. The flux due to a number of turns is simply the sum of the fluxes due to individual turns.

The **reluctance** is analogous to the resistance in an electric circuit, and likewise varies directly as the length and inversely as the cross-section of the circuits. The other term μ , which comes into the expression for reluctance, is a property of the particular substance in the magnetic circuit which determines the extent to which a given M.M.F. or a given field strength H , which is simply $\frac{\text{M.M.F.}}{l}$, can establish a flux density, and so corresponds in inverse

relationship to the "specific resistance" ρ of any particular substance in the electric circuit. Permeability in the magnetic circuit is akin to conductivity in the electric circuit.

The two results are again quoted for comparison :—

$$\text{Reluctance} = \frac{1}{\mu} \frac{l}{A}; \quad \text{Resistance} = \rho \frac{l}{A}.$$

It should, however, be noted that the resemblance is purely formal. As far as we know, there is nothing flowing round a magnetic circuit and composing the flux in the way that electrons flow round an electric circuit and compose the current.

93. If the magnetic circuit be not uniform, the separate parts are treated individually, as in an electric circuit, and their reluctances added.

If the circuit is composed of materials whose dimensions and permeabilities are varying, and, in the same notation as used above, are given by $l_1 A_1 \mu_1, l_2 A_2 \mu_2$, &c., the total M.M.F. required to establish a certain flux Φ through the combined circuit is given by :—

$$\text{M.M.F.} = \Phi \left(\frac{l_1}{\mu_1 A_1} + \frac{l_2}{\mu_2 A_2} + \dots \right).$$

The equation is often written in the form which gives the **ampere-turns** necessary to produce a given flux in the circuit.

$$\text{Since} \quad \frac{4\pi}{10} (\text{IN}) = \Phi \times \frac{l}{\mu A},$$

$$\text{IN} = \frac{10}{4\pi} \cdot \Phi \cdot \frac{l}{\mu A} = \frac{0.8 \Phi l}{\mu A} \text{ approximately.}$$

In the general case,

$$\text{IN} = 0.8 \Phi \left(\frac{l_1}{\mu_1 A_1} + \frac{l_2}{\mu_2 A_2} + \dots \right)$$

As μ varies with B for ferro-magnetic materials, the value of μ must be obtained from a permeability curve, or a B - H curve, in the case of a definite numerical example, where the ampere-turns are required to give a certain total flux, or flux density. An example is appended.

Example 9.

(1) Calculate the ampere-turns necessary to produce a flux of 144,000 lines in a closed ring of mild steel made up of two sections, one (a) of length 100 cms. and square cross-section, the side of the square being 3 cms., and the other (b) of length 60 cms. and square cross-section, the side of the square being 4 cms., using the B - H curve of Fig. 21 (or the μ - B curve).

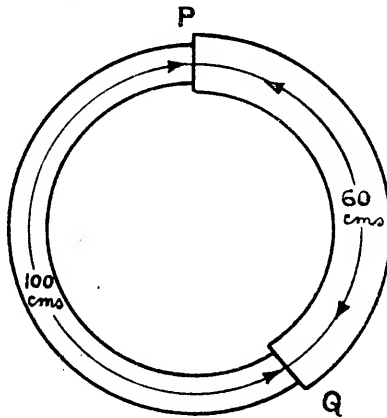


FIG. 24.

The areas of the cross-sections are 9 sq. cms. and 16 sq. cms., respectively.

Flux $\Phi = 144,000$ lines.

$$\therefore \text{Flux density in (a) is given by } B = \frac{\Phi}{A} = \frac{144,000}{9} = 16,000 \text{ lines per sq. cm.}$$

$$\text{Flux density in (b) is given by } B = \frac{\Phi}{A} = \frac{144,000}{16} = 9,000 \text{ lines per sq. cm.}$$

Section (a).

$$l = 100 \text{ cms.}$$

$$A = 9 \text{ sq. cms.}$$

$$B = 16,000 \text{ lines per sq. cm.}$$

$$H \text{ (from } B\text{-}H \text{ curve)} = 54.$$

$$\mu = \frac{16,000}{54} = 296.$$

(A 313/1198)Q

Section (b).

$$l = 60 \text{ cms.}$$

$$A = 16 \text{ sq. cms.}$$

$$B = 9,000 \text{ lines per sq. cm.}$$

$$H \text{ (from } B\text{-}H \text{ curve)} = 12.$$

$$\mu = \frac{9,000}{12} = 750.$$

$$\begin{aligned}
 &\text{Section (a)} && \text{Section (b)} \\
 \text{Reluctance} &= \frac{l}{\mu A} = \frac{100}{296 \times 9} && \text{Reluctance} = \frac{l}{\mu A} = \frac{60}{750 \times 16} \\
 \text{Total reluctance} &= \frac{100}{296 \times 9} + \frac{60}{750 \times 16} \\
 \text{IN} &= 0.8 \Phi \times \text{Total Reluctance} \\
 &= (0.8) (144,000) \left\{ \frac{100}{296 \times 9} + \frac{60}{750 \times 16} \right\} \\
 &= 115,200 \{0.03754 + 0.005\} \\
 &= 115,200 (0.04254) \\
 &= 4,900.
 \end{aligned}$$

This could be produced by 7 amps. flowing through 700 turns or 49 amps. through 100 turns, &c.

(2) Suppose now the ring is cut across at the diametrically opposite points X and Y, both in Section (a), and the two halves are separated so as to leave an air gap of 1 cm. at each of these points, as shown in Fig. 25.

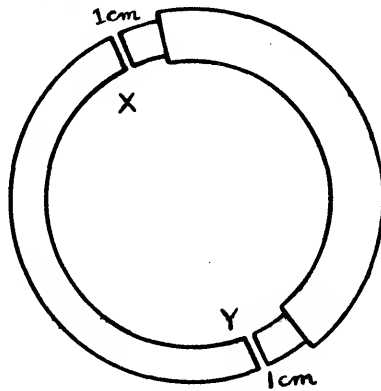


FIG. 25.

The reluctance is found to be greatly increased, because of the small air-gaps. Their total length is small, only 2 cms., but the μ for them is unity.

The additional reluctance is given, as before, by $\frac{l}{\mu A}$, where $l = 2$ cms., $\mu = 1$, $A = 9$ sq. cms.

So that $\frac{l}{\mu A}$ for the air-gaps in series $= \frac{2}{9} = 0.222$.

Total reluctance $= 0.04254 + 0.222 = 0.26454$,

Ampere Turns $= 115,200 (0.26454)$.

$= 30,500$.

Thus, with 100 turns, a current of 305 amperes would be necessary instead of 49 amperes.

This effect of the majority of the reluctance of a magnetic circuit being due to an air-gap, where such is included, is very important in machine construction, and air-gaps are designed to be as short as possible in order to minimise their tendency to diminish flux density.

ELECTROSTATICS AND ELECTRIC FIELDS.

94. It has already been stated, in paragraph 37, that like charges repel one another and unlike charges attract one another.

The forces of attraction and repulsion are called **electric forces** and the space in which they act is called the **electric field**.

Electric fields and magnetic fields, which were treated in the previous section, have many points in common.

95. Unit charge.—It is possible to isolate a “charge” of positive or negative electricity by friction.

If a stick of sealing-wax is rubbed with flannel it acquires the property of attracting small pieces of paper, &c. It has become negatively charged, or, in terms of the electron theory, it has acquired a surplus of electrons. The flannel is left with a deficit of electrons, and is thus positively charged. Two such electrified sticks of sealing-wax will repel one another. A glass rod rubbed with silk will be positively charged, and will attract the sealing-wax. By such simple experiments the mutual action of electrified bodies can be studied, and this is the way in which the first experimenters built up the theory.

It is found that the force between two charged bodies varies inversely as the square of the distance between them.

A system of units based on the mechanical forces acting on electric charges may thus be developed in an exactly similar manner to the system based on the forces between magnetic poles (paragraph 77). This is called the **electrostatic system** of units.

The **electrostatic unit (E.S.U.)** of charge or quantity of electricity is defined as follows:—

Two **unit charges** placed at a distance of one centimetre apart *in vacuo* exert on each other a force of one dyne.

The relation between this unit and the practical unit, the coulomb, is given by:—1 coulomb = 3×10^9 E.S.U.

96. The force between two charges is proportional to their product, so combining this with the result above relating to distance between them, two charges, of Q_1 and Q_2 units respectively, at a distance d cms. apart will exert a mutual force of $\frac{Q_1 Q_2}{d^2}$ dynes.

If the charges are like, the force is one of repulsion; if unlike, one of attraction.

The mutual force is dependent on another factor, a property of the material between the charges called its "**dielectric constant**," or "specific inductive capacity" (S.I.C.), or "permittivity," and denoted by the letter K .

When modified to take account of this property of the medium, the law of force between charges becomes

$$F = \frac{Q_1 Q_2}{Kd^2} \text{ dynes.}$$

Thus the electrostatic system of units is based on the assumption that $K = 1$ *in vacuo*.

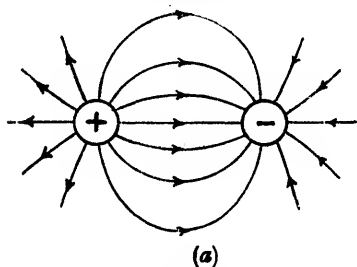
97. Electric Field.—This has already been defined as the space in the neighbourhood of charges where forces of attraction and repulsion are exerted on other charges.

Its **strength** at any point can be measured by the force which would be exerted on a unit charge placed there, and, using the same units as before, the **field strength** is the **force in dynes per unit charge**. It is denoted by X . The field strength at a distance r *in vacuo* from a point charge Q is

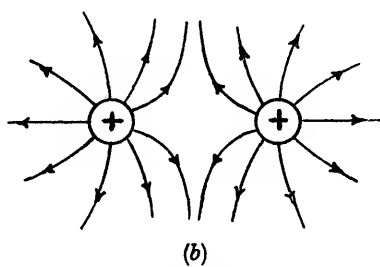
$$X = \frac{Q}{r^2}.$$

In a medium of dielectric constant K , $X = \frac{Q}{Kr^2}$ under the same conditions.

98. Lines of Force.—The direction of the electric field round an isolated electric charge is radial. This can be represented by drawing a series of straight lines outwards from the charge, in which case the direction of the field at any point is along the line passing through that point.



Electric Field between Two Unlike Charges.



Electric Field between Two Like Charges.

FIG. 26.

The direction of the resultant electric field set up by two charges is more complicated. The field between two equal and opposite charges is as shown in Fig. 26 (a). The similarity between this field and that existing between two equal and opposite magnetic poles (see Fig. 14) should be noted.

99. Lines or Tubes of Electric Flux.—In the quantitative study of magnetic phenomena it was found convenient to develop the idea of the magnetic flux through an area in the magnetic field (paragraph 86). The same type of concept is also found useful in electrostatics and is derived in a very similar manner. Lines or tubes of electric flux are supposed to emanate from electric charges. Each line is assumed to start from a positive charge and end on an equal negative charge, as indicated in Fig. 26 (a).

If we assume that these lines or tubes behave like stretched rubber cables, *i.e.*, tend to contract in the direction of their length, a simple explanation of the attraction at a distance between two opposite charges is obtained. The lines, by tending to contract, tend to pull the charges together.

Repulsion effects between like charges may also be explained on this basis if we assume that the tubes of flux in contracting lengthwise expand laterally, *i.e.*, increase in cross-section.

The field in the neighbourhood of two like charges is shown in Fig. 26 (b). The tubes of flux from one of these charges cannot end on the other; and because of their tendency to contract lengthwise, there will be a competition between them to end on the nearest possible negative charges. This brings the tubes from each charge running sideways in the same general direction. Their tendency to expand sideways will then force the two charges apart.

Electric flux is denoted by the symbol ψ (psi). The unit is derived on a slightly different basis from the unit line of magnetic flux, the definition of which led to the result that 4π lines started from a unit pole.

One line or tube of electric flux is defined as the total flux which starts from a unit positive charge (or ends on a unit negative charge).

It may be considered rather difficult to picture the single tube of flux from a unit point charge, as presumably it must spread out to enclose all the space around the point and the idea of "tube" rather loses its significance. It should be remembered, however, that if these tubes are more than a convenient way of obtaining results and if there is anything corresponding to them in reality, there must be at least one tube per electron, *i.e.*, one unit tube, as above defined, must correspond in reality to about two thousand million electron tubes, which makes the flux easier to visualise. It is on account of unitary considerations that the unit tube is defined as above, and it is to be taken merely as the representative of an unknown number.

The **Electric Displacement** or **Flux Density** at any point in a field is the number of lines of flux passing through an area of 1 sq. cm. at right angles to the direction of the field at the point.

Flux density is given the symbol D . Thus by definition $D = \frac{\psi}{A}$, where ψ is the flux (of uniform density D) through an area A .

100. Relation between Field Strength and Flux Density.—The field strength X at a distance r from a point charge Q in a medium of dielectric constant K is given by

$$X = \frac{Q}{Kr^2} \text{ dynes per unit charge.}$$

The total flux through the sphere of radius r described about the charge Q as centre is Q lines.

The surface area of this sphere is $4\pi r^2$, and the flux distribution is obviously uniform through its surface, so that the flux density

$$D = \frac{Q}{4\pi r^2} \text{ lines per sq. cm.}$$

$$\therefore X = \frac{4\pi D}{K}, \text{ and } D = \frac{KX}{4\pi}.$$

These results should be compared with $H = \frac{B}{\mu}$ and $B = \mu H$ respectively. It is seen that they differ by the factor 4π . This is due to the 4π lines of magnetic flux taken to be associated with a unit pole compared with the one line of electric flux attributed to a unit charge.

Though derived from the special case of a point charge, the results above can be shown to be perfectly general and to hold for any distribution of charges.

101. Potential Energy of Electric Field.—It has been shown that a charged body placed in an electric field has a force acting on it and will generally be set in motion, *i.e.*, it will acquire kinetic energy. The only possible source of this energy is the electric field itself. Thus, whenever an electric field is established, there is a certain amount of energy stored up. By allowing the charges which have produced the field to come together under their mutual attractions, this stored energy will be converted to energy of motion of the charges and eventually to heat or light energy when the motion is stopped. Thus we may consider the electric field to be a storehouse of electrical potential energy.

102. Potential Difference and Potential.—When a charged body moves under the action of the field, the work done on it is obtained from the field energy.

Just as in the case of a current flowing in a conductor, this transformation of energy leads to the concept of potential differences between different points in the field.

The potential difference between two points in an electric field is defined as the work done when a unit positive charge moves from one point to the other. If this movement takes place owing to the field forces themselves, the first point is said to be at a higher potential than the second. If the charge has to be moved from one point to the other against the opposition of the field forces, the first

point is at a lower potential than the second. The zero of potential, as before (paragraph 53), is taken as the potential of the earth's surface), thus enabling absolute values (P.D.s. to earth) to be given to the potentials at various points of the field.

The E.S.U. of P.D. is thus the P.D. between two points when the work done in moving one E.S.U. of charge from one point to the other is one erg.

$$\text{i.e., 1 E.S.U. of P.D.} = \frac{1 \text{ erg}}{1 \text{ E.S.U. of charge}}$$

The practical unit of P.D. is the volt, *i.e.*, the P.D. between two points when the work done in moving 1 coulomb from one point to the other is 1 joule.

Since 1 coulomb = 3×10^9 E.S.U. of charge,

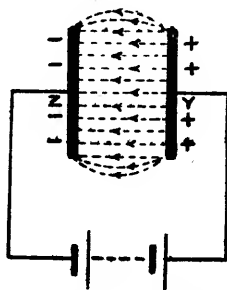
and 1 joule = 10^7 ergs,

we may easily find the connection between the E.S.U. of P.D. and the volt.

$$\begin{aligned} 1 \text{ E.S.U. of P.D.} &= \frac{1 \text{ erg}}{1 \text{ E.S.U. of charge}} = \frac{\frac{1}{10^7} \text{ joule}}{\frac{1}{3 \times 10^9} \text{ coulomb}} \\ &= \frac{300 \text{ joules}}{\text{coulomb}} = 300 \text{ volts.} \end{aligned}$$

The relation between the practical and electrostatic units of other electrical quantities may be similarly obtained.

103. The above ideas will now be illustrated for the case of a uniform electric field, *i.e.*, a field in which the electric field strength has the same value and direction at every point. Such a field is



Electric Field between Two Charged Conductors.

FIG. 27.

obtained approximately in practice by the arrangement shown in Fig. 27, where two large conducting plates are set up parallel to each other with a layer of some dielectric between them, and connected to the terminals of a battery. The P.D. between the plates

is the terminal P.D. of the battery (V). Plate Y has a positive charge, say, $+Q$, and plate Z has an equal negative charge $-Q$. Owing to their longitudinal contraction and lateral expansion the tubes of flux will be arranged as shown. They then have the minimum possible length. Tubes at the edges of the plates are bowed outwards because there is no lateral pressure on the outside of them to counterbalance that of the more central tubes. Thus, in practice, the field is not uniform at the edges. For simplicity we shall neglect this and assume the field to be uniform everywhere between the plates.

The flux starting from plate Y is $\psi = Q$.

If Y has an area of A sq. cms., then $D = \frac{\psi}{A} = \frac{Q}{A}$ is the flux density.

The field strength X is therefore given by $X = \frac{4\pi D}{K} = \frac{4\pi Q}{KA}$, where K is the S.I.C. of the dielectric. This is the force on a unit charge anywhere in the field.

Suppose a unit charge is allowed to move from Y to Z (a distance of d cms., say) under the action of the field. It will move along a line of force, *i.e.*, it travels d cms. under a force X and so the work done on it is Xd .

By definition this is the P.D. (V_{yz}) between Y and Z.

$$\text{i.e., } V_{yz} = Xd,$$

or P.D. between two points = field strength \times distance between the two points.

$$\text{In this case, } V_{yz} = Xd = \frac{4\pi Qd}{KA}.$$

If Z were at earth potential, V would be the absolute potential of Y.

104. The relation between field strength and P.D. just derived for a uniform field may also be written

$$X = \frac{V}{d}$$

It is true for all fields provided we consider two points close enough to each other for us to assume that the field is uniform between them. It may be expressed generally by saying that the field strength is the rate of change of P.D. along a line of flux. This leads to a common practical unit of field strength, the **volt per cm.**, *i.e.*, the field strength is such that the P.D. between two points 1 cm. apart on a line of flux is 1 volt.

Since 1 E.S.U. of P.D. = 300 volts and the centimetre is the same on both systems,

$$1 \text{ E.S.U. of field strength} = 300 \text{ volts per cm.}$$

The electrostatic field strength at a receiving aerial is often quoted in terms of a submultiple of the above unit, the **millivolt per metre**.

$$\begin{aligned} 1 \text{ volt per cm.} &= 100 \text{ volts per metre.} \\ &= 100,000 \text{ millivolts per metre.} \end{aligned}$$

105. Systems of Units.—In the brief description of electrical phenomena given in the previous paragraphs, three systems of units have been derived :—

- (1) The practical system.
- (2) The electromagnetic system.
- (3) The electrostatic system.

The two latter systems are based on the absolute units of length, mass and time (cm., gm. and sec.).

The electromagnetic system, however, assumes that the permeability of a vacuum is unity, while the electrostatic system assumes that the specific inductive capacity of a vacuum is unity, thus introducing another arbitrary element into both systems. It can be shown by comparing the values of the same quantity, *e.g.*, a current, on the two systems that these assumptions are not consistent with each other. It is actually found that for a vacuum

$$\frac{1}{\sqrt{\mu K}} = 3 \times 10^{10} \text{ numerically,}$$

and has the units of centimetres per second, *i.e.*, it is of the nature of a velocity. This velocity of 3×10^{10} cms. per sec. is the measured velocity with which light travels.

It was this fact which led to the idea that light waves were electromagnetic in nature, and gave rise to the attempt to produce similar waves by electrical means which laid the foundations of wireless telegraphy.

106. Following on the description of direct-current phenomena given above, a section on batteries is appended. In the next chapter, the meaning of the electrical quantities which are particularly important in wireless circuits, *viz.*, inductance and capacity, will be developed from the outline of electromagnetism and electrostatics given in this chapter.

PRIMARY CELLS.

107. The existence of an electric current in any circuit means that energy in some form is being liberated at the generating source, and the continuance of the current necessitates the continuous expenditure of energy. In the case where the current is

supplied by a dynamo driven by a steam or gas engine, the source of the supply is the coal, and the place where the energy is being liberated is the furnace. Coal contains a large supply of energy, which it readily liberates in the form of heat, and which, after several transformations, may appear in a circuit in the form of electrical energy, and is there used for lighting, running motors, energising W/T sets, &c.

The coal becomes oxidised or burnt in the process, and the quantity of energy that can thus be obtained is clearly limited by the amount of coal consumed or burnt.

In the case of an ordinary voltaic cell the conversion of the energy of supply into electrical energy is a much simpler and less wasteful process, but the material which acts as the source of supply—in other words, the fuel—is far more expensive.

In most cells the fuel consists of zinc and acid, which are consumed, but which, instead of giving out their energy in the form of heat, give it out directly in the form of current.

A cell is in reality nothing more than a little furnace in which zinc instead of coal is used as fuel, and in which the "burning" is not simple oxidation.

103. Conduction of Electricity through Liquids.—Pure water is a very good insulator, *i.e.*, if a P.D. is applied across two terminals in pure distilled water, no measurable current will flow. It is found, however, that if certain chemical substances are dissolved in water, the solution is conducting, although its conductivity is much less than that of a metallic conductor.

Such chemical solutions are called "electrolytes" and belong mainly to three types of chemical compound, "acids," *e.g.*, sulphuric acid, "bases," *e.g.*, caustic soda, and "salts," which are formed by the interaction of an acid and a base, *e.g.*, common salt, which is formed by the interaction of hydrochloric acid and caustic soda.

Just as gases are ionised under various conditions (*see* paragraph 36), so it is supposed that the molecules of these chemicals are ionised by the process of solution in water. Sulphuric acid may be taken as an illustration. The molecule of sulphuric acid (H_2SO_4) consists of two atoms of hydrogen, one of sulphur, and four of oxygen. On solution, this molecule splits up into three parts, two atoms of hydrogen, which have each lost an electron, *i.e.*, hydrogen ions, and one complex ion containing an atom of sulphur, four atoms of oxygen, and the two electrons from the hydrogen atoms.

This complex ion therefore has a nett negative charge of two electrons. The process may be represented as



where the + and - indices represent loss and gain of one electron respectively.

Electrolytic ions differ from gaseous ions in two respects :—

- (1) They are stable in solution and do not tend to recombine with each other as gaseous ions do.
- (2) Negative ions are never free electrons, but always consist of an atom or group of atoms having more than its normal share of electrons.

If a P.D. is applied across an electrolyte, the negative ions will be attracted to the positive plate and the positive ions to the negative plate. There is thus a passage of electricity through the electrolyte in consequence of an applied P.D., *i.e.*, the electrolyte is a conductor. Electrolytic conductors, like metallic conductors, obey Ohm's Law.

109. If a plate of zinc is dipped into dilute sulphuric acid, chemical action takes place—hydrogen is given off, zinc sulphate is formed in solution and considerable heat is produced, *i.e.*, in addition to the chemical changes there is a conversion of energy from one form to another, the conversion being from chemical energy of constitution to heat energy in this case.

A primary cell is a device by which the energy made available by chemical reactions may be converted to electrical energy instead of heat. This can be brought about by dipping a copper plate into the dilute sulphuric acid in addition to the zinc plate. As long as the zinc and copper rods are not connected, no chemical action takes place, but an E.M.F. is set up between the rods. If they are connected by a wire, a current flows from copper to zinc in the wire and from zinc to copper in the electrolyte. As we are chiefly interested in what happens in the wire or external circuit, the copper is therefore looked on as the **positive "electrode" or "anode"** of the cell and the zinc as the **negative electrode or "cathode."** As soon as flow of current is made possible, the zinc starts to dissolve in the acid. The energy made available by this interaction of zinc and sulphuric acid appears in the form of electrical energy, which maintains a current round the circuit.

110. Polarisation.—The action is accompanied by the liberation of hydrogen at the surface of the copper. This has two bad effects :—

- (1) The conductivity of hydrogen is very much less than that of dilute sulphuric acid, and so the internal resistance of the cell is greatly increased.
- (2) If we could construct a cell with zinc and hydrogen electrodes in the same electrolyte, we should find that although a current still flowed from hydrogen to zinc in the external circuit, *i.e.*, zinc was still the cathode of the cell, the current was much smaller than with a copper anode because the E.M.F. developed between the electrodes was smaller. This is essentially what happens when a hydrogen film collects on the copper plate. Hydrogen becomes the anode of the cell, and its E.M.F. is greatly diminished.

This effect is called "polarisation," and on the method adopted to avoid it, to "depolarise" the cell, depends very largely the efficiency of all primary cells.

111. The essentials of a primary cell are thus :—

- (a) Positive and negative electrodes.
- (b) Electrolyte.
- (c) Depolarising device.

The E.M.F. of a cell depends only on the chemical nature of the electrodes and electrolyte and not at all on their size or quantity. The resistance of the electrolyte, like that of a metallic conductor, varies directly as its length and inversely as its area, and so the larger the surface area of the plates, and the less their distance apart, the smaller is the internal resistance of the cell. Naturally, also, the larger the cell, the longer its life, since a greater quantity of chemicals is available.

The Daniell and Menotti-Daniell cells consist of copper, zinc, dilute sulphuric acid, and copper sulphate solution (depolariser). Their E.M.F.s. are 1.1 volts each. Their resistances depend on their size. They give a steady current when in service.

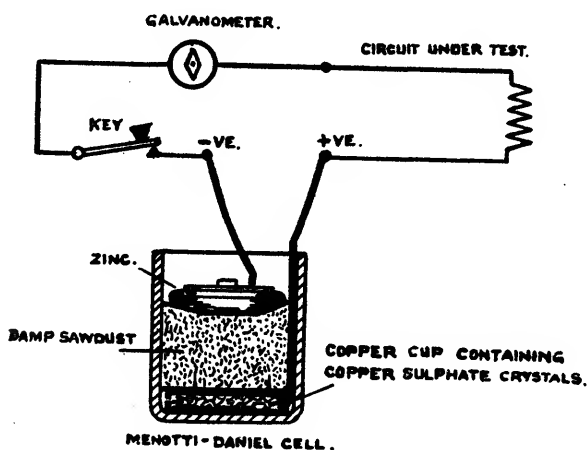


FIG. 28.

The Leclanche cell consists of carbon, zinc, salammoniac solution, and manganese peroxide (depolariser). E.M.F., 1.5 volts. Resistance varies with size.

When in service the current quickly falls, and so these cells are chiefly used for intermittent work such as bells.

Nearly all dry cells are of this type, the salammoniac being contained in a paste or jelly.

The Menotti test battery (Fig. 28) consists of a Menotti-Daniell cell (1 volt and 30 ohm resistance approximately), a key, and a 20-ohm galvanometer. The circuit under test is connected between the other terminal of the galvanometer and the positive battery terminal.

In wireless work it is chiefly used to test for "conductivity." When the key is pressed and a swing of the galvanometer needle is obtained, the circuit is continuous or unbroken. Its high resistance is necessary in testing detonator or gun circuits, so that the current will be small and there will be no risk of firing.

112. As a typical example of the action of a primary cell, the action of a Menotti will be described.

The Menotti consists of a copper cup (positive plate) containing CuSO_4 crystals (depolariser), a zinc slab (negative plate), and damp sawdust. The sawdust makes the cell portable. The electrolyte is dilute sulphuric acid formed by the interaction of water and the copper sulphate crystals.

The chemical action is roughly as follows. The zinc dissolves in the acid, forming zinc sulphate, and hydrogen ions travel to the copper cup. Here, in the simple cell described above, they each collect an electron which has formed part of the current in the external circuit, and become hydrogen atoms, which combine into molecules, giving the film of gas causing polarisation.

In the Menotti, the hydrogen ions have to pass through the moist copper sulphate crystals before they can reach the copper electrode, and a chemical reaction takes place, sulphuric acid and copper being formed. The copper is deposited on the copper cup and the acid replaces that which has been used up in dissolving zinc. The net result is that copper sulphate and zinc are used up and zinc sulphate and copper are produced. No hydrogen thus appears at the copper cup and polarisation is prevented.

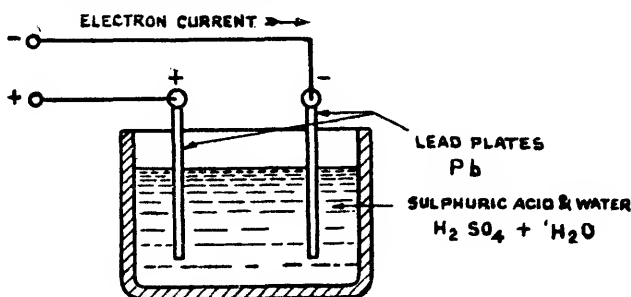
SECONDARY BATTERIES.

113. A secondary battery, or accumulator, is an arrangement from which an electric current may be drawn for a certain time, as from a primary battery; unlike the primary battery, however, when the accumulator is exhausted it may be recharged by having an electric current passed through it.

An accumulator does not store electricity; it stores energy.

When it is being charged the electrical energy imparted to it is transformed into chemical energy, which is stored in the cell. Then, when the cell discharges, that is, when an external circuit is completed through which current can be forced by the E.M.F. of the cell, the stored energy is reconverted into electrical energy.

The simplest accumulator consists of two lead plates immersed in dilute sulphuric acid, as in Fig. 29. The first operation necessary



Primitive form of Accumulator.

FIG. 29.

is to "form" the plates, *i.e.*, to bring about that change in the composition of the electrodes which finally gives the cell its ability to produce an E.M.F.

114. Forming the Plates.—This is done by passing a direct current through the cell from the mains.

The plate at which the current enters the cell is the anode.

The plate at which the current leaves the cell is the cathode.

The actual chemical changes which occur are very complicated and only the principal ones can be considered here.

As explained in para. 108, the electrolyte is dissociated into H^+ and SO_4^{--} ions.

When a P.D. is established between the plates by means of the mains, H^+ ions are attracted towards the negative plate or cathode. As each H^+ ion arrives there, it is neutralised by an electron (supplied from the mains) and becomes a hydrogen atom. Two of these hydrogen atoms form a hydrogen molecule, and the molecules agglomerate into bubbles of hydrogen gas, which are liberated at the surface of the plate. Thus no change takes place in the composition of the cathode.

The SO_4^{--} ions are attracted to the anode. When they arrive they each lose their two electrons to the positive main. The group of atoms SO_4 cannot exist alone and immediately acts on the water present, giving sulphuric acid and oxygen.



The oxygen liberated combines with the lead of the anode to give lead peroxide PbO_2 , a chocolate-brown coloured substance.

The results of this process may be summarised as follows:—

- (1) Cathode—No change.
- (2) Anode—From lead to lead peroxide.

- (3) **Electrolyte**—The hydrogen given off at the cathode, and the oxygen which oxidises the anode, both come from the electrolyte, which therefore loses water (H_2O). As a result, the concentration of sulphuric acid increases.

The end of this stage may be detected by observing that both plates will then "gas" freely—for the oxygen formed at the anode will then be liberated as oxygen gas.

The next step is to reverse the connections to the mains and pass current through the cell in the opposite direction.

The previous cathode, which is still unchanged lead, is now the anode, and so the SO_4^{--} ions repeat upon it their action on the original lead anode, *i.e.*, it ends with a covering of chocolate-brown lead peroxide.

The previous anode, which is covered with PbO_2 , is now the cathode, and the H^+ ions are attracted to it. On arrival, they each collect an electron as before and become neutral atoms, which combine with the oxygen of the lead peroxide to form water. The lead peroxide is thus reduced to metallic lead once more, but the lead thus produced has a porous, spongy texture. This renders it considerably more efficient, and is the object of this "alternate charge" method of forming the plates. The concentration of the acid is also reduced during this stage by the formation of water.

The whole process is now repeated several times, and in their final condition one plate is mostly PbO_2 and chocolate-brown in colour, while the other is mostly spongy lead of a light slate colour.

115. Construction of Accumulators.—In practice, the holding power or capacity is increased by furrowing, grooving, gimping or otherwise increasing the working surface of the lead plates, and the

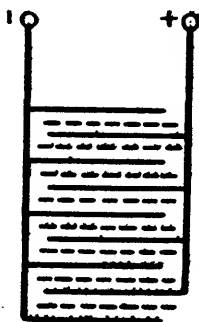


FIG. 30.

prepared surface is filled with the active material—lead peroxide or spongy lead

The greater the number of plates in a cell and the larger their surface, the greater will be the capacity and current output.

The positive and negative plates are arranged alternately, each group being connected at the top by lugs to a lead bar.

The negative group contains one plate more than the positive group, except in two-plate cells.

The plates are placed close together to ensure low resistance, and are kept apart by separators of wood, glass, celluloid, &c. (as indicated in Fig. 30).

116. The Electrolyte.—The sulphuric acid and water must be free from impurities. If distilled water is not available, rain water or melted artificial ice is recommended.

Sulphuric acid, when concentrated, has a "specific gravity" of about 1.84, *i.e.*, it is 1.84 times as heavy as an equal bulk of water.

When the cell is charged the acid strength should be about 1.22 (or, as sometimes written, 1220). Slightly different strengths are stated by different makers.

This strength gives approximately the least specific resistance. A stronger acid, in addition to having a higher specific resistance, attacks the plates to an undesirable degree.

To "break down" the acid it is mixed with about four times its bulk of water, and care must be exercised in the mixing.

When mixed with water a large amount of heat is developed. The acid should be added gradually to the water, stirred meanwhile, and not used till cool.

On no account must water be added to acid.

The strength or "specific gravity" is tested by a "hydrometer"—an instrument with a weighted bulb and a thin graduated stem; when in the acid, the reading is taken at the point on its scale at the surface.

As will be explained shortly, the acid becomes weaker as the cell is discharged, and will fall to about 1.170 at about 1.85 volts—the point at which discharging should be stopped.

Its strength is recovered on recharging.

117. Initial Charge.—The makers issue directions, which must be carefully followed if the battery is to be maintained in a state of efficiency.

The general plan is to give a prolonged first charge immediately after the acid has been poured in.

The acid will fall in specific gravity as soon as it is poured into the cells and will continue to do so for the first twelve or eighteen hours.

During charge it will gradually rise, and the charge should not be considered complete until the voltage and specific gravity show no rise over a period of, say, five hours, and gas is being given off freely from all plates.

At the end of the charge the voltage will have risen to 2.5–2.7 volts per cell.

After the charging current is cut off the voltage per cell will immediately fall to about 2.2 volts, or slightly less, at which it will remain while the battery is left on open circuit.

The battery should not be left unused for more than a week without recharging.

With large cells, in which a hydrometer can be inserted, the specific gravity of the acid is the most reliable guide. With small cells the acid should be tested occasionally by transferring some acid with a syringe to a narrow tube and using the hydrometer in the latter.

Tests for Completion of Charge.

- (1) Appearance of plates : Positive, chocolate brown ; negative, slate grey ; and no trace of whiteness on either.
- (2) Voltage : 2.5 to 2.7 volts.
- (3) Plates gassing freely.
- (4) Specific gravity of acid : About 1220, according to maker.

In the first week the cell should be given plenty of work, and an hour's extra charging on the first few charges.

118. Discharging.—If the plates of an accumulator are joined by an external resistance, it is found that a current flows from the brown plates to the slate-coloured ones through the resistance, and therefore in the opposite direction through the electrolyte.

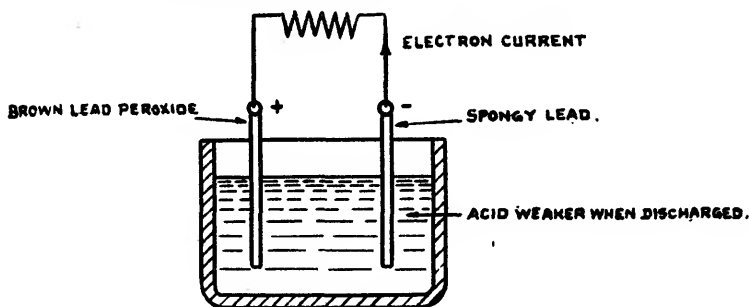
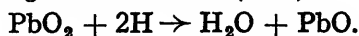


FIG. 31.

H^+ ions thus go to the brown plate and combine, when neutralised, with the oxygen of the lead peroxide. This action does not go so far as when the plates were being formed, and instead of spongy lead being formed, the peroxide molecule loses only one of its two oxygen atoms, forming lead monoxide (PbO).



Lead oxide readily combines with sulphuric acid, forming lead sulphate ($PbSO_4$) and water.

At the spongy lead plate the SO_4 group of atoms combines with the lead, also forming lead sulphate. This is the process which is facilitated by obtaining the lead in a porous condition.

As a result both plates become partly coated with white lead sulphate and the E.M.F. falls. Some sulphuric acid is used up and water is formed, so that the acid concentration is reduced. This is recognised by a fall in the specific gravity of the electrolyte.

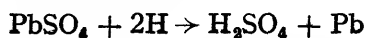
The voltage should not be allowed to fall below about 1.85 volts.

The specific gravity will be about 1.170 at this stage. If the discharge is carried persistently beyond this point, more sulphate will form on each plate, and when each plate is totally covered the voltage will be zero.

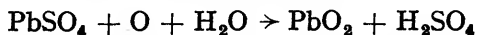
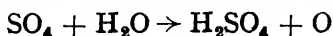
Caution.—A cell must not be left for any length of time in a discharged state.

119. Recharging.—This consists in removing the lead sulphate and restoring the plates to their condition before discharge. It is accomplished by passing current through the cell from the mains.

As before, hydrogen atoms are liberated at the negative plate, and interact with the lead sulphate forming sulphuric acid and reducing the plate to spongy lead.



At the positive plate the SO_4 groups combine with water, forming sulphuric acid and oxygen. The oxygen acts on the lead sulphate, forming brown lead peroxide and sulphuric acid.



It will be seen that sulphuric acid is formed at both plates during recharging and so the specific gravity of the electrolyte rises.

When the cell is fully recharged, *i.e.*, when the sulphate has been entirely removed from the plates, the hydrogen and oxygen atoms can no longer react with the plates, and so bubbles of gas are given off at each plate. This is allowed to continue until the acid reaches the required specific gravity and the E.M.F. is from 2.5 to 2.7 volts.

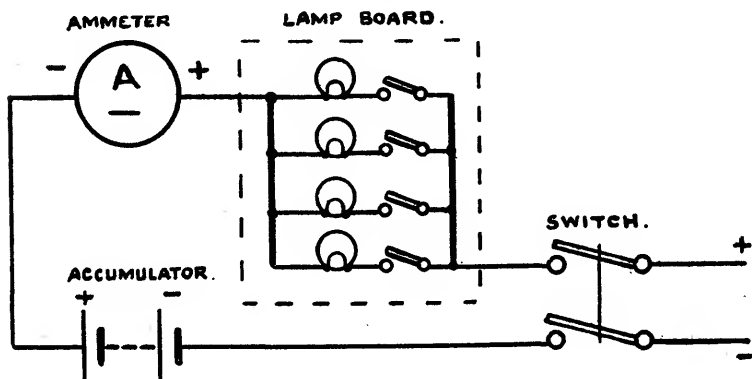
When the cell is fully charged and lying idle the plates are less liable to be attacked by the acid.

The charge in ampere-hours should generally be about 10 per cent. more than the discharge in ampere-hours.

120. Method of Charging.—The positive plate must be connected to the positive main and the negative plate to the negative main. Passing current through the cell in the wrong direction will ruin it.

To determine which is the positive terminal, connect an ammeter in the circuit, as in Fig. 32.

The ammeter terminals are always marked “+” and “-”. If the pointer swings in the correct direction over the scale, the lead connected to the “+” of the ammeter will be that from the positive main.



Accumulator on Charge.

FIG. 32.

According to the size of the cell a certain charging rate or current is necessary; this is stated by the maker. We shall consider how the particular value of the charging current is obtained.

Example 10.—Suppose two small accumulators, each of E.M.F. 1.9 volts and resistance 0.1 ohm, are to be charged by a current of 4 amps. from 220-volt mains through leads of resistance 0.1 ohm.

If they were connected directly across the mains an enormous current would flow.

The voltage of the mains has to overcome the back E.M.F. of the cells in series (3.8 volts) and the resistance of the cells and leads (0.3 ohm altogether). Thus $220 - 3.8 = 216.2$ volts are available to drive current through 0.3 ohm.

$$\text{Current} = \frac{216.2}{0.3} = 721 \text{ amperes,}$$

which would burn up the cell.

In order that just the right amount of current shall flow, some form of resistance must be included in the circuit.

The most convenient form for small charging currents is a "lamp resistance," consisting of lamps, of the voltage of the mains, arranged in parallel (Fig. 32).

The more lamps there are in parallel the greater will be the current that will flow through the circuit, but naturally we wish to use as few lamps as possible, and so we choose high c.p. carbon lamps.

Suppose that the 220-volt lamps available for our use are two 50-c.p., and a number of 16-c.p. carbon lamps.

50 c.p. (4 watts per c.p.) take 200 watts :

$$I = \frac{200}{220} = 0.9 \text{ A.}$$

16 c.p. (4 watts per c.p.) take 64 watts :

$$I = \frac{64}{220} = 0.3 \text{ A.}$$

Thus two 50-c.p. (1.8 amp.) and seven 16-c.p. (2.1 amp.) lamps in parallel will allow a current of $1.8 + 2.1 = 3.9$ amps. to flow.

If a number of 50 c.p. lamps were available, four of these (3.6 amp.) and one 16 c.p. (0.3 amp.) would give 3.9 amperes. This would be preferable, as fewer lamps are required.

When only a few cells in series are being charged, their back E.M.F. is negligible in comparison with the voltage of the mains.

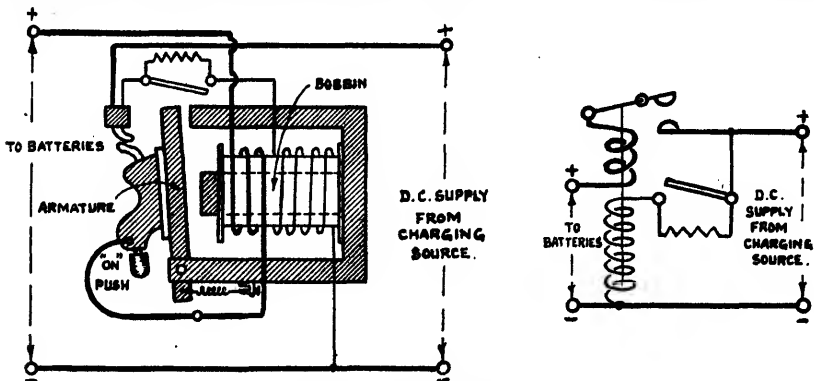
The lamps in the circuit are taking practically their "full brilliancy" current.

High potential batteries (50 volts or 100 volts) have naturally a greater back E.M.F., and the lamps used do not get the full voltage of the mains.

For example, a 150-volt battery used in a valve receiving set requires, for charging, a $2\frac{1}{2}$ c.p. 220-volt carbon lamp in series.

Normally this lamp would take about 0.05 amp., but in this case the applied voltage would only be about $220 - 150 = 70$ volts, and the lamp would not take as much as 0.05 amp., and would not burn at full brilliancy.

121. Reverse Current Switch.—When charging accumulators from a generator whose voltage is approximately the same as that of the fully charged accumulators, there is a possibility that towards the end of the charging process the accumulator voltage may rise temporarily above the generator voltage. The accumulators would



Reverse Current Switch.

FIG. 33.

then start to discharge and try to drive the generator as a motor. To prevent this a reverse current switch is fitted in the charging circuit. This switch is illustrated in Fig. 33. It consists of two coils wound on the same bobbin. One of these, in series with a resistance, which may be short-circuited, is directly across the generator terminals and is called the shunt coil. The other is in series with the batteries to be charged when the "On" push is made, and is called the series coil.

The flux through the bobbin core produced by the shunt coil current is alone sufficient to hold on the soft iron armature and therefore to keep the "On" push made. When the battery circuit is completed current also flows through the series coil. As long as this current is in the correct direction, *i.e.*, the generator voltage is higher than the battery voltage, the flux it produces in the bobbin core assists the shunt coil flux in holding on the armature. If the battery voltage rises above the generator voltage, current flows in the opposite direction through the series coil and so its flux opposes the shunt coil flux.

When the reverse current reaches a specified value the resultant flux is too weak to hold on the armature, the "On" push contact is broken, and the generator is disconnected from the batteries.

The resistance in the shunt coil circuit is short-circuited or not according to the generator voltage and charging current required. The charging current determines the series coil flux. The shunt coil current and flux must be adjusted correspondingly for the contact to fall off at the correct value of reverse current. The resistance allows the P.D. across the shunt coil to be altered to give the correct current through it.

122. Sediment.—In time the active material on the plates gradually disintegrates and a sediment forms at the bottom of the cell. Care must be taken that this does not reach the bottom of the plates to cause short-circuiting. Too much sediment may be produced by charging too much or at too high a rate. Its colour indicates whether the cell is receiving normal treatment; if so, the sediment should be brown.

123. Sulphating.—In the discharge action, described above, we saw that lead sulphate forms on the plates in the ordinary discharging process.

This is not what is known as "sulphating." The latter is due to incorrect treatment—such as cell not charged sufficiently, especially if new, over-discharge, acid too strong, and a discharged cell unattended to.

A coating of hard white lead sulphate forms on the plates, and this is difficult to remove.

The plates in consequence become light in colour and lose their porosity and holding power.

"Buckling" of the plates is also very liable to occur.

The simplest remedy, if the sulphating is not too deep-seated, is prolonged and repeated charging at low rate, say, at half normal charging current, and, when full gassing occurs, at quarter-normal.

124. Change of Acid Strength.—The water in the electrolyte gradually evaporates and must be replaced at intervals to the proper level. The solution should be stirred to prevent the water lying on top, or, in small enclosed cells, the water should be introduced well below the surface by a syringe. This is best performed

just before the charge and the gassing will ensure thorough mixing.

Acid may be lost by over-vigorous gassing ; thus a check must be kept on the electrolyte. It is very inadvisable to add strong acid to a cell to bring up the specific gravity. A better plan is to refill the cell with fresh acid of correct specific gravity.

125. Capacity and Efficiency.—The “capacity,” or holding power, is rated in “ampere-hours” and in “watt-hours.” Service cells are usually rated on their output over a period of five hours.

The output at other rates will be greater or less than the standard rate according as the time of discharge is more or less than five hours, *e.g.*, a 100 amp.-hour cell discharged in one hour would only give about 60 amp.-hours, but if discharged in ten hours would give 120 amp.-hours.

The efficiency denotes the ratio of the capacity output to the input. The “ampere-hour” efficiency is from 80–90 per cent., and the “watt-hour” efficiency from 60–75 per cent.

Thus, if a cell is charged at 10 amps. for 16 hours, the input is 160 amp.-hours.

The output would be about $160 \times \frac{80}{100} = 128$ amp.-hours, which would give, say, a discharge current of 9 amps. for about 14 hours, and an efficiency of $\frac{128}{160} \times 100 = 80$ per cent.

126. Precautions.

Give battery proper initial charge.

Give a new battery plenty of work and liberal charging.

Do not charge too much or too little, or at too high or too low a rate.

Do not run batteries too low in voltage or specific gravity.

Do not allow batteries to stand long completely discharged.

Charge once a week if possible.

Sediment should not reach bottom edges of plates.

Keep plates covered with electrolyte, making up evaporation losses with distilled water.

Test strength of acid periodically.

Keep terminals and top of cell clean and dry, and terminals coated with vaseline.

127. Cells in Series and Parallel.—When cells are arranged in series, the total E.M.F. is the sum of their separate E.M.Fs. and the total resistance is the sum of their separate resistances.

When they are connected in parallel (all positives to one terminal and all negatives to another) their total E.M.F. is that of one cell, and their total resistance that of one cell divided by the number of cells (assuming each to have the same E.M.F. and resistance).

CHAPTER III.

ELECTROMAGNETISM, INDUCTANCE AND CAPACITY.

128. In the last chapter (paras. 79-81) the production of a magnetic field by an electric current was described and some typical fields were discussed. It is now proposed to consider in greater detail the mechanical effects produced by the interaction of these magnetic fields.

These mechanical effects may all be explained by attributing to magnetic lines of flux the same properties as were found to be successful in explaining electrostatic effects in terms of electric lines of flux, viz. :—

- (1) They tend to shorten themselves as far as possible, *i.e.*, to contract in the direction of their length.
- (2) They tend to expand laterally, *i.e.*, to increase in cross-section, and so resist lateral compression.

129. These properties may be applied to the case of two parallel conductors carrying current, as illustrated in Fig. 16. When the current is flowing in opposite directions in the two conductors, it will be seen that where the lines of flux due to the two currents approach each other, their direction is the same. Each current endeavours to extend its flux in the direction of the other so that the lines of flux may be circles, *i.e.*, have their shortest possible length. Where the two sets of flux lines are competing for the same space they will therefore endeavour to push each other back. Their resistance to lateral compression then becomes operative, with the result that the conductors are forced apart.

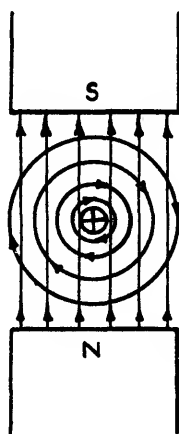
When the two currents are flowing in the same direction, their lines of flux are in opposite directions when they meet each other. They can thus coalesce. Where they are equal and opposite there will be no magnetic field and so no lines of flux. At other points they take the direction of the resultant magnetic field due to the currents. The result is to produce lines of flux encircling both conductors as shown. These lines then endeavour to contract and in doing so bring the two conductors together, *i.e.*, the conductors appear to be attracted towards each other.

The component magnetic fluxes which interact to give these mechanical forces need not be produced by currents. The deflection of a compass needle in the vicinity of a current-carrying conductor has already been mentioned, and is due to the interaction of the conductor flux and the flux from the needle, which is a permanent magnet.

The mechanical forces produced by the interaction of the flux due to a permanent or electromagnet and that due to a current flowing in a conductor are of great importance. Some of the measuring instruments described later in this chapter depend on the motion resulting from these forces. The principle of the electric motor, described in Chapter IV, is also based on them. Because of their origin they are often called **electrodynamic** forces.

The calculation of the magnitude and direction of electrodynamic forces usually presents mathematical difficulties owing to the complicated shape of the circuits involved. A simple case which illustrates the general principles will now be considered.

130. An arrangement which commonly occurs in practice is shown in Fig. 34. A straight conductor carrying a current is placed between two large plane pole faces of a permanent or electromagnet. The component fluxes are as in the figure. The flux due to the magnet is uniform in the gap between the poles, as shown by the equally-spaced parallel straight lines. The conductor lies in a plane parallel to the pole faces, and current is flowing through it into the paper. Its lines of flux are concentric circles.



Illustrating Reaction between Current-carrying Conductor and Magnetic Field.

FIG. 34.

The resultant distribution of flux lines is shown in Fig. 35 (a). On one side of the conductor the component flux lines run in the same direction, and so the field is strengthened as shown by the packing of lines. The resultant field is weak on the other side and the lines are comparatively far apart. The tendency of the lines to shorten (to become straight in this case) forces the conductor to the right in a plane parallel to the pole faces.

If either the field of the magnet or the direction of the current in the conductor were reversed, it will be seen that the conductor would move in the opposite direction, *i.e.*, to the left. If both were reversed, the motion of the conductor would again be to the right.

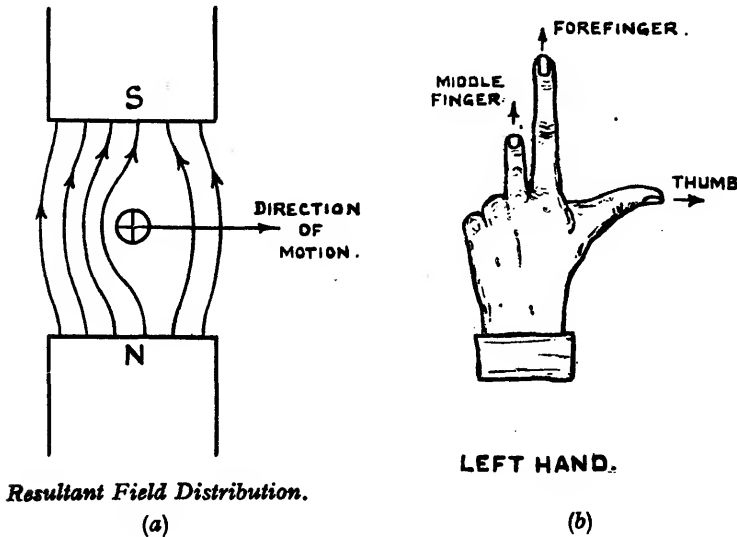


FIG. 35.

The flux due to the magnet, the direction of the current and the resultant motion are in three lines at right angles to each other, but may be in either of the two directions given by such lines according to the particular case, *e.g.*, the current flow may be either into, or out of, the plane of the paper. A convenient method of determining the direction of any one, when the other two are known, is given by **Fleming's Left Hand Rule**.

The thumb, forefinger and middle finger of the left hand are extended in three directions at right angles as shown in Fig. 35 (b). If the **ForeFinger** then represents the direction of the **Flux** and the **mIddle fInger** the direction of the current (**I**), then the **thuMb** gives the direction of **Motion** of the conductor. The letters in bold type provide a mnemonic for remembering this rule. Fig. 35 (b) illustrates the use of the rule for the case considered in Fig. 35 (a).

The magnitude of the force is given by $\frac{BIl}{10}$ dynes, where B is the flux density due to the magnet in lines per sq. cm., I is the current in amperes flowing in the conductor, and l is the length in cms. of the conductor which is in the region of uniform flux density B .

Example 11.

Suppose that the pole faces are squares of side 20 cms. and the flux in the gap is 12,000 lines. If a current of 3 amps. is flowing

through a conductor placed as in Fig. 34, find the mechanical force on the conductor.

$$B = \frac{\Phi}{A} = \frac{12,000 \text{ lines}}{400 \text{ sq. cms.}} = 30 \text{ lines per sq. cm.}$$

$$I = 3 \text{ amps.}$$

$$l = \text{length of conductor in field} = 20 \text{ cms.}$$

$$\therefore \text{Force on conductor} = \frac{30 \times 3 \times 20}{10} = 180 \text{ dynes.}$$

131. The motion of a conductor carrying current when placed in the field of a permanent magnet evidently involves a supply of energy from somewhere to sustain the motion and account for the work done on the conductor. If the motion continued as it starts, the acceleration imparted to the conductor by the electrodynamic force acting on it would result in the conductor attaining larger and larger velocities and possessing correspondingly greater kinetic energy. According to Ohm's Law, all the energy supplied to the conductor by the battery or other source of current is dissipated as heat (I^2R losses). The magnetism of the permanent magnet is also unimpaired, and so the difficulty arises of explaining the source of the vast kinetic energy the conductor would eventually acquire. The solution of this problem will be considered later (para. 148). It may be stated now that the motion of the conductor in a magnetic field gives rise to an E.M.F. opposing the current flow and eventually bringing the current to zero, in which case the electrodynamic force is then also zero and no further acceleration can take place.

INDUCTANCE.

132. The various examples given earlier of magnetic fields produced by electric currents should serve to illustrate the principles that:—

- (1) The field strength at any point is directly proportional to the strength of the current producing the field.
- (2) The value of the field strength depends on the arrangement of the conductors in which the current is flowing.

For instance, it was stated in para. 91 that the field strength along the axis of a solenoid was

$$H = \frac{4\pi NI}{10} \text{ dynes per unit pole.}$$

- (1) H is directly proportional to I .
- (2) The value of H depends on the value of N , the turns per unit length, and the reason that this factor appears as $\frac{4\pi N}{10}$ is because the N turns are wound as in a solenoid.

Another example is the field due to a current of I amps. flowing in a straight conductor. At a distance of r cms. from the conductor, the field is $\frac{2I}{10r}$ dynes per unit pole.

The factor connecting field and current is thus $\frac{2}{10r}$, as compared with $\frac{4\pi N}{10}$ for a solenoid.

In air the flux through any area is directly proportional to the field strength, and so it follows that in air (or non-ferromagnetic materials generally) the total flux produced by a current is directly proportional to the current and otherwise only depends on the shape of the circuit in which the current is flowing.

In ferromagnetic materials the lines produced by the rearrangement of the molecules when the material is magnetised by the current also contribute to the total flux. The effect of this will be considered later (para. 152).

133. Flux-Linkage.—If the flux due to the current flowing in a single loop of wire, as in Fig. 17, is examined, it will be seen that every line must pass through the loop somewhere, or "link" with the loop. The number of lines linking with the loop is called the flux-linkage with the loop. In this case the number of flux-linkages is the same as the total number of lines of flux.

Now consider the solenoid flux shown in Fig. 18. The majority of the lines of flux link with every turn of the solenoid, although there are a number which only link with one or two turns. The total flux-linkage with the solenoid is obtained by considering every turn separately, counting the number of lines of flux that link with it and adding together all the separate results obtained. It is obvious that the total of flux-linkages is much greater than the number of lines of flux.

A simple result for this case may be obtained by neglecting the lines that link with only one or two turns and assuming that every line links with every turn. The field H is $\frac{4\pi NI}{10}$ dynes per unit pole, and so the flux $\Phi = BA = HA$ (in air) $= \frac{4\pi NIA}{10}$ lines, where A is the area of each turn.

If the axial length of the solenoid is l cms., the number of turns is Nl , and as Φ is the flux-linkage per turn, the total number of flux-linkages is $Nl\Phi = \frac{4\pi N^2 l A I}{10}$.

134. Self-Inductance.—Flux-linkage is simply the flux multiplied by some numerical factor depending on the shape of the circuit; and as the total flux is directly proportional to the current, so also

will be the total flux-linkage. We may express this proportionality by writing

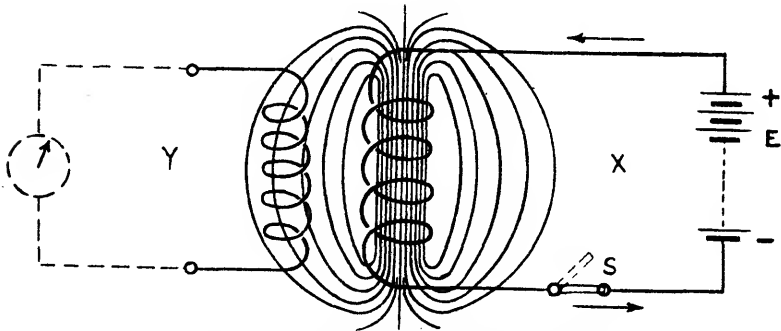
$$\frac{\text{Flux-linkage}}{\text{Current}} = \text{constant for any particular circuit,}$$

the constant being different in circuits of different shapes and sizes. This constant is called the **self-inductance** of the circuit and given the symbol L .

The self-inductance of a circuit is thus a geometrical property of the circuit, *i.e.*, it depends on the shape and arrangement of its various parts and the consequent distribution of the lines of flux in the magnetic field of the circuit. It may be defined as the number of flux-linkages with a circuit when unit current is flowing.

The word "self" is often omitted and the flux-linkage per unit current referred to simply as the "inductance" of the circuit.

Thus, when a current I is flowing in a circuit of inductance L , the number of flux-linkages with the circuit is LI .



Linkage of Flux between Adjacent Circuits.

FIG. 36.

135. Mutual Inductance.—Fig. 36 shows two circuits X and Y, both represented as coils of wire. Coil Y is supposed to be on open circuit for the moment. In coil X a current is maintained by the battery E, and some of the lines of flux due to this current are illustrated. It will be seen that a number of them link with coil Y, this number depending on :—

- (1) The current flowing in coil X.
- (2) The shape and size of coil X.
- (3) The shape and size of coil Y.
- (4) The positions of coil X and coil Y relative to each other.

The number of flux-linkages with coil Y due to the current flowing in coil X is thus determined mutually by the two circuits. It is called the **mutual** flux-linkage between the two circuits.

If the situation be reversed, *i.e.*, if coil X be put on open circuit and the same current established in coil Y as was previously flowing

in coil X, it can be shown that the number of mutual flux-linkages is exactly the same as before. It is immaterial in which circuit we consider the current to be flowing.

As the mutual flux-linkage is proportional to the current flowing in either circuit, the procedure that was used in dealing with the flux-linkage of one coil due to its own current may again be adopted, *i.e.*, we may write

$$\text{Mutual Flux-linkage} = MI,$$

where I is the current in either circuit and M is a constant depending on the configurations and relative position of the two circuits. M is called the **mutual inductance** between the two circuits.

It may be defined as the number of mutual flux-linkages between two circuits when unit current is flowing in either circuit.

(Note.—Owing to the various units employed, the equations,
Number of flux-linkages = MI or LI,

are only correct provided electromagnetic units are used. This is fully explained in paras. 142 and 144.)

Since both self and mutual flux-linkage are proportional to the current flowing, whenever the current alters the flux-linkage must also alter in conformity. The effects produced when the flux-linkage of a circuit is altering are of fundamental importance and will now be considered.

136. Mutual Induction.—These effects were first investigated by Faraday, and he used a circuit similar to that shown in Fig. 36. The circuit of coil X contained a switch S and the terminals of coil Y were connected to a galvanometer (shown dotted). As long as a steady current was maintained in coil X, *i.e.*, as long as the mutual flux-linkage remained constant, no deflection of the galvanometer pointer was observed. Whenever the switch S was made or broken the galvanometer pointer was deflected. In other words, whenever the mutual flux-linkage between the two circuits was changing, a current flowed round the circuit of coil Y. Further the pointer deflections were in opposite directions according as switch S was being made or broken, *i.e.*, the current flowed in opposite directions round the Y circuit according as the mutual flux-linkage was increasing (S being made) or decreasing (S being broken).

This phenomenon is called **electromagnetic induction** and the current in the Y circuit is called an **induced current**. The flow of a current in circuit Y implies that an E.M.F. must be acting in it. This E.M.F. is called an **induced E.M.F.**

The production of an induced E.M.F. is the primary phenomenon occurring. The induced current flows because the E.M.F. is being developed in a closed circuit. If coil Y is on open circuit, an E.M.F. is produced in it as before, but, of course, no current can flow.

Faraday found that the **magnitude** of the E.M.F. was proportional to the **rate** of change of mutual flux-linkage between the circuits.

The more quickly the current in coil X was altered, the greater was the induced E.M.F.

It has been stated that the **direction** of the induced E.M.F. depends on whether the change in mutual flux-linkage is an increase or a decrease. The induced current flowing in coil Y will set up a magnetic field of its own, giving rise to mutual flux-linkage with coil X. These linkages may either increase or decrease the total mutual flux-linkage between the two coils, according to the direction of the current in Y. It is found that when the mutual flux-linkage due to the current in X is decreasing (S being broken) the current in Y flows in such a direction as to increase the mutual flux-linkage ; and conversely, when the current in X is increasing, the direction of the induced current in Y is such that it produces mutual flux-linkage to decrease the total. This may be summed up by saying that the current in Y tries to keep the mutual flux-linkage constant. If the latter tends to increase due to an increase in the current in X, the direction of the current in Y is determined by the fact that it endeavours to produce mutual linkages to nullify the increase ; if the mutual flux-linkage decreases, current flows in Y to endeavour to bring it up to its former value. If Y is on open circuit and no current can flow, the direction of the induced E.M.F. is still governed by the same considerations, *i.e.*, it is such that if the circuit were closed the E.M.F. would tend to send current through Y in that direction which kept the mutual flux-linkage constant.

The direction of the induced E.M.F. is taken to be positive when, assuming that it could cause a current to flow, the mutual flux-linkage due to the induced current would be in such a direction as to increase the original mutual linkage, *i.e.*, a decrease of the original mutual flux-linkage produces a positive E.M.F. Conversely, an increase in the initial mutual linkage produces a negative E.M.F., by which is meant an E.M.F. in the opposite direction to a positive E.M.F. as above defined.

137. Self-Induction.—The production of an E.M.F. of mutual induction naturally leads to the question as to whether any corresponding effect occurs when the flux-linkage of a circuit due to its own current is altered. Thus, in Fig. 36, whether Y is present or not, the coil X has the same number of flux-linkages for a given current. It is found that when the current in X is altering there is an E.M.F. produced in X itself which tends to keep the flux-linkage of X constant. It is exactly similar in nature to the mutually induced E.M.F. in Y. It is proportional to the rate of change of current in X ; and if the flux-linkage of X is increasing (*e.g.*, when the switch S is being made), the self-induced E.M.F. acts so as to oppose the increase in flux-linkage, *i.e.*, it opposes the battery E.M.F. which is endeavouring to send current through coil X. For this reason it is often called the “back E.M.F. of self-induction.” Conversely, when the switch S is broken and the current in X falls to zero, an

E.M.F. is induced in X which tends to prevent this decrease of flux-linkage. The E.M.F. in this case is therefore in the same direction as the battery E.M.F., so as to keep the current flowing as before and preserve the flux-linkage unaltered.

138. It may easily be seen that these inductive effects are in accordance with the principle of conservation of energy. It was shown in Chapter II that energy must be associated with a magnetic field, for magnetic poles are set in motion in such fields and acquire kinetic energy. If the magnetic field is due to a current, then the energy of the field must have been derived from the source that causes the current to flow, *i.e.*, from chemical energy in the case of a battery or mechanical energy in that of a dynamo. As long as the current is steady, the magnetic field, and therefore the magnetic energy associated with the current, remain constant and no energy need be taken from the battery for this reason. This is in accordance with Ohm's Law, which is merely another way of saying that the chemical energy taken from the battery is transformed into heat energy in the circuit. If the battery E.M.F. is increased, the current should, according to Ohm's Law, increase in proportion. This involves an increase in the magnetic energy associated with the current, and while this transformation from chemical to magnetic energy is occurring, all the chemical energy cannot be converted to heat energy. Thus the current does not assume instantaneously the new value it should have if calculated by Ohm's Law, *i.e.*, on the basis that all the energy supplied by the battery is converted to heat. It thus appears as if a back E.M.F. is acting in the circuit and limiting the current to a smaller value. When the increase in magnetic energy is supplied, the current assumes its larger constant value, as calculated from battery E.M.F. divided by resistance, and the back E.M.F. disappears.

Similarly, when the switch is broken, according to Ohm's Law the current should fall instantaneously to zero, for no more energy is being supplied from the battery. But, as the current decreases, the magnetic energy associated with it must also decrease in proportion, *i.e.*, it must be transformed into another form of energy. It is actually converted into heat energy in the conductors forming the circuit and this transformation takes the form of an E.M.F. tending to keep the current flowing and hence to allow the conversion to heat energy to take place. The direction of the induced E.M.F. is thus accounted for. Obviously, the more quickly the current changes, the more quickly must the corresponding changes of magnetic energy take place, *i.e.*, the greater will be the induced E.M.F.

139. These phenomena exhibit a close analogy with the mechanical phenomena which are observed when the velocity of a body is altered, and the mechanical analogy is correspondingly helpful in understanding inductive effects.

It is shown in the Mechanics Appendix that any alteration in the velocity of a body is equivalent to altering its kinetic energy and that in the process work must be done either by, or on, the body. It is common experience, for instance, that much less exertion is required to keep a railway truck steadily moving along the line than to start it in motion, and that, if the pull on the truck is removed, it will not come instantaneously to rest, but will travel some distance along the line under its own "way." If it were not for the retarding friction between the rails and its wheels, the truck would go on indefinitely on a level line at the speed that had been imparted to it. The property of a body whereby it tends to oppose any change in its motion is called its "**mass**" or "**inertia**." In estimating the change of motion produced in a given time by mechanical forces, it is found convenient to focus attention on the **momentum** or "way" of the body, which is defined as its mass multiplied by its velocity. A body tends to resist any change in its momentum; to effect such a change, **force** must be applied and the rate of change of momentum is proportional to the force. In analysing the rate of change of momentum, the mass generally remains constant and only the velocity changes. Thus rate of change of momentum is equivalent to mass multiplied by rate of change of velocity, *i.e.*, mass multiplied by acceleration; and the applied force is therefore proportional to mass multiplied by acceleration (Appendix C).

In the electrical case, the quantity that opposes any change in its value is the flux-linkage of a circuit, and so flux-linkage may be compared to momentum. Just as momentum consists of the product of two factors, mass and velocity, so we have seen (para. 134) that flux-linkage may be naturally analysed into two factors:—

- (1) The current, which, as it is the rate of flow of a quantity of electricity, may be compared to velocity in mechanics.
- (2) The inductance, self or mutual, of the circuit, which does not depend on the current, but on the mutual arrangement of parts of the circuit. Similarly, mass in mechanics does not depend on velocity, but on the material and size of the body. Inductance may thus be compared to inertia or mass.

The analogy may be carried further by the consideration that if current corresponds to velocity, rate of change of current may be compared with rate of change of velocity, *i.e.*, acceleration; and since the inductance is constant, rate of change of flux-linkage may be analysed into inductance multiplied by rate of change of current, corresponding to the analysis of rate of change of momentum into inertia multiplied by rate of change of velocity. As the force which must be applied gives in mechanics a measure of a body's opposition to change of its momentum, so in the electrical case the induced E.M.F. gives a measure of the opposition exerted

by an electrical circuit to change in its flux-linkage. It should be noticed, however, that although induced E.M.F. may in this instance be formally compared with force, the two quantities are defined in different ways (para. 48), and that, in general, E.M.F. corresponds more closely to the idea in mechanics of potential energy per unit mass.

140. In completing this descriptive account of electromagnetic induction, it may be emphasised that when the flux-linkage of a circuit is changing, every new linkage that is produced, or every old one that disappears, must in the process pass through the material of the conductors in the circuit if it is to make any contribution to the induced E.M.F. In the circuit of Fig. 36 it will be seen that this must be the case, for the mutual flux linkages have started from coil X and must have cut coil Y when they were being established. Similarly, as the flux due to the current in X collapses when the circuit is opened by breaking the switch, every mutual linkage must cut coil Y in disappearing. This gives another way of estimating the induced E.M.F. in any part of a circuit, viz., by the number of lines of flux which pass through it or "cut" it. It is particularly useful in the case of a straight conductor, where the idea of "inter-linking" flux lines is not so obvious.

This explains why flux-linkage and not flux is the important factor in induction. The greater the number of conductors which are cut by a line of flux in disappearing, (i.e., the greater its original flux-linkage), the larger is the induced E.M.F.

141. The mathematical formulation of Faraday's Law will now be considered. The law states that the induced E.M.F. is proportional to the rate of change (with time) of flux-linkage.

Flux-linkage may be denoted by $N\Phi$ to indicate that it is measured by lines of flux multiplied by a purely numerical factor (para. 133). In the notation of the calculus, rate of increase of

flux-linkage with time is then expressed as $\frac{d(N\Phi)}{dt}$, rate of decrease

as $-\frac{d(N\Phi)}{dt}$ (Appendix B). It has already been stated that the E.M.F. (E) is to be considered positive when it is due to a decrease of flux-linkage. Thus Faraday's Law is expressed by the formula

$$E = K \times -\frac{d(N\Phi)}{dt} = -K \frac{d(N\Phi)}{dt}$$

where K is the factor of proportionality.

Also it has been seen that flux-linkage is equal to inductance multiplied by current,

i.e., $N\Phi = LI$, (if we consider self-inductance),

and so $\frac{d(N\Phi)}{dt} = \frac{d(LI)}{dt} = L \frac{dI}{dt}$, since L is constant.

$$\therefore E = -KL \frac{dI}{dt}.$$

142. The units of E.M.F. and current have already been chosen from direct current considerations, but no unit of inductance has so far been specified, and we are at liberty to choose any unit that simplifies the above formula. The unit of inductance is defined so that $K = 1$ in the expression of Faraday's Law when the E.M.F., current and inductance are measured in electromagnetic units. The law then becomes

$$E = -L \frac{dI}{dt},$$

and defines the E.M.U. of **self-inductance** as follows:—

The **self-inductance** of a circuit is **one E.M.U.** if the E.M.F. induced in it is one E.M.U. when the current is changing at the rate of one E.M.U. of current per second.

The E.M.U. of **mutual inductance** is obtained in a similar manner. As $N \Phi = MI$ in this case, Faraday's Law becomes

$$E = -M \frac{dI}{dt}$$

if the E.M.U. of mutual inductance is defined to correspond with that of self-inductance.

The **mutual inductance** of two circuits is one E.M.U. if, when the current in one of them is changing at the rate of one E.M.U. of current per second, the induced E.M.F. in the other is one E.M.U.

The E.M.U. of self or mutual inductance is called the centimetre. The centimetre of inductance has, of course, no relationship to the centimetre of length.

As $\frac{d(N \Phi)}{dt}$ has been taken equal to $L \frac{dI}{dt}$ and $M \frac{dI}{dt}$ in the deriva-

tion of the above units, this is really equivalent to defining what is to be considered one flux-linkage on the electromagnetic system. In a circuit of unit self-inductance, when the current is changing at

the rate of one E.M.U. per second, *i.e.*, $L = 1$, and $\frac{dI}{dt} = 1$,

$$L \frac{dI}{dt} = 1, \text{ and therefore } \frac{d(N \Phi)}{dt} = 1.$$

In other words, the flux-linkage changes by one whenever the current changes by one E.M.U. and so a current of one E.M.U. flowing in a circuit whose inductance is one E.M.U. must produce one flux-linkage on the electromagnetic system, or one E.M.U. of flux-linkage.

Thus $N \Phi = LI$, or $N \Phi = MI$, provided each quantity is expressed in electromagnetic units. It will be seen in para. **144** that this equation is modified by a numerical factor when the quantities are expressed in practical units.

143. Faraday's Law in Practical Units.—In this case the constant K is again made equal to unity by a suitable choice of the practical units of inductance, *i.e.*, the law is again written as

$$E = -L \frac{dI}{dt} \text{ or } E = -M \frac{dI}{dt}$$

The **practical unit of inductance** is called the **Henry** and the above law involves its definition as follows :—

- (1) **Self-Inductance.**—A circuit has a self-inductance of one henry if the E.M.F. induced in it is one volt when the current is changing at the rate of one ampere per second.
- (2) **Mutual Inductance.**—Two circuits have a mutual inductance of one henry if the E.M.F. induced in one of them is one volt when the current in the other is changing at the rate of one ampere per second.

In the circuits used in wireless telegraphy, the henry is an inconveniently large unit, and inductance is usually given in the following submultiples :—

1 millihenry (mH) = one thousandth (10^{-3}) of a henry ;

1 microhenry (μH) = one millionth (10^{-6}) of a henry.

The microhenry is often abbreviated to the "mic."

144. The relation between the E.M.U. of inductance (the cm.) and the henry is easily obtained from the corresponding relations for current and E.M.F.

$$\begin{aligned} 1 \text{ henry} &= \frac{1 \text{ volt}}{1 \text{ amp. per sec.}} = \frac{10^9 \text{ E.M.U. of E.M.F.}}{\frac{1}{10} \text{ E.M.U. of current per sec.}} \\ &= 10^9 \times \frac{1 \text{ E.M.U. of E.M.F.}}{1 \text{ E.M.U. of current per sec.}} = 10^9 \text{ cms.} \end{aligned}$$

Thus 1 mic. = 10^8 cms.

No corresponding practical unit of flux-linkage is used. The E.M.U. of flux-linkage is the only unit employed. One flux-linkage is associated with a circuit of inductance 1 cm. when a current of one E.M.U. is flowing, and so 10^9 flux-linkages are associated for the same current with a circuit whose inductance is one henry. If the current is one amp. (one-tenth of an E.M.U.) the number of flux-linkages is therefore $\frac{10^9}{10} = 10^8$ for a circuit of inductance 1 henry.

Thus when L , or M , and I are expressed in practical units, the flux-linkage is given by

$$\begin{aligned} N\Phi &= LI \times 10^8 \text{ and } N\Phi = MI \times 10^8, \\ \text{or } LI &= N\Phi \times 10^{-8} \text{ and } MI = N\Phi \times 10^{-8}. \end{aligned}$$

It follows that the rate of change of flux-linkage with a circuit must be 10^8 flux linkages per second to induce an E.M.F. of one volt.

Thus Faraday's Law may be written in terms of induced E.M.F. and rate of change of flux-linkage as

$$(1) E = - \frac{d(N\Phi)}{dt} \text{ E.M.U. of E.M.F.}$$

$$\text{or } (2) E = - \frac{d(N\Phi)}{dt} \times 10^{-8} \text{ volts.}$$

Example 12.

The current is increasing uniformly at the rate of 2 amps. per second in a circuit of inductance 0.5 henry. Find

- (1) The induced E.M.F. in volts ;
- (2) The rate of change of flux-linkage.

$$(1) \frac{dI}{dt} = 2 \frac{\text{amps.}}{\text{sec.}} ; L = 0.5 \text{ henry ;}$$

$$\therefore E = - L \frac{dI}{dt} = - 0.5 \times 2 = - 1 \text{ volt.}$$

The minus sign indicates that the E.M.F. is acting to oppose the **increase** of current, *i.e.*, it is a back E.M.F.

(2) The flux-linkage produced by a current of 2 amps. flowing in a circuit of inductance 0.5 henry is given by

$$N\Phi = 0.5 \times 2 \times 10^8 = 10^8.$$

As the current is increasing by 2 amps. every second, the flux-linkage must therefore be increasing by 10^8 units per second.

The application of Faraday's Law in its flux-linkage expression also shows that this gives rise to a back E.M.F. of one volt.

$$E = - \frac{d(N\Phi)}{dt} \times 10^{-8} = - 10^8 \frac{\text{units}}{\text{sec.}} \times 10^{-8} = - 1 \text{ volt.}$$

Example 13.

The field in an air-cored coil of 300 turns, axial length 15 cms. and area 50 sq. cms., decreases uniformly from 600 gauss to 400 gauss in 0.005 second. Assuming that the field is uniform everywhere inside the coil, find

- (a) The E.M.F. induced ;
- (b) The self-inductance of the coil.

$$(a) \text{ When } H = 600 \text{ gauss ; } N\Phi = 300 \times 600 \times 50 = 9 \times 10^6 \text{ E.M.U.}$$

$$\text{When } H = 400 \text{ gauss ; } N\Phi = 300 \times 400 \times 50 = 6 \times 10^6 \text{ E.M.U.}$$

$$\therefore \frac{d(N\Phi)}{dt} = - \frac{3 \times 10^6}{0.005} = - 6 \times 10^8 \text{ E.M.U. per sec.}$$

$$\therefore \text{ Induced E.M.F. } = - \frac{d(N\Phi)}{dt} \times 10^{-8} \text{ volts} \\ = 6 \times 10^8 \times 10^{-8} = 6 \text{ volts.}$$

The E.M.F. appears as positive, *i.e.*, it is trying to keep the current and therefore the field up to its initial value.

(b) For a solenoid, $H = \frac{4\pi NI}{10}$, where I is the current in amps. and N = turns per cm. $= \frac{300}{15} = 20$ in this case.

When $H = 600$ gauss, $I = \frac{10H}{4\pi N} = \frac{6,000}{80\pi}$ amps. and $N \Phi = 9 \times 10^6$.

In practical units $LI = N \Phi \times 10^{-8}$,

$$\begin{aligned}\therefore L &= \frac{9 \times 10^6 \times 10^{-8} \times 80\pi}{6,000} \text{ henry} \\ &= 3.77 \times 10^{-3} \text{ henry} = 3.77 \text{ mH.}\end{aligned}$$

Alternatively :—

$$H = 600 \text{ gauss, } I = \frac{6,000}{80\pi} \text{ amps.}$$

$$H = 400 \text{ gauss, } I = \frac{4,000}{80\pi} \text{ amps.}$$

$$\therefore \frac{dI}{dt} = - \frac{2,000}{80\pi \times 0.005} \text{ amps. per sec.}$$

$$E = -L \frac{dI}{dt}$$

$$\text{i.e., } 6 = -L \times - \frac{2,000}{80\pi \times 0.005}$$

$$\therefore L = \frac{6 \times 80\pi \times 0.005}{2,000} \text{ henry} = 3.77 \text{ mH.}$$

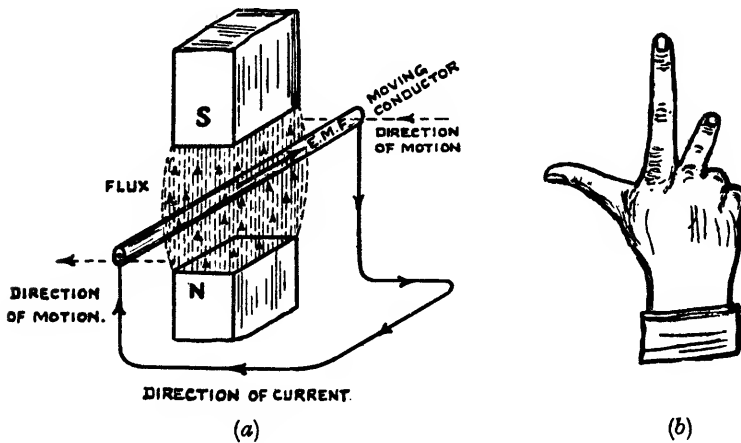
It should be noted that for an actual coil of these dimensions these results are rather high, as in practice, with an air core, every line of flux does not link with every turn (*cf.* Fig. 18).

145. There are various ways, other than alteration of current, by which the flux-linkage of a circuit may be changed. Thus, in Fig. 36 the coil Y might be moved about in the magnetic field of coil X. Such movements would obviously alter its mutual flux linkage and an E.M.F. would be induced in it. Again, the magnetic field need not be directly due to a current. It may be the field of a permanent magnet. The methods for inducing E.M.Fs. may be classified as follows :—

- (1) Variable flux and stationary conductor.
- (2) Moving flux and stationary conductor.
- (3) Stationary flux and moving conductor.
- (4) Variable flux and moving conductor.

Case (1) is that which has already been considered. The other cases occur in the production of E.M.Fs. in dynamos and alternators. Case (3) will now be considered. It is illustrated in Fig. 37 (a).

146. It is more convenient in this instance to consider the flux cut by the conductor in its motion. It is at once evident that lines of flux are only cut across by the conductor, provided its motion has some component at right angles to the direction of the field. If the conductor is moved straight from one pole face to the other it is moving parallel to the lines of flux and does not cut them. Thus no E.M.F. is induced in it (para. 140). If the conductor is moving in any other direction it must cut flux lines and so have an E.M.F. induced. The simplest case to consider is that in which the motion of the conductor is at right angles both to its own length and to the direction of the field (assumed to be uniform).



Illustrating Fleming's "Right Hand Rule."

FIG. 37.

If the velocity of the conductor is v cms. per second and its length in the field is l cms., then the area it traverses at right angles to the field in one second is lv sq. cms. If the flux density is B lines per sq. cm., the flux Φ through this area is Blv lines. The conductor thus cuts Blv lines per second, and so by Faraday's Law the E.M.F. induced in it is $Blv \times 10^{-8}$ volts.

More generally, if the motion of the conductor makes an angle θ with the direction of the flux, instead of being perpendicular to it, the distance the conductor travels perpendicular to the flux is $v \sin \theta$ cms. per second. It thus sweeps out an area of $lv \sin \theta$ sq. cms. perpendicular to the field every second and so cuts $Blv \sin \theta$ lines per second. Hence the induced E.M.F. is $Blv \sin \theta \times 10^{-8}$ volts.

In the particular case when $\theta = 90^\circ$, *i.e.*, the conductor is moved perpendicular to the field, $\sin \theta = 1$ and the induced E.M.F. is $Blv \times 10^{-8}$ volts, as was directly calculated above for this case.

Example 14.

A conductor 25 cms. long is moved at right angles to its own length across a magnetic field whose flux density is 2,000 lines per square centimetre with a velocity of 50 feet per second, the direction of motion being such that it makes an angle of 30° with the direction of the field. Find the E.M.F. induced in the conductor.

E.M.F. in volts = $10^{-8} \times$ rate of change of flux-linkage. The rate of change of flux-linkage is the rate at which the conductor cuts across the lines of flux, *i.e.*, it is given by the formula above,

$$E = Blv \sin \theta \times 10^{-8} \text{ volts.}$$

$$B = 2,000 \text{ lines per sq. cm., } l = 25 \text{ cms., } v = 50 \times 30.48 \text{ cms. per sec.}$$

$$\sin \theta = \sin 30^\circ = 0.5.$$

$$\therefore E = 2,000 \times 25 \times 50 \times 30.48 \times 0.5 \times 10^{-8} \text{ volts} \\ = 0.381 \text{ volt.}$$

147. Fleming's Right Hand Rule.—When the motion of the conductor is at right angles both to its own length and to the magnetic field, this rule provides a convenient method of determining the direction of the induced E.M.F. The thumb, forefinger and middle finger of the **right** hand are extended at right angles to each other as shown in Fig. 37 (b). The hand is then arranged so that the **thumb** points in the direction of **Motion** of the conductor, and the **Forefinger** in the direction of the **Flux**. The **Middle Finger** then gives the direction of the induced **E.M.F.** It can easily be verified that if either the flux or the motion is reversed in direction, the E.M.F. is also reversed in direction. If both flux and motion are reversed, the direction of the E.M.F. is unaltered.

The principle behind this rule is the same as that which has already been derived in considering E.M.Fs. induced by varying currents, *viz.*, that the circuit opposes any alteration in the electrical conditions and that the induced E.M.F. is an attempt to preserve the *status quo*. Suppose the ends of the moving conductor are joined by a wire so as to form a closed circuit. A current will flow owing to the induced E.M.F. In Fig. 37 (a), the direction of this current is indicated by arrows.

Reference to Fig. 35 shows that the flux due to a current in this direction interacts with the flux of the magnet to produce an "electrodynamic force" on the conductor in the **opposite** direction to its motion, and therefore tending to arrest the motion of the conductor which is causing the induced E.M.F. Thus if the motion of the conductor is to continue, an extra force must be applied to overcome the electrodynamic force. This involves the expenditure of more work on the conductor, and this extra work is the source of the energy which is converted to heat when the induced E.M.F. is acting in a closed circuit and current is able to flow. If the moving conductor is on open circuit, the induced E.M.F. is still in a direction such that if the circuit were closed and current could flow, the

electrodynamic force set up would oppose the motion. In the open circuit case no extra work is necessary to preserve uniform motion of the conductor, as no conversion of energy is taking place. A force will, of course, be necessary to overcome mechanical resistance to motion such as friction and air resistance.

These "inertial" tendencies of electrical circuits are conveniently summed up for conductors moving in magnetic fields in Lenz's Law.

Lenz's Law.—The direction of the induced E.M.F. produced by the motion of a conductor in a magnetic field is such that if induced current could flow, it would produce a force opposing the motion.

148. We are now in a position to discuss the solution of the problem indicated in para. 131, viz., when placed in a magnetic field, a conductor carrying current is acted on by an electrodynamic force proportional to the current and is therefore accelerated with consequent increase in kinetic energy. What is the source of the energy and will the velocity of the conductor increase indefinitely?

We shall assume the same arrangement as in Fig. 35, and suppose that the source of current in the conductor is a cell connected across its ends by means of long extensible leads whose resistance we may neglect. Thus when placed between the poles of the magnet, the current-carrying conductor has an acceleration to the right. But as soon as it starts in motion, it cuts lines of flux and so has an E.M.F. induced in it. The application of Fleming's Right Hand Rule to Fig. 35 shows at once that the E.M.F. is in a direction opposing the flow of current in the conductor. The current falls, and the electrodynamic force, being proportional to the current, is correspondingly decreased. As the current is now less than its original value, (battery E.M.F. + resistance), this indicates that the energy taken from the battery is no longer being completely converted into heat energy in the conductor. If at any moment I is the current flowing, E the E.M.F. of the battery, and R the resistance of the conductor circuit, then the resultant E.M.F. is

$$E - \text{induced E.M.F.},$$

and by Kirchhoff's Law this is equal to the fall of potential RI round the circuit,

$$\text{i.e., } E - \text{induced E.M.F.} = RI.$$

$$\therefore EI - \text{induced E.M.F.} \times I = RI^2.$$

EI is the energy taken from the battery per second.

RI^2 is the heat energy produced in the circuit per second.

There is thus an amount of energy, equal to induced E.M.F. $\times I$ per second, which is being used up in some other way. It is this energy which supplies the kinetic energy acquired by the conductor.

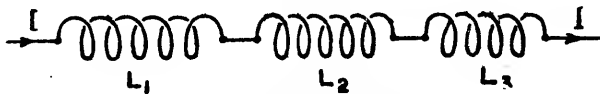
As the electrodynamic force decreases, so the acceleration of the conductor decreases, i.e., the amount of kinetic energy it acquires per second decreases, but as long as any current flows there will be a

force and therefore an acceleration. As the velocity of the conductor increases, however, the induced E.M.F., which is proportional to the velocity (para. 146), also increases, so that eventually a time will come when it is equal to the battery E.M.F. The current in the conductor then falls to zero, and the latter experiences no further acceleration, but goes on with constant velocity. No energy is then being taken from the battery, ($I = 0$), as we should expect, for there is no heat produced in the conductor, nor is its kinetic energy increasing.

In a practical case, of course, this limiting condition is not reached, as sufficient current must flow to give a force overcoming friction and air resistance. The velocity of the conductor thus becomes constant before the current in it falls to zero and the induced E.M.F. never quite becomes equal to the battery E.M.F.

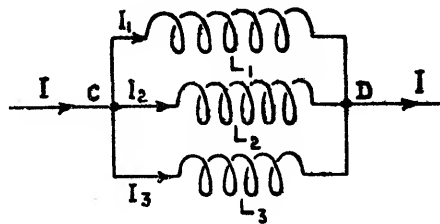
The above argument gives the essential theory of the electric motor (Chapter IV). We shall now return to the discussion of self and mutual inductance.

149. Inductances in Series.—Three inductances, L_1 , L_2 and L_3 , are shown in series in Fig. 38 (a). In this arrangement the same current (I) flows through each of them, by Kirchhoff's First Law. If the inductances are in henries and the current in amps., the flux-linkages associated with the three coils are $L_1 I \times 10^8$, $L_2 I \times 10^8$, and $L_3 I \times 10^8$ respectively (para. 144).



Inductances in Series.

(a)



(b)

Inductances in Parallel.

FIG. 38.

Thus the total flux-linkage is $(L_1 + L_2 + L_3) I \times 10^8$ and so the equivalent inductance L producing the same number of flux-linkages ($LI \times 10^8$) is given by

$$L = L_1 + L_2 + L_3$$

It will be seen that the procedure for finding the equivalent inductance of a number of inductances in series is the same as in the corresponding arrangement for resistances.

It should be noted that the above result depends on the assumption that none of the flux produced in one of the inductances links with either of the others. If this occurred, the equivalent inductance of the three coils would be less or greater than the sum of their individual inductances, according to the amount of such mutual flux linkage and its direction.

150. Inductances in Parallel.—The arrangement is shown in Fig. 38 (b). The equivalent inductance is found most simply in this case by a consideration of the back E.M.Fs. induced when the current is increasing. With a steady current (I) flowing, the currents flowing in the three parallel paths are taken as I_1 , I_2 and I_3 .

By Kirchhoff's First Law

$$I = I_1 + I_2 + I_3$$

$$\therefore \frac{dI}{dt} = \frac{dI_1}{dt} + \frac{dI_2}{dt} + \frac{dI_3}{dt}$$

Suppose now that the current I is increased. I_1 , I_2 and I_3 must increase to keep their sum equal to I . While this is happening, back E.M.Fs. are induced in them of values

$$L_1 \frac{dI_1}{dt}, L_2 \frac{dI_2}{dt} \text{ and } L_3 \frac{dI_3}{dt} \text{ respectively.}$$

The potentials at C and D at any given moment can only have one value each, and so the P.D. between C and D must be independent of whichever path it is reckoned for. Thus the back E.M.Fs. in the three paths must be equal to each other at every moment,

$$\text{i.e., } L_1 \frac{dI_1}{dt} = L_2 \frac{dI_2}{dt} = L_3 \frac{dI_3}{dt} = E \text{ (back E.M.F.).}$$

If the equivalent inductance is L , then this coil should give the same back E.M.F., E , when the total change of current takes place through it,

$$\text{i.e., } L \frac{dI}{dt} = E$$

$$\therefore \frac{E}{L} = \frac{dI}{dt} = \frac{dI_1}{dt} + \frac{dI_2}{dt} + \frac{dI_3}{dt}$$

$$= \frac{E}{L_1} + \frac{E}{L_2} + \frac{E}{L_3}$$

$$\therefore \frac{1}{L} = \frac{1}{L_1} + \frac{1}{L_2} + \frac{1}{L_3}$$

Thus inductances in parallel behave like resistances in parallel. It must again be emphasised that no **mutual** linkage must take place

between the fluxes produced in the separate coils for this proof to be correct.

(This proof is repeated in Chapter V for the case of alternating current, where the expressions $\frac{dI}{dt}$ etc., can be evaluated more exactly.)

151. Types of Inductance.—The result above for the equivalent self-inductance of a number of inductances in series leads to a consideration of the various ways in which a length of wire may be wound to give the greatest self-inductance. If the wire is straight, the lines of flux are circles round it, and so when they change with changing current, any new lines in being formed will only cut a small part of the wire. The case is thus equivalent to a number of small inductances in series with no mutual linkage and the self-inductance of the whole wire is merely the sum of the self-inductances of its parts.

The circumstances are completely altered when the wire is wound in the form of a coil. The self-inductance of the complete coil is much larger than the sum of the self-inductances of each turn; for the current flowing in any one turn produces flux lines which not only link with that turn but also with many of the other turns, *i.e.*, there is mutual inductance between the turns. The total self-inductance of the coil is the sum of the self-inductances of all the turns plus the sum of all the mutual inductances between the turns. Thus for a given length of wire, the inductance is greatly increased by winding it in solenoidal form.

152. The self-inductance of a solenoid is easily deduced from the result obtained in para. 133, *viz.*,

$$\text{Flux-linkage} = \frac{4\pi N^2 A I l}{10}$$

$$\text{But flux-linkage} = L I \times 10^9$$

$$\therefore L = 4\pi N^2 A l \times 10^{-9} \text{ henry.}$$

It will be recalled that this result was based on the assumption that every line of flux produced linked with every turn. This is hardly justifiable for air-cored solenoids, as reference to Fig. 18 will show. It is much more nearly the case if the solenoid has an iron core, for the lines of flux find a much easier path in iron and do not tend to "leak" to the outside of the solenoid and complete themselves after linking with only a few turns, as happens with the air-cored solenoid. In addition, when a current flows in the coil, the iron becomes magnetised and adds its own flux lines to those produced directly by the current. The total flux-linkage is thus much greater in an iron-cored solenoid than in an air-cored solenoid of

the same dimensions. The self-inductance of such a solenoid per centimetre length may be calculated as follows :—

$$H = \frac{4\pi NI}{10} \text{ dynes per unit pole.}$$

$$\therefore \Phi = \mu HA = \frac{4\pi N \mu AI}{10} \text{ lines}$$

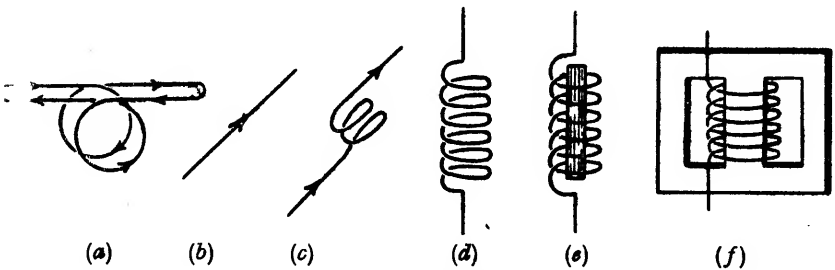
$$\therefore \text{Flux-linkages per cm.} = \frac{4\pi N \mu AI}{10} \times N = \frac{4\pi N^2 \mu AI}{10}$$

$$= LI \times 10^8$$

$$\therefore L \text{ (per cm.)} = 4\pi N^2 \mu A \times 10^{-9} \text{ henry.}$$

Since μ is variable and depends on the current flowing in the coil (para. 88), the inductance of an iron-cored coil is likewise variable, a point which is of considerable importance in the design of such coils for wireless telegraphy circuits.

153. It may be desired to wind a coil (or arrange a length of wire), so that it has as little self-inductance as possible. This is necessary, for instance, in winding standard resistances, *e.g.*, for measuring instruments and rheostats (variable resistances). It may be achieved as shown in Fig. 39 (a). The wire is doubled back on itself, before winding, or in a bight as shown. Thus the current flows in opposite directions in adjacent turns and no resultant field and flux can be set up. Such a coil is said to be “non-inductively wound.”



Various forms of Inductance.

FIG. 39.

The circuits in Fig. 39 are shown in order of increasing inductance from left to right, to illustrate the above remarks. In Fig. 39 (f), a complete path in iron for the lines of flux is provided; there is thus practically no leakage, and every line links with every turn.

In a dynamo or motor the flux lines produced by the current flowing in the field coils are established in material that is nearly all ferromagnetic, the only air they traverse being in the narrow gaps between the poles and the armature. The field coils, which may have many turns and carry large currents, are thus highly

inductive, and very large induced E.M.Fs. are developed when the current is altering. The effect of this is not important on making the field circuit, the current merely taking longer to rise to its final value, but undesirable results may follow on breaking the circuit. The induced E.M.F. may be large enough to produce a spark or arc at the break or across the insulation of various turns and so damage the machine. Provision must be made in the design for the current to die away slowly enough to limit the induced E.M.F. to a reasonable amount.

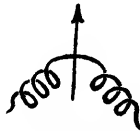
154.—Inductances met with in Wireless Telegraphy Circuits.—These may be classified as :—

(a) Inductances having a maximum of inductance (of the order of henries) in a minimum of space, and large current-carrying capacity. These will have iron cores. Such are armatures and field coils of motors, dynamos and alternators, transformers, induction coils, etc.

(b) Receiving Inductances.

Inductances used in receiving circuits usually consist of a number of insulated turns of wire, closely wound on a cylinder of insulating material. Owing to the fact that voltages and currents are small in receiving circuits, the insulation need not be excessive, and such inductances can be very compact. Ohmic resistance must be kept small, and this is secured by using wire made of interwoven insulated strands. Either single layer or multiple layer coils can be used. Variations in the value of the inductance can be ensured in two ways, either by having a sliding contact which only brings in a certain number of turns into the circuit, or by employing a variometer.

A variometer inductance is composed of two coils in series so arranged that their fields can be made to assist or to oppose each other. One coil is fixed and the other can be rotated inside the



Variometer.

FIG. 40.

fixed one. In one limiting position, the direction of the windings of the two coils is such that the field produced by one annuls nearly all the field due to the other, and the inductance of the two coils is a minimum. When the moving coil is rotated through 180° the fields are additive, and the inductance is a maximum. In intermediate positions the combined inductance can be made to vary

continuously between one extreme and the other according to the angle between the axes of the coils.

Thus a variometer affords a very delicate variation of inductance.

A variometer is indicated diagrammatically as in Fig. 40.

The values of inductances used in receiving circuits may be of the order of hundreds or thousands of mics.

(c) **Transmitting Inductances.**

In general the difference between receiving and transmitting inductances may be summed up in the statement that the latter are

- (1) of far larger dimensions,
- (2) not always capable of continuous variation.

They have to carry much greater currents and have much greater voltages induced across them. Thus they are generally constructed of copper tubing or multi-stranded wire, with well-spaced turns and high insulation. Inductances used in the primaries of spark oscillators are of small value, of the order of mics., while those used in the aerial coils of spark and continuous wave sets are of considerably larger value, though still of the same order.

155. Practical Calculation of Self-Inductance.—The limitations of the formulæ obtained for the self-inductance of air and iron-cored solenoids have already been mentioned. For a toroidal coil, *i.e.*, a coil wound on an iron anchor ring, the formula for an iron-cored solenoid, *viz.*, $L = 4\pi N^2 \mu A \times 10^{-9}$ henry per cm. length is very nearly correct, if the cross-section of the ring is circular and small in dimensions compared with the length of the ring.

Large corrections may have to be made to fit the formula for practical calculation in other cases, but the practical formula which will now be developed for air-cored coils is based on the theoretical result found for such a coil in para. 152,

$$\text{viz., } L = 4\pi N^2 Al \times 10^{-9} \text{ henry.}$$

N is the number of turns per cm., and so the number of turns in a length " l " cms. is $n = Nl$.

Also in a circular coil $A = \pi r^2$, where r is the radius of the coil.

$$\text{Hence we may write } L = \frac{4\pi^2 n^2 r^2}{l} \times 10^{-9} \text{ henry.}$$

In the practical formula the factor n^2 is retained and one of the " r "'s in r^2 . The remaining factors are assimilated into a "form factor" F , which depends on the dimensions of the coil, and which is determined from the curve in Fig. 41 in a way to be described. The formula then becomes

$$L = r \times n^2 \times F \text{ mics.,}$$

where r = mean radius of coil in inches,
 n = number of turns in coil,
 F = form factor of coil.

To obtain F , the ratio $\frac{r}{l+d}$ is first determined

where l = winding length in inches

d = depth of the winding in inches, or thickness of the wire used if the coil is single layer.

The corresponding value of F is then read off from the graph of F against $\frac{r}{l+d}$ in Fig. 41.

The above data can be obtained in the following manner:—

- (1) Measure the radius of the former and add half the depth of the winding (Fig. 42), or half the diameter of the wire used; or, measure the circumference of the former, divide by 2π and add half the depth of the winding. This gives " r ."

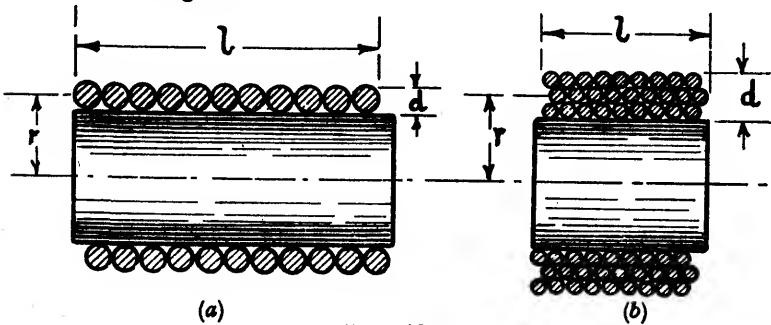


FIG. 42.

- (2) Measure the total winding length of the coil in inches. This gives " l ."
- (3) Count the total number of turns in the coil; or, if there are a great many turns, count the number in, say, one inch of winding length and multiply by l . This gives " n ."
- (4) Determine " d ," the depth of the winding. In a single-layer coil this will be the diameter of the wire used.
- (5) Work out the ratio $\frac{r}{l+d}$
- (6) Find the value of F from the curve corresponding to the ratio $\frac{r}{l+d}$, being careful to work on the right part of the curve, and to read the scale very accurately.

For values of $\frac{r}{l+d}$ which are between 4 and 8, the value of F can be found thus: divide $\frac{r}{l+d}$ by 2; look up the value of F corresponding to this, add 0.0220 to this value, and the answer will be the correct value of F .

- (7) Work out the formula

$$L = r \times n^2 \times F \text{ mics.}$$

Example 15.

It is required to determine the inductance of a single layer coil of 64 turns of wire 0.08 in. in diameter, wound on a former of 2.65 in. radius, for a winding length of 16.2 inches.

(1) Radius of former	= 2.65 in.
Half diameter of wire	= 0.04 in.
Mean radius of coil	= 2.69 in.

(2) $l = 16.2$ inches.

(3) $n = 64$ turns.

(4) $d = 0.08$ inches.

(5) $\frac{r}{l+d} = \frac{2.69}{16.2+0.08} = \frac{2.69}{16.28} = 0.1652.$

(6) $F = 0.0145$ (from the lowest curve).

(7) $L = 2.69 \times 64^2 \times 0.0145 = 160$ mics.

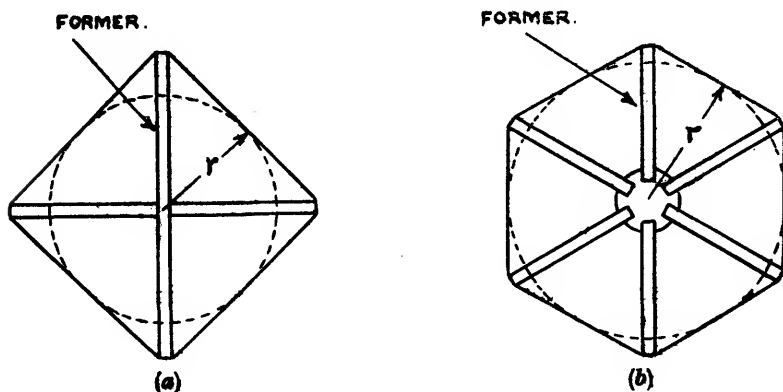


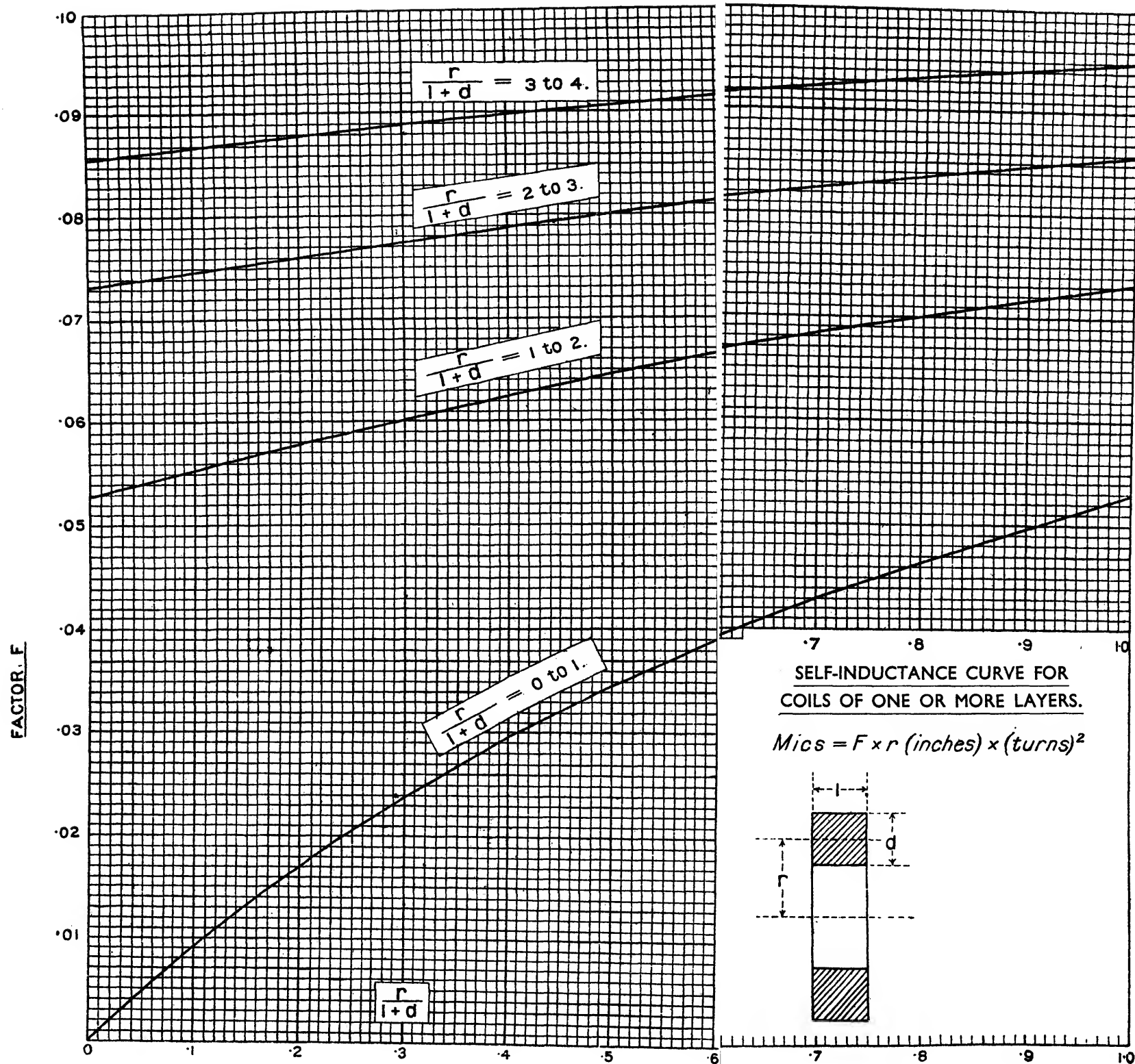
FIG. 43.

If the coil is wound on a former having a square cross-section (Fig. 43 (a)) instead of on a cylinder, its self-inductance will be from 22 per cent. to 27½ per cent. greater than that of the corresponding cylindrical coil which would be bounded by the circle inscribed in the square that forms the boundary of the coil.

If the coil is wound on a hexagonal (six-sided) former (Fig. 43 (b)) its self-inductance will be about 10 per cent. greater than that of the corresponding cylindrical coil which would be bounded by the circle inscribed in the hexagon bounding the coil.

In order to obtain the maximum self-inductance with the minimum length of wire, the diameter of the coil should be 2.414 times the winding length.

156. To wind a coil of given Self-inductance.—The problem of calculating self-inductance is usually presented to us in another form.



Instead of having to find the inductance of a given coil of wire, we have usually to **wind a coil of given self-inductance.**

Having first selected a former and the size of wire necessary, we wrap some of the wire round the former with the same spacing (if any) that we are going to use in the finished coil. We then measure the axial length occupied by 10, 20 or more turns, taking the measurements in inches.

Suppose we have an idea that somewhere between 100 and 200 turns will be correct for our particular inductance.

Assume 100, 130, 160 and 200 turns, working out the inductance in each case. Probably none of these numbers will be the exact number of mics we require.

By making a rough curve of mics and turns we can then pick out fairly exactly the number of turns necessary.

From the formula given above it may be deduced that the more closely the turns are spaced, and the greater the diameter of the coil, the greater will be its inductance.

It is not so easy a matter to calculate the inductance of a coil wound on an iron core, since the reluctance of the various magnetic paths has to be taken into account.

The method of making this calculation is given in any standard electrical textbook.

157. The Magnetic Energy associated with a Current may be expressed in terms of the current and the self-inductance of the circuit in which the current is flowing. It is not possible to do this in an elementary manner, and those who are unable to follow the argument must merely accept the result.

The magnetic energy associated with a current of I amps. flowing in a circuit of self-inductance L henries is $\frac{1}{2} LI^2$ joules.

This formula may be derived as follows :—

It has already been mentioned several times that the current does not rise immediately to its final value in an inductive circuit. While it is increasing from zero to I , a back E.M.F. is established. Let the value of the current, at a time " t " seconds after it has started to flow, be " i " amps. The back E.M.F. at this moment is $-L \frac{di}{dt}$ volts and the applied E.M.F. necessary to overcome this is $+L \frac{di}{dt}$ volts. Thus the power or rate of working of the applied E.M.F. at this particular moment " t " is

$$Ei = Li \frac{di}{dt} \text{ watts}$$

and the energy taken from the source of E.M.F. in a short interval of time " dt " seconds is

$$Ei \cdot dt = Li \frac{di}{dt} \cdot dt = Li \cdot di \text{ joules.}$$

This is the amount of energy which is stored in the field as magnetic energy during the interval " dt ." The total energy stored in the field of the current will be obtained by adding together the separate small amounts derived as above until the current reaches its final steady value I , when the back E.M.F. falls to zero and no more magnetic energy is produced, *i.e.*, by integrating $Li \cdot di$ between the current limits 0 and I .

$$\int_0^I Li \cdot di = \left[\frac{1}{2} Li^2 \right]_0^I = \frac{1}{2} LI^2 \text{ joules.}$$

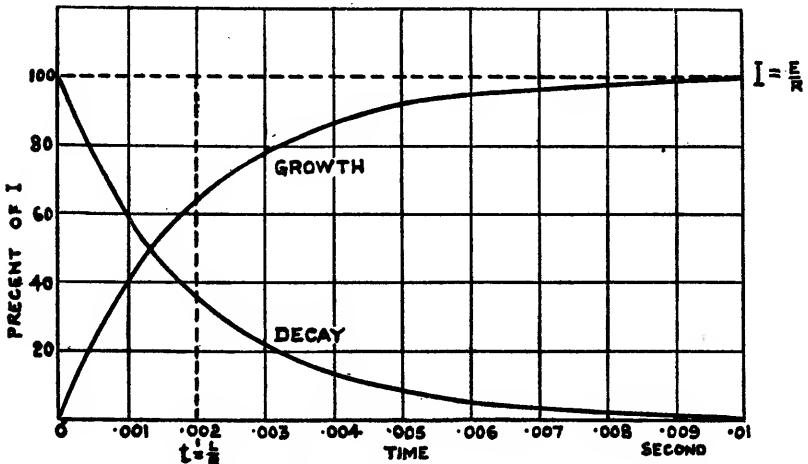
It will be seen that the time does not appear in the result. The total magnetic energy is the same whether the current is established quickly or slowly.

Example 16.

What is the magnetic energy in the field due to a current of 2 amps. flowing in a circuit of inductance 500 mics.

$$\begin{aligned} L &= 500 \text{ mics.} = 5 \times 10^{-4} \text{ henry} \\ \therefore \text{Energy} &= \frac{1}{2} LI^2 = \frac{1}{2} \times 5 \times 10^{-4} \times 4 \\ &= 10^{-3} \text{ joule or } \frac{1}{1,000} \text{ joule.} \end{aligned}$$

A descriptive account of the growth and decay of currents in inductive circuits has already been given, and the practical effects



Growth and Decay of Current in a Circuit containing only Inductance and Resistance.

FIG. 44.

noted. The mathematical investigation will now be considered and the exact way in which the current depends on the inductance and resistance examined. The complete derivation is given; those who are unable to follow it should concentrate attention on the results and their graphical illustration (Fig. 44).

***158. Growth of Current in a Circuit containing Inductance and Resistance.**—The circuit is shown in Fig. 45. A battery of E volts is available to send current round the circuit when the switch S is

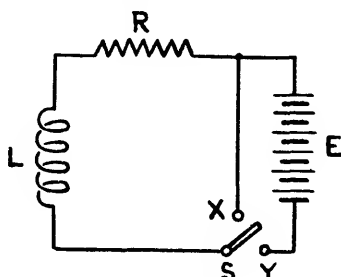


FIG. 45.

made to Y . R is the total resistance of the battery, coil and leads, in ohms. L is the self-inductance of the coil in henries. Zero time is the moment at which the switch is made, and " t " seconds afterwards the current is " i " amps.

Thus at $t = 0$, $i = 0$.

The P.D. across the resistance at time " t " is iR .

By Kirchhoff's Law this is equal to the resultant E.M.F. acting in the circuit, which is the applied E.M.F., E , less the back E.M.F.

$$L \frac{di}{dt}.$$

$$\therefore E - L \frac{di}{dt} = iR$$

The solution of this equation for i in terms of t determines the current at any instant after the circuit is closed.

$$\text{Rearrangement gives } \frac{di}{E - iR} = \frac{dt}{L}$$

Integrating both sides, $-\frac{1}{R} \log_e (E - iR) = \frac{t}{L} + \text{a constant term}$, or $\log_e (E - iR) = -\frac{Rt}{L} + \text{a constant term}$.

Rewriting this in exponential form,

$$E - iR = Ce^{\frac{-Rt}{L}}$$

where C is a constant to be determined from the initial conditions $i = 0$ at $t = 0$.

Substituting these values gives

$$\begin{aligned} E &= C \text{ (since } e^0 = 1), \\ \text{so that } iR &= E \left(1 - e^{\frac{-Rt}{L}} \right) \\ \text{and } i &= \frac{E}{R} \left(1 - e^{\frac{-Rt}{L}} \right) \end{aligned}$$

$\frac{E}{R}$ is the steady value I which the current eventually takes according to Ohm's Law. Hence the relation between the instantaneous value i of the current at any time t and its final value I is given by

$$i = I \left(1 - e^{-\frac{Rt}{L}} \right)$$

In theory $e^{-\frac{Rt}{L}}$ does not become zero, *i.e.*, the current does not reach its final value I , until t is infinite, but, practically speaking, $e^{-\frac{Rt}{L}}$ becomes small enough to be negligible in a very short time.

Example 17.

Find the percentage of the final value of current reached in 0.01 second, when a circuit for which $L = 1$ henry and $R = 600$ ohms is made.

It is only necessary to find the value of the term

$$\left(1 - e^{-\frac{Rt}{L}} \right)$$

In this case the value is $(1 - e^{-0.006}) = 1 - 0.00248 = 0.99752$.

So that, in this short space of time, the current rises to 99.75 per cent. of its maximum value.

In a time $t = \frac{L}{R}$, the current rises to $(1 - e^{-1}) = 1 - 0.368 = 0.632$, or 63.2 per cent., of its final value.

This time $t = \frac{L}{R}$ is known as the **Time Constant** of the circuit, and is a measure of the rapidity of growth of current in different circuits.

***159. Decay of Current in a Circuit containing Inductance and Resistance.**—When steady conditions have been reached in the

circuit of Fig. 45, *i.e.*, a current $I = \frac{E}{R}$ is flowing, let the switch S

be made to X . This removes the battery from the circuit, and if there were no inductance the current would fall instantaneously to zero. In the inductive circuit the magnetic energy in the field has to be converted to heat energy and so a current continues to flow while this is occurring. Let i be this current in amps, t seconds after the switch is put over, *i.e.*, $t = 0$, $i = I$. The induced E.M.F.

at this time is $-L \frac{di}{dt}$, and the P.D. across the resistance is iR . By

Kirchhoff's Law these must be equal, *i.e.*,

$$-L \frac{di}{dt} = iR$$

$$\therefore \frac{di}{i} = -\frac{R}{L} dt$$

Integrating both sides $\log_e i = -\frac{Rt}{L} + \text{constant}$

$$\therefore i = Ce^{-\frac{Rt}{L}}$$

At $t = 0, i = I, \therefore I = C,$

$$\text{and finally } i = Ie^{-\frac{Rt}{L}}.$$

The graph of i against t is shown in Fig. 44. It is the growth curve inverted. Theoretically the current should take an infinite time to decay, but in practice it dies away in a very short time. Thus, in Fig. 44, which might correspond to a circuit of inductance 2,000 mics. and resistance 1 ohm ($\frac{L}{R} = 0.002$), the current has fallen to about 1 per cent. of its original value in 0.009 second.

After a time equal to the time constant, $\frac{L}{R}$, of the circuit, the percentage of the initial current still flowing is

$$e^{-1} \times 100 \text{ per cent.} = 36.8 \text{ per cent.}$$

160. The time constant $\frac{L}{R}$ is a measure of the rate at which current either grows or decays in an inductive circuit. From its form the following general deductions may be made:—

- (1) In a circuit of given resistance the rate of growth or decay is directly proportional to the inductance of the circuit. This is to be expected on general grounds.
- (2) The current may be made to grow or decay more quickly in a circuit of given inductance by **increasing the resistance** of the circuit. This is of practical importance in the design of brushes for electrical machines (para. 220).

161. Mutual Inductance.—It has already been seen that the total self-inductance of a complex circuit such as a coil depends very largely on the mutual inductance between its component parts, *e.g.*, the individual turns of the coil. When all the turns are wound in the same way the mutual inductance is such that it increases the total self-inductance. The “non-inductively wound” coil gives an example of the opposite effect; the mutual inductance between the various turns is exactly equal and opposite to their individual self-inductances, and so the total self-inductance of the coil is zero.

We may thus look on the **mutual inductance** between parts of a circuit, or between different circuits, as being **either positive or negative**.

The mutual inductance between two circuits is taken as positive when the flux-linkage with the second circuit due to the current in the first circuit is in the same direction as the flux-linkage with the second circuit produced by the current flowing in the second circuit itself.

Conversely, when the mutual flux-linkage between the two circuits is in the opposite direction to the flux-linkage with either due to its own current, the mutual inductance is said to be negative.

Thus, in the solenoid, the mutual inductances between the various turns are positive; in the non-inductively wound coil the various mutual inductances are negative.

Another example is provided by the variometer, in which the mutual inductance is either positive or negative according to the angle through which the moving coil is turned with respect to the fixed coil.

A case in wireless telegraphy, in which the sign of the mutual inductance is of fundamental importance, arises in the type of valve transmitter where oscillations are maintained by mutual inductance between the grid and anode circuits (para. 610).

162. Calculation of Mutual Inductance.—It will readily be realised that the calculation of the mutual inductance of two circuits is an even more complicated problem than the calculation of self-inductance, except in very simple cases.

One case which lends itself to calculation is that of two coils wound on the same former. If we make the assumption that when a current flows in either coil, all the flux lines associated with the current link with both coils, a simple result may be obtained. The limitations of this assumption in practice have already been mentioned (para. 152). An air core will also be assumed.

Let one coil have N_1 turns per cm. Its self-inductance L_1 per cm. length is given by $L_1 = 4\pi AN_1^2 \times 10^{-9}$ henry, where A is the area of the former.

If the corresponding quantities for the other coil are N_2 and L_2 ,

$$L_2 = 4\pi AN_2^2 \times 10^{-9} \text{ henry.}$$

If a current of I amps. is established in coil L_1 , the flux is

$$\Phi = HA = \frac{4\pi AN_1 I}{10}.$$

This links with N_2 turns per cm. of the coil L_2 .

Therefore mutual flux-linkage per cm. ($N\Phi$) = $\frac{4\pi AN_1 N_2 I}{10}$.

But $MI = N\Phi \times 10^{-8}$,

where M is the mutual inductance per cm. length of the two coils.

$$\therefore M = 4\pi AN_1 N_2 \times 10^{-9} \text{ henry.}$$

163. Coupling and Coupling Factor.—When two circuits are arranged so that some of the flux lines due to one circuit link with the other they are said to be coupled to each other. There are several ways of coupling circuits (Chapter V), and this particular method is called "mutual magnetic," "mutual inductive," or simply "mutual" coupling. According to the proportion of the total flux

produced by the first circuit which links with the second, the coupling is said to be "tight" (or "close"), or "loose." Thus, on the assumptions made in the last paragraph, the coupling between the two coils L_1 and L_2 is as tight as it possibly can be, for all the flux produced by one coil links with the other. This case is made the basis of a criterion of the tightness of coupling between any two circuits. It was found that

$$M = 4\pi AN_1 N_2 \times 10^{-9} \text{ henry per cm.}$$

$$\therefore M^2 = 16\pi^2 A^2 N_1^2 N_2^2 \times 10^{-18} = (4\pi AN_1^2 \times 10^{-9}) \times (4\pi AN_2^2 \times 10^{-9}) \\ = L_1 L_2,$$

$$\text{and } M = \sqrt{L_1 L_2}.$$

This is the maximum possible mutual inductance between the circuits, and so M can never be greater than $\sqrt{L_1 L_2}$. In any practical case, M is less than $\sqrt{L_1 L_2}$, for it is impossible to arrange that all the flux produced by one circuit links with another; there is always some "leakage" of flux, *i.e.*, some lines complete themselves without passing through every turn.

The ratio $\frac{M}{\sqrt{L_1 L_2}}$ is taken as a criterion of the closeness of the coupling between two circuits. It is called the "coupling factor," and is denoted by K .

$$\text{Thus } K = \frac{M}{\sqrt{L_1 L_2}}, \text{ or } M = K \sqrt{L_1 L_2}.$$

It is obvious that K is a pure number and has no units. Its maximum value is unity, which corresponds to the ideal case discussed above ($M = \sqrt{L_1 L_2}$). Its value in any given case is a measure of how many lines of flux from one circuit are linking with the other compared with the case when all the flux-linkages are mutual.

Another term used is "percentage coupling," which is $100K$. The ideal case above has 100 per cent. coupling between the coils.



FIG. 46.

Fig. 46 is a diagrammatic representation of two coils in which the mutual coupling becomes progressively tighter from left to right; the two coils wound on the iron core have tightest coupling because the core minimises flux-leakage.

Mutual coupling is further discussed in Chapter V.

CAPACITY.

164. An account of the simpler electrostatic effects was given in Chapter II. It was there seen that the behaviour of charged bodies could be rendered intelligible by the idea of lines of electric flux, which tended to longitudinal contraction and lateral expansion; the meaning of field strength and P.D. was explained, and the electric field was seen to be a storehouse of potential energy.

The existence of an electric field depends, of course, on the presence of electric charges. Remove the charges and the field disappears. The nature of the field and the direction of the lines of electric flux depends on the value of the charges and their distribution. For instance, the field due to an isolated point charge is different from that between two parallel plates with equal and opposite charges.

When a charge moves in an electric field its motion involves the transformation of energy. Work must be done on the charge if it is moved against the field forces; work is done by the field forces if the charge moves under their influence. This led to the ideas of the P.D. between two points in the field, and the absolute potential at a point in the field. The charged body producing the field is, of course, in the field and so has a potential, viz., the work necessary to bring a unit positive charge up to it from earth (or from a point where there is no electric field due to the charge).

It is found that in all cases the potential of a charged body is proportional to its charge. Thus in the case of two parallel plates, if the negatively-charged plate is earthed, the potential of the positive plate is $\frac{4\pi dQ}{KA}$ (para. 103), and so is directly proportional to Q , the charge on the plate.

In general, for any charged body we may write $Q \propto V$, or $Q = CV$, where C is the constant of proportion.

This relation is of great importance and the constant C is given a special name. It is called the **capacity** of the charged body.

$$\text{Hence capacity} = \frac{\text{charge}}{\text{potential}}$$

165. Condensers.—Every charged body has a certain capacity, but, unless special arrangements are made, it will not be of large amount. In other words, an electric charge given to the body will raise it to a high potential, sufficient perhaps to break down the insulation of the material surrounding it. The charge which a body can hold for a given potential may be increased by concentrating the region over which its field extends. Devices in which this takes place are called **condensers**. The electric field is confined to a small space and so can be made much more intense without raising the body to too high a potential. The condenser may also be regarded as a device for concentrating electrostatic field energy

Increase of field strength means that there is a greater density of electric flux, *i.e.*, the flux lines are packed more closely together. They resist this packing because of their tendency to lateral expansion, which forces them apart. The extra work necessary to accomplish the packing appears as an increase of energy in the field.

166. Parallel-Plate Condenser.—One of the commonest forms of condenser consists of two parallel metal plates with some dielectric between them. The lines of flux from one of these plates would normally end on any convenient points, *e.g.*, the walls of the room. If the other plate is earthed and brought near the first one, the lines of flux can shorten themselves considerably, and most of them will now end on the earthed plate, thus concentrating the field.

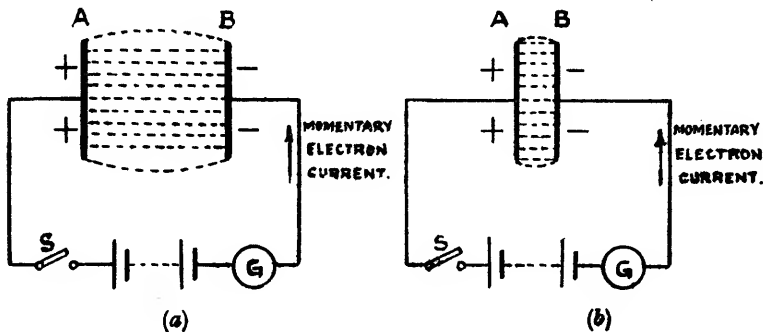


FIG. 47.

The operation of charging such a condenser may be considered from the electron standpoint. Fig. 47 shows a simple charging arrangement. The two plates A and B are connected to a battery and galvanometer (G) through the switch S.

(a) When the plates are well separated, as soon as the switch S is closed the battery will cause a momentary rush of electrons round the circuit from plate A, which is connected to the positive terminal of the battery, through the battery towards plate B, where they will crowd together; the galvanometer pointer will be deflected. The current flowing will stop as soon as the potential difference between the plates is equal to the terminal P.D. of the battery. The two plates will now be oppositely charged. Plate B will have a surplus of electrons which will repel others trying to arrive. Some of the atoms of plate A have lost electrons, and are therefore positive ions, which, by their attraction, prevent any more electrons from moving away. Equilibrium is thus established.

These two oppositely-charged plates produce an electric field in the dielectric separating them, and a displacement of electrons, or displacement current, will occur within the atoms in the dielectric, in the direction from the negatively-charged plate B to the positively-charged plate A.

(b) Let the two plates be suddenly brought closer together.

The negative charge on plate B overcomes to some extent the effect of the positive charge on plate A and *vice versa*, so that the potential difference between the two plates is momentarily lowered, the forces holding the electrons in check are decreased, and another momentary electron movement takes place until equilibrium is again established, and the P.D. between the plates is again equal to the terminal P.D. of the battery.

Thus the closer the plates are together the greater is the charge which may be concentrated on one or other for the same P.D. between them.

(c) Suppose we break the switch S. Each plate is now left charged with a certain quantity of electricity, A positively and B negatively. The sum of these charges is algebraically zero. When the term "charge on a condenser" is used, it refers to the charge on **one** of the plates.

(d) If the battery B is now short-circuited and the switch S again closed, the electrons will rush back from the negative plate to the positive plate until they are equally distributed round the circuit, and there will be a deflection of the galvanometer pointer in the reverse direction.

The electrons in the dielectric, which were strained towards the positive plate, will return to their normal orbits.

There will thus be a momentary conduction current (para. 40) round the circuit, and a displacement current (para. 41) through the dielectric.

Clearly, the greater the voltage applied to the condenser the greater will be the electric flux density in the dielectric.

The greater the flux-density the greater will be the amount of electrical energy stored in the condenser.

167. A condenser is illustrated diagrammatically as in Fig. 48.

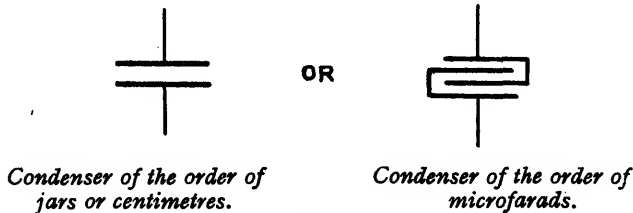


FIG. 48.

A useful mechanical analogy to a condenser is furnished by a steel spring. When the spring is compressed or extended, potential energy is stored up in it and is liberated when the spring is allowed to return to its normal size. Similarly, in the case of a charged condenser, lines of electric flux fill the space between the plates. These lines are in a state of tension and try to make themselves

as short as possible. When the plates are joined by a wire, the ends of these lines can come together in the wire. While they are thus shortening, the potential energy they represent is given up and sustains a current in the wire.

168. Units of Capacity.—The formula for capacity

$$C = \frac{Q}{V}$$

will give different units for capacity according to the units in which charge and potential (or P.D.) are measured.

In the electrostatic system a charged body has unit capacity if its potential is one E.S.U. of potential when it is given a charge of one E.S.U.

This unit is called the centimetre.

In the practical system, charge is measured in coulombs and potential in volts. The practical unit of capacity derived on this system is called the Farad (symbol F).

Definition.—A charged body has a capacity of one farad if it is raised to a potential of one volt when it is given a charge of one coulomb.

A condenser thus has a capacity of one farad if the P.D. between its plates is one volt when the charge on either plate is one coulomb.

A farad is, for convenience, subdivided into the following smaller units :—

$$\begin{aligned} 1 \text{ farad} &= 10^3 \text{ (a thousand) "millifarads" (mF).} \\ &= 10^6 \text{ (a million) "microfarads" (}\mu\text{F).} \\ &= 10^9 \text{ "millimicrofarads" (m}\mu\text{F).} \\ &= 10^{12} \text{ "micromicrofarads" (}\mu\mu\text{F).} \\ &= 9 \times 10^8 \text{ (nine hundred million) jars.} \\ &= 9 \times 10^{11} \text{ absolute units or cms.} \end{aligned}$$

$$\text{For 1 cm.} = \frac{1 \text{ E.S.U. of charge.}}{1 \text{ E.S.U. of P.D.}}$$

$$= \frac{1}{3 \times 10^9} \frac{\text{coulomb}}{\text{volts}} \quad (\text{see paras. 95 and 102})$$

$$= \frac{1}{9 \times 10^{11}} \text{ farad.}$$

It will be seen that 1 jar = 1,000 cms.

Also 1 μF = 900 jars.

$$1 \text{ jar} = \frac{1}{9 \times 10^8} \text{ F.}$$

The jar is the Service unit, and is very useful when dealing with the small capacities met with in ordinary wireless practice.

Example 18.

Find the quantity of electricity in a condenser of 500 jars capacity when charged to a P.D. of 10,000 volts.

$$Q = CV, \text{ where } \left. \begin{array}{l} Q \text{ is in coulombs} \\ C \text{ in farads} \\ V \text{ in volts} \end{array} \right\}$$

$$Q = \frac{500 \times 10,000}{9 \times 10^9} = \frac{500}{9 \times 10^4} \text{ coulomb}$$

$$= \frac{5}{900} \text{ coulomb} = \frac{1}{180} \text{ coulomb.}$$

The same quantity introduced into a condenser of 1,000 jars capacity (twice as much) would create a P.D. of only half the amount, *viz.* 5,000 volts.

169. Specific Inductive Capacity.—It was seen in para. 97 that the field strength due to a charge depended on the material surrounding the charge, and this was expressed in the formulæ derived there by the appearance of the factor K , called the specific inductive capacity (S.I.C.) of the material.

In a material whose S.I.C. is K , the field strength at any point is $\frac{1}{K}$ times the field strength that would be produced *in vacuo* by the same arrangement of charges.

The potential difference between any two points in the field is thus reduced in the same ratio, *i.e.*, it is $\frac{1}{K}$ times what it would be *in vacuo* for the same charges.

It follows that the capacity of the arrangement of charges, which is equal to charge divided by P.D., is K times as great in a material whose S.I.C. is K as it would be *in vacuo*.

The S.I.C. of an insulator may thus be defined as the ratio of the capacity of a condenser with the insulator as dielectric, to the capacity of the same condenser with a vacuum as dielectric. In fact, the S.I.C. of an insulator is found in practice by measurements of capacity.

For the same P.D. between the condenser plates in the two cases the charges on the plates would be K times as great with the insulator as dielectric as with a vacuum as dielectric.

K is equal to unity for a vacuum. In all other materials it is greater than unity. It does not appear to vary appreciably with temperature at ordinary temperatures. For dry air at 0°C. and 760 mm. pressure it is 1.00059. It is greater in moist air, but, generally speaking, it may be taken as unity in air to a good approximation.

Approximate S.I.C. values for some dielectrics at ordinary temperatures are as follows :—

Solids :—

Ebonite = 2—3·2 (fairly constant).

Vulcanite fibre = 2·5.

Glass = 4—10. (Very variable. Pebble glass 5 (about); plate glass 7—8.)

Shellac = 2·75—3·73.

Dry paper = 1·5.

Mica = 5.

Ice = 71 (at $-7\cdot5^{\circ}\text{C.}$).

Indiarubber = 2·12—2·34.

Porcelain 4·4—6·8.

Liquids :—

Distilled water $K = 80$, but its efficiency is very low.

Paraffin = 2—2·3 (variable).

Finest vaseline oil = 2 (constant).

Service insulating oil = 2·217.

Petroleum and turpentine = 2·2.

170. Capacity of a Parallel Plate Condenser.—The calculation of the P.D. between two parallel plates has already been given (para. 103), but is recapitulated here for convenience.

In a condenser consisting of two parallel plates the charge on either plate may be considered as being uniformly distributed over the area of the plates. The lines of force are therefore parallel and the flux density will be uniform.

Let V be the P.D. between the plates, Q the charge on either plate, d cms. the distance between the plates, and A sq. cms. the area of the plates. Q and V are supposed to be measured in electrostatic units.

The flux density D is $\frac{Q}{A}$ lines per sq. cm. The relationship between flux density D and field strength X is given by the formula of para. 100, *i.e.*, in the case of a dielectric whose S.I.C. is K ,

$$X = \frac{4\pi D}{K}.$$

The P.D. in electrostatic units between the plates is given by the field strength multiplied by the distance between them (para. 103).

$$\begin{aligned}\text{So that } V \text{ (in E.S.U.)} &= d \times \frac{4\pi D}{K} \\ &= \frac{4\pi Qd}{AK}.\end{aligned}$$

But if C is the capacity in E.S. Units or centimetres,

$$V = \frac{Q}{C}$$

$$\therefore C = \frac{AK}{4\pi d} \text{ centimetres.}$$

$$\begin{aligned} \text{Hence, in practical units, } C &= \frac{AK}{4\pi d \times 9 \times 10^{11}} \text{ farads.} \\ &= \frac{AK}{4,000 \pi d} \text{ jars.} \end{aligned}$$

If A is expressed in square inches, and d in inches,

$$\begin{aligned} C &= \frac{A \times (2.54)^2 K}{4,000 \pi d \times (2.54)} \text{ jars.} & (1 \text{ inch} = 2.54 \text{ cms.}) \\ &= \frac{2.54 AK}{4,000 \pi d} \text{ jars.} \\ &= \frac{2.54 AK}{12,570 d} \text{ jars} = \frac{AK}{5,000 d} \text{ jars approximately.} \end{aligned}$$

In practice, condensers of large capacity are built up by separating a series of thin flat conducting plates by thin layers of insulating material. Alternate plates are joined together to form one conductor; the remainder, similarly connected together, form the second plate. Such a condenser is equivalent to two large parallel sheets of conductor separated by a large thin sheet of insulating material.

If the number of sheets of dielectric under strain in such a condenser is N , the formula then becomes

$$C = \frac{AKN}{4,000 \pi d} \text{ jars,}$$

where A and d are in sq. cms. and cms. respectively.

Example 19.

Find the capacity of a condenser of 16 tinfoil plates, 15 cms. by 10 cms., separated by ebonite of S.I.C. = 2.5 and thickness 0.2 mm.

$$N = 16 - 1 = 15 \text{ dielectrics under strain.}$$

$$A = 150 \text{ sq. cms.}$$

$$d = 0.02 \text{ cm.}$$

$$\begin{aligned} C &= \frac{150 \times 15 \times 2.5}{4,000 \times 3.14 \times 0.02} \text{ jars} = \frac{150 \times 37.5}{80 \times 3.14} = \frac{562.5}{25.12} \\ &= 22.4 \text{ jars.} \end{aligned}$$

171. Energy Stored in a Condenser.—Just as when a magnetic field is created round an inductance a certain amount of energy is stored up, so when a condenser, capacity C , is charged up to a voltage V , energy is stored in the creation of the electric field

between the plates. When the condenser discharges, this energy is returned to the circuit.

If the condenser is perfectly efficient, there is no expenditure of energy, *i.e.*, all the charge put into the condenser is returned by it.

The energy stored in a magnetic field is comparable to kinetic energy, being associated with current flow, or motion of electrons; in a condenser it is comparable to potential energy, being dependent on the P.D. set up across the plates.

At any time during the charging of the condenser let the P.D. be v . A small charge dq introduced into the condenser at this voltage will mean an amount of work done $= vdq$.

The total work done in charging the condenser to its maximum voltage V is found by adding up these small amounts of work done.

Now

$$q = Cv.$$

$$\therefore dq = Cdv,$$

and so

$$vdq = Cvdv.$$

The total work done, which is the energy stored in the condenser $= \int_0^V Cvdv = \frac{1}{2} CV^2$.

If C is in farads, and V in volts, the energy is $\frac{1}{2}CV^2$ joules.

The same result can be obtained in a less general manner as follows:—

The total quantity of electricity in coulombs introduced into a condenser of C farads charged to a maximum potential of V volts is given by $Q = CV$ coulombs.

The charging current, if the condenser is charged by a steady current in t seconds, has a value of $\frac{CV}{t}$ amperes.

The average voltage during this time is $\frac{V}{2}$ volts. Therefore the average rate at which work is done is $\frac{CV^2}{2t}$ watts.

$$\begin{aligned} \text{The total work done in time } t &= \frac{CV^2}{2t} \text{ watts} \times t \text{ seconds} \\ &= \frac{1}{2}CV^2 \text{ joules.} \end{aligned}$$

This is also the amount of energy stored in the condenser when charged.

172. Power Taken in Charging a Condenser.—If a condenser is charged N times a second, a power of $\frac{1}{2}CV^2 \times N$ joules per second will be required. But joules per second = watts.

Hence the power required to charge a condenser N times per second $= \frac{1}{2}CV^2N$ watts.

173. The capacity of a condenser is a **fixed quantity**.

A condenser may be compared with an iron gas cylinder. To increase the quantity of gas pumped into it an increased pressure must be applied.

At first the gas passes easily, but the more gas is pumped in the harder it is to force in more.

At any moment the internal pressure equals the applied pressure.

***174. Charge and Discharge of a Condenser through a Resistance.**

—When a condenser is joined up to a source of steady E.M.F. and the circuit contains resistance, the condenser does not instantaneously acquire a P.D. equal to that of the source; in other words, it does not acquire its full charge immediately. The investigation of the gradual building-up of the charge to its maximum amount is **very similar to the case of para. 158**, where we considered the way in which current in an inductive circuit reaches its maximum value.

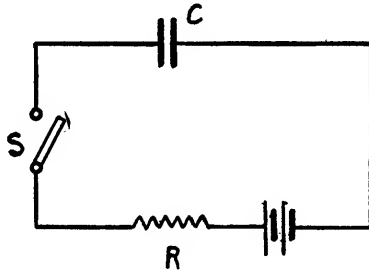


FIG. 49.

In the charging case, suppose the switch just made and a steady voltage V applied to the circuit. The charging current is given by $\frac{V}{R}$ at the beginning of the action, because the condenser has no charge and hence exerts no back E.M.F. As the condenser charges up, however, its voltage acts in opposition to the impressed E.M.F., so that at any instant it is the difference between these voltages that is effective in sending the current through the resistance,

i.e., $V - V_c = iR$, where $V_c =$ condenser voltage.

Let q be the instantaneous charge on the condenser.

Then $V_c = \frac{q}{C}$ and $i = \frac{dq}{dt}$.

Hence $V = \frac{q}{C} + R \frac{dq}{dt}$.

This may be written as

$$\frac{dq}{q - CV} = - \frac{dt}{CR}.$$

Integrating both sides,

$$\log_e (q - CV) = -\frac{t}{CR} + K,$$

where K is a constant depending on the initial conditions.

These are, in this case, that at $t = 0$, $q = 0$.

$$\therefore K = \log_e (-CV).$$

$$\therefore \log_e \frac{q - CV}{-CV} = -\frac{t}{CR}$$

$$\text{or } \frac{CV - q}{CV} = e^{-\frac{t}{CR}}$$

$$CV - q = CV e^{-\frac{t}{CR}}$$

$$q = CV \left(1 - e^{-\frac{t}{CR}}\right).$$

CV is the final value of the charge, and so, theoretically, this is only reached after an infinite time.

In any practical case this value is nearly reached in a very short time.

As in para. 158, we can define a **time constant** for this circuit.

In a time $t = CR$, the charge reaches $(1 - e^{-1})$, or 63.2 per cent. of its final value. This time, $t = CR$, is the time constant of the circuit.

$$\text{Since } i = \frac{dq}{dt}, i = \frac{V}{R} e^{-\frac{t}{CR}}.$$

So i has the value $\frac{V}{R}$ at $t = 0$, as stated before, and in a time given by the time constant CR it falls to e^{-1} , or 36.8 per cent., of its maximum value.

The charging current decreases to zero as t increases.

Graphs indicating the growth of the charge in the condenser and the decay of the charging current are exactly similar to the growth and decay curves of Fig. 44.

Case of Discharge.—If the two plates of a charged condenser are connected by a conductor of resistance R , a current starts to flow, and continues until the P.D. across the condenser is zero. The stored-up energy is expended in I^2R losses in the resistance.

The equation representing the action can be easily seen, from previous work, to be

$$R \frac{dq}{dt} + \frac{q}{C} = 0.$$

(This is simply the general equation with $V = 0$.)

The solution, assuming the initial conditions that at $t = 0$, $q = CV$, V being the voltage to which the condenser was charged, is given by

$$q = CV e^{-\frac{t}{CR}}.$$

This shows that, as might be expected, the charge on the condenser decreases continuously, and in a time $t = CR$, the **time constant** of the circuit, diminishes to e^{-1} , or 36.8 per cent. of its original value.

The discharging current is given by $i = \frac{dq}{dt}$,

$$\text{i.e., } i = -\frac{V}{R} e^{-\frac{t}{CR}}.$$

The general shape of the curves of charge and current flowing are therefore both similar to the decay curve of Fig. 44.

The negative sign in the expression for i simply means that the current is in the opposite direction to that during charging.

From the solution $q = CVe^{-\frac{t}{CR}}$, the time may be found in which the voltage of the condenser diminishes from V to, say, V_1 .

At the latter voltage

$$\begin{aligned} q &= CV_1 = CVe^{-\frac{t}{CR}} \\ \therefore \frac{V_1}{V} &= e^{-\frac{t}{CR}}, \text{ or } \frac{V}{V_1} = e^{+\frac{t}{CR}} \\ \log_e \frac{V}{V_1} &= \frac{t}{CR} \\ \text{or } t &= CR \log_e \frac{V}{V_1}. \end{aligned}$$

Thus the time required for the voltage to drop from V to V_1 is proportional to CR .

This result is useful in valve theory later on, when considering the leak away of the accumulated charge in a condenser through a grid leak.

175. Dielectric Strength.—If the electric strain in a condenser rises beyond a certain point the dielectric is punctured.

In the case of liquids or gases the wound so created heals itself (though, in oil, contamination will occur), but in solids the insulation is punctured, *i.e.*, a hole is formed which only offers the insulation of air, instead of the insulation of the solid dielectric.

The voltage corresponding to that at which the rupture of a plate 1 mm. thick takes place is called the **dielectric strength of the insulator**.

A thin sheet of dielectric is **proportionally** stronger than a thicker one of the same material.

That is, a sheet of $\frac{1}{2}$ in. thickness is not twice as strong as one of $\frac{1}{4}$ in., but something considerably less.

The reason for this appears to be that, as the thickness of the dielectric increases, it becomes more difficult to keep the field strength uniform between the plates. Thus, in parts of the field,

the field strength becomes considerably greater than that calculated on a basis of uniform field strength, and the insulation breaks down.

For this reason, large condensers are usually built up of several different parts—or sections—connected in series; this saves space.

The strength of dielectrics is compared with that of air.

A spark of 1 mm. in air at atmospheric pressure between flat metallic surfaces requires 4,300 volts. A spark of 2 mm. would only require 7,400 volts.

Two plates 1 cm. apart in air require 30,000 volts applied to spark across.

With spark balls the voltage is less, the less the diameter of the balls. Between sharp points the voltage required is much less.

The dielectric strengths of various dielectrics are given in the following table:—

Dielectric.	Dielectric Strength.	Corresponding sparking distance between plates in air at atmospheric pressure.
Crystal Glass.. ..	28,500 volts ..	9 mm.
Indiarubber	40,000 „ ..	13 mm.
Ebonite	50,000 „ ..	16 mm.
Mica	60,000 „ ..	20 mm.
(0.1 mm.) of mica ..	10,000 „ ..	3 mm.
Oil (vaseline).. ..	6,000–8,000 volts.	2.4 mm.

It is usual in building condensers to allow a factor of safety of from three to six times the dielectric strength, since the above only gives voltages at which the dielectric is bound to puncture; it may puncture at a much less voltage if any brush discharge burns the plate or sparking takes place between the contacts.

Dielectrics puncture at lower voltages if subjected to alternating E.M.Fs. as compared with steady E.M.Fs., and the higher the frequency of alternation the lower is the puncturing voltage.

176. Dielectric Efficiency.—Dielectric efficiency is the ratio of the energy output to the energy input, thereby taking account of any waste of energy that occurs during the charge and discharge of a condenser.

If a dielectric is perfect there should be no waste of energy in it.

In most condensers, however, this does not hold good. The term “Hysteresis” is used to cover all the losses, and these may be summed up as:—

(a) **Conductor losses**, or true resistance losses due to the resistances of the plates and leads.

These are easily kept low in parallel plate condensers. They are equivalent to a resistance in series with the condenser.

(b) **Chemical action.**—This may take place if damp is present. Hence condenser cases must be properly sealed.

(c) **Leakage losses.**—These may be due to faulty insulation or to corona discharge from points and edges of the condenser plates.

Faulty insulation, which might enable a considerable current to flow through the dielectric, is equivalent to a resistance in parallel with the condenser.

Leakage over the plate edges may be prevented by immersing the condenser in oil and keeping the plate edges well away from the sides of the condenser tank.

Moisture should not be allowed to collect on the surface of a solid dielectric.

(d) Most important is **dielectric absorption.**

When a condenser is charged, the initial rush of current is followed by a relatively small more gradual current, which appears to "soak in" to the dielectric. The charge on the condenser is therefore dependent on the time during which it is connected to the source of E.M.F., quite apart from considerations mentioned in the last paragraph concerned with the resistance of the charging circuit. Similarly, when such a condenser is short-circuited, the initial heavy discharge, which should leave it practically uncharged, is an incomplete one. If the short-circuit is removed and the condenser set aside and again short-circuited a few minutes later, a second discharge can be obtained. This is due to the charge which "soaks into" the dielectric during charging. If the condenser is charged and discharged periodically, this absorption causes heat to be generated in the dielectric and can be looked upon as a resistance in series with the condenser. It can be shown that, the higher the frequency, the less is the loss due to this cause, due to the fact that there is less time available for absorption to take place.

This last cause of loss of efficiency, dielectric absorption, is also responsible for variations in the value of K (the S.I.C.). At high frequencies, when the condenser is charged and discharged many times per second, the inability of the condenser to absorb its full charge each time can be expressed by saying that K is reduced; the apparent capacity being considerably less at high frequencies. The value of K is as much as 10 per cent. more for direct current than for "low frequency" alternating current, and may with certain dielectrics be twice as much for direct current as for alternating current of high radio frequency.

The loss due to dielectric absorption only occurs in solid or liquid dielectrics. In air or gases it is negligible.

Reckoning the efficiency of an air condenser as 100 per cent., other dielectrics have the following efficiencies:—

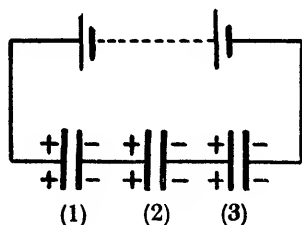
Ebonite: 70 per cent. about (is more efficient for thick than thin plates; very much less efficient if ebonite is very thin).

Glass: 60 per cent. (best plate glass may be more).

Mica : very variable. Good quality mica is now being made, having an efficiency as high as 90 per cent. Ordinary mica of older make may be as low as 40 per cent. Good mica is very expensive, and has to be made up in sheets clamped together to get efficiency as well as sufficient dielectric strength.

Oil : nearly 100 per cent., but only if it contains no moisture. If a receptacle containing oil is left unsealed, the oil will readily absorb moisture from the atmosphere, and its efficiency will be impaired.

This moisture can be evaporated by heating the oil—say, by placing electric radiators on either side of the receptacle. It is also very advisable to strain oil through a piece of chamois leather—placed fluffy side up—in order to remove any impurities from it.



Condensers in series.

FIG. 50.

177. Condensers in Series.—Fig. 50 shows three condensers connected in series, and joined up to a battery. The left-hand plate of No. 1 will be positive, being connected to the positive pole of the battery. Let the charge on it be $+Q$ units. This charge, acting through the dielectric, will attract the free electrons in the conductors comprising the right plate of No. 1, the connecting wire, and the left plate of No. 2, and these will constitute a negative charge on the right-hand plate of No. 1 equal to $-Q$ units, the charges on opposite plates being equal and opposite.

Since the right-hand plate of No. 1, the connecting wire, and the left-hand plate of No. 2, are as a whole uncharged, the left-hand plate of No. 2 is left with a positive charge of $+Q$ units; and so on for any number of condensers in series, the right-hand plate of the last one, in this case No. 3, having a negative charge of $-Q$ units. Thus the distribution of charges is as shown in the figure.

The voltage drop across the three condensers is the sum of the voltages across the individual condensers.

If the total voltage of the battery is V , and these individual voltages are V_1 , V_2 , V_3 , then $V = V_1 + V_2 + V_3$.

Now, for each condenser in turn, their capacities being C_1, C_2, C_3 , the equations

$$Q = C_1 V_1, Q = C_2 V_2, Q = C_3 V_3, \text{ hold.}$$

$$V = V_1 + V_2 + V_3 = \frac{Q}{C_1} + \frac{Q}{C_2} + \frac{Q}{C_3} = Q \left(\frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} \right)$$

$$\text{or } Q = V \left(\frac{1}{\frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}} \right).$$

If the **equivalent capacity** of the three is written as C , *i.e.*, the capacity of a single condenser such that for the same voltage the same charge is acquired, then $Q = CV$.

So that

$$C = \frac{1}{\frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}}$$

$$\text{or } \frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}.$$

Hence, for condensers in series, the reciprocal of the equivalent capacity is the sum of the reciprocals of the individual capacities.

Also the voltages across the individual condensers are in inverse ratio to their capacities.

It follows from the above result that the equivalent capacity of a number of condensers in series is always less than the capacity of the smallest individual condenser.

For let C_1 , say, be the smallest condenser.

$$\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} + \dots$$

Therefore $\frac{1}{C}$ is greater than $\frac{1}{C_1}$, and so C_1 is greater than C .

Example 20.

Two condensers of capacities 2 jars and 3 jars are in series and a voltage of 900 volts is applied. Find the equivalent capacity, the charge in each condenser, and the division of the voltage between them.

$$(1) C = \frac{1}{\frac{1}{C_1} + \frac{1}{C_2}} = \frac{1}{\frac{1}{2} + \frac{1}{3}} = \frac{6}{5} = 1.2 \text{ jars.}$$

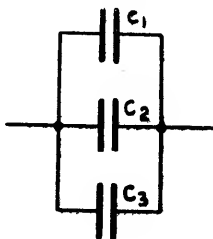
$$(2) Q = CV = 1.2 \text{ jars} \times 900 \text{ volts} \\ = \frac{1.2 \times 900}{9 \times 10^6} \text{ coulombs} = 1.2 \text{ micro-coulombs.}$$

$$(3) V_1 = \frac{Q}{C_1} = \frac{1.2 \times 10^{-6}}{\frac{2}{9 \times 10^8}} = \frac{1.2 \times 9 \times 100}{2} = 540 \text{ volts.}$$

$$V_2 = \frac{Q}{C_2} = \frac{1.2 \times 10^{-6}}{\frac{3}{9 \times 10^8}} = 360 \text{ volts.}$$

It may be noted that, with only two condensers in series, C is given by the simple formula

$$C = \frac{C_1 C_2}{C_1 + C_2}.$$



Condensers in parallel.

FIG. 51.

178. Condensers in Parallel.—Fig. 51 shows three condensers in parallel, all the positive plates being joined together, and similarly all the negative plates.

If a common P.D., V , is applied to them, and the individual capacities are C_1 , C_2 , C_3 , the charges introduced into each are C_1V , C_2V , C_3V .

Thus $Q_1 = C_1 V$, &c.

The total charge on the condensers is

$$Q_1 + Q_2 + Q_3 = V (C_1 + C_2 + C_3).$$

Let C be the equivalent capacity defined as before, the capacity of a single condenser such that for the same voltage the same total charge is required.

$$\text{Then} \quad Q = Q_1 + Q_2 + Q_3 = CV.$$

$$\therefore C = C_1 + C_2 + C_3.$$

Hence with condensers connected in parallel the equivalent capacity is the sum of the individual capacities.

Condensers may be joined both in series to give adequate dielectric strength, and in parallel to give increased capacity.

Example 21.

Given a number of condensers, each capable of standing 1,000 volts and each with a capacity of 1 jar. Find an arrangement

suitable for giving a condenser of capacity 2 jars, across which 2,000 volts may be applied.

Two condensers in series will stand the voltage of 2,000 volts, but their capacity will be only $\frac{1}{2}$ jar.

Four such groups in parallel will give a capacity of 2 jars.

Therefore 8 condensers are needed, 2 in series and 4 groups in parallel.

179. Condensers used in Wireless Telegraphy Circuits.—The condensers used in wireless telegraphy fall under three headings:—

- (a) **Natural capacity circuits** in which the capacity exists between the wires of the circuit itself, or between the circuit and earth.
- (b) **Artificial capacities**, in which a built-up condenser, generally of the parallel plate type, is used.
- (c) Combinations of (a) and (b).

(a) **Natural Capacity Circuits.**—Every ordinary electric circuit possesses capacity. In an electric cable the conductor forms one plate of the condenser, the insulation is the dielectric, and the outer lead casing, or the earth, the other plate.

In general, any two wires which are adjacent to each other have capacity to each other.

Thus there is always a certain amount of capacity between the turns of a coil of wire. The combined effect of all the small capacities between turns is spoken of as the “**self-capacity**” of the coil.

Again, a suspended wire has a capacity to earth, the air being dielectric.

The capacity of the wire is therefore very complex, being the sum of the capacities of each portion of it to the earthed points in the neighbourhood as indicated in Fig. 52 (a).

This capacity may be increased by arranging wires to form a roof, as in Fig. 52 (b).

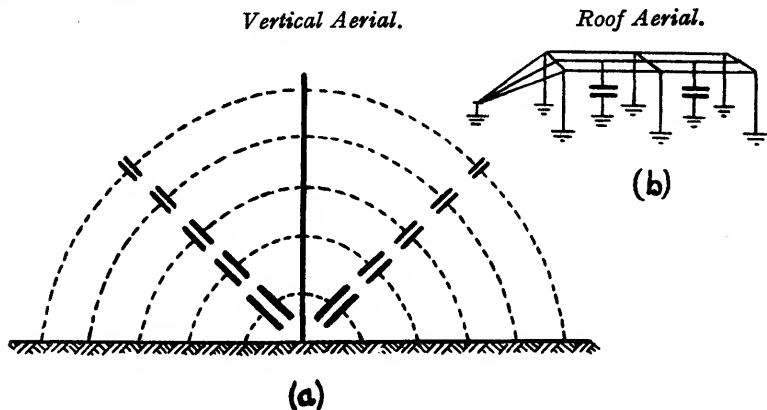


FIG. 52.

The total capacity is called the "**natural capacity**" of the wire to earth, and is denoted in the Service by the letter " σ " (sigma).

(b) **Artificial Capacities.**—These are used very extensively in wireless telegraphy, in transmitting, receiving, wavemeter circuits, etc. They will be dealt with in due course.

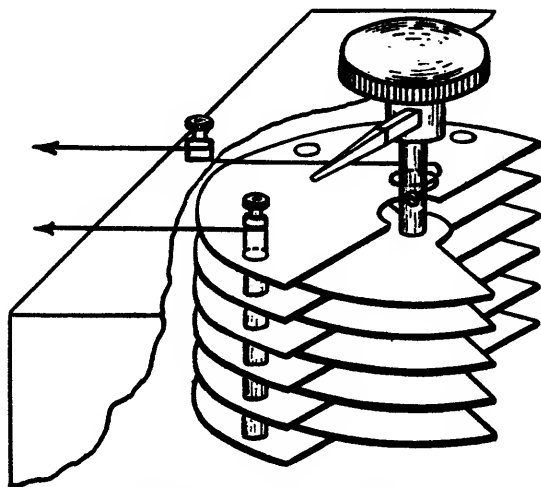
(c) In every circuit containing an artificial condenser there must also always be a natural capacity to earth or between leads. This capacity is in parallel with the artificial capacity and must therefore be added to it.

If the artificial condenser is large the capacity of the leads may be neglected, whereas if the condenser is small the latter may be a considerable factor.

If an artificial condenser be inserted in an earthed wire having a natural capacity to earth, it is in series with that capacity and reduces the total capacity value.

180. Receiving Condensers.—Receiving condensers are designed to have a maximum dielectric efficiency, and to occupy a minimum of space. Generally air dielectric is used because of its efficiency. Mica may be used if a large capacity is required.

Such condensers may have a **fixed** or a **variable** value of capacity.



Simple Receiving Condenser.

FIG. 53.

In a **fixed** condenser the plates, or sets of plates, are rigidly mounted with respect to each other, so that the capacity value does not vary.

In a **variable** condenser there are two sets of plates, one set being fixed, mounted one above the other at equal fixed intervals, and attached to a rigid support. The other set is mounted one above

the other at equal intervals on a rotating spindle, and arranged to be rotated into the spaces between the fixed plates so as to overlap them more or less.

The greater the overlap the greater the capacity.

The commonest type of such a variable condenser has plates cut in the form of semi-circular segments, as indicated in Fig. 53.

The amount of overlap of area is directly proportional to the angle of overlap in this case, and can be measured by a pointer travelling over a scale of degrees and attached to the spindle of the rotating plates.

When the condenser is in its minimum position and the plates not overlapping at all the capacity does not fall quite to zero, owing to the fact that a small capacity exists between the edges of the two sets of plates. This is known as edge effect. A curve, known as a calibration curve, may be drawn for such a condenser, showing the capacity graphed against angle of overlap. It is a straight line curve, except for small angles, where, on account of edge effect, it flattens out somewhat.

A typical curve of this type is shown below (Fig. 54).

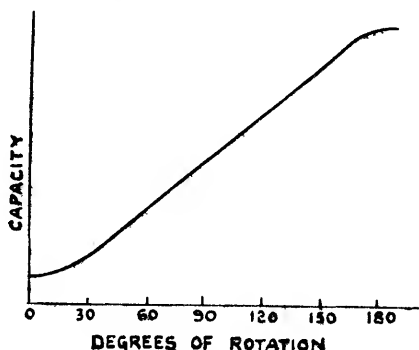


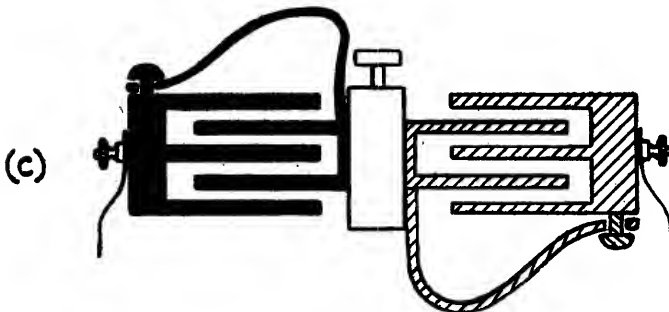
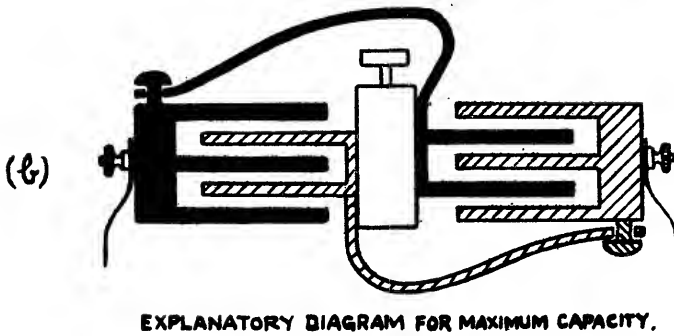
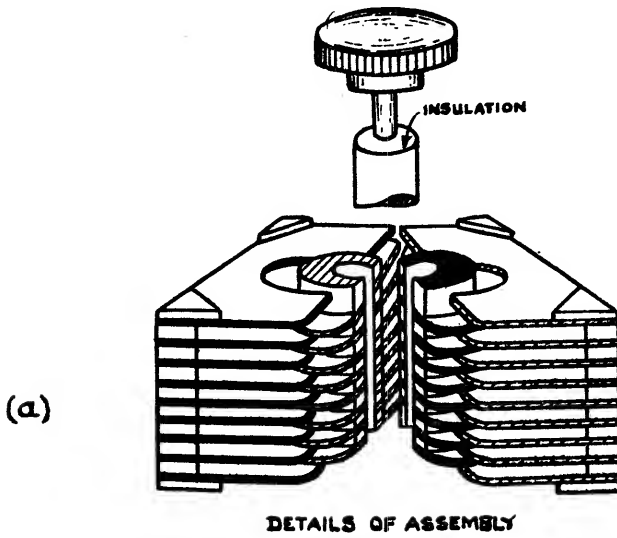
FIG. 54.

If it is desired to increase the capacity of a condenser of this type it may be immersed in oil, which has a S.I.C. higher than that of air.

There are other types of variable condenser besides that described above. It is sometimes found convenient to have a condenser whose capacity varies as the square of the angle of overlap, and such a condenser is designed by constructing the plates in a different shape from a semicircle (*cf.* Chapter XX).

181. Die-Cast Condenser.—The type of variable capacity condenser used in the Navy is more economical in space than that illustrated in Fig. 53. It is known as the die-cast condenser.

In this type, illustrated in Fig. 55, there are two sets of moving and two sets of fixed plates, one set of moving plates being connected



Die-cast Condenser.

to one set of fixed plates. Thus, in the position shown in the bottom figure of Fig. 55, the capacity is zero, since the rotating plates are in this case entirely contained between the fixed plates to which they are made common as regards potential.

A rotation through 180° gives the position shown diagrammatically in Fig. 55 (b), in which the capacity is obviously a maximum. The capacity can be made variable to any degree required up to its maximum value by a proportionate rotation.

The great advantage of this form of construction is that, with a condenser of the same superficial area and the same depth, a maximum capacity can be obtained of **double** the value possible with only one set of fixed and one of moving plates.

In addition, the depth of the condenser and hence its capacity value can be increased by adding on further units.

182. Transmitting Condensers.—The general considerations that determine the design of transmitting condensers as compared with receiving condensers may be summed up as follows :—

- (a) They are much larger in dimensions.
- (b) Variability in value is secured generally by methods of joining up separate fixed value condenser units in series and parallel, instead of having the condensers themselves continuously variable by rotation.

(a) The increase in the dimensions is accounted for, firstly, by the fact that it is usual to employ larger capacities in transmitting circuits, and secondly, because the question of dielectric strength prohibits the reduction of the distance between the plates to less than a certain amount. To achieve a large capacity value, therefore, the area of the condenser plates must be larger and dielectrics of high S.I.C. must be employed.

(b) An example of such an arrangement is given below. Condensers joined in series can stand higher voltages without danger of rupture of the dielectric, and condensers joined in parallel give an increase in total capacity.

Transmitting Condensers for Spark sets have dielectrics of ebonite, mica or glass, and are generally arranged with "sections" in series, to give adequate dielectric strength. If a variation of capacity is necessary, the sections are arranged in groups or "elements," whose terminals are brought out of the tank, so that by suitable switches these groups—termed "Elements"—may be arranged in various combinations.

The plates of ebonite or glass condensers are generally immersed in oil, to prevent brushing.

Fig. 56 illustrates the construction of such a condenser.

It can be seen that each section comprises, in the case illustrated, six active plates and five active dielectrics under strain.

Between sections are placed ebonite separators, generally of thicker material.

The whole element is clamped up between metal plates, which are earthed.

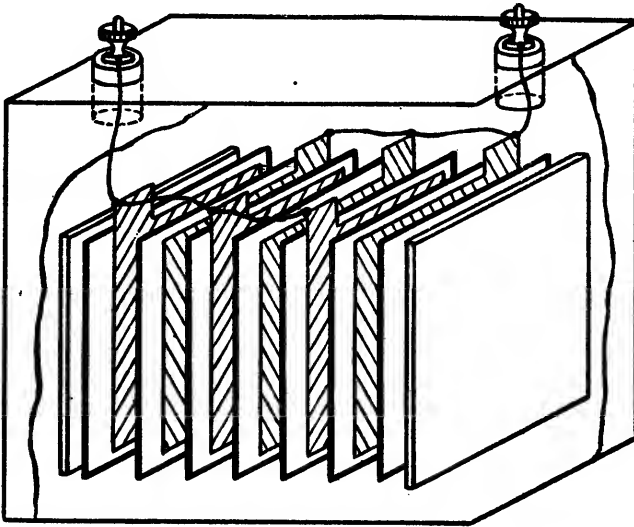


FIG. 56.

Next the active plate at each end is placed a sheet of dielectric, and next it a conducting plate in electrical connection with the earthed clamping plate, so as to keep the capacity to earth constant.

The following is an arrangement used in the Navy for securing a certain amount of variability of capacity value, as referred to in (b) above. The separate elements are two condensers, each having

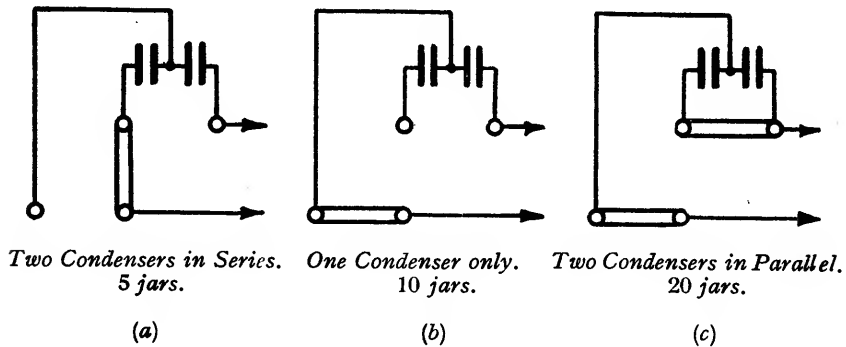


FIG. 57.

a capacity of 10 jars. Leads are taken from these condensers to four terminals, as shown in Fig. 57, and from two of these terminals connections are made to the remainder of the circuit.

By different arrangements of two movable links, which can be inserted between any pair of the terminals, different capacity values can be obtained.

Thus, in Fig. 57 (a), the two condensers are in series, and the total capacity is $\frac{10}{2} = 5$ jars.

In Fig. 57 (b), one condenser only is in circuit, and the capacity is 10 jars. In Fig. 57 (c), the two condensers are in parallel and the total capacity is 20 jars.

Example 22.

You have the following materials available: ebonite $\frac{1}{8}$ in. thick (S.I.C. = 2.5) and dielectric strength sufficient to stand 5,000 volts, and tinfoil sheets 12 in. \times 12 in.

You wish to construct a condenser (a) to give a total capacity of 234 jars with its elements arranged in parallel, and (b) to stand 15,000 volts with its elements arranged in series.

(i) How many plates would you require for (a) ?

(ii) What would be the capacity of the condenser in (b) ?

For (b) we shall require three elements in series to stand 15,000 volts.

Hence capacity of each element, to give 234 jars when the three are joined in parallel will be :—

$$\frac{234}{3} = 78 \text{ jars.}$$

$$C = \frac{AKN}{5,000d} \quad \text{Hence } N \text{ (the number of dielectrics required)}$$

will be equal to $\frac{C \times 5,000 \times d}{AK}$

$$= \frac{78 \times 5,000}{12 \times 12 \times 2.5} \times \frac{1}{32} = \frac{390,000}{11,520} = 33.8 = 34 \text{ dielectrics.}$$

Hence the number of plates required for each element = 34 + 1 = 35.

Total number of plates = 3 \times 35 = 105 plates.

(ii) The capacity of three elements of 78 jars joined in series will be $\frac{78}{3} = 26$ jars.

183. The units of the quantities dealt with in Chapters II and III are summarised in the following table, and the conversion factor from one system to another is given in each case :—

TABLE 7.

Quantity.	Symbol.	Practical Unit.	1 E.M.U.	1 E.S.U.	1 E.M.U. in E.S.U.
			In Practical Units.		
Energy	W	1 joule	1 erg = $\frac{1}{10^7}$ joule ..		1
Power	P	1 watt	1 erg per sec. = $\frac{1}{10^7}$ watt		1
Quantity of elec- tricity	Q	1 coulomb ..	10	$\frac{1}{3 \times 10^9}$	3×10^{10}
Potential differ- ence or E.M.F.	V or E	1 volt	$\frac{1}{10^8}$	3×10^8	$\frac{1}{3 \times 10^{10}}$
Electric field strength	X	1 volt per cm. ..	$\frac{1}{10^8}$	3×10^8	$\frac{1}{3 \times 10^{10}}$
Electric flux den- sity	D	1 E.S.U. ..	3×10^{10}	1	3×10^{10}
Electric flux ..	Ψ	1 E.S.U. ..	3×10^{10}	1	3×10^{10}
Magnetic pole strength	m	1 E.M.U. ..	1	3×10^{10}	$\frac{1}{3 \times 10^{10}}$
Magnetic field strength	H	1 gauss (E.M.U.)	1	3×10^{10}	$\frac{1}{3 \times 10^{10}}$
Magnetic flux density	B	1 line per sq. cm. (E.M.U.)	1	3×10^{10}	$\frac{1}{3 \times 10^{10}}$
Magnetic flux ..	Φ	1 line (E.M.U.) or maxwell	1	3×10^{10}	$\frac{1}{3 \times 10^{10}}$
Magnetic reluct- ance	S	1 oersted (E.M.U.)	1	$\frac{1}{9 \times 10^{10}}$	9×10^{10}
Magneto Motive Force	G	1 gilbert (E.M.U.)	1	$\frac{1}{3 \times 10^{10}}$	3×10^{10}
Electric Current ..	I	1 ampere ..	10	$\frac{1}{3 \times 10^9}$	3×10^{10}
Resistance ..	R	1 ohm	$\frac{1}{10^9}$	9×10^{11}	$\frac{1}{9 \times 10^{10}}$
Capacity	C	1 farad = 9×10^9 jars	10^9	$\frac{1}{9 \times 10^{11}}$	9×10^{10}
Inductance ..	L	1 henry	$\frac{1}{10^9}$	9×10^{11}	$\frac{1}{9 \times 10^{10}}$

D.C. MEASURING INSTRUMENTS.

184. In electrical circuits the quantities that commonly require measurement are the currents flowing in, and the potential differences across, various parts of a circuit. The instruments used for these purposes are called ammeters and voltmeters respectively.

If it is also desirable to measure power directly, an instrument called a wattmeter, which combines the functions of voltmeter and ammeter, may be used. Wattmeters are not at present used with Naval wireless equipment.

The principles on which both ammeters and voltmeters operate are the same, and the differences that occur are due only to their different functions. The ammeter, being required to measure current, must be inserted in series in a circuit so that the current to be measured will flow through it. It follows also that it must be of low resistance compared with the rest of the circuit or the extra resistance will alter appreciably the current flowing. This does not apply if the ammeter is permanently in the circuit, but the resistance should still be kept as low as possible to cut down the power loss, (I^2R), in the instrument.

The voltmeter measures P.D. and so must be connected in parallel between the two points in a circuit across which the P.D. is to be measured. It must therefore have a high resistance compared with that of the circuit. This keeps down its power loss (V^2/R) and, if it is not permanently in circuit, prevents its insertion from making an appreciable change in P.D.

An ammeter is thus converted into a voltmeter simply by putting a large resistance in series with the actual measuring part of the instrument.

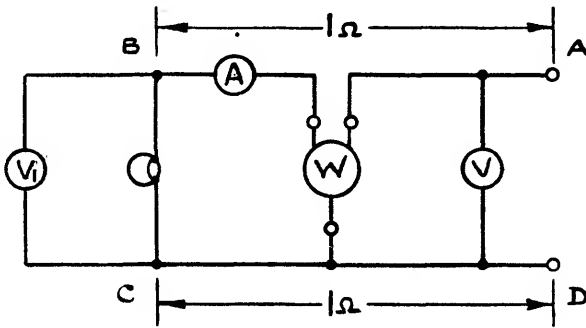


FIG. 58.

Example 23.

Between the two mains AB and CD a lamp of resistance 108 ohms is joined.

The resistance of line A to B is 1 ohm, and the resistance C to D is 1 ohm. The P.D. between A and D is 110 volts.

Then the voltmeter V will read 110 volts.

The current flowing will be $I = \frac{E}{R} = \frac{110}{108 + 2} = 1$ ampere.

This will be indicated by the ammeter A (which could equally well have been joined in the circuit between C and D).

As the voltage drop A to B and C to D in each case $= I \times R = 1 \times 1 = 1$ volt, the P.D. between B and C $= 110 - 2 = 108$ volts; this will be indicated by the voltmeter V_1 .

The wattmeter W would read $110 \times 1 = 110$ watts.

185. According to the size of the currents they are designed to measure, current measuring instruments are described as ammeters, milliammeters, microammeters and galvanometers.

Their range of measurement may be greatly extended by the use of shunts, *i.e.*, by adding a resistance in parallel with the instrument (para. 67).

Shunts are usually made of a material which has a very small temperature coefficient of resistance, *e.g.*, an alloy like manganin. This lessens inaccuracies due to temperature changes, which, by altering its resistance, will alter the readings of the instrument. This also applies to the series resistance in voltmeters.

The ammeter itself then carries only a fraction of the total current, depending on its resistance compared with that of the shunt.

The ammeter is graduated to read the total current flowing in the circuit, and so the scale reading will have a different value for every shunt that can be inserted.

Ammeter and voltmeter terminals are always marked + and -, and must be correctly joined in a circuit, *i.e.*, positive terminal to positive lead.

Example 24.

An ammeter of resistance 2 ohms can take a maximum current of 0.5 amp. How can it be adapted to operate as :—

- (a) An ammeter reading to 5 amps. ?
- (b) A voltmeter reading to 100 volts ?

(a) It must be shunted by a resistance which takes 4.5 amps. when the ammeter is taking 0.5 amp. The value of the resistance

is therefore $\frac{0.5}{4.5} = \frac{1}{9}$ of the ammeter resistance, *i.e.*, $\frac{2}{9}$ ohm.

(b) The P.D. across the ammeter when taking 0.5 amp. is $2 \times 0.5 = 1$ volt. A resistance must therefore be put in series which has a P.D. of 99 volts across it when 0.5 amp. is flowing in it, *i.e.*,

$$R = \frac{99}{0.5} = 198 \text{ ohms.}$$

186. Types of Instrument.—These may be classified as :—

- (a) Hot wire.
- (b) Moving coil.
- (c) Moving iron.
- (d) Electrostatic.

With the exception of (*d*), which is only used as a voltmeter, these may all be adapted for use either as ammeters or voltmeters.

Hot-Wire Instrument.—This is shown diagrammatically in Fig. 59. A is a taut wire, through which flows the current to be measured. The wire is of some material of high melting-point and high resistivity, usually platinum-silver. One end of a phosphor-bronze wire F is attached to some point in A near the centre, and its other end to an insulated block E. A silk fibre is attached to F as shown and passes round a pulley P on the spindle carrying the pointer. The fibre is held taut by the pull of the light spring R. When a current flows through A it is heated and expands, allowing the wire F to sag. As a result, the silk fibre in being pulled taut by the spring R rotates the pulley P, and the pointer moves over the scale.

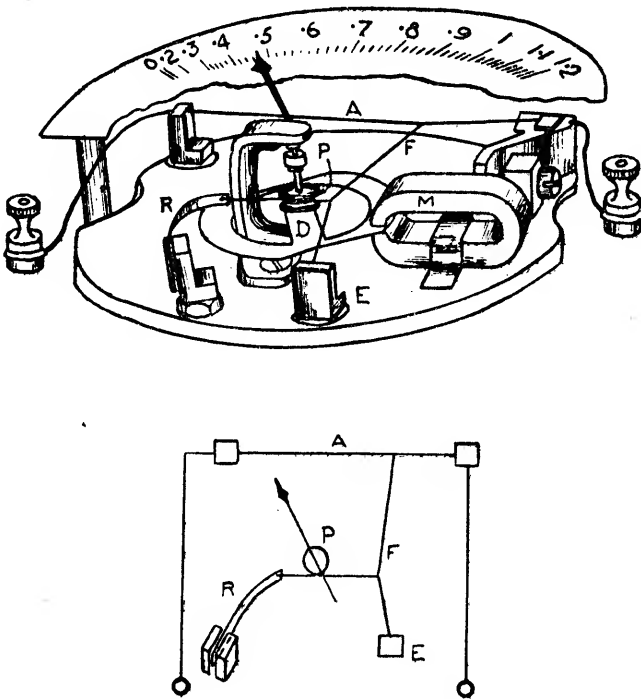


FIG. 59.

The aluminium disc D, attached to the spindle of the pointer and placed between the poles of the permanent magnet M, acts as a damping device. Movement of the pointer causes movement of the disc, thus producing eddy currents, which oppose the motion causing them and prevent oscillations of the pointer before it takes up its final position.

Since the heating effect, and therefore the expansion of A, is proportional to the square of the current, the scale is not uniform, being crowded at the lower end and open at the upper end.

The principal defect of a hot wire instrument is wandering of the zero, due to changes in the surrounding temperature. These changes produce different expansions in the wire itself and the metal block on which it is mounted, and so the wire may sag and a pointer deflection be produced even if no current is flowing.

It is possible to compensate for this to some extent if the base on which the hot wire is mounted is arranged to have the same coefficient of expansion as the wire itself.

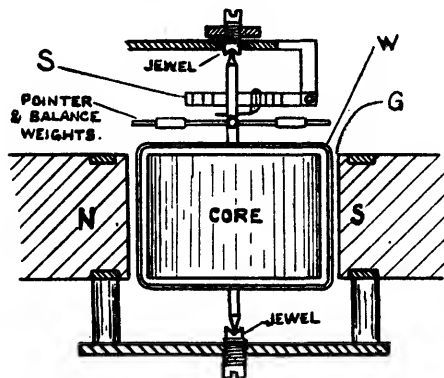
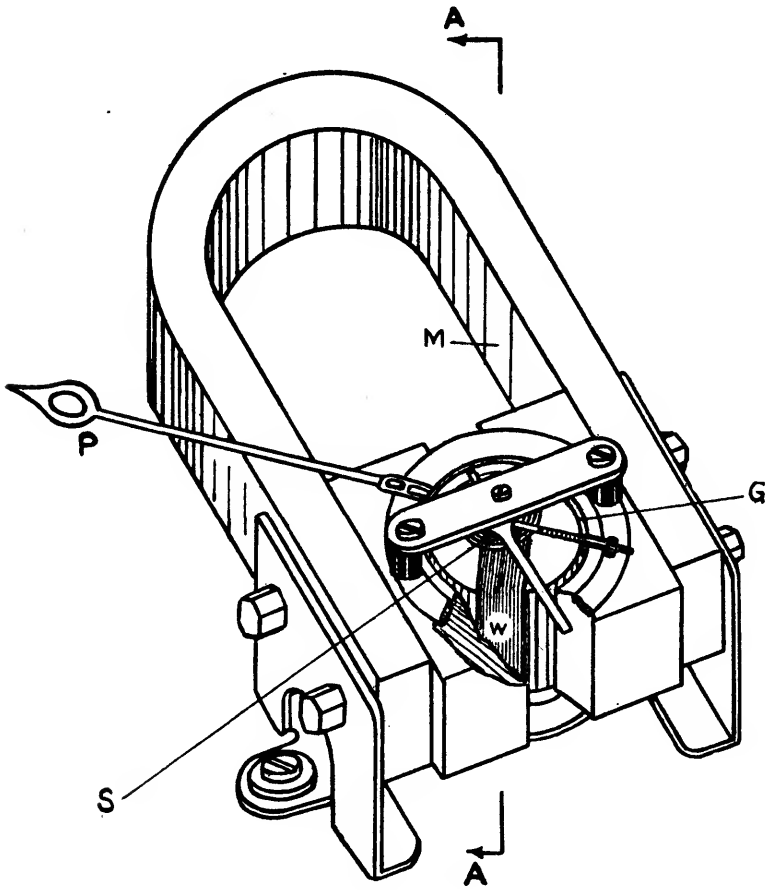
Hot-wire instruments are also sluggish in action owing to a time lag between the current flowing and the expansion it produces.

187. Moving Coil Instrument.—If a coil carrying a current is placed in a magnetic field a mechanical force will act on the coil, (para. 129). A typical instrument making use of this principle is shown in Fig. 60. The coil W is wound on a former of copper or aluminium, inside which is a fixed iron cylindrical core. The former is mounted on pivots so that it can rotate between the poles of a permanent magnet M. This leaves only a narrow air gap G between the poles and the former, so that the field of the magnet is radial and uniform for any position of the coil. Current is led into and away from the coil by one or two phosphor bronze springs S.

Equal forces in opposite directions act on the two halves of the winding, as can be seen by applying Fleming's Left Hand Rule. The coil thus tends to turn about its axis and, as the magnetic field is the same for all coil positions, the coil is subject to a constant torque or turning moment proportional to the current flowing in it. In the zero position there is no twist of the controlling springs. When the coil starts to rotate the springs become twisted and set up a torsional torque opposing the motion and proportional to the angle through which the coil has turned. The coil therefore takes up an equilibrium position in which the resisting torque due to the springs just balances the electrical torque due to the current. Since these torques are directly as the angle of rotation and the current respectively, it follows that the deflection of a pointer attached to the coil is directly proportional to the current and so the scale is uniform.

When in motion the metal former and the winding will have induced E.M.Fs. set up in them, causing currents tending to oppose the motion (Lenz's Law) and so acting as a damping device. The coil and pointer also have very small inertia. The motion of the pointer is, in consequence, "dead-beat."

As the magnetic field in the air gap is strong, the instrument is very little affected by stray magnetic fields. The permanent magnets are specially "aged" so that their strength remains



Sectional view at A.A.

FIG. 60.

constant for a long period. In consequence, moving coil instruments are the most accurate and satisfactory type for all D.C. measurements.

188. Moving-Iron Instruments.—Though not as accurate as the best moving-coil instruments, this type is usually simpler and of more robust construction. Two kinds are in common use, the attracted iron type and the repulsion type.

The operation of these instruments depends on the magnetisation of soft iron by a current and the consequent mechanical forces of attraction or repulsion.

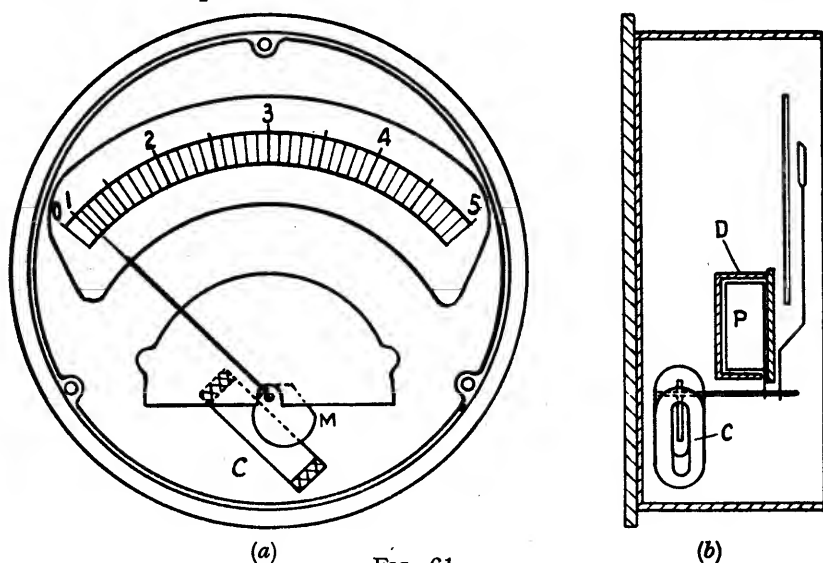


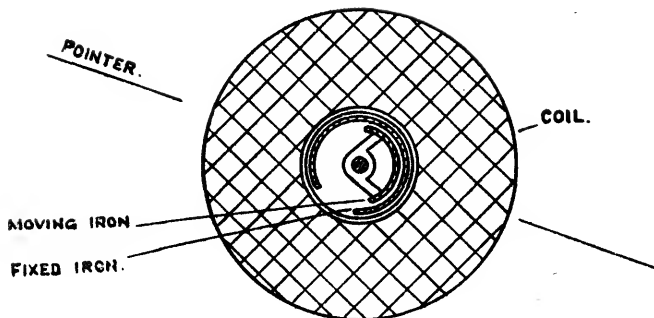
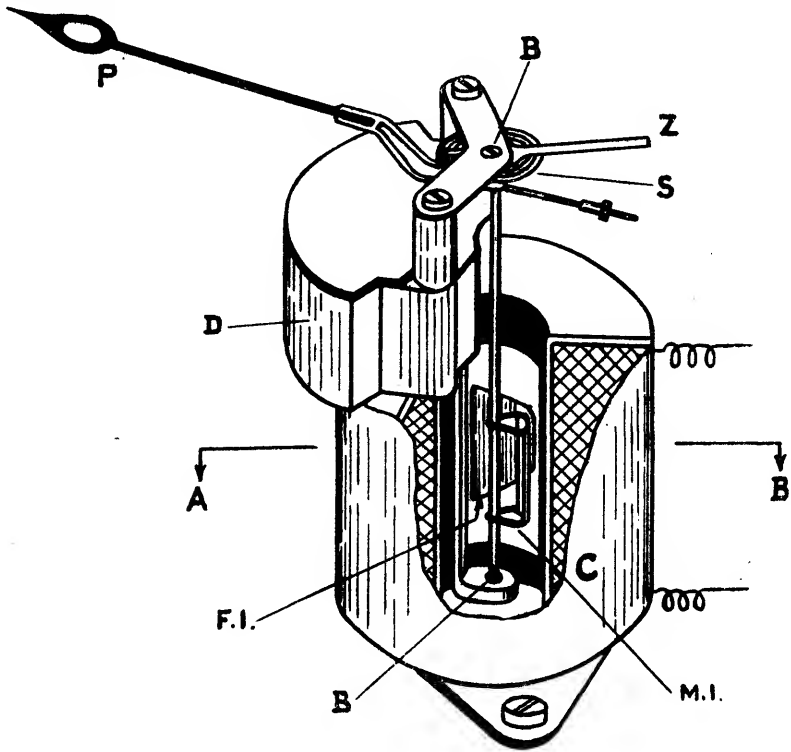
FIG. 61.

Fig. 61 (a) illustrates the **attracted iron type**. C is a fixed coil carrying the current to be measured and M is a disc of soft iron, eccentrically pivoted and carrying a pointer. When current flows in the coil, M is magnetised. Not being pivoted at its C.G., it is attracted into the interior of the coil, thus causing the pointer to move. The controlling torque may be provided by a spring or a weight.

An air damping device to render the movement "dead-beat" is shown in Fig. 61 (b). The block D is hollowed out so that a piston P, mounted on the axis of the moving iron, can travel into it. The compression by the piston of the air in the hollow provides sufficient resistance to the motion to prevent oscillation of the moving parts.

Provided the disc is not near saturation, its magnetisation is roughly proportional to the current flowing in the coil. The force of attraction, being proportional to the product of disc magnetisation and coil current, is therefore proportional to the square of the

current to be measured. As in the case of the hot-wire ammeter, the scale will not be uniform, being crowded at the lower end and more open at higher current values.



Sectional plan at A.B.

FIG. 62.

The **repulsion type** is shown in Fig. 62. The spindle pivoted at BB, which carries the pointer P, runs down the axis of a cylindrical coil C carrying the current to be measured. Attached to the spindle is a wedge-shaped piece of soft iron MI, the moving iron. Inside the coil and bent so as to be parallel to its circumference is the fixed iron FI. This covers a larger arc than the moving iron and is shaped in such a way that it is much narrower at one end than the other. When current flows in the coil both irons become magnetised longitudinally in the same direction. The magnetic effect of the fixed iron is greater at its broader end, and so the moving iron is repelled towards the narrower end, causing movement of the pointer. The motion is controlled by the spring S and rendered dead-beat, as in the attraction type, by an air dashpot, shown at D. Z is a zero-adjusting lever.

The force of repulsion is proportional to the product of the pole strengths of the two irons, and as each of these is proportional to the current, when working away from saturation, the movement is proportional to the square of the current. The non-uniformity of scale which this produces may be minimised over the working region by suitably shaping the irons.

Moving iron instruments are affected by external magnetic fields, and are also liable to hysteresis errors which cause them to read low with increasing currents and high when the current is decreasing. In modern instruments, these errors can be made very small by shielding from external fields in an iron case, and by using for the "irons" a nickel-iron alloy which has negligible hysteresis effects. Particularly in the attracted iron type, the coil C can also be made small, thus minimising temperature effects.

189. Electrostatic Voltmeter.—The action of this instrument depends on the force of attraction between two opposite charges, insulated from each other. A spindle mounted between pivots carries a pointer and a light aluminium vane V, which can swing between two electrically common brass plates BB. The whole instrument is enclosed in a metal case on an insulating base and provided with a glass window for observing the scale. Apart from the window, the instrument is thus shielded from the electrostatic effects of external charges.

The moving vane is generally in conducting communication with the case *via* its spindle and the brass plates are insulated from the case.

In the uncharged position the long axis of the moving vane is set at some angle to that of the brass plates, as shown in the diagram. When a P.D. is established between the moving and fixed parts they are equivalent to the plates of a condenser and acquire equal and opposite charges, the electrostatic attraction between which draws the moving vane into the space between the brass plates and causes the pointer to move over the scale. The motion is controlled

by a spring, and the pointer comes to rest in such a position that the opposing torque due to the spring balances the moment of the electrostatic forces causing motion. The equal charges on the moving and fixed plates are proportional to the P.D. and the force

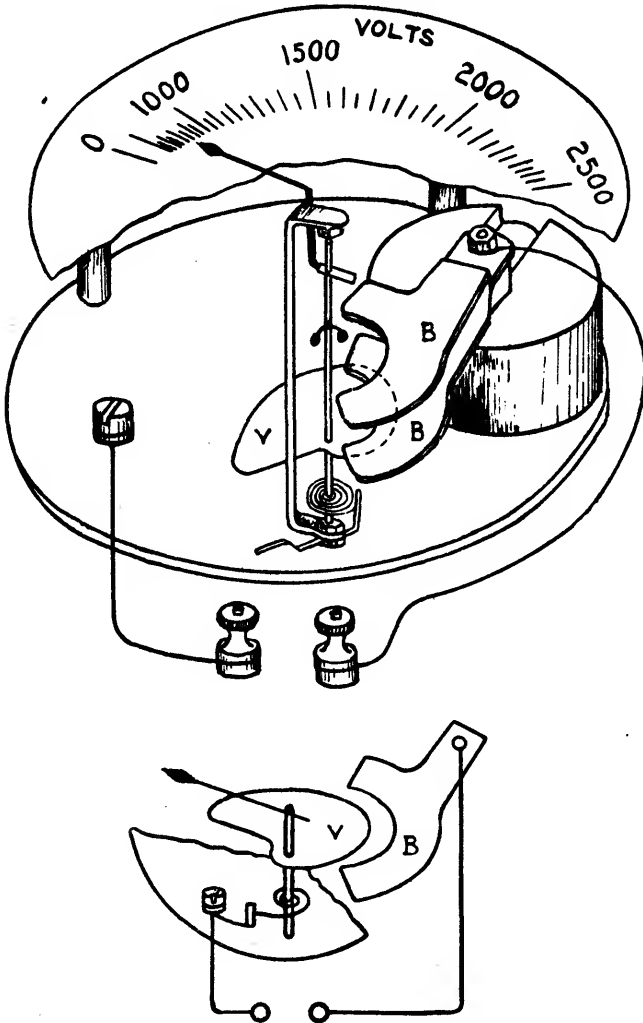


FIG. 63.

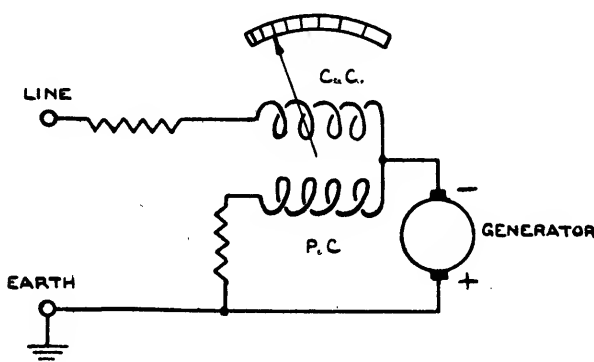
of attraction is proportional to the product of the charges. The movement of the pointer is thus proportional to the square of the P.D. and the scale is not uniform.

In a ship, the instrument should work in any position, but the weight of the moving vane would have a different moment in

different positions and so would alter the zero. To prevent this the vane is counterpoised by two small weights on the other side of the axis of rotation.

The great advantage of the electrostatic voltmeter is that it takes no current, and so wastes no energy and does not suffer from temperature defects. It is also unaffected by stray magnetic fields, but the unavoidable use of a glass window renders it liable to errors from external electrified bodies and particularly from electrification of the glass itself, such as is produced by cleaning it. This is avoided to some extent by coating the glass with transparent conducting varnish and fixing on it a strip of metallic foil to conduct electrostatic charges to earth.

190. Measurement of Resistance.—In Service wireless practice, it is often necessary to test the insulation between a high potential point and earth. The instrument used for this purpose is called a Megger and reads directly on a scale the resistance between the two points at which it is connected.



Megger Setting.

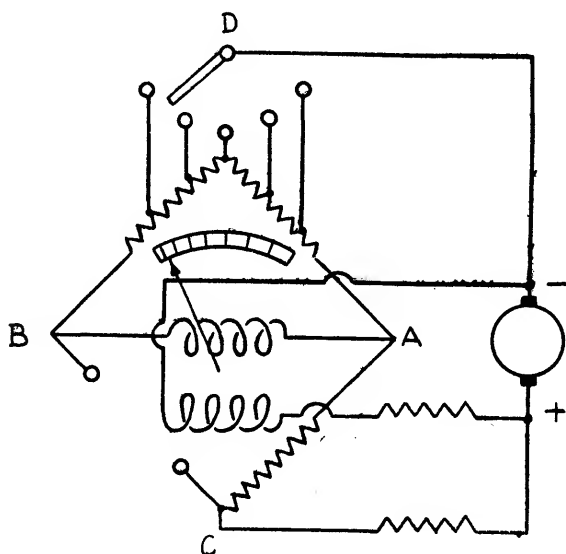
FIG. 64.

The principle of the Megger is shown in Fig. 64. The measuring system consists of two coils mounted at right angles to each other on a common shaft and called the Current or Deflecting Coil and Pressure or Control Coil respectively. They are pivoted so as to be free to move in the magnetic field of a permanent magnet, and wound so that when carrying current they tend to rotate the movement in opposite directions. The pointer thus comes to rest in an equilibrium position determined by the relative values of the currents flowing in the two coils.

The Pressure Coil is connected, in series with a fixed high resistance to adjust the range, directly across the generator. One generator terminal is earthed and the other is connected through the Current Coil and a fixed resistance to the point whose insulation

to earth is to be tested. Thus the Current Coil is in series with the unknown insulation resistance across the generator terminals.

When the Current Coil is on open circuit, *i.e.*, when the unknown resistance is infinite, current will flow only in the Pressure Coil and the pointer will take up a definite position which is marked "Infinity" on the scale. When the Line-Earth terminals are short-circuited, *i.e.*, the unknown resistance is zero, the pointer will assume another position, marked "zero" on the scale. Intermediate values of resistance will cause the pointer to take up positions on the scale between these two limits, and these positions may be determined by inserting known resistances. The instrument is thus calibrated to read resistance directly.



Bridge Setting.

FIG. 65.

The generator armature is rotated by hand from a handle outside the instrument. A constant-speed clutch is interposed between the handle and the armature, so that the armature cannot rotate above a certain rate (160 r.p.m. in the Service instrument), no matter how fast the handle is turned. In addition, if the handle is stopped suddenly, a free wheel allows the armature to come to rest slowly so that the generator is not damaged.

The lowest resistance which can be measured accurately on this instrument is about 10,000 ohms. The range may be extended to 0.01 ohm by means of a change-over switch, which gives the circuit of Fig. 65, known as the Bridge Setting.

The principle employed is that of Wheatstone's Bridge (para. 73). The unknown resistance forms one arm of the bridge, the fixed resistance in series with the current coil forms another, and a known resistance with various tappings provides the other two arms as shown.

The current coil takes the part of the galvanometer in the ordinary Wheatstone Bridge arrangement. The bridge is balanced when no current flows in this coil. As shown above, the pointer in this case indicates "infinity" on the scale, so that the procedure is to adjust the tapping point until this reading is obtained.

CHAPTER IV.

ALTERNATORS, GENERATORS AND MOTORS.

191. The discovery that an E.M.F. is induced in an electrical circuit which moves so as to alter its flux-linkage with a magnetic field (para. 145), provides a method of generating E.M.Fs. on a commercial scale. The most convenient method of obtaining the continual relative motion of conductor and field necessary for continual change of flux-linkage is to have the conductor in the shape of a loop, and either to rotate the conductor in a fixed field, or to keep the conducting loop fixed and rotate the field. In either case, as will be seen below, the E.M.F. induced in the loop is alternating, *i.e.*, it changes its direction round the loop periodically.

The loop E.M.F. may be applied directly to an external circuit, in which case the current in the external circuit will also be alternating. Such machines are called Alternating Current Machines or Alternators.

It is also possible to arrange that the E.M.F. applied to the external circuit is direct, *i.e.*, always in the same direction round the circuit, in spite of the alternating character of the loop E.M.F. Machines of this type are called Direct Current Machines, or Dynamos, or Generators.

192. The Alternator.—Fig. 66 (a) shows a conducting loop arranged in a magnetic field, with a collector ring electrically connected to each side of the loop and provided with two metallic brushes, each so mounted as to make electrical contact with one of the collector rings.

The external circuit is connected to the two brushes as shown.

Call the conductor on one side of the loop A, and that on the other side B.

In Fig. 66 (b), (c), (d) and (e) the loop is shown in section at successive positions, at angles of 0° , 30° , 60° , and 90° with the vertical.

Now consider the changes in flux enclosed by the coil. In Fig. 66 (b) the coil is enclosing 19 lines; in Fig. 66 (c) 15 lines, a decrease of 4 lines; in Fig. 66 (d) 8 lines, a decrease of 7 lines; and in Fig. 66 (e) no lines, a decrease of 8 lines.

It is not possible in a figure such as this to draw sufficient lines to get any accurate results, but at any rate we may deduce that when the loop is moved a very small way away from the vertical position the decrease in flux interlinked by the coil is small, while the nearer the coil gets to the mid-position between the poles the more rapid is the decrease in the flux enclosed.

After the mid-position has been passed, the flux enclosed by the coil increases, at first rapidly and then more and more slowly, till the vertical position is reached once more.

The magnitude of the E.M.F. induced in the coil depends upon the rate of change of flux enclosed by the coil.

Hence, when the coil is moving from 0° to 90° , the E.M.F. induced will increase from zero to maximum, and when moving through the next quadrant it will fall from maximum to zero.

193. As regards the direction of the induced E.M.F., one need only apply Fleming's Right-Hand Rule (para. 147). The bar B

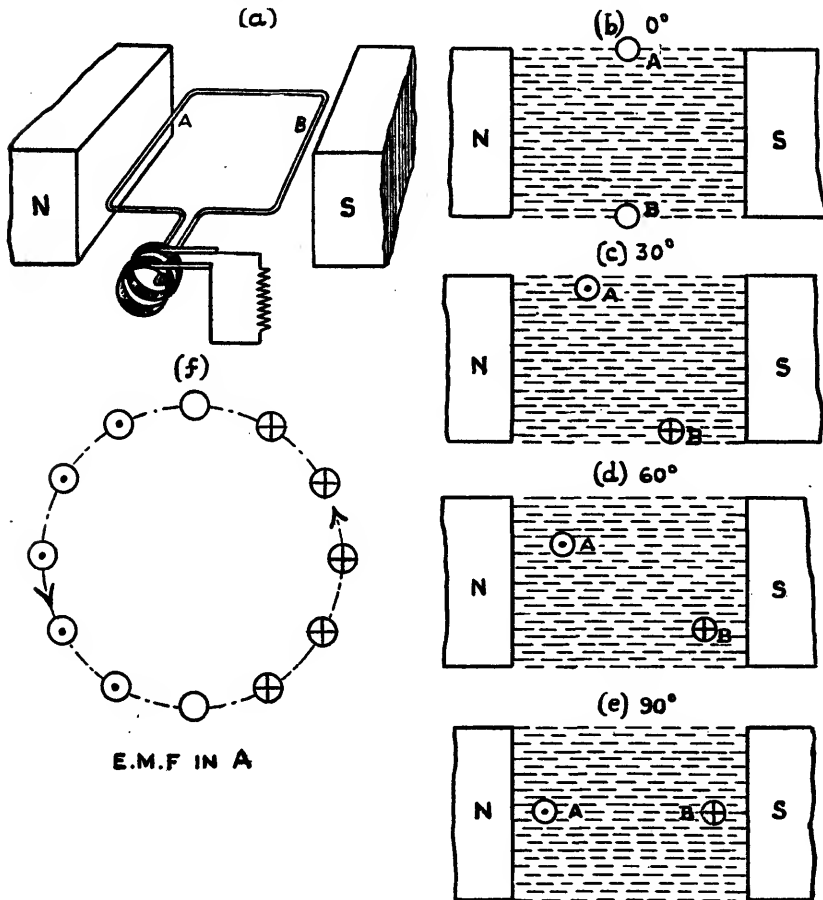


FIG. 66.

is cutting lines of flux in an upward direction and the field is from left to right; hence the direction of the E.M.F. will be inwards.

The bar A is cutting the same field in a downward direction; hence the direction of the induced E.M.F. will be outwards.

Thus the induced E.M.F.s in the two bars will combine to drive a current round the circuit.

The total E.M.F. acting round the loop will be double that generated in either bar.

When the loop has turned through half a revolution and conductor A begins to move up on the right, and B down on the left, the directions of the E.M.F.s in the two bars will be reversed.

Fig. 66 (f) illustrates the directions of the E.M.F.s at successive instants.

Hence the E.M.F. induced in any one armature bar A varies from a maximum in one direction when the bar is opposite the centre of one pole, to zero when it is on the neutral line—midway between the poles. Then the direction of the E.M.F. reverses, rises to a maximum in the opposite direction, and falls to zero once more.

194. The Sine Curve.—It is required to show that the voltage induced in any loop, rotating at a uniform speed in a uniform field, varies as the sine of the angle through which the loop has been turned from the neutral plane which lies at right angles to the field.

The loop AB is being rotated in the uniform field shown in Fig. 67.

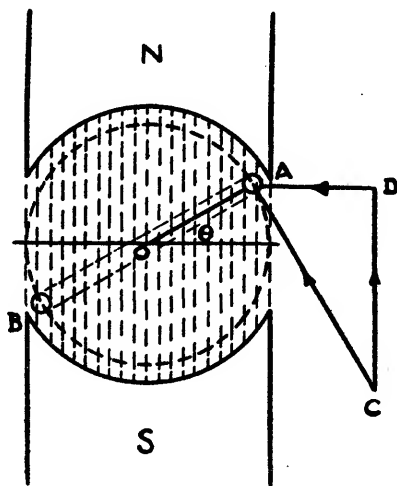


FIG. 67.

The line AC drawn at right angles to OA (that is, as a tangent to the circular path in which A and B are revolving) will represent the direction of motion of the conductor at the instant shown, *i.e.*, after it has revolved through an angle θ .

Its length, drawn to any convenient scale, can be taken to represent the velocity of the bar A.

We draw the line AD horizontally from the point A, and the line CD vertically from the point C.

This gives a triangle of velocities (Appendix B). The velocity line CA has been resolved into two components, viz., AD representing the horizontal velocity of the bar A, and DC representing its vertical velocity.

From the point of view of generating E.M.F. we are only interested in the horizontal movement of A, since it is only when moving horizontally that it cuts across the magnetic field, and generates an E.M.F.

The length of AD at any instant will, therefore, be a measure of the rate of cutting of lines of flux by A, and therefore of the voltage generated.

The vertical component CD represents the motion of A parallel to the field, which motion has no effect in generating an E.M.F.

We therefore wish to investigate how AD will vary during a revolution of the bar A.

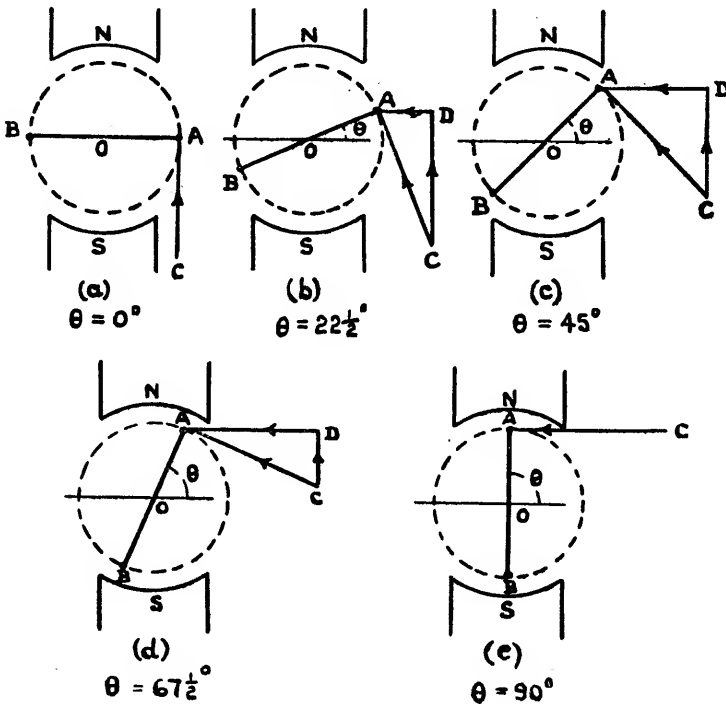


FIG. 68.

195. Since CD is perpendicular to the initial position of the conductor, and AC is perpendicular to its direction AB after it has turned through an angle θ , it follows that angle ACD is equal to θ .

Fig. 68 shows the bar in successive positions as the angle θ varies from 0° to 90° .

(a) $\theta = 0^\circ$, $DC = AC$, and $AD = 0$; no lines of force are being cut and no voltage is being generated.

(b), (c) and (d). The line AD , and therefore the voltage generated, steadily increases.

(e) $\theta = 90^\circ$: $AD = AC$: therefore the voltage generated is a maximum;

and so on.

Now $\frac{AD}{AC} = \text{sine of the angle } ACD$.

Therefore $AD = AC \times \sin ACD = \text{a constant} \times \sin ACD$.
 $\qquad \qquad \qquad = \text{a constant} \times \sin \theta$.

AD is proportional to v , the voltage induced at any instant.

Therefore $v = \text{a constant} \times \sin \theta$.

If $v = \mathcal{V}$ when A is opposite the centre of the north pole, *i.e.*, when $\theta = 90^\circ$, it follows that the constant in the above expression is equal to \mathcal{V} , for $\sin 90^\circ = 1$. \mathcal{V} is the maximum value assumed by v in a revolution.

Hence

$$v = \mathcal{V} \sin \theta \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

Taking figures, let us suppose that \mathcal{V} is 10 volts.

Then from a table of sines we find that v will go through the following values:—

θ : 0° ,	10° ,	20° ,	30° ,	40° ,	50° ,	60° ,
v : 0,	1.74,	3.5,	5.2,	7,	7.6,	8.6,
θ : 70° ,	80° ,	90° ,	100° ,	110° ,	120° ,	&c.
v : 9.4,	9.8,	10,	9.8,	9.4,	8.6,	

A convenient method of plotting the curve $v = \mathcal{V} \sin \theta$ geometrically is shown in Fig. 69.

Let the line OA represent \mathcal{V} . Then the line $AM = OA \sin \theta = \mathcal{V} \sin \theta$.

We may now take a horizontal line DE to represent degrees and plot on it the various lengths and directions of the line AM as the point A revolves.

Fig. 69 is a graph of the various values of AM as the line OP revolves through 360° . The dotted lines indicate how the curve is constructed, and should need no further explanation.

When AM comes above the line CD we call it positive in sign. This will be for values of θ lying between 0° and 180° . For values of θ between 180° and 360° , A will be below the line and AM will be negative.

After joining up all the points we have the curve as in Fig. 69.

A curve constructed in this manner is termed a "Sine Curve."

The maximum height of the curve, called its "amplitude," will be equal to OA , which equals \mathcal{V} .

At all times the height of the curve = $\mathcal{V} \sin \theta$, so that :—

When $\theta = 0^\circ$ or 180° , $\mathcal{V} \sin \theta = 0$: when $\theta = 90^\circ$ or 270° ,
 $\mathcal{V} \sin \theta = \mathcal{V} \times 1 = \mathcal{V}$, or $\mathcal{V} \times -1 = -\mathcal{V}$.

The variation of the curve from zero, to a maximum positive, through zero to maximum negative, and back to zero again, is termed a "Cycle."

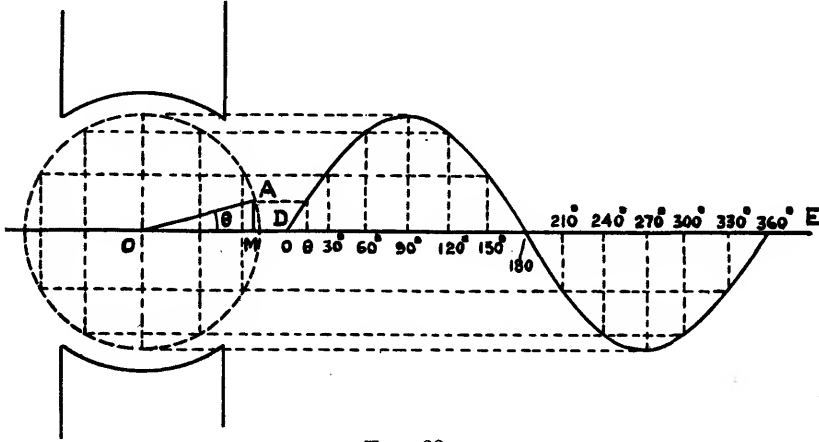


FIG. 69.

196. Frequency.—In every revolution of the armature bar, a cycle of the E.M.F. is completed.

Hence, if the bar is revolving at f revolutions per second, f cycles are completed per second.

"Cycles per second" is termed "Frequency."

In the two-pole machines shown, the frequency = revolutions per second.

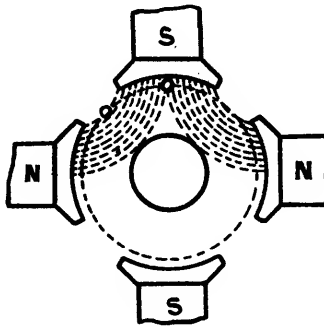


FIG. 70.

If the alternator has more than one pair of poles (as indicated in Fig. 70), then a cycle occurs for each pair of poles passed in the course of the revolution.

Hence, frequency = revs. per sec. \times number of pairs of poles.

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197. It will be convenient to express the angle θ in circular measure (*see* Appendix B).

Consider a two-pole machine.

In one second f revolutions take place, each of 360° :
but $360^\circ = 2\pi$ radians.

Therefore in one second f revolutions take place, each of 2π radians.

In one second, $2\pi f$ radians are swept out in all.

So $2\pi f$ is the number of radians swept out per second.

This is called the "angular velocity."

For brevity, the expression $2\pi f$ is denoted by the letter " ω " (a Greek letter called "omega").

Thus $\omega = 2\pi f = 6.28 \times f$.

In a time t secs., the angle θ in radians swept out by the conductor is $\theta = \omega t$ (Appendix C).

Formula (1) on page 156 thus becomes

$$v = \mathcal{V} \sin \omega t \text{ volts} \quad \dots \quad (2)$$

If two pairs of poles are fitted, although the armature sweeps out only 360 actual degrees, we have two complete cycles, which require on the curve 720 electrical degrees.

By using " ω " to represent angular velocity, we mean "electrical" angles, without reference to any definite number of poles.

***198.** The above results may be obtained more directly as follows:—

Let the initial position of the loop be on the neutral axis midway between the poles, and after a time t , let it have turned through an angle θ from its initial position. The angular velocity is ω and so

$$\theta = \omega t.$$

Let B be the flux density, and

A be the area of the loop.

The flux linkages in the initial position = BA . In its position at time t , the area of the loop projected at right angles to B is $A \cos \theta$.

$$\begin{aligned} \therefore \text{Flux linkages at time } t &= BA \cos \theta \\ &= BA \cos \omega t. \end{aligned}$$

$$\begin{aligned} \therefore \text{Induced E.M.F. } v &= - \frac{d}{dt} (\text{flux linkages}) \\ &= - \frac{d}{dt} (BA \cos \omega t) = \omega BA \sin \omega t. \end{aligned}$$

The maximum value \mathcal{V} of the induced E.M.F. is when $\sin \omega t = 1$, i.e., $\mathcal{V} = \omega BA$, and we can write

$$v = \mathcal{V} \sin \omega t.$$

Thus the induced E.M.F. is a simple sine alternating E.M.F. and goes through one cycle per electrical revolution.

199. Alternating Current.—Now let us take this sinusoidal voltage and apply it to the ends of a circuit, shown in Fig. 66 (a).

Suppose, at first, that the circuit contains resistance, but that it is non-inductively wound and has no capacity.

A current will flow, which will rise, fall and reverse in step with the voltage impressed on the ends of the circuit, and we can employ Ohm's Law to find the strength of the current.

Hence, the current at any moment

$$= \frac{V}{R} \sin \omega t = I \sin \omega t.$$

200. The Alternator.—A single armature bar would have to be revolved at a tremendous speed in a very dense magnetic field in order to generate an E.M.F. that would be of any use to us.

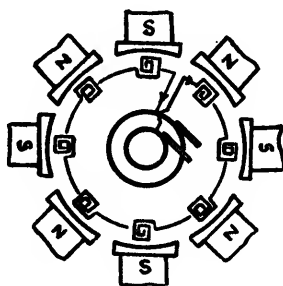


FIG. 71.

An alternator armature winding therefore consists of several groups of windings arranged in series, separated by the same distance that separates the poles, and so wound that the E.M.F.s induced in all the coils act in the same direction, as in Fig. 71.

In Fig. 72 is illustrated a portion of an armature winding in successive positions as it passes under a pair of poles.

When it starts (a) it is not cutting any flux, and so is generating no E.M.F.

As it moves under the pole pieces, the E.M.F. induced in it rises, till in Fig. 72 (c) it is generating a maximum E.M.F. (as shown by the curve above it) because it is cutting across the field at right angles.

As it moves away from the central position the generated E.M.F. will fall off, till it is zero again as in Fig. 72 (e).

The right-hand side of the coil will then come under the influence of the north pole and the left-hand side under that of the next

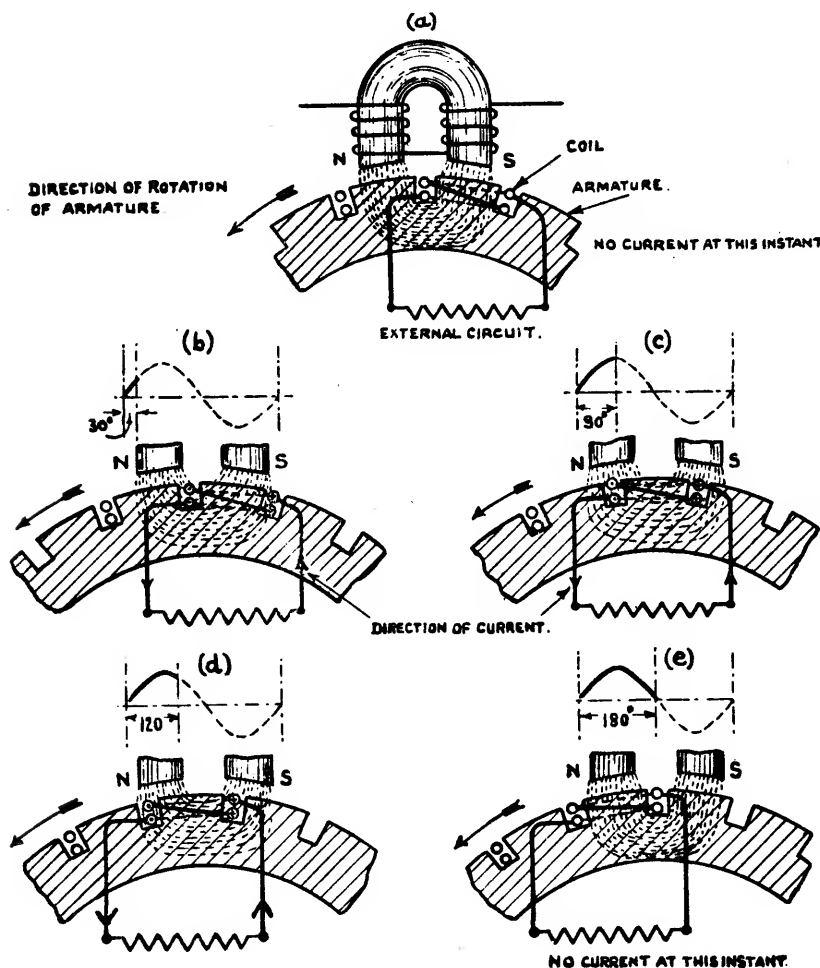


FIG. 72.

south pole (not shown in the illustration), and the E.M.F. will start to rise in the opposite direction.

201. The Armature.—We have previously shown that for a given magnetising force, the total number of magnetic lines produced in a magnet will depend upon the reluctance of the magnetic circuit. The greater part of the reluctance in a magnetic circuit of ordinary dimensions is due to the air gap. It is clear that by reducing the air gap between the poles, we can get a greater density and therefore a greater total number of magnetic lines with the same magnetising force.

This air gap can be reduced in two ways; firstly, by shaping the pole faces in a curve so that they are parallel to the path of rotation of the conductors, and, secondly, by filling up the space inside the path of the conductors—that is to say, filling up the core of the armature—with iron.

Such an arrangement is shown in Fig. 73 (a), where, as is usual in most dynamos, the conductors are shown embedded in slots in the iron core, thus reducing the gap between the iron of the pole face and the iron of the armature core to a minimum.

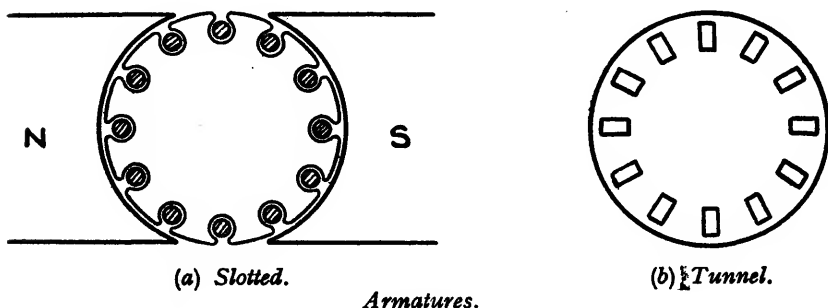


FIG. 73.

Besides increasing the density of the magnetic field, this will, to a certain extent, alter the distribution of the magnetic lines of force, with the result that the E.M.F. generated by rotating the conductor through a complete revolution will not exactly follow the sine curve as shown in Fig. 69.

Alternators designed for wireless telegraphy purposes, however, are generally arranged to give what is practically a sinusoidal voltage curve.

This is sometimes arranged for by inserting the conductors through holes completely enclosed by the iron of the armature as illustrated in Fig. 73 (b). An armature wound in this manner, is known as tunnel-wound.

202. Pole Winding.—Unless permanent magnets be used for the field of an alternator, some arrangement must be made for producing a magnetic flux through the pole pieces.

Except for special machines (such as a “magneto”), permanent magnets are unsuitable; in the first place, the magnetic flux density is low, thereby necessitating a large amount of steel to produce a given amount of flux; secondly, any current taken from the armature tends to demagnetise the poles owing to reaction; and, thirdly, they are expensive. (Quite recently, however, it has become possible to obtain much higher flux densities with permanent magnets.)

It is usual, therefore, to use electro-magnets for the fields of a dynamo, *i.e.*, coils of wire wound round cores, and supplied with direct current as illustrated in Fig. 74.

If it is required to regulate the current flowing through the field winding, an adjustable resistance may be inserted in series with the mains; such a resistance is termed a "Field Regulator."

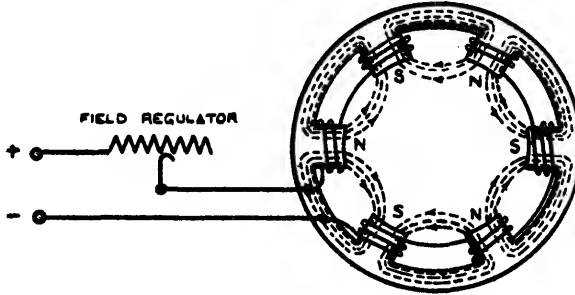


FIG. 74.

203. The Slip Rings.—Since the armature of the type of machine being described must necessarily be kept rotating to generate an E.M.F., a method must be devised for connecting the windings of the armature to any desired outside circuit.

This is usually accomplished by the use of "slip rings" and "brushes."

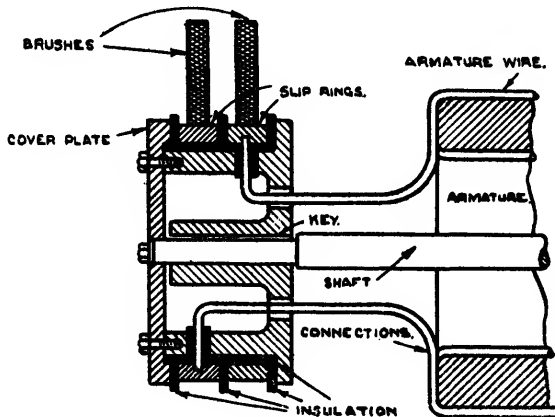
*Slip Rings.*

FIG. 75.

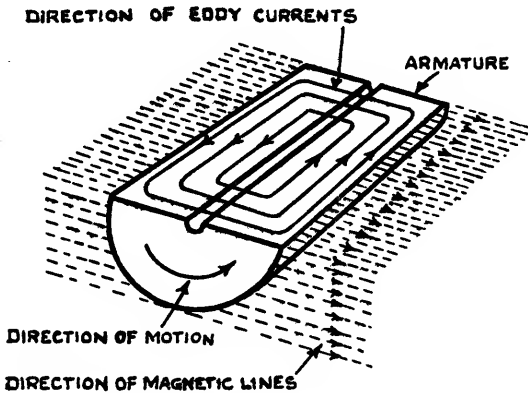
Two brass rings are carried on the shaft of the armature and carefully insulated from each other and from the shaft, as shown in Fig. 75.

These rings rotate with the armature, but as they have a smooth surface, connection can conveniently be made to them from a fixed part of the machine by means of carbon "brushes" pressing lightly on the surface of the rings.

One end of the armature winding is then connected to one slip-ring and the other end to the other slip-ring, while the outside circuit to which it is desired to connect the alternator is connected to the two brushes.

204. Eddy Currents.—It has already been seen that the conductors on the armature are embedded in an iron core in order to reduce the air gap in the magnetic circuit of the alternator.

Obviously, this iron core revolves with the conductors in the magnetic field. Since the iron is also a conductor of electricity and is unavoidably cutting the lines of force induced by the field magnets, the result is that E.M.F.s are generated in the iron body of the armature and cause currents to circulate continually in the metal.



Eddy Currents in Armature.

FIG. 76.

These currents are known as Eddy Currents, and since they cannot be utilised they only represent so much wasted energy, and in addition heat up the iron of the armature core to the detriment of the running of the machine. Means must therefore be found of reducing them to a minimum.

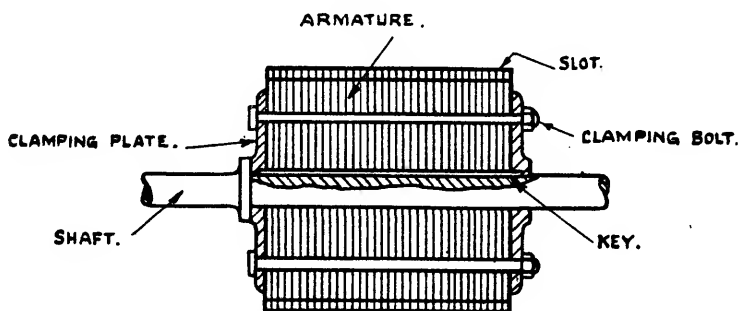
The direction of these currents will be found by applying Fleming's "Right Hand Rule," and will be as shown in Fig. 76; that is to say, round the core at right angles to the lines of force.

205. Lamination.—To prevent these currents flowing, armature cores are built up of a large number of thin circular plates of iron, separated from each other by very thin paper or varnish. The

plates are threaded on to the armature shaft and are clamped together by some suitable means, such as that illustrated in section in Fig. 77.

It will be seen that these sheets of paper, being non-conducting, offer a large resistance to any currents which tend to flow in the iron core in a direction parallel to the shaft, as indicated in Fig. 76.

At the same time they do not increase the reluctance of the magnetic circuit to any great extent, as the lines of force can pass freely down each plate of the armature core without passing through the paper.



Lamination of Armature.

FIG. 77.

206. Alternator Construction.—Practical alternating-current generators may be divided into three classes, thus :—

- (1) Rotating armature machines.
- (2) Rotating field machines.
- (3) Inductor-type machines.

207.—(1) Rotating Armature Machines.—These are of the type previously described, and are spoken of as having a “Rotor” (or rotating) armature, and a “Stator” (or stationary) field.

A revolving armature carries the winding in which the alternating current is generated, and poles projecting inwards from a yoke in the form of a ring carry the field windings, which are supplied with direct current from an outside source.

(2) Rotating Field Machines (Fig. 78).—These have a Rotor field and a Stator armature, *i.e.*, the magnet coils are carried on poles projecting from a hub (or ring in larger sizes) and are supplied with direct current through two slip-rings, while the armature winding in which the alternating current is generated is wound in slots on the inside of a cylinder enclosing the rotor. Fig. 78 (*a*) and (*b*) show diagrammatically an elevation and plan view respectively of part of the armature winding.

The direction of the induced E.M.F. is obtained by Fleming’s Right Hand Rule, but if, as in Fig. 78, the field is moving to the

right, this is equivalent to keeping the field stationary and moving the armature conductors to the **left**. This must be allowed for in applying the rule, in which the thumb gives the direction of relative

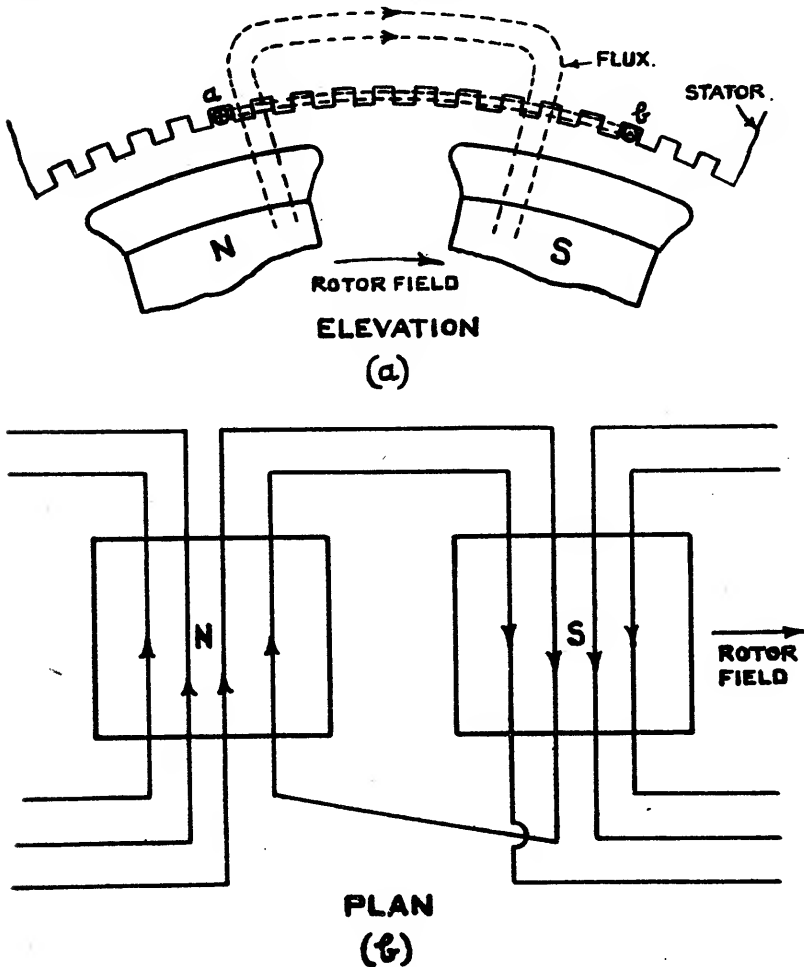


FIG. 78.

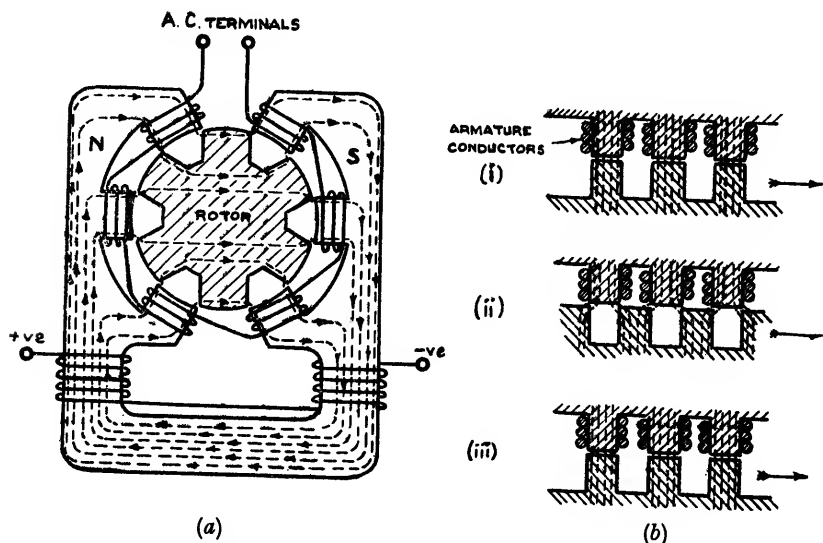
motion of the **conductor**. The E.M.F. induced into conductor *a* (Fig. 78 (a)) is into, and in conductor *b* out of, the paper. In Fig. 78 (b) the flux is coming out of the paper from the north pole and going into the paper again in the south pole.

This is a more convenient method of generating a high-voltage alternating current, since the conductors and slip rings of the rotor have to stand only the voltage of the direct current supplied for magnetising the field, while it is much easier to insulate the stator winding, which does not require slip-rings.

308.—(3) The Inductor Type.—In the “Inductor” type of alternator both the armature winding and the field magnet winding are wound on projections inside the stator, while the rotor consists of a drum carrying projections of steel or iron material.

Fig. 79 (a) illustrates diagrammatically a machine of this type. The armature winding is wound on projections of the stator, while, for clearness, the field winding is shown on the legs of the field magnet system. The rotor is simply a soft iron cylinder with projections whose width corresponds to the pole pitch of the stator.

Fig. 79 (b) illustrates the action. The flux is formed into “tufts” each time a rotor pole is opposite a stator pole (i) and (iii): when the poles are in the midway position (ii) the flux is fairly evenly distributed across the air gap. Therefore, considering any one stator pole, the flux is continually spreading out and gathering in again, the spreading out causing one alternation and the gathering in a reverse alternation, and the two together forming one complete cycle.



Alternator—Inductor Type—diagrammatic.

FIG. 79.

An E.M.F. is developed in the stator winding and is in the form of a sine curve.

The frequency is given by the number of revolutions per second of the rotor, multiplied by the number of its projections.

The generation of an E.M.F. in this case may also be considered as a result of the variable reluctance of the path of the flux lines, as the air gaps between the stator and rotor poles alter in length

during a revolution. The ampere-turns producing the field are a constant quantity and so therefore is the M.M.F. (para. 92). Variation in the reluctance of the magnetic circuit thus produces a variation in the number of lines linking with the armature circuit and an E.M.F. is generated.

THE DIRECT CURRENT GENERATOR, OR DYNAMO.

209. Exactly the same principles hold good as regards the E.M.F.s produced in the conductors of the armature of a continuous current dynamo. That is to say, alternating E.M.F.s are produced in the conductors themselves, but, as we shall show later, these alternating E.M.F.s, instead of being brought straight to the outside circuit through slip-rings, as in the case of the alternator, are taken through an apparatus for automatically reversing their direction, so far as the outside circuit is concerned, at definite intervals.

In order to produce a continuous current in the outside circuit, or, as it may be better considered, in order to produce a continuous E.M.F. at the brushes of the dynamo, an arrangement is provided for reversing the connections of the armature coils at the brushes at the moment when the E.M.F. induced in the coils reverses.

This arrangement is known as the Commutator, and its action is described in the following paragraphs.

210. The Commutator.—Let us take the simplest case of a single coil being rotated, as shown in Fig. 80.

If the two ends of the coil, instead of being connected to two slip rings, be connected one to each half, A and B, of a divided ring which rotates with the armature, and if the two brushes C and D

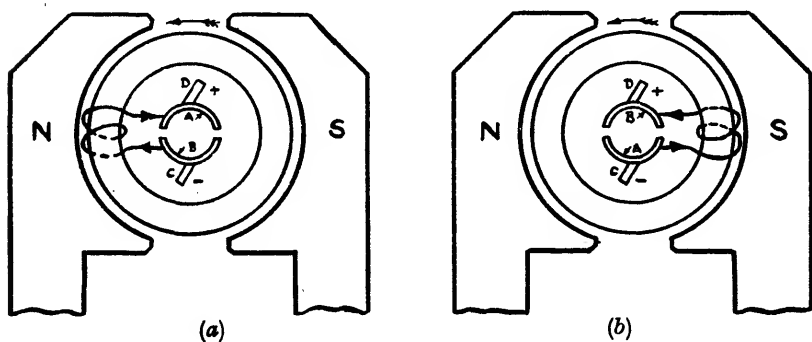


FIG. 80.

be fixed in the position shown, it is evident that while the coil is travelling under the N pole of the magnet the half ring A will be in contact with the brush D and the half ring B with the brush C (Fig. 80 (a)), and, similarly, while the coil is travelling under the

S pole of the magnet, A will be in contact with C, and B with D, as shown in Fig. 80 (b).

By tracing the directions of the E.M.F. generated, it will be seen that while the coil is travelling under the N pole, the half ring A will be positive and B will therefore be negative, and, similarly, while the coil is travelling under the S pole, A will be negative and B positive. It follows, therefore, that the brush D will always be in contact with whichever half ring is positive and the brush C with whichever half ring is negative throughout the revolution.

Since the wave form of the E.M.F. generated in the active conductors of the coil takes the form shown in Fig. 69, it is obvious that the curve showing the value of the E.M.F. at the brushes, when the coil is thus connected to a split ring, will take the form shown in Fig. 81.

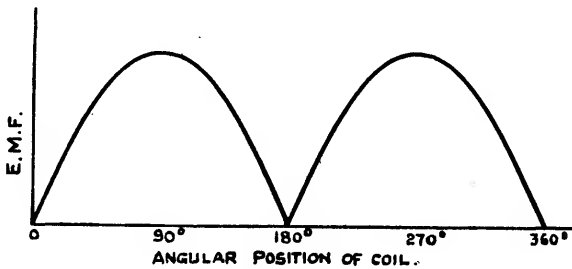


FIG. 81.

Let us develop this arrangement a little further and take a commutator with four segments, connected to four points on the armature winding, as in Fig. 82.

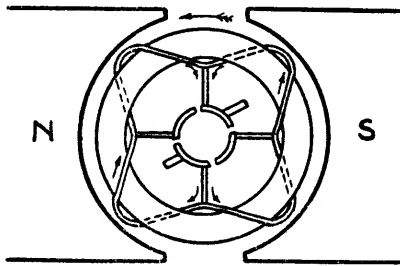


FIG. 82.

If the position of the brushes is adjusted as shown, it will be observed that each brush short-circuits two segments when the coil attached to these segments is generating no E.M.F. With respect to the brushes, the coils are arranged in two sets of two coils in series, the sets being in parallel with each other. The E.M.F. between the brushes is thus the sum of the E.M.F.s in the

two coils in series on one half of the armature. The individual E.M.F.s in these coils are shown dotted in Fig. 83. One lags on the other by a quarter of a period, as is evident from Fig. 82. The total E.M.F. of the machine is the sum of these two E.M.F.s, indicated by the full line curve in Fig. 83.

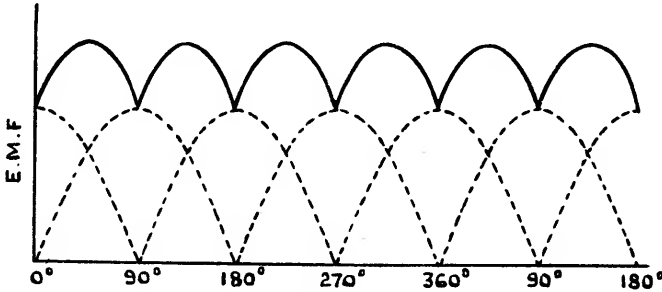


FIG. 83.

211. Armature Windings.—In practice, an armature winding consists of a great many conductors arranged in slots on an iron core. Each conductor is connected to one segment of the commutator.

The conductors are not in practice arranged as in Fig. 82, which illustrates a “ring-wound” armature, but are wound entirely on the outside of the armature, as in Fig. 84.

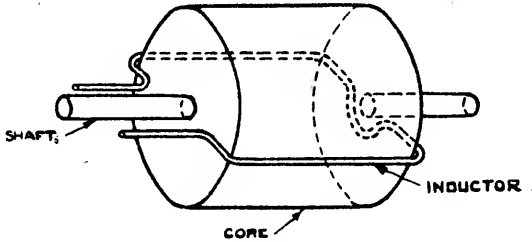


FIG. 84.

All the coils are so arranged that they, together with the commutator segments to which they are connected, form a closed circuit upon themselves, and each coil always comprises part of the circuit; consequently the P.D. between the brushes is half the sum of the average E.M.F.s induced in all the conductors.

An illustration of one coil, and the way coils are built up on an armature, is given in Fig. 85 (a) and (b).

There are two main types of windings:—

- (1) Lap (or parallel); and
- (2) Wave (or series), with simple and complex forms of each.

The reader is referred to any standard electrical textbook for a description of the various methods of winding an armature.

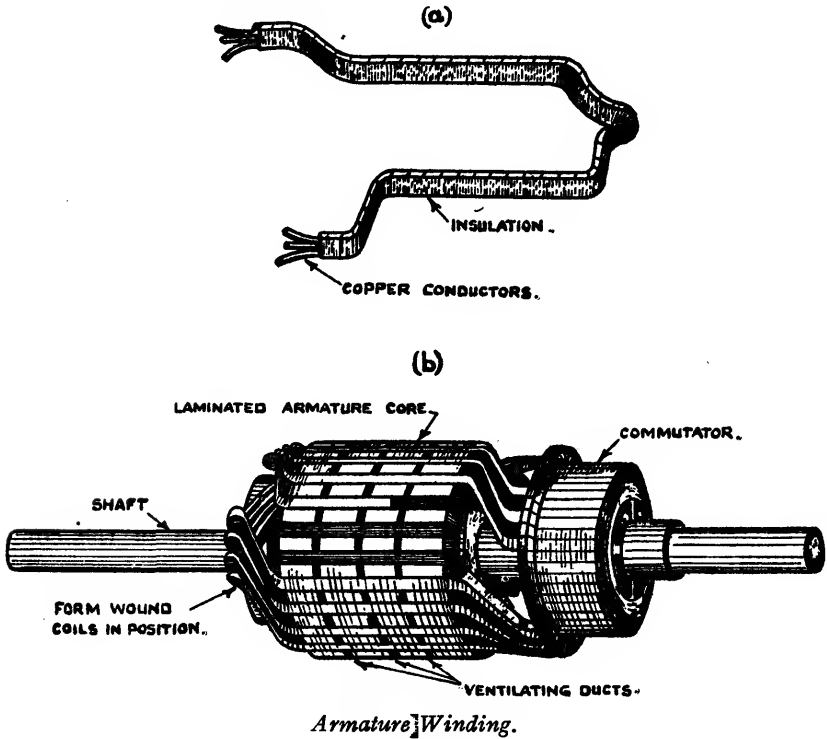


FIG. 85.

212. Commutator and Brushes.—The commutator consists of strips of copper insulated—usually by mica—from each other and from the shaft, about which it is built in the form of a cylinder.

The brushes are usually of graphitic carbon, which has a high resistance and keeps down sparking, keeps the commutator clean, is comparatively soft, and does not wear out or groove the commutator, but takes its shape.

They are sometimes copper-plated at the point where they make contact with their holders, although this practice is being discontinued in machines for wireless purposes, and definite electrical connection is made by means of a flexible copper wire.

They are held in brush-holders and the latter are fixed to a “brush rocker,” enabling the brushes to be shifted all together round the commutator and fixed in position.

213. E.M.F. of Machine.—However the armature is wound, there is always a number of conductors in series between the brushes, each conductor generating an E.M.F. in the same direction.

The E.M.F. between the brushes at any instant is the sum of the instantaneous E.M.F.s in these conductors, just as the E.M.F. of a number of cells in series is the sum of their individual E.M.F.s. The E.M.F. of the individual conductors is changing continually, but whenever a conductor is passing through a fixed position relative to the brushes, it always has the same E.M.F. momentarily induced, so that there will be, on the average, always the same E.M.F. between the brushes. This will be equal to the average E.M.F. induced in any one conductor in its passage from brush to brush, multiplied by the number of conductors in series between the brushes. It is called the E.M.F. of the machine. As is suggested by a comparison of Fig. 81 and 83, the fluctuation about this mean value becomes smaller as the number of conductors in a series zone becomes greater.

There will be a number of such series zones of conductors in parallel between the brushes; *cf.* Fig. 82, where there are two such parallel paths. Each series zone produces the same E.M.F., which is the E.M.F. of the machine, and is unaffected by the number of parallel paths; but, as in a series parallel arrangement of cells, the internal resistance of the armature winding is decreased, and its current-carrying capacity increased, by increasing the number of such parallel paths.

214. Calculation of Dynamo E.M.F.—As explained above, the method of doing this is to find the average E.M.F. induced in one conductor in its passage from brush to brush, and multiply the result by the number of conductors between brushes.

Let Φ be the flux per pole. Then in passing under a N pole and the succeeding S pole, a conductor cuts 2Φ lines. If there are p pairs of poles alternately N and S round the armature, a conductor therefore changes its flux linkages by $2p\Phi$ lines in one revolution, and at an armature speed of N revolutions per second, the change of flux linkages per second for each conductor will be $2p\Phi \times N = 2pN\Phi$. Therefore the average induced E.M.F. in any conductor is $2pN\Phi \times 10^{-8}$ volts.

If n is the number of conductors in series between two brushes, the E.M.F. of the machine is thus $E = 2pnN\Phi \times 10^{-8}$ volts.

n and p are constants which depend only on the construction of the machine, and so we may write

$$E = KN\Phi$$

as a formula to cover all machines, the constant K varying with the particular machine considered.

Thus in all cases the E.M.F. of a dynamo is proportional to

- (a) the flux per pole, *i.e.*, the field;
- (b) the speed.

The general formula for calculating the E.M.F. of a particular machine is

$$E = \frac{pZN\Phi}{a} \times 10^{-8} \text{ volts,}$$

where Z is the total number of armature conductors, " a " is the number of parallel conducting paths through the armature, and the other symbols have the same significance as before.

215. Energy Changes in Armature.—So far, we have been discussing the dynamo purely as a generator of E.M.F. We have now to consider what happens when the E.M.F. is applied to an external closed circuit, and current starts to flow. While the machine is on open circuit no electrical power is being provided, and the horse-power supplied to the armature shaft by the steam engine or other source of mechanical power has only to spin the armature against the torque or resistance to rotation offered by friction, air resistance, etc. As soon as a current starts to flow, electrical power is being supplied to the external circuit, this additional power supply being proportional to the current flowing. The power which must be supplied from the mechanical source thus increases with the current taken from the dynamo.

We may consider this conversion of mechanical power to electrical power from two points of view.

216. On closed circuit a current flows through the armature windings, and so they are current-carrying conductors placed in a magnetic field. Thus each conductor is acted on by a force (para. 130) which, by Fleming's Left Hand Rule, is seen to be in the opposite direction to the direction of rotation. This "electrodynamic torque" has to be overcome by the mechanical drive, and so additional power must be supplied for this purpose by the source of power.

We can find the value of the "electrodynamic torque" as follows:—

If the torque is T foot-pounds, the power on the shaft required to overcome it at N r.p.s. is $\frac{2\pi NT}{550}$ H.P. This mechanical power is completely converted to electrical power $E.I_a$ watts, where E is the E.M.F. generated by the dynamo and I_a is the armature current.

Therefore, expressing mechanical and electrical power in the same units (1 H.P. = 746 watts), we have

$$\frac{2\pi NT}{550} = \frac{EI_a}{746}$$

or

$$T = \frac{550 EI_a}{746 \times 2\pi N}$$

We have seen that $E = KN\Phi$ (para. 214) and substituting this value gives

$$T = \frac{550 KN\Phi I_a}{746 \times 2\pi N} = \frac{550K}{746 \times 2\pi} \times I_a \Phi$$

$$= K_1 I_a \Phi,$$

where K_1 is a constant depending on K , i.e., on the particular machine.

This shows that the "electrodynamic torque" is independent of the speed, and depends only on the field flux and the armature current.

217. Armature Reaction.—We can also consider the problem from the point of view of the flux distribution. The current flowing in the armature windings produces a magnetic field, and, along with the original field, this produces a resultant field in which the flux

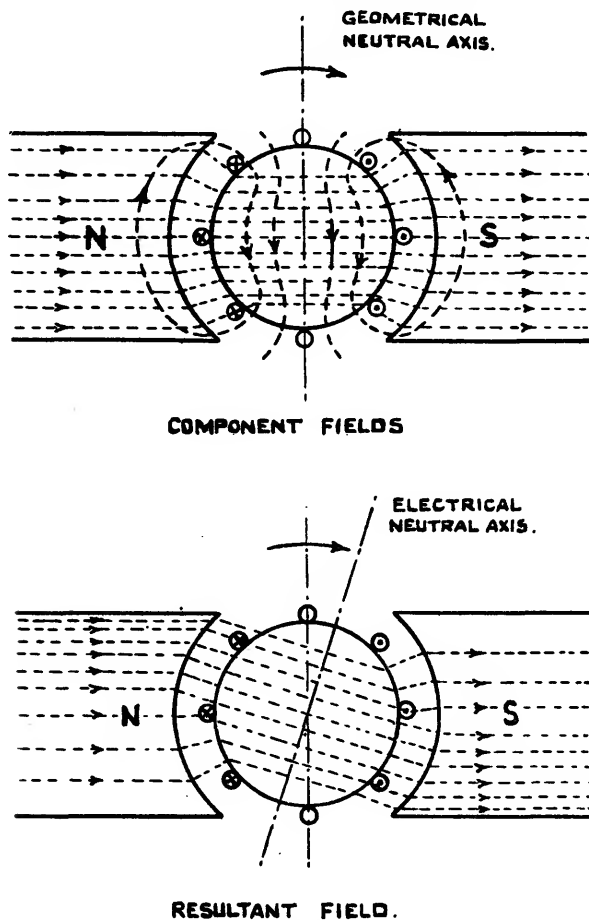


FIG. 86.

lines are distorted from their original direction. This distortion in the case of a dynamo is shown in Fig. 86. The flux, instead of being approximately uniform over the pole surfaces, tends to crowd into the forward pole tip in the direction of rotation, and is correspondingly weakened in the hindward tip. The line along which any coil is enclosing maximum flux and generating no E.M.F., the "electrical neutral axis," has advanced relatively to the axis of

symmetry between the poles, or "geometrical neutral axis," through a certain angle in the direction of rotation.

We have seen (para. 210) that the aim in commutation is to reverse the current in coils when no E.M.F. is being produced in them, *i.e.*, in coils on the "electrical neutral axis." The brushes must therefore be advanced in the direction of rotation from their original position on the geometrical neutral axis.

This will change the distribution of current in the armature windings, and so the flux distribution and electrical neutral axis will alter again. Thus the brushes must be advanced still further in the direction of rotation until the field due to the armature current is perpendicular to the resultant field.

218. Fig. 87 represents the original state of affairs. AB is the flux due to the field magnets, BC is the flux due to the armature current. We advance the brushes through an angle CBC' , so that the armature field is perpendicular to AC, the original resultant field. The armature field, however, retains its original strength and is now represented by BC' , which is not perpendicular to AC' , the new resultant field. Thus the brushes must be advanced through another angle $C'BC''$, until the armature field is perpendicular to the final resultant field AC'' , as shown in Fig. 87.

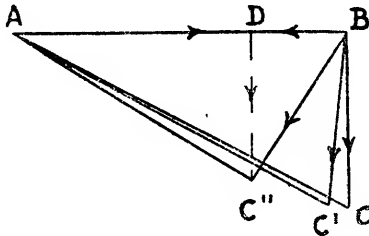


FIG. 87.

The amount of this "armature reaction" is obviously proportional to the armature field, and so varies with the current taken from the machine.

The armature field BC'' can be resolved into two components (*see Mechanics Appendix*) :—

- (a) A cross-magnetising component DC'' . It is this component which produces the flux distortion.
- (b) A demagnetising component BD , in opposition to AB . Thus the field magnet flux is weakened as the current increases and the E.M.F. generated by the machine falls off.

219. **Sparking.**—Sparking at the brushes is caused by the self-inductance of the armature windings. Fig. 88 (a) shows some of the coils in the neighbourhood of the positive brush, joined to their corresponding commutator segments. Fig. 88 (b) shows the instant

just before commutation of coil B commences. The current in bb' is upwards. Fig. 88 (c) shows the instant just after commutation of coil B is finished. The current in bb' is now in the opposite direction. In each case the current is half the armature current (in a two-brush machine), the other half, in the same direction to the brush, but in the opposite direction in the armature, being

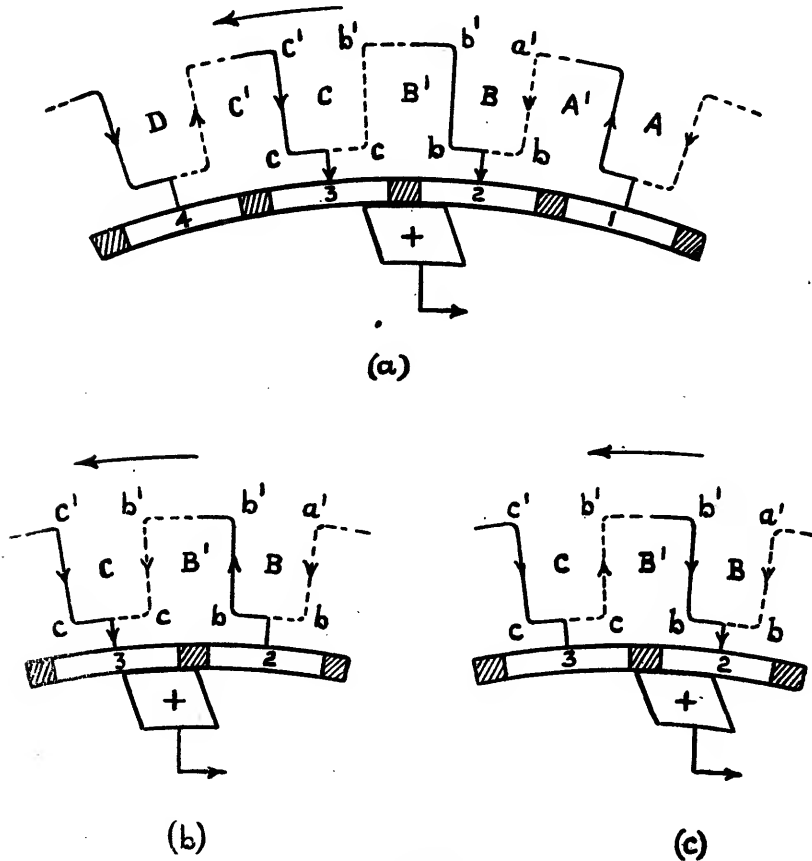


FIG. 88.

provided *via* coil C before commutation and *via* coil A after commutation of coil B. Thus the current in coil B has to be changed by an amount equal to the whole armature current during the time that the brush is short-circuiting the junction B' of coils B and C. Fig. 88 (a) shows the middle instant of this interval.

Under ideal conditions, half the armature current would be flowing upwards in bb' in Fig. 88 (b), zero current in Fig. 88 (a), and half the armature current downwards in Fig. 88 (c)—the current would then have been reversed uniformly in the interval in which the brush passes over the insulating segment of the commutator.

In practice, owing to the self-inductance of the armature winding, this reversal is delayed. At the instant just after that shown in Fig. 88 (b), when the current from b to b' starts to decrease, a back E.M.F. is set up which prevents the current from falling to zero in Fig. 88 (a). When B' is short-circuited, instead of having zero current flowing, there is still a current in the direction $bb'c$, and at the instant shown in Fig. 88 (c), instead of half the armature current flowing from b' to b , the current is considerably less.

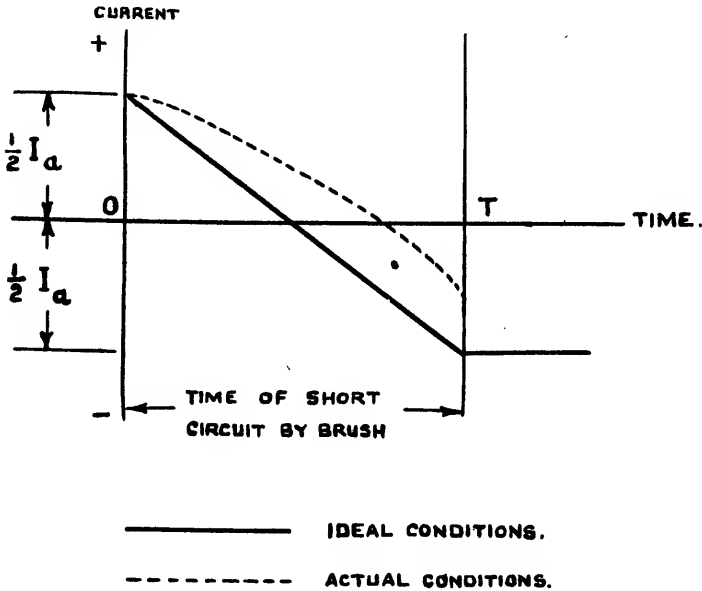


FIG. 89.

Fig. 89 shows how the current actually changes compared with the ideal case. At T the current is less than half I_a , and has to take this value abruptly. This sudden change in current causes a big induced E.M.F., and a spark passes from segment 3 over the insulation to the brush.

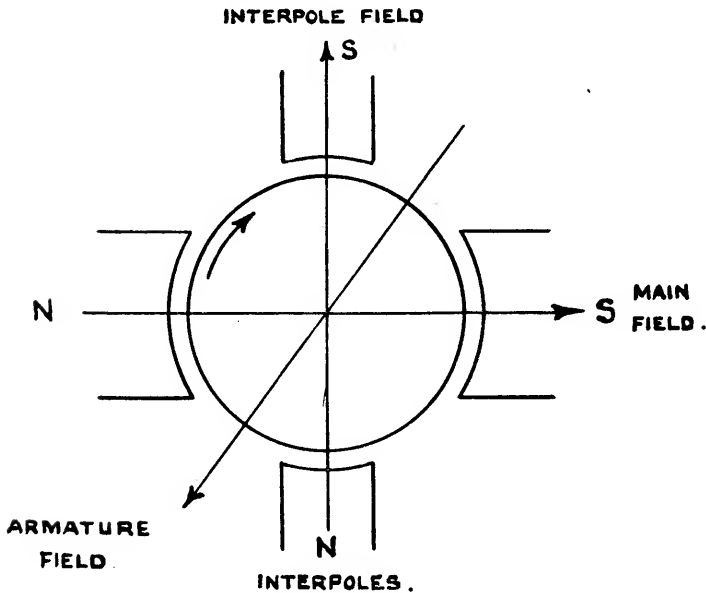
220. Apart from the obvious damage it causes to the commutator, sparking has the further disadvantage from the W/T point of view that H.F. oscillations are produced, which interfere with the working of the sets.

It may be prevented or mitigated in various ways :—

- (a) By using carbon brushes. The inclusion of a high resistance in an inductive circuit lessens the time taken for the current to decay.
- (b) By advancing the brushes beyond the electrical neutral axis. Under these conditions the coil undergoing commutation

is cutting flux and has an E.M.F. induced in it which is in the direction of the current after reversal, thus balancing the E.M.F. of self-induction at commutation. As the latter E.M.F. is proportional to the current, the appropriate position for the brushes will alter with the load. For this reason the method is not employed on modern machines.

- (c) By means of interpoles. These are small poles placed half-way between the main poles and having their field coils in series with the armature windings, so that the flux they produce is proportional to the armature current. Thus the induced E.M.F. in coils passing under them is proportional to the armature current, and automatically balances the reactance volts of the coil being commutated, if the interpoles are of correct strength and their flux



linkages are in the correct direction, *i.e.*, so as to produce an E.M.F. in the coil in the direction of its current after reversal. Interpoles must therefore be wound so that a N interpole comes before a N main pole in the direction of rotation.

Interpoles also serve another useful purpose, as the flux they produce balances that due to the current in the armature windings, thus neutralising armature reaction, and mitigating the fall in E.M.F. at large currents due to the demagnetising component.

221. Types of Machine.—Dynamos are usually classified according to the way in which the magnetic flux is produced. The simplest method is to use permanent magnets, but the difficulty of constructing these to produce sufficient flux for large machines confines their use to the small dynamos called "magnetos." All large machines employ electromagnets. They may be separately-excited or self-excited. Separate excitation involves the use of an independent source of E.M.F. for the field coils, and is now mainly confined to alternators and low-voltage generators. For Naval wireless purposes, most D.C. generators are separately excited.

D.C. machines are usually self-excited, *i.e.*, the E.M.F. generated by the machine itself is used to obtain a current in the field coils. This is possible owing to the retentivity of iron and steel. Some residual magnetism is always present, and, when the armature is rotated, a small E.M.F. is generated. If the field coils are connected so that the field of the current due to this E.M.F. reinforces the residual field, the E.M.F. will build up.

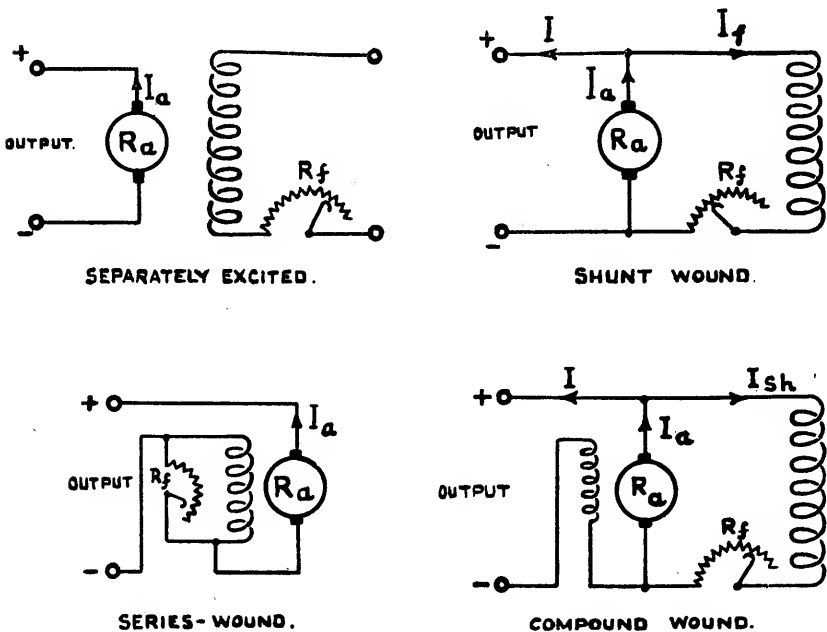


FIG. 91.

The field windings may be connected in series or in parallel with the armature windings, or there may be a mixture of both, giving three types of self-excited machine. Series windings alone are never used on generators for Naval wireless purposes.

222. Types of Self-excited Machine.

- (1) **Shunt wound.**—In this case the armature current divides into two branches, one through the field and one through the external circuit. It is thus advantageous to keep the shunt current I_f as small as possible, getting the ampere turns for the required flux by using a large number of turns. As only a small current is carried by the shunt winding, it thus consists of many turns of fine wire.
- (2) **Series wound.**—Here the whole armature current I_a goes through the field windings and the external circuit; thus for the same ampere turns, a much smaller number of coils is necessary, *i.e.*, a series winding consists of a number of turns of thick wire, capable of carrying a large load.
- (3) **Compound wound.**—This is a mixture of shunt and series windings designed to combine the advantages of both.

The potential difference V at the terminals of the machine will depend in each case on the current I_a taken from the armature. If the armature resistance is R_a , the potential drop in the armature is $I_a R_a$. In addition, there is a drop of potential of one or two volts due to the resistance of the brushes. This varies to some extent with the electrical contact they make. We may call this potential drop V_B .

$$\therefore V = E - I_a R_a - V_B,$$

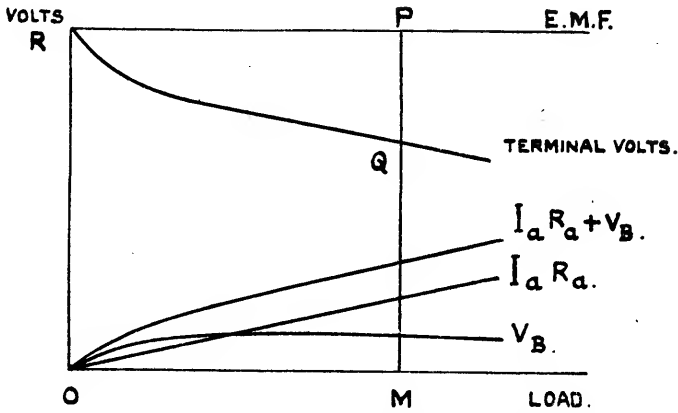
where $E = KN \Phi$ is the E.M.F. produced by the machine.

223. In a **separately-excited machine or magneto** the flux is independent of variations of the armature current, except for the effect of the demagnetising component of the armature reaction, which, as we have seen, can be neutralised by interpoles and other devices.

Thus, at constant speed, the E.M.F. will be a constant independent of the load current, which is the same as the armature current in this case. The curve showing how the terminal volts of the machine at any given speed vary with the current in the load circuit is called a characteristic curve of the machine. For a separately-excited machine this will be nearly a straight line, as shown in the figure, except near the origin, owing to the irregular variations of V_B in this region. Thus the terminal volts fall off slowly as the load current increases. The machine has a "falling characteristic."

A generator is designed not to be run above a certain load, called the "full load" of the machine. If we assume this to be OM in Fig. 92, then the fall in terminal voltage from no load (OR) to full load (MQ) is PQ. This is called the "regulation" of the machine;

and is said to be "good regulation" when PQ is small and "poor regulation" when PQ is large.



Characteristics of Separately excited Machine.

FIG. 92.

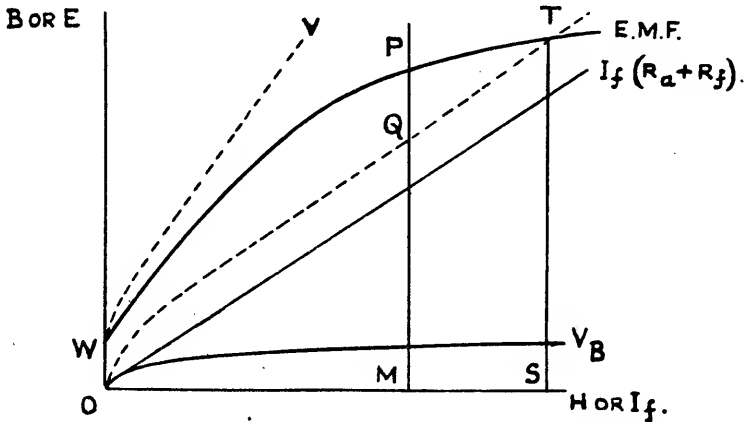
224. Shunt-wound Machine.—The exact nature of the building up of the E.M.F. in a self-excited machine can be found out by drawing the magnetisation curve, *i.e.*, the B-H curve, at any given speed. It will be somewhat as in Fig. 93. Owing to retentivity, B has a definite value when the exciting field H is zero. Hence, an E.M.F. is produced which sends current through the field windings, and so produces an exciting field H and an increase in B. As the E.M.F. is proportional to B at a given speed, and H is proportional to the shunt current I_f , the B-H curve is also the E- I_f curve if the scales are suitably altered. The machine is supposed to be on open circuit in this discussion, so that all the armature current goes through the field windings. The size of I_f thus depends on the resistance of the armature, R_a , and shunt windings, R_f . If V_B is the brush drop, the fall of potential round the circuit is $V_B + I_f(R_a + R_f)$. As long as the generated E.M.F., E, is greater than this, the current will tend to increase to that value which makes

$$E = V_B + I_f(R_a + R_f).$$

If we plot $V_B + I_f(R_a + R_f)$ against I_f , we shall get a curve OQT, as shown in Fig. 93. It is practically a straight line except near the origin, and its slope depends on $R_a + R_f$. The only thing we can vary is R_f , and so the slope of the line alters as we alter R_f . The line will have a steeper slope if R_f is increased.

Consider the value of I_f shown by OM in the figure. For the particular value of R_f considered, PM is the generated E.M.F. and $QM = V_B + I_f(R_a + R_f)$. Thus the E.M.F. is greater than the fall of potential round the circuit by PQ, and so I_f will increase, and

E will increase with it. This goes on till the point T is reached. At T the generated E.M.F. is just sufficient to keep the current OS flowing in the field coils, for it is exactly balanced by the fall of



Internal characteristic of Shunt-wound Machine.

FIG. 93.

potential round the circuit. Thus there is no tendency for the field current to increase over OS, and the corresponding E.M.F., ST, is the steady E.M.F. to which the machine will build up. For any given machine it is obvious that the position of T depends on the slope of OQ, *i.e.*, on the value of R_f . As we increase R_f , the slope of OQ increases, and the point in which it cuts OP moves nearer to the origin, *i.e.*, the steady E.M.F. built up becomes smaller. It is obvious that there is a limiting value of R_f above which the machine will refuse to build up any E.M.F. beyond that due to the residual magnetism. OWV shows the slope of the fall of potential line for a value of R_f above this limiting value. The E.M.F. is limited to OW.

225. Thus in starting up a shunt-wound generator two points should be observed :—

- (a) The field regulator resistance should be adjusted to such a value that the slope of OQ is small enough for the machine to excite. Generally machines excite even with all the resistance in the circuit.
- (b) No external circuits should be connected across the terminals until the E.M.F. has built up. Otherwise part of the armature current is diverted from the field circuit, thus reducing I_f . For instance, if the output terminals were short-circuited, no current whatever would flow through the field coils, and the generator would not excite.

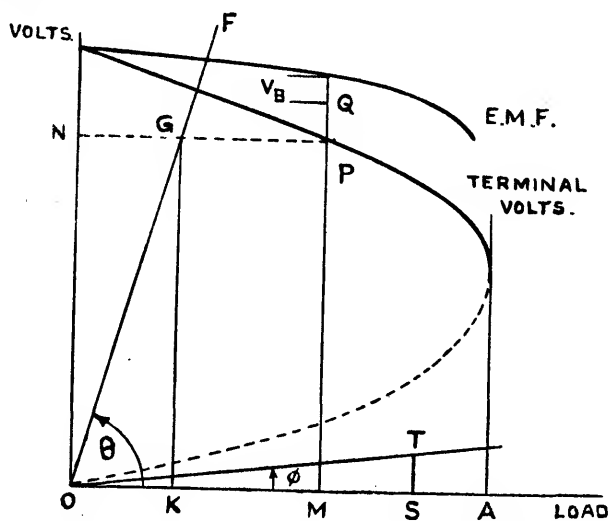
226. The above curve is called the "internal characteristic" of the machine. Of more importance from the point of view of studying the behaviour of the machine is its "external characteristic." This is a curve showing the relation between the terminal P.D. and the external load current taken from the machine at some particular speed. We have seen that on open circuit the E.M.F. builds up to a steady value, depending on the field resistance. If now an external circuit be connected across the terminals, a part of the armature current will flow through it.

The armature current will increase to cope with this, and more power will be supplied by the prime mover to provide the increase in electrical power caused by the increase in armature current. The fall of potential in the armature windings ($= R_a I_a$) will increase, and so the P.D. at the terminals will decrease for the same generated E.M.F. This is the P.D. across the field windings, and so the field current will be lessened. Hence the flux, and therefore the E.M.F. generated by the machine, will fall off. Due to this there will be a further decrease in terminal volts, and the cycle of events will repeat itself until a steady state is reached, in which both the generated E.M.F. and the terminal volts have smaller values than at no-load, the actual values depending on the armature current. The curve showing how the terminal volts vary with the external current can be plotted from experimental measurements of these two quantities for various loads, and is shown in Fig. 94.

When the external current becomes very large, *i.e.*, if the machine is being overloaded, the terminal volts fall off rapidly, and increased difficulty is experienced in getting a field current large enough to produce an E.M.F. which will keep the terminal volts up. Eventually this adjustment becomes impossible (at current OA in Fig. 94), and the terminal volts and therefore the external current fall to zero, *i.e.*, the machine shuts down. This does not occur until the machine is carrying a load well above its specified full load, which should, of course, never be the case in practice. The E.M.F. generated at any load can easily be found when the terminal volts V and external current I are known, as it is $E = V + R_a I_a + V_B$, and $I_a = I + I_f$, where $I_f = \frac{V}{R_f}$.

227.—It can be obtained graphically from the external characteristic as follows. Consider the particular working point P on the characteristic (Fig. 94). The machine is delivering an output current OM or NP and the terminal volts are MP or ON. The field current is obtained from the relation $MP = R_f I_f$. If we draw a straight line OF, making an angle θ with the current axis, where $\tan \theta = R_f$, it cuts NP at G; $GK = PM$ and $GK = OK \tan \theta = R_f \times OK$. Thus OK represents, to scale, the field current for terminal volts PM. If we make $MS = OK$, then $OS = OM + OK = I + I_f = I_a$, the total armature current. We now draw another

resistance line OT, which makes an angle ϕ to the current axis such that $\tan \phi = R_a$. TS = OS $\tan \phi = I_a R_a$ = potential drop in armature.



External characteristic of Shunt-wound Machine.

FIG. 94.

$\therefore E = PM + TS (= PQ) + V_B$ for the particular external current OM. In this way every point on the E.M.F.—external current characteristic may be plotted, and a curve obtained as shown.

228. Series-wound Machine.—In this machine the field current is also the external current, or at least proportional to it. (The field coils may have a rheostat in parallel for regulating purposes.)

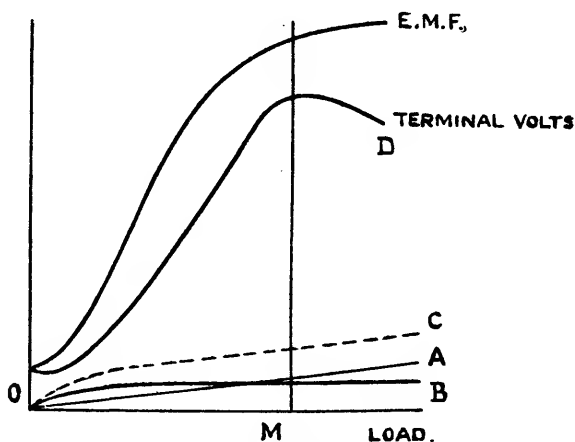


FIG. 95.

Thus the curve connecting generated E.M.F. and external current at any particular speed will have the same shape as the magnetisation curve of the machine. A typical curve is shown in Fig. 95. To find the lost volts in the machine, draw a line OA, making an angle, whose tangent is $(R_a + R_b)$, with the current axis. Any point on this line gives the armature drop for the corresponding current. If to this be added the voltage drop at the brushes given by the corresponding ordinate of curve OB, we get the curve OC giving the resultant lost volts in the machine. The terminal p.d. V = generated E.M.F. (E), less lost volts, and so is given by the curve OD. This is the external characteristic of the machine, which would be obtained by experimental test. It is a "rising characteristic."

229. Compound-wound Dynamo.—In many cases it is desirable that the terminal volts should be independent of the external current being taken by the machine, *i.e.*, that it should have a "level characteristic." As a shunt-wound machine has a falling characteristic and a series-wound machine a rising one, it is obvious that a suitable combination of both will give either a practically level characteristic or a very slowly-rising one. The former is called a "level compounded" machine and the latter an "over-compounded" one.

It is also possible to obtain a level characteristic from a shunt-wound machine by suitably adjusting the series winding on the interpoles.

230. Voltage Regulation of Self-excited Machines.—The terminal volts of a well-designed compound-wound machine are independent of the external current, *i.e.*, the regulation is automatically nearly perfect. The same effect can be obtained for shunt and series-wound machines by altering (*a*) the speed, (*b*) the resistance in the field winding. Generally the speed is kept constant at all loads by means of a "governor," and method (*b*) is used to adjust the voltage.

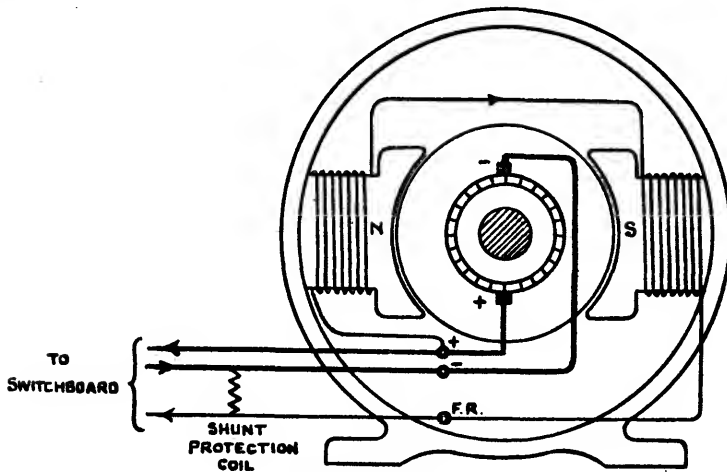
For a shunt-wound machine, the rheostat is in series with the field windings. As resistance is cut out I_f increases, and therefore the flux and generated E.M.F. increase.

Fig. 96 illustrates a shunt dynamo, showing windings and connections.

In parallel with the field regulator, a shunt protection coil is sometimes provided on the dynamo. Its object is to prevent the circuit of the highly inductive field magnet winding from being broken, as might happen in the leads to the field regulator or in the latter itself, thus obviating the voltage failing altogether, and sparking or arcing at the break, with risk of fire and possibly danger to life.

A series-wound machine may have a rheostat in parallel with its field, called a "diverter." The smaller the amount of diverter

resistance included, the less current flows through the field and the smaller is the E.M.F. generated.



Shunt Dynamo.

FIG. 96.

231. Losses in D.C. Machines.—The power losses are of the same nature in both dynamos and motors, and may be classified as follows :—

- (a) **Copper losses**, due to the ohmic resistance of the armature and field windings, and also to the P.D. across the brushes, multiplied by the current flowing through them.
- (b) **Iron losses** caused by :—
 - (i) Hysteresis (para. 89) in the armature core and pole pieces. The iron goes through p cycles of magnetisation per revolution, and so through pN hysteresis loops per second. Each loop represents a definite power loss.
 - (ii) Eddy currents (para. 204).
- (c) **Mechanical losses** due to :—
 - (i) Friction at the bearings and between the commutator and brushes.
 - (ii) Air resistance to the motion of the armature, which is increased by the boring of the armature for ventilation purposes. This is usually called "windage loss."

232. Dynamo Efficiency.—This may be regarded from various points of view. First, we have a conversion from mechanical power in the prime mover to electrical power in the machine. This gives the "mechanical efficiency"

$$= \frac{\text{Total watts generated}}{\text{Mechanical power supplied.}}$$

Of this total electrical power generated, a part is lost due to the causes considered in para. 231. The rest is available for the external circuit. From this is derived the idea of "electrical efficiency"

$$= \frac{\text{Watts available in external circuit.}}{\text{Total watts generated}}$$

Finally, we have the overall or "commercial efficiency"

$$= \frac{\text{Watts in external circuit.}}{\text{Mechanical power supply}}$$

It will be easily seen from these definitions that commercial efficiency = mechanical efficiency \times electrical efficiency.

The mechanical power supply will generally be given in H.P., and to perform the above calculations it must be converted to watts by the formula 1 H.P. = 746 watts.

In a good dynamo the commercial efficiency may be as much as 95 per cent. Where the source of power is an electric motor, this efficiency is usually of the order of 75 per cent.

233. Calculation of Efficiency.—The quantities which can be obtained experimentally are the H.P. on the armature shaft, the terminal volts V and output current I , and the resistances of the armature and field windings, R_a and R_f , respectively. The efficiencies should therefore be expressed in terms of these quantities.

Let E be the E.M.F. generated, I_a the armature current, and I_f the field current :—

(1) **Series Machine.**

$$I_f = I_a = I,$$

$$\text{and } E = V + I(R_a + R_f).$$

$$\text{Mechanical power supply} = \text{HP} \times 746 \text{ watts.}$$

$$\text{Total watts generated} = EI$$

$$= VI + I^2(R_a + R_f).$$

$$\text{Watts in external circuit} = VI.$$

$$\therefore \text{mechanical efficiency} = \frac{VI + I^2(R_a + R_f)}{\text{HP} \times 746}$$

$$\text{electrical efficiency} = \frac{VI}{VI + I^2(R_a + R_f)}$$

$$\text{commercial efficiency} = \frac{VI}{\text{HP} \times 746}$$

(2) **Shunt Machine.**

$$I_a = I + I_f \text{ and } I_f = \frac{V}{R_f}$$

$$E = V + R_a I_a = V + R_a \left(I + \frac{V}{R_f} \right)$$

$$\text{Mechanical power supply} = \text{HP} \times 746 \text{ watts.}$$

$$\text{Total watts generated} = EI_a$$

$$= VI + \frac{V^2}{R_f} + R_a \left(I + \frac{V}{R_f} \right)^2$$

Watts in external circuit = VI .

$$\therefore \text{mechanical efficiency} = \frac{VI + \frac{V^2}{R_f} + R_a \left(I + \frac{V}{R_f} \right)^2}{HP \times 746}$$

$$\text{electrical efficiency} = \frac{VI}{VI + \frac{V^2}{R_f} + R_a \left(I + \frac{V}{R_f} \right)^2}$$

$$\text{commercial efficiency} = \frac{VI}{HP \times 746}$$

Example 25.

What horse-power is required to drive a 150-kilowatt dynamo when it is developing its full-rated load, if the machine has a full-load commercial efficiency of 91.5 per cent. ?

$$\frac{91.5}{100} = \frac{150 \times 1,000}{746 \times \text{H.P.}}$$

$$\therefore \text{H.P.} = \frac{150 \times 1,000 \times 100}{746 \times 91.5} = 220.$$

234. Rating of Dynamos.—The rating of a dynamo is the kilowatt-power-load that the machine will carry continuously without excessive (1) heating, (2) sparking, or (3) internal voltage drop.

Thus, if a maker puts a label on a machine he sells :—

500 amps.
100 volts.
2,400 revs.

he infers—

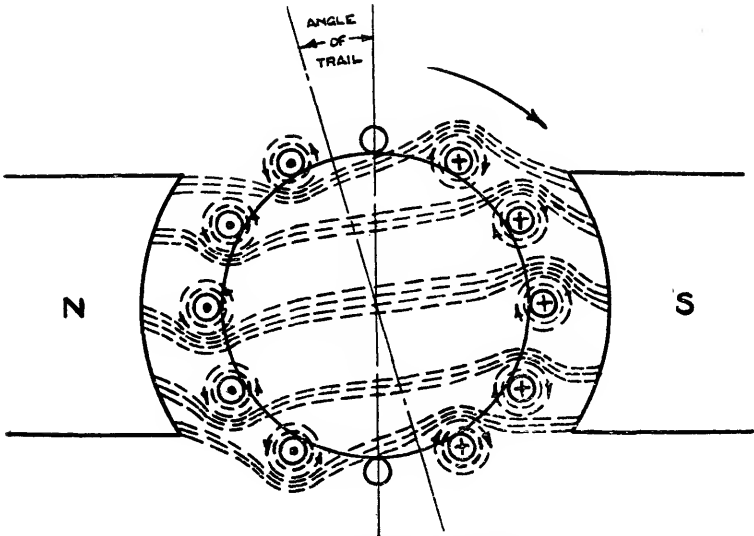
- (a) that if the machine is kept revolving at 2,400 r.p.m. it will always generate a terminal P.D. of 100 volts ;
- (b) that it will stand a maximum current output of 500 amps. without developing any of the faults referred to above.

THE MOTOR.

235. The electric motor is a machine for the conversion of electrical energy into mechanical energy, and so its function is exactly the reverse of that of a generator.

When a conductor carrying a current is placed in a magnetic field, it experiences a mechanical force, the direction of which is given by Fleming's Left Hand Rule (para. 130). The armature conductors of a D.C. generator are situated in the magnetic field of the poles, so that if a current is passed through them by applying an external E.M.F. to the brushes, they will be acted on by mechanical forces, and the armature will be set in rotation. By the use of belting or other devices, the rotation of the armature may be caused to turn a flywheel or do other useful mechanical work,

the ultimate source of which is the electrical energy supplied by the external E.M.F.

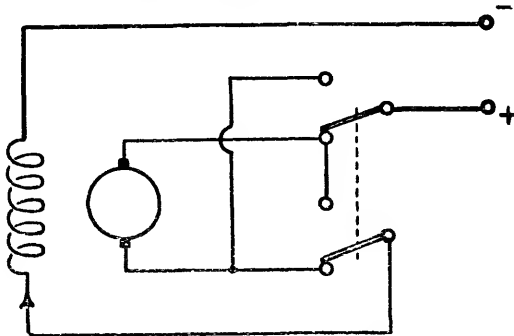


Illustrating Action of a Motor.

FIG. 97.

236. The same machine can therefore be made to act as a dynamo by supplying it with mechanical power, or as a D.C. motor by supplying it with electrical power; as regards types of armature and field windings and arrangements we have thus exactly the same classification for motors as for generators.

The application of Fleming's Left Hand Rule shows that, if both the current and the external field are reversed, the direction of the mechanical force is unchanged; in order to reverse the direction of rotation of a motor, the leads must be interchanged either in the armature circuit or the field magnet circuit, but not in both.



Reversing Switch for Small Series Motor.

FIG. 98.

237. Torque.—We have already calculated the electrodynamic torque on an armature carrying a current I_a when the flux per pole is Φ , and found it to be proportional to ΦI_a (para. 216). This is therefore the value of the driving torque in a motor.

As the flux is proportional to the field current I_f and the permeability μ , we may also write $T \propto \mu I_a I_f$.

238. Back E.M.F.—As soon as the armature starts rotating, its conductors will begin to cut flux, and so an E.M.F. will be induced in them just as in a generator.

Lenz's Law tells us that the induced E.M.F. will be in such a direction as to oppose the rotation which is producing it; the rotation is caused by the external current supply to the armature, and so the induced E.M.F. will act so as to cut down the current, *i.e.*, it will act in opposition to the applied E.M.F. This can also be seen by applying the Right Hand Rule to any conductor. This "Back E.M.F." has the same value as if the machine were running as a generator, *i.e.*, $E = KN \Phi$ (para. 214) and so is proportional to the speed N .

239. The effect of the torque is to produce an angular acceleration of the armature, *i.e.*, to increase N . This causes an increase in the back E.M.F. and cuts down the armature current, thus decreasing the torque.

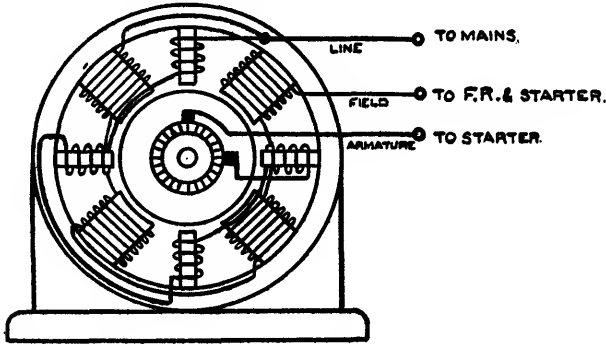
The torque opposing rotation, due to friction and air resistance, also increases with the speed. Thus the nett torque available to accelerate the armature decreases as the speed increases. Eventually a speed is reached at which the frictional torque exactly balances the driving torque (decreased by the decrease in armature current), and there will be no further tendency to accelerate. The motor will run at this steady speed until the conditions change.

This speed may be altered in three ways:—

- (1) The terminal volts may be altered. This is unlikely, as the supply is usually constant voltage.
- (2) The resistance in the field regulator may be altered.
- (3) The opposing mechanical torque may be altered by causing the rotating armature to do mechanical work, *e.g.*, by running a belt round the shaft, which drives a pulley or flywheel.

240. Armature Reaction in Motors.—Just as in generators, the flux due to the armature current combines with the field magnet flux to produce a resultant field which is not symmetrical about the geometrical neutral axis. Fig. 97 shows that in the case of the motor the electrical neutral axis lags behind the geometrical neutral axis in the direction of rotation. Hence, in motors, the brushes must be set at a lagging angle, as opposed to a leading angle in the case of generators. To effect sparkless commutation the lagging

angle must be increased or interpoles may be used. For a motor, the polarity of an interpole must be the same as that of the main pole immediately behind it in the direction of rotation.



Shunt Motor with Interpole Windings.

FIG. 99.

241. Energy Conversion in Armature.—In the case of a motor we have to consider the efficiency of conversion of electrical energy into mechanical energy. The following notation will be used:—

- V = applied volts across armature.
- I_a = armature current.
- R_a = armature resistance.
- I_f = field current producing flux.
- T = driving torque on shaft.
- E = back E.M.F.
- N = speed.

The electrical power supplied to the machine = VI_a .

By Kirchhoff's Law, $V = I_a R_a + E$.

$$\therefore VI_a = I_a^2 R_a + EI_a.$$

$I_a^2 R_a$ is the copper loss in the armature. The residue of power EI_a represents the electrical power directly converted to mechanical power.

The efficiency of the motor is

$$\frac{\text{mechanical power output}}{\text{electrical power input}} = \frac{EI_a}{VI_a} = \frac{E}{V}$$

It is therefore greater the more nearly equal the back E.M.F. is to the applied volts, *i.e.*, the smaller the armature current. Thus a motor is most efficient when running light and taking least power (VI_a) from the mains. As the load increases, the efficiency falls off.

242. Motors, like dynamos, are classified according to the nature of the magnetic field excitation. Thus we may have separately excited, shunt, series and compound-wound motors.

Separately excited and shunt motors behave very similarly in practice, for in each case the field coils are directly across the applied voltage and so the flux is independent of the motor variables.

As mentioned in para. 239, the speed (N) of a motor may be altered by variations of torque or load (T), field flux (Φ), and applied voltage (V).

A detailed investigation of the effect on the speed of variation of these three quantities is fairly complicated. We may simplify the argument by considering what happens in each case if two of them are kept constant and only one is varied at a time. A further simplification in obtaining a general idea of how the speed of a motor varies under these conditions is to assume that the back E.M.F. (E) remains an approximately constant proportion of the applied voltage. This is justifiable in the practical case, because it was seen in the last paragraph that for efficient running the back E.M.F. (E) must be nearly equal to the applied E.M.F. (V). The variation in back E.M.F. is much smaller than the other variations considered. Since the power converted to mechanical power is $E I_a$ watts, we may thus assume that power output $P \propto I_a$. Actually there is a slight decrease in proportional power output as I_a increases, for the back E.M.F. falls off slightly. Expressed in mechanical quantities the power output is equal to the torque on the shaft multiplied by the speed of the motor, and so $P \propto NT$.

It follows therefore that $NT \propto I_a$.

The other general relations we have already obtained are

$$T \propto I_a I_f$$

and

$$E \propto N \Phi \propto N I_f.$$

243. Speed Variation of Shunt Motor.

(a) **Variation of flux (and therefore I_f) only.**—Since E remains approximately constant and $E \propto N I_f$ it follows that N varies inversely as I_f . Thus if the field current is decreased, the speed increases and *vice versa*.

The action is approximately as follows. Suppose that while the motor is running at a steady speed the field current I_f is reduced. The back E.M.F. falls momentarily and the armature current I_a therefore increases. This increase in I_a more than makes up for the fall in I_f , and so the torque increases and the motor is speeded up. This increases the back E.M.F. again and diminishes I_a and therefore the torque. When the torque is reduced to its former constant value the motor is thus running at a higher speed than before. The back E.M.F. is nearly the same as before, the fall in I_f being nearly compensated by the increase in speed.

The field current is decreased by increasing the field resistance and *vice versa*. Thus increase of field resistance increases the speed; decrease of field resistance decreases the speed.

(b) **Variation of load (torque T) only.**—From the relation $T \propto I_a I_f$ it follows that if I_f is constant, then $T \propto I_a$. The general relation $NT \propto I_a$ therefore shows that in this case N is constant. Owing to the slight decrease in back E.M.F. as I_a increases, the speed falls off slightly for large armature currents, but practically the shunt motor may be looked upon as a constant-speed machine for varying load.

(c) **Variation of applied voltage (V) only.**—If I_f and T are kept constant, it follows from the relation $T \propto I_a I_f$ that I_a remains constant and so the back E.M.F. (E) must increase as the applied voltage increases. $E \propto NI_f$; thus for constant I_f , the speed increases with the back E.M.F. and therefore with the applied voltage.

This is a possible method of varying the speed in a separately excited motor, but is hardly practicable in a shunt motor, for any increase of V increases I_f correspondingly, and the strengthened flux counterbalances the effect on the speed of the increase in applied voltage.

244. Series Motor.—The flux in this case depends on the armature current, which is also the field current, or proportional to it. The flux therefore varies according to the magnetisation curve of the machine. At saturation and above, the flux is approximately constant, as in the shunt motor, so that for large armature currents the behaviour of the series motor approximates to that of the shunt motor.

Below saturation the flux Φ may be taken as approximately proportional to the field current I_f , which, in the case of the series motor, is proportional to I_a , the armature current. Thus the relation $T \propto I_f I_a$ becomes $T \propto I_a^2$. As the general relation $NT \propto I_a$ still holds, it follows that N must be inversely proportional to I_a , and therefore inversely proportional to the square root of the load (\sqrt{T}). In other words, as the load increases, the speed decreases and *vice versa*.

When running light (I_a and therefore T small), N will be very large. Hence, in order to avoid dangerous starting speeds, a series motor should always be started on load. By keeping N small, this also gives a big starting torque, so that a series motor is used where quick starting up on load is desirable, e.g., in electric traction.

To increase the speed with a constant load, the relations $T \propto I_f I_a$ and $NT \propto I_a$ show that N varies inversely as I_f . To keep the load constant, I_a must increase as I_f decreases, so that a method has to

be devised of varying the field current without affecting the armature current correspondingly. A controller is used which changes the field windings from a series to a parallel arrangement. For instance, in a four-pole machine, the four field windings in series gives the lowest speed. For successive increases of speed the field windings are arranged two in series, two in parallel, and finally all in parallel. A resistance in parallel with the field windings, called a "Diverter," may also be used to decrease the field flux.

Example on Speed Variation (26).

The particulars of a shunt motor are shown in Fig. 100. It is required to find the speed of this motor when a resistance of 2.5 ohms is inserted in the field regulator, the load remaining unaltered, and assuming that the flux is proportional to the field current.

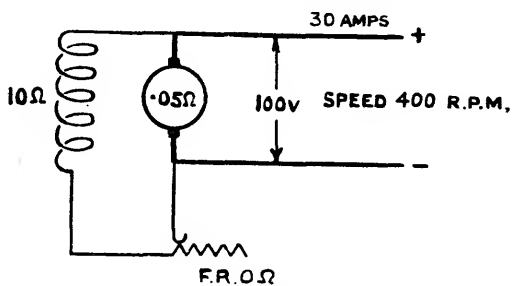


FIG. 100.

At 400 r.p.m., $I_f = 100/10 = 10$ amps.

$$\therefore I_a = 30 - 10 = 20 \text{ amps.}$$

$$\therefore R_a I_a = 20 \times 0.05 = 1 \text{ volt.}$$

$$\therefore \text{Back E.M.F.} = 99 \text{ volts.}$$

$$\text{Back E.M.F.} = K N I_f = 400 \times 10 \times K.$$

$$\therefore K = \frac{99}{4,000}.$$

At final speed, $I_f' = 100/12.5 = 8$ amps.

Load is unaltered, i.e., $T \propto I_a \Phi \propto I_a I_f$ is constant,

$$\text{i.e., } I_a' I_f' = I_a I_f$$

$$I_a' \times 8 = 20 \times 10.$$

$$\therefore I_a' = 25 \text{ amps.}$$

$$\therefore R_a I_a' = 0.05 \times 25 = 1.25 \text{ volts,}$$

and new back E.M.F. = $100 - 1.25 = 98.75$ volts.

$$\text{i.e., } K N' I_f' = 98.75$$

$$\frac{99}{4,000} \times 8 \times N' = 98.75.$$

$$\therefore N' = \frac{98.75 \times 4,000}{8 \times 99} = 500 \text{ r.p.m. nearly.}$$

245. Starting Resistance.—The armature resistance of a motor is made as small as possible, to diminish the power loss in the armature, and is only designed to carry the armature current at the normal running speed. At this speed the back E.M.F. ($= KN \Phi$) is nearly as large as the applied voltage, so that the armature current is small. When the motor is starting up there is no back E.M.F., the full supply voltage will be across the armature, and the consequent heavy current will burn out the armature winding. This is prevented by inserting a large resistance in series with the armature winding when starting the motor, and cutting it out as the speed increases. The arrangement for doing this is called a motor starter. In addition to performing this primary function it also embodies, in general, other safety devices. The starting resistance may be cut out by hand or by automatic methods, giving two types of starter.

Example 27.

Let the armature resistance of a certain machine be $\cdot 05$ ohm and the applied voltage 100 volts. Suppose at full speed the back E.M.F. developed = 98 volts.

$$\text{Then the armature current} = I_a = \frac{100-98}{\cdot 05} = \frac{2}{\cdot 05} = 40 \text{ amps.}$$

If the full 100 volts were applied to the armature when it was at rest, a current of $\frac{100}{\cdot 05} = 2,000$ amps. would flow, which would certainly burn it out if it did not blow a fuse.

Consequently we must use a starting resistance (R_s) to limit this current to (say) 60 amps., as follows:—

$$I_a = \frac{V}{R_a + R_s} \text{ or } R_s = \frac{V}{I_a} - R_a = \frac{100}{60} - \cdot 05 = 1\cdot 66 - \cdot 05 = 1\cdot 61 \text{ ohms.}$$

Hence we shall need a resistance in the starter of 1.61 ohms, which will be cut out gradually as the machine gathers speed and starts generating a back E.M.F.

A current is needed at starting rather larger than when the machine has reached its normal speed, on account of the inertia of the armature.

The power taken from the mains $= V \times I$. Of this a power of $I_a^2 \times R_a$ will be expended in heating losses in the armature.

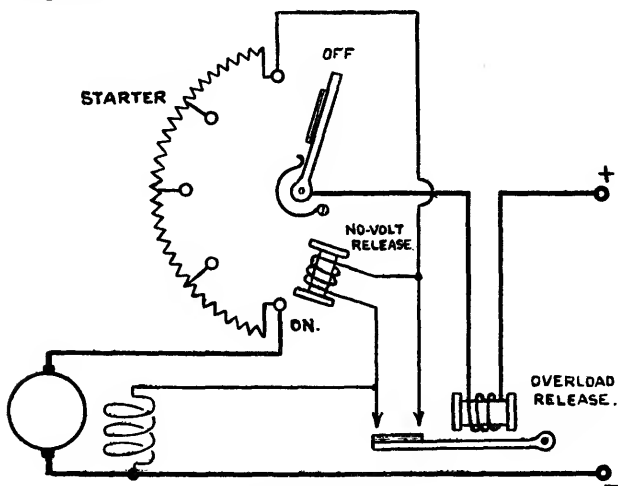
The power developed by the motor, including that available at the motor shaft for driving the load and expended in friction, windage, eddy current and hysteresis losses $= E \times I_a$.

In the above example, the power taken from the mains will be $100 \times 40 = 4,000$ watts.

Of this, $40^2 \times \cdot 05 = 80$ watts is expended in heating the armature.

The remainder, $98 \times 40 = 3,920$ watts, represents the total power developed by the motor, neglecting the heating losses in the field magnet windings.

246. Hand Starter.—A typical hand starter for a shunt motor is shown in Fig. 101.



Motor Starter.

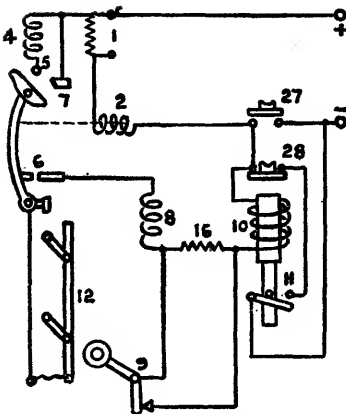
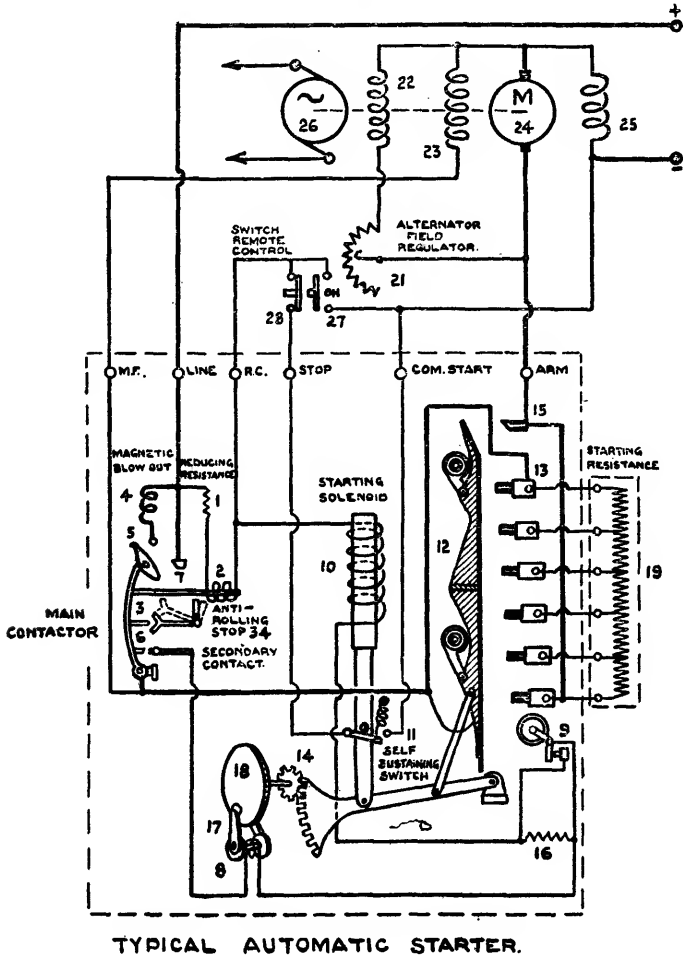
FIG. 101.

As the starter arm is moved downwards, it cuts out resistance in the armature circuit. In the type shown this resistance is inserted in the field circuit instead, but as the field resistance is considerably greater than the starting resistance, this is unimportant. When the starter arm is at "ON" the starting resistance is completely cut out of the armature circuit, and so by this time the motor should be near its normal speed. A spring control prevents the starter arm from being moved over too fast. At "ON" the arm is held against this spring by a small electromagnet energised by the field circuit, and called the "no-volts" coil. Should the supply fail, this coil ceases to be energised, and the arm returns to the "OFF" position under the action of the spring, thus disconnecting the motor from the supply.

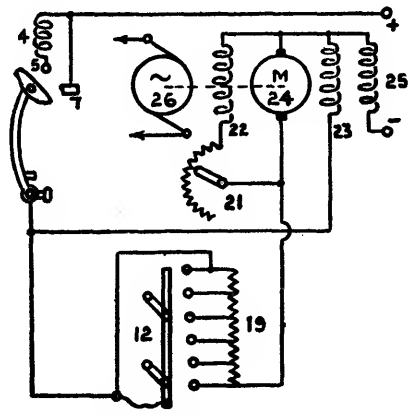
If the supply current exceeds a definite value, the "overload release" coil attracts a pivoted conducting arm, which short-circuits the no-volts coil, thus disconnecting the motor as above.

In neither case can the motor be restarted without the starting resistance being in the armature circuit, and so damage to the armature windings is prevented.

247. Automatic Starter.—The automatic starter enables a machine to be switched on or off from one or more positions remote from the starter by simply making or breaking a switch. A typical



Operating Circuit.



Motor Starting Circuit.

FIG. 102.

Service automatic starter is shown in Fig. 102. The starter arm (12) cuts out the starting resistance (19) from the armature circuit, but does not insert the resistance (19) in the motor field circuit, as in the case of the hand starter discussed above.

Action.—When the “ON” push (27) is made by the operator the circuit is completed from positive, through line terminal, reducing resistance (1) and main contactor solenoid (2) to negative. The reducing resistance (1) adjusts the value of the current in the contactor solenoid (2) according to the input voltage used, and is therefore of different value for varying voltages.

When the main contactor solenoid (2) is energised, it attracts the armature of the anti-rolling stop (34) and allows the main contactor to be pulled on. The anti-rolling stop (34) prevents the contactor from making accidentally when the ship rolls, or by concussion during gun fire, &c.

Three contacts are closed when the contactor is pulled on.

The magnetic blow-out contact (5) makes first, as it is required to break last and allow the blow-out coil (4) to perform its function of blowing out any arcing which may occur at the main contact (7).

The secondary contact (6) makes next and completes the circuit from the contactor through the eddy current brake field coil (8), the economy resistance switch (9), the starting solenoid bobbin (10), and the “ON” push (27). At this stage the economy resistance (16) is short-circuited by the economy resistance switch (9).

The main contact (7) of the contactor makes last and short-circuits the magnetic blow-out coil (4), completes the circuit directly to the motor field (23), and through the whole of the starting resistance (19) to the motor armature (24).

The starting solenoid (10) being energised, the core moves upwards, performing the following functions:—

- (a) The self-sustaining switch (11) is allowed to close and thus provides an alternative path through the “OFF” push (28) for the starting solenoid current, and allows the “ON” push (27) to be released. The “ON” and “OFF” pushes are operated against the tension of a spring. In the normal position the “ON” push (27) is broken and the “OFF” push (28) made.
- (b) By the link mechanism shown, the contact arm (12) is forced sideways against the stops of the starting resistance (19), first making contact with the uppermost stop (13), and then with the other stops in order, and so cutting out the starting resistance (19) in steps.
- (c) The gearing between the horizontal link and the pinion on the axle of the copper armature (18) sets the armature in rotation. It moves between two pole faces (17), energised by the coil (8), and so has eddy currents

induced in it, which tend to stop its motion. It thus acts as a brake on the upward motion of the starting solenoid core and governs the rate at which the starting resistance is cut out. The strength of the braking action is adjusted by varying the distance apart of the pole faces (17).

- (d) In its final motion the starter arm (12) at its lower end makes contact with the roller contact of the economy switch (9), and thus short-circuits the eddy current brake coil (8). At the same time the other contact of the economy switch is broken, thus taking the short-circuit off the economy resistance (16), which is then in series with the solenoid coil. This prevents overheating of the coils. At its upper end the arm makes contact with a separate stop (15), giving a complete short-circuit of the starting resistance (19) through the contact arm (12), in case of bad contact with the intermediate stops.

When it is desired to switch off the motor, the "OFF" push (28) is pressed and the contacts on the main contactor break in the reverse order to that in which they made. The main contact (7) breaks first so that the starting solenoid circuit (made through the secondary contact (6)) remains energised for an instant and the starter arm (12) is held on for a short time. Thus any arcing due to the sudden stoppage of current in a highly inductive circuit takes place at the main contact (7), and damage from arcing between the starter arm (12) and the stops is prevented. As the magnetic blow-out contact (5) breaks last, arcing at the main contact (7) is blown out, the field of the blow-out coil (4) bowing the arc out to a length at which it cannot persist.

In starters handling smaller power, a separate contactor is dispensed with, and the magnetic blow-out functions directly at the starter arm.

An overload release coil may also be included if no provision for it has been made at another part of the circuit. Its operation is similar to that in the hand starter, but as, obviously, a no-volt release coil is not necessary in an automatic starter, the overload release coil in this case breaks the solenoid coil circuit.

248. Motor Generators.—Even when the main power supply for W/T purposes is electrical, that which is directly available may be at an unsuitable voltage, or it may be D.C. power when A.C. power is necessary. Under these conditions the main supply is utilised to drive an electric motor, which in turn supplies mechanical power to operate a D.C. generator or alternator suited to the required conditions. Such an arrangement is called a motor-generator, or motor-alternator, or rotary converter, according as the output is D.C. or A.C.

In the D.C. case the motor and generator may—

- (i) Be entirely separate and have their shafts coupled together ;
or
- (ii) Be built in one casting, but have separate pole-pieces, armatures and commutators ; or
- (iii) In smaller types be arranged in one casting with one set of pole-pieces and one armature, the latter having two distinct windings each with its own commutator at either end ("Dynamotor").

The step-up or step-down in voltage relative to the voltage of the ship's mains is arranged for by :—

- (a) The relative number of armature bars in each armature ; or
- (b) The relative strength of magnetic fields ; or
- (c) Both.

The motor being shunt-wound—as is usually the case—runs at a practically constant speed, and the generator voltage is not dependent on this factor.

Fig. 103 shows a general arrangement of electrical connections.

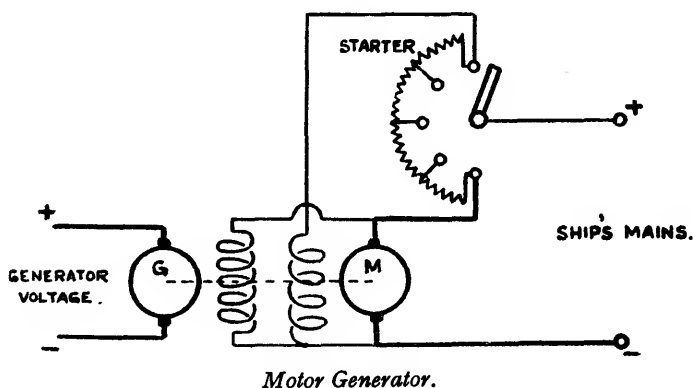


FIG. 103.

Applications of motor generators in the Service are, among others—to supply the low power switchboard (20 volts) for bells, telephones, fire-control, etc. ; for transmitting valves and Poulsen arcs (step-up in voltage) ; for supplying constant current to search-lights at reduced voltage ; for charging secondary batteries, etc.

249. The Motor Alternator.—The motor alternator consists of a motor and alternator on one shaft, as illustrated diagrammatically in Fig. 104.

The motor is fitted with a starter and sometimes with a field regulator if variation in speed (and therefore frequency) is required : a separate regulator is supplied for the alternator field current.

The motor field regulator controls the speed of the motor, hence the speed of the alternator, and hence the alternating frequency, and also the voltage to a certain extent.

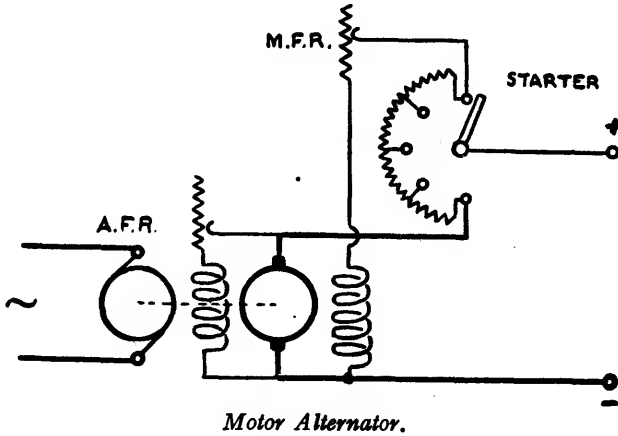


FIG. 104.

The alternator field regulator controls the density of the alternator flux and hence the alternating voltage independently of frequency.

250. Motor-Booster.—This is a particular type of motor generator, consisting of a motor and a generator (the “booster”), keyed to the same shaft. The generator is in series with the supply mains, and so the voltage generated is proportional to the supply current, and will assist or oppose the supply voltage according to the method of connecting up.

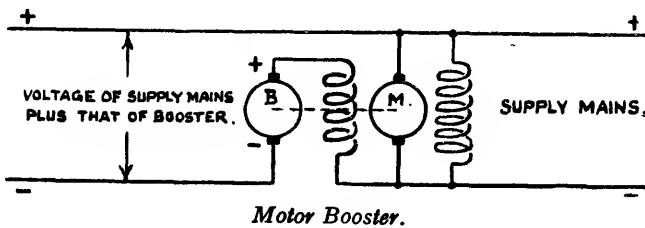


FIG. 105.

The commoner uses of the booster are :—

- (1) To make up for the voltage drop in a long transmission line ;
- (2) To provide the additional voltage across transmitters in ships whose main supply voltage is insufficient for this purpose.

These two uses are essentially the same, and are illustrated by the arrangement shown in Fig. 105.

It will be seen that the positive terminal of the booster is connected to the negative main, so that the booster acts just like an additional cell in a battery. Further, as the current in the leads increases, the booster field, and therefore the booster E.M.F., also increases, compensating for the increased IR drop in the leads.

251. Automatic Voltage Control of Alternators.—The output voltage of an alternator depends on the speed of the machine, and the flux, *i.e.*, the field current. Variations in either or both of these are caused by:—

- (a) Fluctuations of supply voltage. These alter both the speed and the field current, and may be large in ship practice.
- (b) Variations in output current such as are produced, for instance, by switching over from two valves to one in a transmitter. This alters the reaction of the alternator armature and therefore the flux.

The object aimed at in automatic control is to produce such variations of the alternator field current as will automatically compensate for these effects.

The necessary variations of field current are obtained by means of the “reversing booster” and “contactor” circuits shown in Fig. 106.

The reversing booster is a small generator with two oppositely-wound fields. When the supply voltage and alternator output are steady these fields have equivalent ampere-turns and neutralise each other, so that no booster E.M.F. is generated.

If the alternator voltage rises above normal, the “opposing” booster field is made to take a larger current by the operation of the contactor, and the booster generates an E.M.F. acting against the supply voltage to the alternator field. The alternator field current is thus reduced and the output voltage falls. If it drops below normal, the reverse occurs; the “assisting” booster field is made to predominate and the booster E.M.F. augments the supply voltage to the alternator field. The alternator field current rises, and therefore the output voltage.

252. These processes are made automatic by means of the contactor circuit shown inside the dotted rectangle in Fig. 106.

The outer ends of the booster fields are brought to two contacts A and B, across which is the combined resistance of R_{3a} and R_{3b} . This ensures that the circuit to either booster field is never broken during the operation of the contactor. The common point of R_{3a} and R_{3b} is joined to the pivot F of a conducting arm DE, which rocks about F so that either D makes contact with A, in which case R_{3a} is short-circuited, or E makes contact with B, short-circuiting R_{3b} .

The movement of the contactor arm is mechanically controlled by a spring. It is electrically controlled by an electro-magnetic system across the alternator output terminals. The pull of this system thus varies with the alternator output. The other details of the contactor circuit perform subsidiary functions which are explained below.

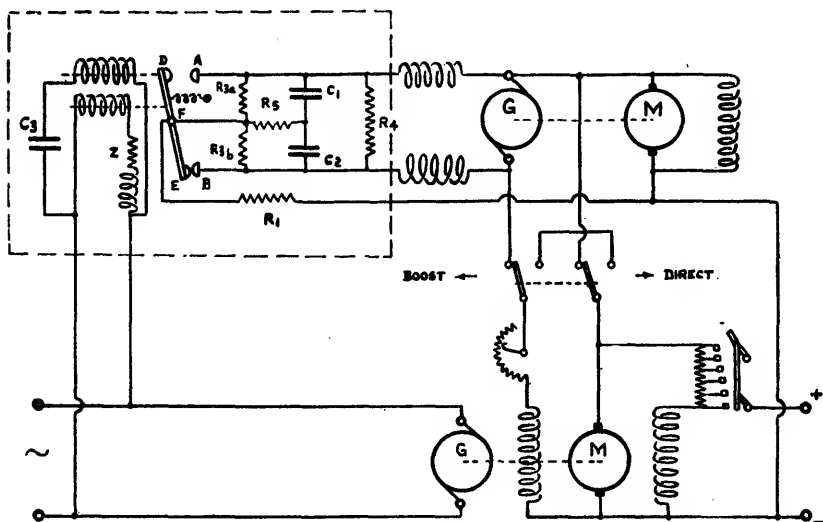


FIG. 106.

Action.—Suppose the supply voltage is switched on. Contacts AD will then be closed. R_{3b} is in the path through the opposing field, but R_{3a} in the parallel path through the assisting field is short-circuited. Hence the assisting field predominates, and the corresponding increase in alternator field current quickly raises the output voltage. When it rises above normal, the attraction of the electromagnetic system overcomes the pull of the spring, and the contactor arm swings over to the position shown in diagram, breaking contact at A and making it at B. Thus R_{3b} is short-circuited and R_{3a} is brought into circuit. The opposing field is now the stronger, and the booster generates an E.M.F. against the supply voltage.

The alternator field current falls and the output voltage likewise. The pull of the spring now overcomes the attraction of the magnet, the contactor arm swings back to its former position, and the sequence of events is repeated. Thus, under working conditions, the contactor arm is continually and rapidly vibrating and the output voltage is kept approximately constant.

The booster circuit is unsymmetrical, as one of the parallel paths contains the armature resistance and the booster E.M.F. Hence, for correct adjustment of the currents in the two field windings, the

resistances R_{3a} and R_{3b} are not equal. Their most suitable values are found by experiment when the design is being arranged.

253. C_1 , C_2 and R_5 .—Whenever contact is broken by the contactor arm rocking, a large current is suddenly brought to zero, and, owing to the self-inductance of the circuit, arcing is liable to occur at the gaps. To obviate this the condensers C_1 and C_2 are put in parallel with the gaps at A and B respectively. It will be seen that, together with the self-inductances of their respective circuits, these condensers constitute oscillatory circuits and may produce undesirable oscillatory currents if sparking takes place at the gaps. As R_{3a} and R_{3b} are in parallel with these oscillatory circuits, their equivalent series resistances ($\frac{1}{\omega^2 C_1^2 R_{3a}}$ and $\frac{1}{\omega^2 C_2^2 R_{3b}}$ respectively) do not provide sufficient damping. A resistance R_5 in series in both oscillatory circuits is therefore added, large enough to make the condenser discharges non-oscillatory.

R_1 and R_4 are auxiliary resistances, which keep the voltage generated by the booster within suitable limits.

R_1 is in series with the booster and contactor circuits, and is chosen of such a value that a suitable fraction of the supply voltage is applied across these circuits.

R_4 is in parallel with the contactor circuit and limits the current changes in the booster field windings as R_{3a} and R_{3b} are short-circuited in turn. If R_4 were itself a short-circuit, alterations in the resistance of the contactor circuits would have no effect on the currents in the booster fields, while if R_4 were removed such alterations would have their maximum possible effect. R_4 is selected so that the actual effect has a suitable intermediate value between these extremes.

C_3 and Z .—Variations in the speed of the main motor will vary the frequency of the alternator voltage, and therefore the current in the magnet field windings. The pull of the magnetic system would thus alter with the frequency. To prevent this the windings are "frequency compensated." One winding has an inductive resistance Z in series and the other has a condenser C_3 . When the frequency increases the current in the inductive circuit falls, but that in the capacitive circuit rises. The reverse occurs when the frequency decreases. The total magnetising current is thus independent of frequency changes.

254. The Rotary Converter.—A rotary converter is a machine with one set of field magnets, usually shunt wound and excited with direct current.

It has only one armature with a uniformly distributed winding (as in D.C. machines), which has connections to a commutator at one end, and, if single phase, tapings to two slip rings at the other.

In a two-pole machine these tapplings are taken at points 180° apart, in a four-pole machine 90° apart, in an eight-pole machine 45° apart, &c.

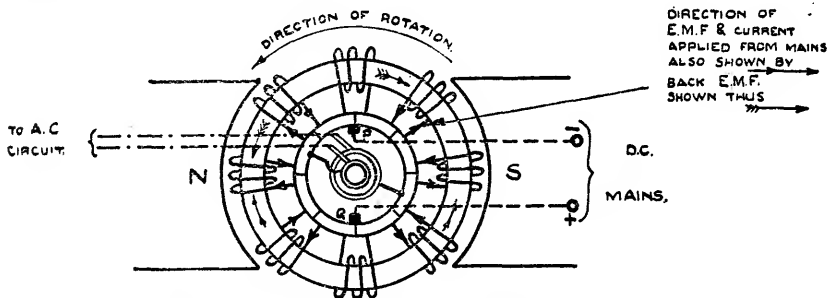
Its uses are :—

- Supplied with D.C. at the commutator end, it runs as a D.C. motor, and A.C. is tapped off from the slip rings. This is the Service application.
- Supplied with A.C. at the slip rings, it runs as a motor, and D.C. is obtained from the commutator end. This is the common application in commercial work.
- If the armature is revolved by an engine, both A.C. and D.C. may be obtained simultaneously. It is then called a "Double Current Generator."

255. Action.—Running as a D.C. motor and delivering A.C. (the Service application).

It is necessary to understand clearly how the back E.M.F. of a motor varies, so we recapitulate the statements given in para. 238.

The "Gramme ring" type of armature is illustrated (Fig. 107) for simplicity, its winding being easier to represent than that of a drum armature.



Diagrammatic Representation of a Rotary Converter.

FIG. 107.

When a motor is running the armature conductors have E.M.F.s induced in them (the dynamo action), which by Lenz's Law oppose the E.M.F. applied by the mains.

The sum of the instantaneous E.M.F.s induced in all the inductors under either pole is called the "back E.M.F."; this is of constant value for a given field magnet flux, speed and load.

The sets of inductors under each pole, at any instant, are in parallel with respect to the D.C. brushes bearing on the commutator (this should be traced out in Fig. 107).

The back E.M.F. (E) will be a little less than the applied voltage (V)—say, 98 volts with 100-volt mains; *i.e.*, 98 volts of those applied balance the back E.M.F. and the other two volts supply the ohmic drop in the armature resistance.

Thus the applied voltage $V = E + I_a R_a$.

256. The variations of P.D. between the tapping points A and B (which is the voltage applied to the A.C. circuit) as the armature rotates may now be considered.

Fig. 108 is a further simplification of Fig. 107, drawn to assist in the following explanation. Only four armature bars are shown, namely, the pair lettered A and B, which are connected through the slip rings to the A.C. circuit, and the pair lettered P and Q, which are in connection with the D.C. brushes at any instant.

For the purpose of this elementary treatment of the rotary converter, the IR drop in the armature will be neglected; in most cases it will be very small owing to the low armature resistance.

Thus we will assume that $E = V$.

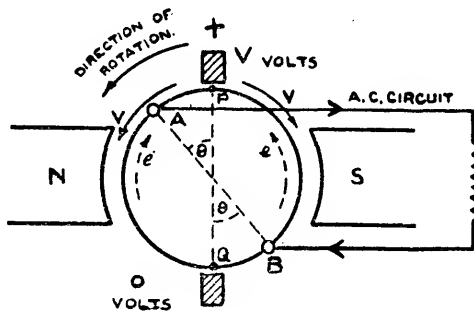


FIG. 108.

When A and B are on the neutral line they will obviously have the P.D. of the mains, (V) or (E), and maximum current will flow in the A.C. circuit.

When A and B have moved through an angle θ from the neutral line, the potential of A with respect to Q will be

$$V - (\text{back E.M.F. in AP} + \text{IR drop in AP}).$$

Ignoring the IR drop, this becomes

$$V - E_{AP} = E - E_{AP}.$$

Similarly, the potential of B with respect to Q will be

$$\begin{aligned} &\text{back E.M.F. in BQ} \\ &= E_{BQ}, \text{ which is equal to } E_{AP}. \end{aligned}$$

Thus the P.D. between A and B

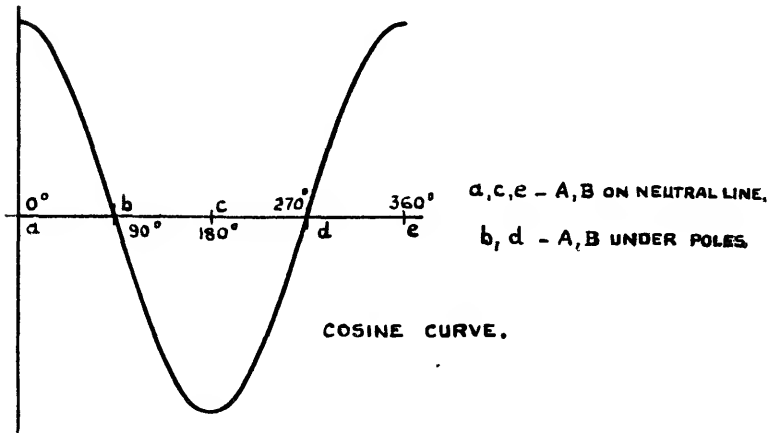
$$= (E - E_{AP}) - E_{BQ} = E - 2E_{AP}.$$

This can be shown mathematically to be equal to $E \cos \theta$.

Fig. 109 shows $E \cos \theta$ plotted and represents the variations in the P.D. between A and B for one revolution. This curve is a cosine curve, which is merely a sine curve moved 90° to the left, so that the P.D. follows a sine law, tracing out one cycle in each revolution.

The following points should be specially noted :—

- (1) When A and B are under the D.C. brushes, $\theta = 0$; therefore $E_{AP} = 0$, and the P.D. $= E = V$.
- (2) When A and B are centrally under the poles ($\theta = 90^\circ$) E_{AP} will be equal to $\frac{1}{2}E$, so that $E - 2E_{AP} = E \cos 90^\circ = 0$.
- (3) A, when between 0° and 90° , has a higher potential than B.
- (4) When $\theta = 180^\circ$ and A is under the negative brush, the P.D. $= -E = -V$, and so on for the next half revolution.



Voltage between Slip Rings.

FIG. 109.

257. As regards the currents : in the position of A and B shown in Fig. 108, the motor current will follow the course ... positive brush, PAQ and PBQ in parallel, to the negative brush ; the alternating current will follow the course ... positive brush, PA, external circuit, BQ, to negative brush.

Both the motor and the alternating currents are supplied by the mains.

Thus the portions of the winding AP and BQ will carry, at this stage, more current than the other portions, causing additional and varying field distortion and armature reaction.

When A has passed beyond the centre of the left pole piece it will be negative to B, and the course of the alternating current will be :—

Positive brush, PB, external circuit, AQ, to negative brush.

258. Armature Reaction.—The armature reaction of a rotary converter, as in dynamos and motors, has important effects in practice.

The A.C. circuit may consist of :—

- (1) A pure resistance load ;
- (2) A combined resistance and capacity load ;
- (3) A combined resistance and inductive load ;
- (4) A combined resistance, capacity and inductive load.

The combined armature reactions are rather complicated to follow, and no attempt will be made here to describe them.

Their effects, in so far as their results affect the practical running of machines is concerned, may be summarised as follows :—

- (1) A pure resistance load causes a small resultant cross-magnetising field and a slight decrease of speed.
- (2) A purely inductive load causes a demagnetising field and consequently an increase of speed.
- (3) A pure capacity load causes a magnetising field and consequently a decrease of speed.

Combinations of these loads give a resulting speed retardation or acceleration, depending on which preponderates.

259. Frequency, Voltage and Current.—The frequency of the alternating current delivered by a rotary converter, as with an alternator, is equal to $\frac{\text{R.P.M.}}{60} \times \text{pairs of poles}$.

The maximum value of the voltage will be a little less than that of the D.C. mains and can never exceed it.

The "R.M.S. value" (*see* next Chapter) will be a little less than $\cdot 707$ of that of the D.C. supply, *e.g.*, in the case of a 100-volt D.C. supply, the R.M.S. alternating voltage which will be given by a rotary converter is a little less than 70·7 volts.

The relations between the alternating and direct currents can be found approximately from considerations of power.

If there were no losses in the machine the alternating output would equal the direct current input (in watts).

Assume, for the sake of argument, a perfectly efficient machine running at 15 amperes on 100 volts direct.

The input is 1,500 watts or 1·5 kilowatts.

The alternating volts are 70·7 R.M.S., so that $70\cdot7 \times \text{R.M.S. current} = \text{output in watts} = \text{input} = 1,500 \text{ watts}$.

The alternating current has therefore an R.M.S. value $\frac{1,500}{70\cdot7} = 21\cdot23$ amperes.

This is greater than the direct current, a result which at first sight appears peculiar. It is accounted for by the fact that current is taken from the mains as well as from the rotary converter. The excess of output current over input will not be quite so marked as is shown by this example, owing to the efficiency of the machine not being 100 per cent., but it is at any rate a factor which may have to be taken into account.

260. With a rotary converter, a normal-type starter and a motor field regulator are provided.

As with a motor, when resistance is inserted in series with the field, the machine speeds up ; thus the frequency of the alternating voltage will be increased.

The value of the voltage remains approximately the same, since it depends on the applied voltage, and is approximately 65 per cent. of that of the D.C. supply.

261. Advantages and Disadvantages of the Rotary Converter.—For W/T purposes, rotary converters are used for moderate power and frequency only.

They are lighter than motor alternators, as they have one armature only, but labour under the disadvantages :—

- (a) That they can only give an R.M.S. voltage less than that of the D.C. supply ;
- (b) That it is very difficult to make them for high frequencies ;
- (c) That the output is not insulated from the input.

THREE-PHASE CURRENT.

262. The following brief description of three-phase currents, and of the motors and generators used with them, is inserted because they are sometimes used in the supply of power to shore stations and to transmitting valve sets.

Three-phase currents are very widely used in the commercial world in the transmission of power over long distances, on account of the fact that they require conductors of a less total cross-section for the conveyance of equivalent power than single-phase currents.

263. It will be noticed in the diagrams of alternators in Figs. 71 and 72 that there are portions of the armature which do not carry any winding.

In a three-phase alternator we fit three separate and distinct windings into the armature, spacing them symmetrically round its circumference, *i.e.*, with similar poles 120° apart.

We thus generate three separate alternating currents. These three currents will not be in step with one another, but will rise to their maximum values in succession, as shown in Fig. 110.

They are said to differ in phase by 120° .

An inspection of this curve will show that when No. 1 is zero, Nos. 2 and 3 are equal and opposite ; when No. 1 is maximum Nos. 2 and 3 are each of half the amplitude of No. 1, and are both of opposite sign to No. 1 ; in fact, that at any moment the sum of the three currents is zero. In consequence, as will be explained

later, only three wires are required to carry them instead of six, as might have been expected.

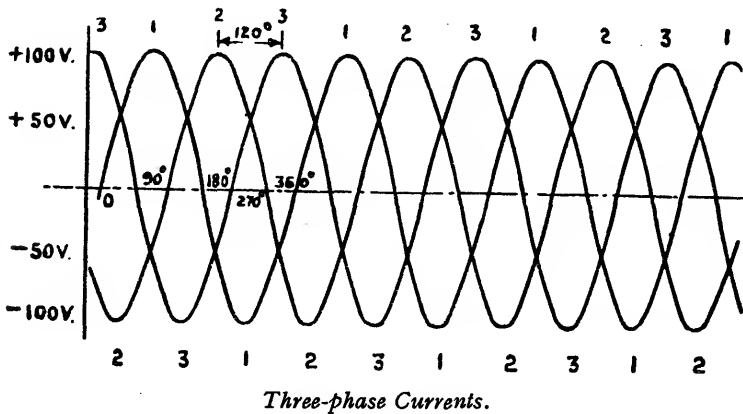
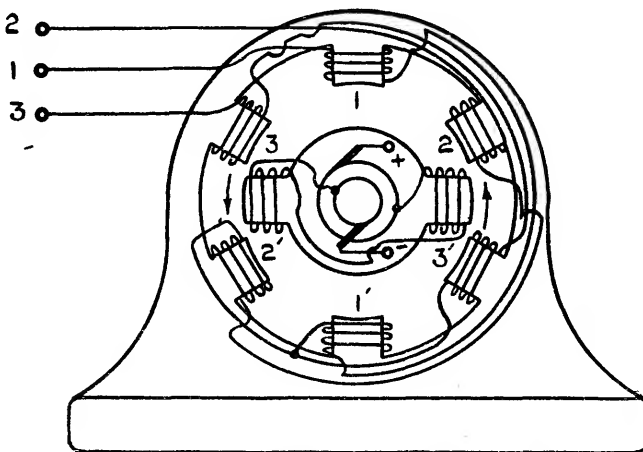


FIG. 110.

264. Three-phase generators are usually of the revolving field type (para. 207). Fig. 111 gives a diagrammatic picture of a typical machine.



Three-phase Generator, or Synchronous Motor.

FIG. 111.

It will be noticed that the poles of the armature are wound alternately right and left-handed, and that similarly wound poles are 120° apart.

The direction of the voltage induced as the field sweeps round will depend on the direction in which each pole is wound.

Let us take the rotor in successive positions 30° apart, and check the direction of the voltage with Fig. 110 :—

- (1) Rotor poles between stator poles 2 and 3 ; result, a falling positive voltage in 3, a rising negative voltage in 2, and zero voltage in 1.
 - (2) Rotor poles opposite 2 ; result, a maximum negative voltage in 2, equal positive voltages in 3 and 1.
 - (3) Rotor poles between 2 and 1 ; result, a rising positive voltage in 1, a falling negative voltage in 2, and zero voltage in 3.
 - (4) Rotor poles opposite 1 ; result, maximum positive voltage in 1, equal negative voltages in 2 and 3 ;
- and so on.

The rotor field must be imagined as spreading out a good deal more than would be expected from Fig. 111, and as influencing adjacent poles ; actually in practice the stator would have more than three pairs of poles, and the rotor more than one.

265. Three-phase Connections.—Both the armature windings of motors and generators, and the loads joined between the wires, may be connected up in one of two ways, viz. :—

- (a) The Delta or Mesh connection.
- (b) The Star or Y connection.

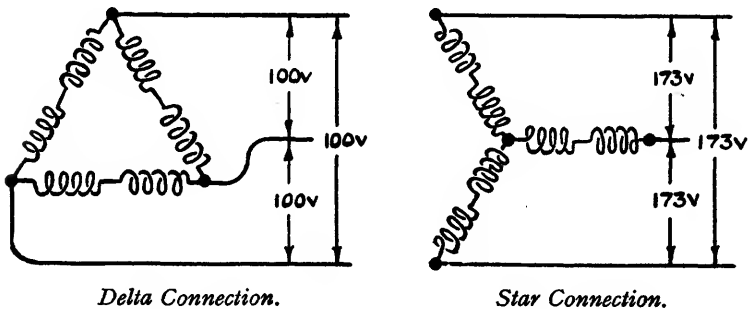


FIG. 112.

With the Delta connection the maximum voltage between any pair of wires is equal to that generated in one armature winding.

With the Star connection the maximum voltage between any pair of wires is equal to $\sqrt{3}$, or 1.732 times that generated in one armature winding.

266. In order to explain why no return wire is needed with the three-phase system, Fig. 113 has been inserted.

The generator windings, and the three 10-ohm resistances joined across them as a load, are both star-connected.

At the moment illustrated the voltage across No. 1 winding is assumed to be 100 volts positive; at the same moment the voltages across Nos. 2 and 3 windings will be -50 volts, as can be seen from Fig. 110.

If a fourth wire were joined up as shown dotted, there would be a current of 10 amps. flowing from right to left in it, counter-balanced by the two currents of 5 amps. due to windings 2 and 3. Consequently no current would flow along the dotted wire, and it may be dispensed with.

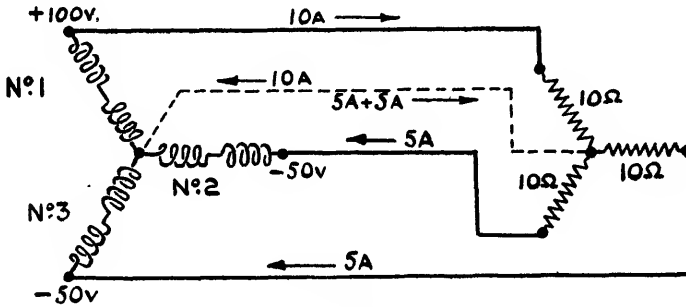


FIG. 113.

This really means that each wire acts as a common return for the other two in turn.

267. The Rotating Field.—A special advantage that three-phase currents possess is the fact that they can be made to produce a rotating field.

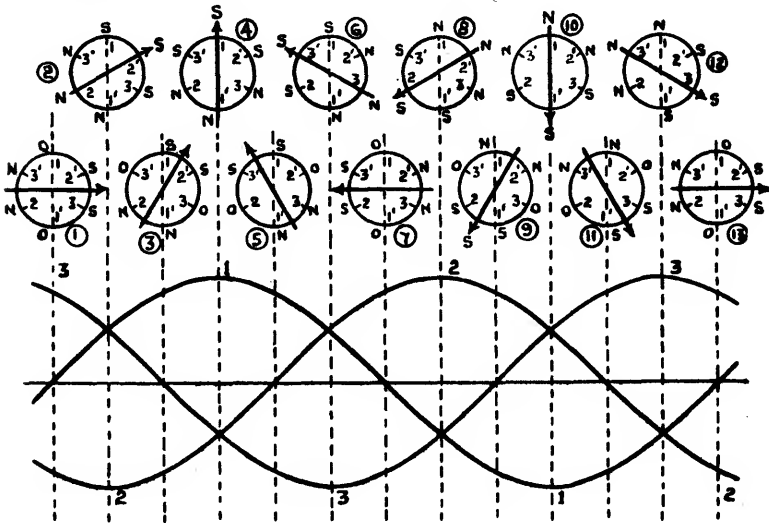


Diagram of Rotating Field.

FIG. 114.

Just as they are generated by the use of a rotating field, so when they are applied to a motor they produce a rotating field.

Consider the effect of the three-phase currents shown in Fig. 114, as applied to the field of the Induction Motor shown in Fig. 115; remember that a positive current through a right-hand pole will produce the same polarity as a negative current through a left-hand pole.

Moment 1. Field of poles 1, 1' zero; fields of poles 2, 2' and 3, 3' equal; resultant midway between 2, 2' and 3, 3'.

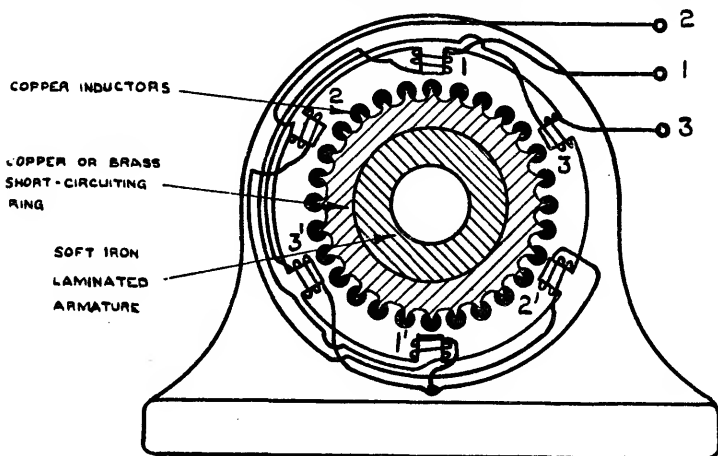
Moment 2. Field of poles 2, 2' maximum; fields of poles 1, 1' and 3, 3' symmetrical on either side; resultant across 2, 2'.

Moment 3. Field of poles 3, 3' zero; fields of poles 2, 2' and 1, 1' equal; resultant midway between 2, 2' and 1, 1'.

Moment 4. Field of 1, 1' maximum; fields of poles 2, 2' and 3, 3' symmetrical on either side; resultant across 1, 1'; and so on.

Thus the position of maximum field strength will rotate at the same frequency as that of the applied alternating current.

268. The Induction Motor; Squirrel-cage Type.—A very simple type of motor is that illustrated in Fig. 115. The rotor simply consists of a number of copper inductors joined together at each end.



Induction Motor, Squirrel-cage Type.

FIG. 115.

Action.—The effect of the rotating field will be to induce alternating currents in the rotor inductors.

The reaction of the fields produced by these currents with the rotating field of the stator will cause the rotor to move, and it will speed up until it is revolving at nearly the same pace as the stator field.

This is in accordance with Lenz's Law, which, put colloquially, says that all inductive effects are suicidal in tendency.

The rotor would like to run at exactly the same speed as the stator field, but then there would be no currents induced in it. It therefore runs a little more slowly, and "slip" is set up. "Slip" increases slightly with the load.

269. The Induction Motor : Wound Rotor Type.—The squirrel-cage type is very useful for comparatively small loads, but it fails when heavy loads have to be started up from rest.

For starting against heavy loads a "Wound Rotor" type is used. (In England the majority of motors of 10 H.P. and over are furnished with wound rotors.)

In this type the rotor is wound with a three-phase winding, connected to three slip rings.

The starter used is a group of three resistances connected in star or delta and joined between the slip rings.

As the machine gathers speed these resistances are cut out till eventually they are out of circuit, the windings are short-circuited on themselves and the brushes are raised. The machine then behaves in the same manner as the squirrel-cage.

270. The Synchronous Motor.—In cases where an absolutely constant speed is required, without slip, a "Synchronous Motor" is used.

This is very similar in design to the generator shown in Fig. 111, and the same diagram may be used.

Its drawback is that it is not self-starting. To start it we must disconnect the three-phase supply from its stator, and rotate it mechanically by the use of (say) an induction motor on the same shaft, until its rotor field is revolving at the same frequency as that of the three-phase supply.*

If then the three-phase supply is switched on to the stator, the rotating field of the stator will drive the rotor round without any slip.

If, however, current is switched on without these precautions, a heavy short-circuit current flows which may damage the machine.

271. It may be useful to remember that three-phase generators and synchronous motors with stator armatures have two slip rings supplied with D.C. ; wound rotor induction motors have three slip rings, and squirrel-cage induction motors have no slip rings.

* Synchronous motors are sometimes started by connecting them directly to the mains, the eddy currents induced in their field systems and in copper grids fitted in slots in the pole faces being sufficient to run them up to speed, as induction motors, on no load. This involves taking a very large current from the mains, and for machines of large size, auto-transformers (para. 370) are employed to reduce the voltage applied to the stator windings on starting.

CHAPTER V.

ALTERNATING CURRENT.**GENERAL PRINCIPLES.**

272. In the preceding chapter we saw that electric currents are generated in two different forms, viz., direct current and alternating current.

The former is generated by means of a D.C. dynamo, and the latter for W/T purposes by means of a (motor) alternator or a rotary converter.

In this chapter we are particularly concerned with the latter—alternating current—its measurement, and its behaviour under various conditions.

273. Radio and Audio Frequencies.—So far we have been speaking of alternating current as being produced by an alternator, and its frequency as depending upon the number of poles of the alternator and its speed of revolution.

Such currents are spoken of as “**audio-frequency**” currents.

In wireless telegraphy, we meet with alternating currents ranging from, say, 10,000 cycles per second, up to, say, twenty to thirty million per second, produced by means of transmitting arcs and transmitting or receiving valves, by methods which will be described later.

Such currents are spoken of as “**radio-frequency**” or “**oscillatory**” currents. The division between radio- and audio-frequency currents is not at all sharply defined, and both kinds obey exactly the same laws, if we assume that they follow, or approximate to, the Sine Law.

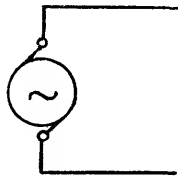


FIG. 116.

In Chapter VII, a method of producing a high frequency oscillation by means of a “**spark oscillator**” is described. The high frequency current there described is slightly different in form, since it starts at a maximum amplitude and dies away to zero, but it also obeys the laws laid down in this chapter.

For our immediate purpose, then, we shall consider that we can obtain currents at any frequency we like, without explaining how they are produced in practice.

In diagrams, the source of alternating current will be represented as in Fig. 116, which indicates "a source of alternating E.M.F. of any required frequency."

274. Measurement of Alternating Current.—In measuring the value of an alternating current, it is not convenient to measure its **maximum** value, *i.e.*, the value it reaches at the top of each half cycle of the current curve.

If an alternating current were passed through a D.C. measuring instrument whose pointer deflection was proportional to the first power of the current flowing, the rapid alternations in value would not be followed by the pointer of the instrument; it would simply register the average value of current, which would be zero, as is obvious from the shape of the curve which represents the variation of an alternating quantity with time. The value of any current may, however, be measured by its heating effect upon a conductor of given resistance, and therefore a hot wire ammeter (described in Chapter III) is suitable for our purpose.

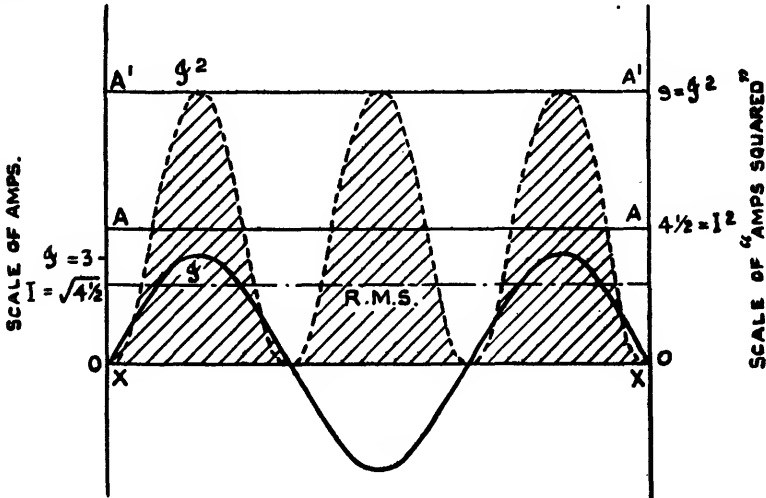


FIG. 117.

The heating effect is independent of the direction in which current is flowing, positive or negative, and so, even although the actual value of the current is continuously changing, an indication will be given by the instrument of the average heating effect, and hence of the magnitude of the current.

We shall deal first of all with a current whose wave form is assumed to be sinusoidal, and is therefore represented by the formula

$$i = I \sin \omega t.$$

In Fig. 117 is drawn a curve showing an alternating current of this type for which I , the maximum value in each half-cycle, is 3 amperes.

The power expended in producing heat by a current of i amperes in a resistance of R ohms is $i^2 R$ watts.

Let the resistance in this case be 1 ohm.

The instantaneous rate at which heat is produced is given by $i^2 \times 1$ watts, *i.e.*, i^2 watts. So that, if we draw the dotted curve, which represents i^2 and so rises each half-cycle to a maximum value of 9, it will represent the variations in the power expended in watts.

The total amount of heat produced, or the total **energy** dissipated, is given by summing up the small amounts of energy dissipated at each instant, *i.e.*, by summing the products of the instantaneous powers multiplied by the small intervals of time during which they are available.

This is simply, therefore, the area contained between the dotted curve and the axis, which area gives the total energy in joules, the axis being an axis of time.

If now we had been considering the total energy which would be dissipated by a direct current of 3 amps. during the same time t , it would be given by $9t$ joules. Graphically, it would be the area enclosed between the horizontal line $A'A'$, where $OA' = 9$, and the axis.

Now the area enclosed between the curve and the axis is exactly **half** this rectangle $A'A'XX$.

Therefore the energy dissipated by the alternating current is half that dissipated by the direct current which has the same maximum value.

The direct current which would give the **same** heating effect would be such that the rectangle $AAXX$ represented the energy, where $AX = 4\frac{1}{2}$.

In other words

$$I_{dc}^2 \times R = I_{dc}^2 \times 1 = 4\frac{1}{2}$$

$$\text{or } I_{dc} = \sqrt{4\frac{1}{2}} = \sqrt{\frac{9}{2}} = \frac{1}{\sqrt{2}} \times 3$$

$$= \frac{1}{\sqrt{2}} \times \text{the maximum value of the alternating current.}$$

In general, the direct current giving the same heating effect is such that

$$I_{dc}^2 R t = \frac{1}{2} J^2 R t$$

$$\text{or } I_{dc} = \frac{1}{\sqrt{2}} J = 0.707 J.$$

This equivalent direct current, which is equivalent in heating effect to an alternating current whose amplitude is J , is known as the **effective value**, or **virtual value**, or **root mean square** value of the alternating current, and is 0.707 of the maximum value.

"Root mean square" is usually written R.M.S. The name is derived from the fact that it is the square root of the average (or

mean) value of the squares of all the different values the current can take during a complete cycle. The standard notation is that R.M.S. values are written in "square" capitals, as I , maximum values in "curly" capitals, as \mathcal{I} , and instantaneous values in small letters, as i .

$$I = 0.707\mathcal{I}, \text{ and } \mathcal{I} = \sqrt{2} I = 1.414 I.$$

It is obvious that a sinusoidally varying quantity is determined in magnitude by either its maximum value or its R.M.S. value.

Alternating voltages are likewise measured by either maximum values or R.M.S. values, the relationship between them being as before.

An A.C. voltmeter measures R.M.S. values only, so that if it reads 200 volts, the voltage is rising and falling between zero and a maximum value of $200\sqrt{2} = 282.8$ volts.

For power calculations and for finding the sizes of cables necessary to carry currents we are concerned with R.M.S. values of current and voltage; while for determining the thickness of insulation necessary, the strength of a dielectric, or the instant at which a spark gap breaks down, we are concerned with maximum values of voltage.

Remember that in any calculations where current and voltage are interdependent, if we start with a maximum value of voltage the answer will be given as a maximum value of current, and *vice versa*.

***275. Alternating Current Measurement (continued).**—The results of the last paragraph can be quickly and accurately obtained by methods utilising the calculus, and also extended to cases where the waveform is not sinusoidal.

(a) **R.M.S. value of a pure sine-wave alternating current.**

Let the current be $i = \mathcal{I} \sin \omega t$.

The energy dissipated in a resistance R ohms during a small time $dt = i^2 R dt$

$$= \mathcal{I}^2 R \sin^2 \omega t dt.$$

Total energy dissipated in a complete cycle

$$\begin{aligned} &= \int_0^{2\pi} \mathcal{I}^2 R \sin^2 \omega t dt \\ &= \mathcal{I}^2 R \int_0^{2\pi} \frac{1 - \cos 2\omega t}{2} dt \\ &= \mathcal{I}^2 R \left[\frac{t}{2} - \frac{\sin 2\omega t}{4\omega} \right]_{t=0}^{t=\frac{2\pi}{\omega}} \\ &= \mathcal{I}^2 R \cdot \frac{2\pi}{2\omega} = \frac{\mathcal{I}^2 R}{2} \times \text{time of complete cycle.} \end{aligned}$$

A direct current of value I would dissipate as much energy in the same time if

$$I^2 R \times \text{time of complete cycle} = \frac{J^2 R}{2} \times \text{time of complete cycle.}$$

$$\therefore I^2 = \frac{J^2}{2},$$

$$\text{and } I = \frac{J}{\sqrt{2}} = 0.707J.$$

(b) **Average Value of Current per half-cycle.**—This is sometimes required, and the value is obviously

$$\begin{aligned} \frac{\int_0^{\frac{\pi}{\omega}} J \sin \omega t \, dt}{\frac{\pi}{\omega}} &= \frac{J \left[-\frac{\cos \omega t}{\omega} \right]_0^{\frac{\pi}{\omega}}}{\frac{\pi}{\omega}} \\ &= J \times \frac{2}{\pi} = 0.637J. \end{aligned}$$

(c) **Alternating Current not sinusoidal.**—Many alternating quantities met with in A.C. work are not pure sine waves in form, although repeating themselves at definite intervals. By a well-known theorem called Fourier's Theorem, any such wave form can be represented by an equation (taking the alternating quantity as a current) of the form:—

$$i = I_0 + J_1 \sin (\omega t + \phi_1) + J_2 \sin (2\omega t + \phi_2) + \text{etc.}$$

The term I_0 simply indicates that there is a preponderance of current in one direction, and superimposed on this are sinusoidal variations of different amplitudes, different periods and different phases.

As we are only considering alternating quantities, it is sufficient to take the terms

$$i = J_1 \sin (\omega t + \phi_1) + J_2 \sin (2\omega t + \phi_2) + \text{etc.,}$$

each of which has an average value of zero over a certain period of time, just as a single sine wave has.

These frequencies are seen to be made up of a frequency $\frac{2\pi}{\omega}$ and multiples of it, twice as much, three times as much, etc.

The frequency $\frac{\omega}{2\pi}$ is known as the "fundamental" or "first harmonic" frequency,

$$2 \times \frac{\omega}{2\pi} \text{ as the "second harmonic,"}$$

$$3 \times \frac{\omega}{2\pi} \text{ as the "third harmonic," and so on.}$$

The second, fourth, sixth, etc., are known as the "even harmonics"; the others as the "odd harmonics."

The result of combining a fundamental wave and its second harmonic is shown in Fig. 118 (a), and that of combining a fundamental wave and its third harmonic (*i.e.*, an odd one) is given in Fig. 118 (b).

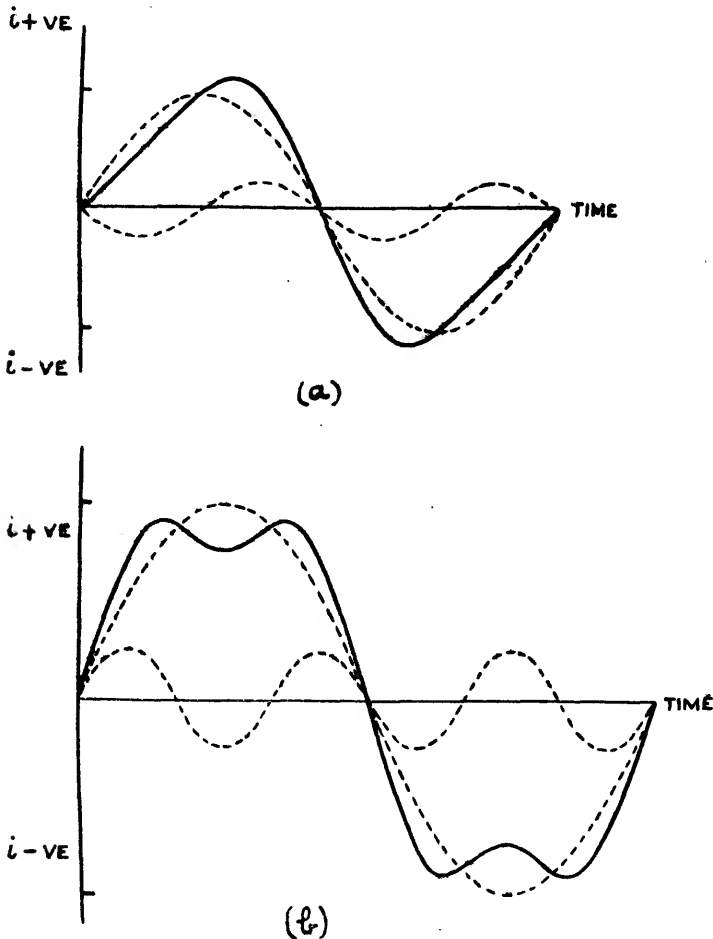


FIG. 118.

The R.M.S. value of such complicated waveforms can be worked out as in section (a) of this paragraph.

R.M.S. value = square root of the mean value of the square of the current = square root of

$$\frac{\int_0^{2\pi} [J_1 \sin(\omega t + \phi_1) + J_2 \sin(2\omega t + \phi_2) + \dots]^2 dt}{\frac{2\pi}{\omega}}$$

which works out to be the square root of

$$\frac{J_1^2}{2} + \frac{J_2^2}{2} + \dots$$

$$\therefore I = \sqrt{\frac{J_1^2}{2} + \frac{J_2^2}{2} + \frac{J_3^2}{2} + \dots} = \sqrt{I_1^2 + I_2^2 + I_3^2 + \dots},$$

where I_1, I_2 , etc., are the separate R.M.S. values of the fundamental and the harmonics.

276. Vector Representation of Alternating Quantities.—

Alternating currents and voltages have so far been represented by sine curves, which show the instantaneous value of the quantity with respect to an axis of time. A much simpler method is to represent alternating quantities by the use of **vectors**.

A vector quantity is one which has magnitude and direction, *e.g.*, velocity, acceleration, force, current, etc.

Any of these may be represented by a straight line whose length represents the magnitude of the quantity, and whose direction represents the direction of the quantity at the moment illustrated.

In the special application of vectors to alternating quantities (varying in accordance with a sine law) the length of the line represents the maximum value of the quantity, while the line may be supposed to rotate in a counter-clockwise direction, the projection along a given direction representing the instantaneous value.

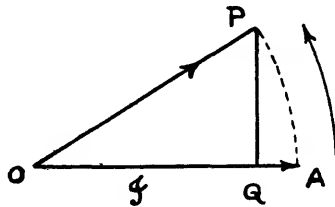


FIG. 119.

For instance, in the figure shown, to represent $i = J \sin \omega t$, $OA = OP = J$. If the line OP rotates in the direction shown with angular velocity ω radians per second, then the perpendicular PQ , or the projection of the moving line perpendicular to OA , represents the instantaneous value at a given time t_1 , *i.e.*, $J \sin \omega t_1$, where angle $POA = \omega t_1$. Furthermore, the idea of phase difference can be brought out by the same method. In the figure above, if angle

POA = θ , OP can be taken to mean an alternating quantity of the same amplitude as OA, but leading it by an angle θ ,

i.e., if OA represents $J \sin \omega t$,

OP represents $J \sin (\omega t + \theta)$.

Thus, for instance, two alternating currents, which have the same frequency, can be added by using the parallelogram law.

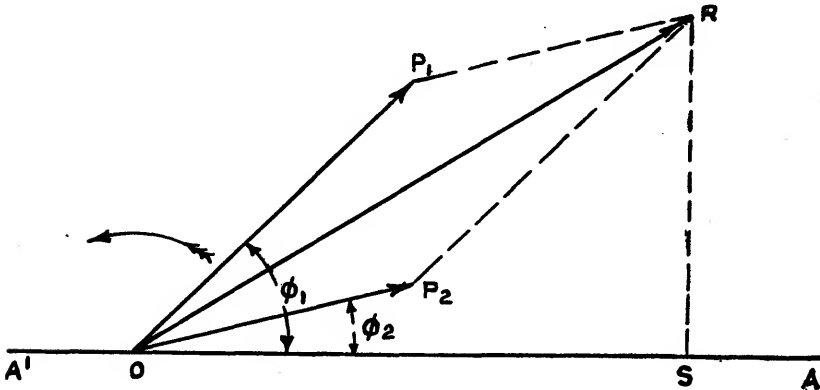


FIG. 120.

If $J_1 \sin (\omega t + \phi_1)$ is represented by OP_1 , and $J_2 \sin (\omega t + \phi_2)$ is represented by OP_2 , and the parallelogram is completed, then OR represents the sum of these quantities in magnitude and phase.

Actually

$$OR^2 = OS^2 + RS^2$$

$$= (J_1 \cos \phi_1 + J_2 \cos \phi_2)^2 + (J_1 \sin \phi_1 + J_2 \sin \phi_2)^2$$

and

$$\tan ROA = \frac{J_1 \sin \phi_1 + J_2 \sin \phi_2}{J_1 \cos \phi_1 + J_2 \cos \phi_2}.$$

By trigonometry,

$$\begin{aligned} & J_1 \sin (\omega t + \phi_1) + J_2 \sin (\omega t + \phi_2) \\ = & J_1 \sin \omega t \cos \phi_1 + J_1 \cos \omega t \sin \phi_1 + J_2 \sin \omega t \cos \phi_2 \\ & + J_2 \cos \omega t \sin \phi_2 \\ = & \sin \omega t (J_1 \cos \phi_1 + J_2 \cos \phi_2) + \cos \omega t (J_1 \sin \phi_1 + J_2 \sin \phi_2). \end{aligned}$$

Write

$$J_1 \cos \phi_1 + J_2 \cos \phi_2 = X.$$

$$J_1 \sin \phi_1 + J_2 \sin \phi_2 = Y.$$

\therefore The above = $X \sin \omega t + Y \cos \omega t$

$$\begin{aligned} & = \sqrt{X^2 + Y^2} \left(\sin \omega t \frac{X}{\sqrt{X^2 + Y^2}} + \cos \omega t \frac{Y}{\sqrt{X^2 + Y^2}} \right) \\ & = \sqrt{X^2 + Y^2} \sin \left(\omega t + \tan^{-1} \frac{Y}{X} \right) \end{aligned}$$

∴ The maximum value of the resultant current is $\sqrt{X^2 + Y^2}$ which is the same as the result by vector, and the phase angle is $\tan^{-1} \frac{Y}{X}$, again the same.

It will be found, in the rest of this chapter, that the voltages and currents in alternating current circuits are usually not in phase with each other, and vector diagrams are very useful for showing their relationships.

The vectors may be used to represent, not only the maximum values of the alternating quantities, but their R.M.S. values, but it must be kept in mind that in the latter case the projections along the direction perpendicular to the axis do **not** represent instantaneous values. Phase differences, as before, are shown by angles between the vectors. Angles of **lead** are measured in a counter-clockwise direction; angles of **lag** in a clockwise direction.

A very important special case of phase difference is that of "rate of change" of a sinusoidally varying quantity (Appendix B).

The time rate of change $\frac{di}{dt}$ of a quantity $i = \mathcal{I} \sin \omega t$ is $\mathcal{I} \omega \cos \omega t$, which may be written $\omega \mathcal{I} \sin \left(\omega t + \frac{\pi}{2} \right)$, and **leads** by 90° on the quantity i itself.

Vectorially, of course, the two quantities i and $\frac{di}{dt}$ are represented by two lines at right angles to each other, the vector representing $\frac{di}{dt}$ being in length ω times that representing i .

RESISTANCE, INDUCTANCE, CAPACITY, AND SERIES COMBINATIONS OF THESE IN ALTERNATING CURRENT CIRCUITS.

277. Resistance in an Alternating Current Circuit.—If between the slip rings of an alternator giving an alternating voltage of sine form, represented by $v = \mathcal{V} \sin \omega t$ volts, we join a resistance of R ohms, as in Fig. 121 (a), then the current flowing through the circuit (neglecting any inductance of the armature winding) will be given by $i = \frac{v}{R}$ amperes.

$$i = \frac{v}{R} = \frac{\mathcal{V}}{R} \sin \omega t.$$

If we put \mathcal{I} for $\frac{\mathcal{V}}{R}$, the maximum value of the current, the current waveform is $i = \mathcal{I} \sin \omega t$.

The current and voltage rise and fall simultaneously, as illustrated in Fig. 121 (b), and obviously they are in phase with each other. The thin curve represents the voltage, drawn to a scale of volts, and the thick curve the current, drawn to a scale of amperes.

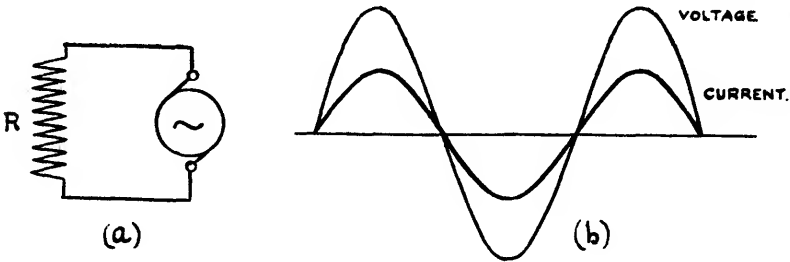


FIG. 121.

Represented vectorially, the two vectors would be laid off along the same line.

278. Effect of Inductance on an Alternating Current.—We shall now discuss the effect of inductance in an alternating current circuit.

Let us first recall what we have already learnt about the effects of inductance.

If current is switched on to an inductance, the inductance sets up a counter E.M.F. which opposes the rise of the current.

When the current is switched off, the inductance sets up a counter E.M.F., which tends to make the current continue flowing.

Hence the counter E.M.F. **opposes the rise** and **opposes the fall** of a current.

The reason for this action is that as the current increases through the coil, a magnetic field is set up round it, and work has to be done on the coil in order to create this magnetic field.

When the current is stopping, the magnetic field ceases to be maintained by the current, and the energy that was stored in the magnetic field is restored to the circuit.

The unit of inductance is the **Henry**, which is the inductance of a coil of such a form that when the current through the coil is increasing or decreasing at the rate of one ampere per second, the induced E.M.F. is equal to one volt.

279. Rate of Change.—It follows from the above that what we are concerned with is the **rate of change of the current**: when the current is changing most quickly the greatest E.M.F. will be induced, and when it is not changing at all, there will be no E.M.F. induced.

Let us join up a coil (L), which is supposed to have inductance without resistance, to a source of alternating E.M.F., as shown in

Fig. 122, and assume that an alternating current is flowing through the inductance as shown in Fig. 123 (a).

Fig. 123 (a) shows a curve where time is plotted horizontally and current strength vertically; hence, if the current makes a big change in a short time, the slope of the curve during that period will be steep. On the other hand, if the current makes but little change in a long time, the slope will be slight; thus the slope of the current curve is an indication of its rate of change.

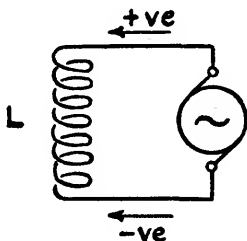


FIG. 122.

From an inspection of Fig. 123 (a), it can be seen that at moments 1, 3, 5 and 7, the slope is steepest. At moments 2, 4 and 6, the current is not changing at all—neither increasing nor decreasing—and the **slope** of the curve is zero, as indicated by the dotted line.

Fig. 123 (b) is a curve giving the **rate of change of current** in amperes per second, drawn to any suitable scale.

At moment 1, Fig. 123 (a), the current is increasing very rapidly in a positive direction.

This is indicated at 1 in Fig. 123 (b), which shows a maximum rate of change in a **positive** direction (above the axis) at this moment.

At moment 2, Fig. 123 (a), the current has just stopped increasing and is just going to decrease. Exactly on the top of the curve it is not changing at all.

This is indicated at 2, Fig. 123 (b), *i.e.*, zero rate of change.

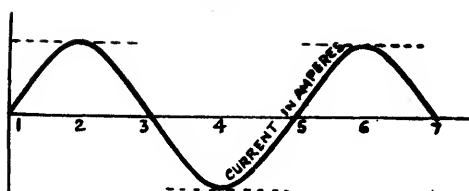
At moment 3, Fig. 123 (a), the current is falling very rapidly through zero, and rising very rapidly in the other direction. That is, the current is changing very rapidly in the opposite direction to that at moment 1.

This is indicated at 3, Fig. 123 (b); and so on.

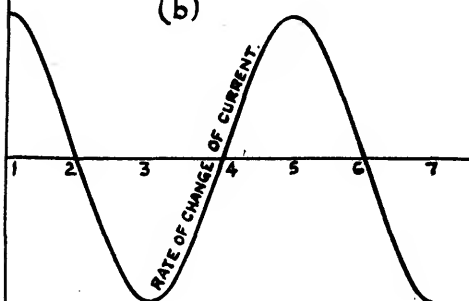
Hence Fig. 123 (b) is a curve indicating the rate of change of the current shown in Fig. 123 (a).

280. Induced E.M.F.—Since the E.M.F. induced in any inductance, through which the current of curve (a) is flowing, depends on the rate of change of that current, this E.M.F. will rise and fall in time with curve (b), being maximum when curve (b) is maximum, and zero when curve (b) is zero.

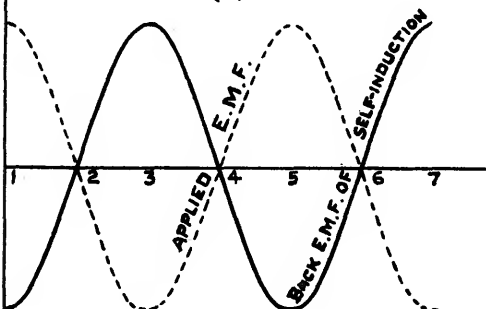
(a)



(b)



(c)



(d)

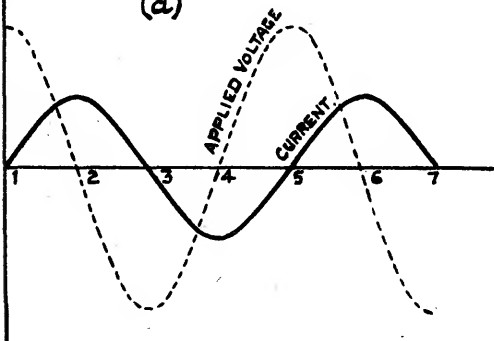


FIG. 123,

Its direction—positive or negative—can easily be determined by remembering that it always opposes the rise and fall of current through the inductance.

In curve (c) the full line curve represents this induced E.M.F.

At moment 1 it is maximum because the rate of change of the current (curve (b)) is maximum, and as the current (curve (a)) is trying to rise in a positive direction, the induced E.M.F. is acting in a negative direction, trying to prevent it from rising.

At moment 2 the rate of change of the current is zero, so the induced E.M.F. is zero also.

Between moments 2 and 3 the current wants to fall, so the induced E.M.F. acts in the opposite direction to that in which it was acting between moments 1 and 2.

Between moments 3 and 4, the current is trying to rise in a negative direction. Therefore the induced E.M.F. acts in a positive direction trying to prevent it from rising; and so on.

Hence curve (c) shows the relative strength and direction of the E.M.F. of self induction at any instant.

281. Applied E.M.F.—If the induced E.M.F. had its own way, it would prevent the current from rising and falling at all.

In order, therefore, to make the current flow through the inductance, the source of alternating current has to apply a voltage which is equal and opposite to the induced E.M.F.; that is to say, a voltage which is equal to the induced E.M.F. at any instant, and is acting in a positive direction when the induced E.M.F. is acting in a negative direction, is zero when the induced E.M.F. is zero, and is maximum negative when the induced E.M.F. is maximum positive.

The applied voltage from the source of alternating current will then be as shown by the dotted curve shown in Fig. 123 (c). It can be seen at once that this voltage is equal in magnitude and opposite in direction to the induced E.M.F. at any instant.

In addition, a small voltage will be required from the supply source to overcome the small resistance which must be present in the coil. But as the latter has been supposed to be negligible, we shall neglect this small additional voltage for the present.

Fig. 123 (d) shows the combination of curves (a) and (c). That is to say, it shows the voltage applied by the alternating source and the resulting alternating current flowing through the inductance L.

From this diagram it can be seen that in a circuit containing nothing but inductance, the current and voltage do not rise and fall together, as was shown in Fig. 121 (b), but the current always comes to its maximum value a quarter cycle later than the alternating voltage.

When the current and voltage rise and fall together, as in Fig. 121 (b), they are said to be "in phase."

When they do not rise and fall together, they are said to be "out of phase."

When the alternating current reaches its maximum value after the applied voltage, as in Fig. 123 (d), the current is said to **lag** behind the voltage.

Conversely, when it reaches its maximum **before** the applied voltage, it is said to be a **leading** current.

282. An almost parallel example is given by the flow of water into and out of harbour. Let us call the level of water in a harbour at half tide the normal or zero value, and the level at lowest ebb the maximum negative value.

At the top of the flood and at lowest ebb the level of water in the harbour is maximum positive or maximum negative; at these moments the flow of water into or out of harbour is zero.

At half-tide—the moment of normal or zero level of water in the harbour—the ebb or flow current is maximum.

Thus curves representing these two variables—the current at the mouth of the harbour, and the level of water in the harbour—would be a quarter of a cycle out of phase, and just like Fig. 123 (d).

283. Voltage and Current in an Inductive Circuit.—The results obtained in the preceding five paragraphs as regards phase relationship of current and voltage in a purely inductive circuit can be arrived at much more quickly by using a mathematical treatment, and at the same time, a formula giving the relationship between current and voltage amplitudes can be found.

The problem, as before, is to investigate the voltage necessary to send an alternating current through an inductance, where self-induction exercises a continuous effect.

Let the current be $i = J \sin \omega t$, where $\omega = 2\pi f$.

From Chapter III, the induced E.M.F. is given by $-L \frac{di}{dt}$, that is, $-L \times$ rate of change of current; the result will be in volts, if L is in henries, i in amperes, and t in seconds.

$$\begin{aligned} \text{In this case } -L \frac{di}{dt} &= -\omega L J \cos \omega t \\ &= \omega L J \sin \left(\omega t - \frac{\pi}{2} \right). \end{aligned}$$

The E.M.F. of self-induction **lags**, therefore, by $\frac{\pi}{2}$, or 90° , on the current flowing.

Now, in the case we are considering, where the resistance is neglected, the applied E.M.F. has just to overcome this counter E.M.F. to keep the current flowing.

Thus the applied E.M.F. $= +\omega L J \cos \omega t = +\omega L J \sin \left(\omega t + \frac{\pi}{2} \right)$ **leading** by 90° on the current flowing. It is itself sinusoidal, and

has the same frequency as the current. This applied E.M.F. has an amplitude $\omega L \mathcal{I}$ volts, L being in henries and \mathcal{I} in amperes.

From the other point of view, if a sinusoidal voltage whose amplitude is \mathcal{V} volts is applied to an inductive circuit, whose inductance is L henries, at a frequency of f cycles per second, the resulting current flowing will be, in amplitude,

$$\mathcal{I} = \frac{\mathcal{V}}{\omega L} = \frac{\mathcal{V}}{2\pi f L} \text{ amperes,}$$

and will lag 90° behind the voltage.

Because of the constant relationship between maximum and R.M.S. values for both alternating currents and voltages, we may conclude that a similar formula will hold for R.M.S. values.

Thus a voltage of R.M.S. value V volts, under the same conditions as above, will give a current of R.M.S. value I amperes, where

$$V = \omega L I = 2\pi f L I, \text{ or } I = \frac{V}{\omega L} = \frac{V}{2\pi f L}.$$

In phase relationship, the current lags 90° on the voltage.

284. Inductive Reactance.—This relationship between voltage and current resembles the relationship expressed by Ohm's Law. The expression " ωL " takes the place of R and has the same effect in determining the value of the current flowing. As, however, the energy expended in creating the magnetic field round an inductance is restored when the magnetic field collapses, there is no expenditure of energy involved in the introduction of an inductance into a circuit, such as there would be were resistance introduced.

ωL , or $2\pi f L$, is termed "**Inductive Reactance**," and is denoted by the symbol X_L .

The unit in which it is measured must be the same as the unit of resistance, the ohm, since it is a relationship between voltage and current. In its physical sense it is not, of course, a true ohmic resistance, but it is convenient in calculations to give it simply the name "ohm."

Example 28.

Find the value of the current flowing through an inductance of 0.5 henry, of negligible resistance, if an alternating voltage of 200 volts is applied at a frequency of 50 cycles per second.

$$I = \frac{V}{2\pi f L} = \frac{200}{2 \times 3.14 \times 50 \times 0.5} = \frac{4}{3.14} = 1.27 \text{ amps.}$$

Example 29.

Find the value of the current flowing through an inductance of 200 mics, with an applied voltage of 1,000 volts at a frequency of 500,000 cycles per second.

$$200 \text{ mics} = \frac{200}{10^6} \text{ henries.}$$

$$X_L = \omega L = 6.28 \times 500,000 \times \frac{200}{10^6} = 6.28 \times 5 \times 10^5 \times \frac{200}{10^6} \\ = 628 \text{ ohms.}$$

$$I = \frac{V}{X_L} = \frac{1,000}{628} = 1.6 \text{ amperes.}$$

285. Vectorial Representation.—The method of vector representation of alternating quantities can be used to show the preceding results with greater simplicity than in Fig. 123.

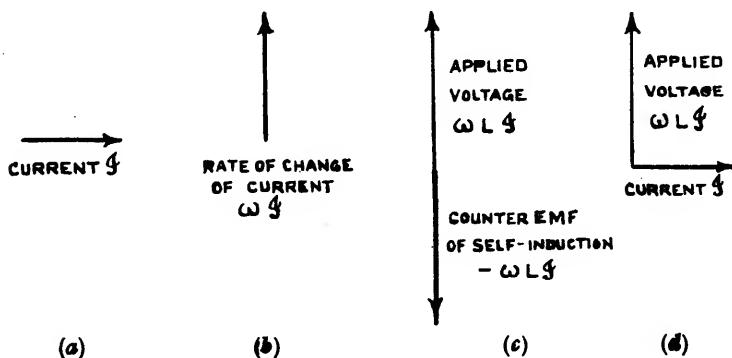


FIG. 124.

In Fig. 124 (a) above, the alternating current $i = \mathcal{I} \sin \omega t$ is represented simply by a line whose length represents \mathcal{I} amperes.

The rate of change of current $\frac{di}{dt} = \omega \mathcal{I} \sin \left(\omega t + \frac{\pi}{2} \right)$ is shown in Fig. 124 (b) by a line of length $\omega \mathcal{I}$, leading the current by 90° .

The E.M.F. of self-induction, $-L \frac{di}{dt}$, or $-\omega L \mathcal{I} \sin \left(\omega t + \frac{\pi}{2} \right)$, or $\omega L \mathcal{I} \sin \left(\omega t - \frac{\pi}{2} \right)$, is represented by a line, length $\omega L \mathcal{I}$, lagging 90° on the current.

The applied voltage, which must be equal and opposite at each instant, *i.e.*, $+\omega L \mathcal{I} \sin \left(\omega t + \frac{\pi}{2} \right)$, is shown also in Fig. 124 (c).

Fig. 124 (d) shows the applied voltage and current flowing in an inductive circuit on one diagram, the current **lagging** 90° on the voltage.

286. Inductances in Series and Parallel (*cf.* paras. 149 and 150).

(a) Series.

If two inductances L_1 and L_2 are joined in series, the total applied voltage necessary to send a current \mathcal{I} through both inductances will

be given by $\omega L_1 \mathcal{I} + \omega L_2 \mathcal{I}$, the two separate voltages being simply additive, because each leads 90° on the current.

$$\mathcal{V} = \omega (L_1 + L_2) \mathcal{I}.$$

The two inductances in series are therefore equivalent to one inductance L , where $L = L_1 + L_2$, i.e., inductances in series are additive.

(b) Parallel.

If two inductances L_1 and L_2 are joined in parallel and an alternating voltage \mathcal{V} is applied to them (Fig. 125), then currents of \mathcal{I}_1 and \mathcal{I}_2 will flow through them respectively, these currents both lagging by 90° on the voltage and hence in phase with one another, and additive.

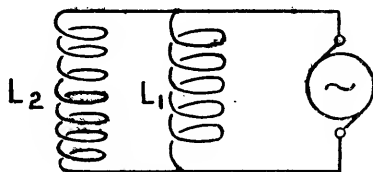


FIG. 125.

The total current

$$\mathcal{I} = \mathcal{I}_1 + \mathcal{I}_2 = \frac{\mathcal{V}}{\omega L_1} + \frac{\mathcal{V}}{\omega L_2} = \frac{\mathcal{V}}{\omega} \left(\frac{1}{L_1} + \frac{1}{L_2} \right).$$

The inductances in parallel are therefore equivalent to a single inductance L , where

$$\frac{1}{L} = \frac{1}{L_1} + \frac{1}{L_2}, \text{ or } L = \frac{L_1 L_2}{L_1 + L_2}.$$

Hence, inductances in parallel are additive by the reciprocal law, as is the case with resistances.

287. Resistance and Inductance in Series.—So far, we have only considered two cases:—

(a) where the circuit contains nothing but **resistance**. Here

$$\mathcal{I} = \frac{\mathcal{V}}{R}, \text{ and current and voltage are in phase;}$$

(b) where the circuit contains nothing but **inductance**. Here

$$\mathcal{I} = \frac{\mathcal{V}}{\omega L}, \text{ and the current lags } 90^\circ \text{ behind the applied voltage.}$$

Now, although in practice we often get cases where the inductive reactance is so small compared with the resistance, or the resistance is so small compared with the inductive reactance, that the smaller item can be neglected, yet, as a rule, we have to consider both the inductance and the resistance of the circuit.

In the case shown in Fig. 126 the applied voltage has to do two things :—

- (1) overcome the iR drop in the circuit ;
- (2) overcome the counter E.M.F. of the inductance of L henries.

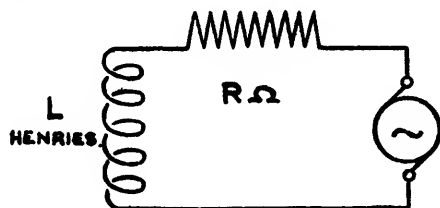


FIG. 126.

Fig. 127 (b) shows the conditions under these circumstances. The thick line curve indicates the rise and fall of the current.

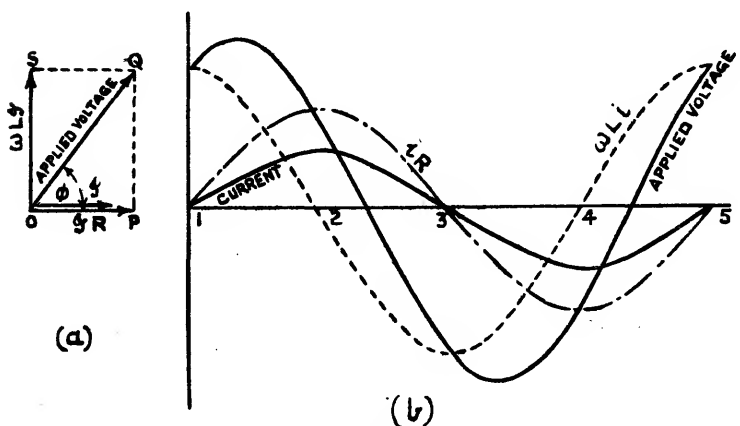


FIG. 127.

The voltage forcing the current through the resistance R is indicated by the curve $v = iR$. It is in phase with the current.

The voltage overcoming the counter E.M.F. of the inductance L is indicated by the curve $v = \omega Li$, which leads the current by 90° . The necessary total voltage from the alternator in order to supply these two voltages simultaneously can be found by adding the ordinates of the two curves together at every instant, taking account of their directions ; the result is shown by the curve marked "Applied voltage."

It will be seen that :—

- (a) At moment 1 the iR curve is zero and the ωLi curve is maximum, so that the applied voltage required $= \omega Li$.

- (b) At moment 2 the iR curve is maximum and the ωLi curve is zero, so that the applied voltage $= iR$.
- (c) Between moments 1 and 2 both the iR and the ωLi curves are positive.
- (d) Between moments 2 and 3 the ωLi curve is increasing in a negative direction, while the iR curve is decreasing, but is still positive; hence the resultant curve is found by subtracting the ordinates of the ωLi curve from those of the iR curve.
- (e) Between moments 3 and 4 the iR curve and the ωLi curve are both negative; hence their ordinates must be added to find the resultant.

It will be seen by comparing maximum positive values that the current is lagging behind the applied voltage by some angle less than 90° .

288. Value of Applied Voltage.—Instead of the laborious graphical method above, the results, trigonometrical and vectorial, of para. 276, can be utilised to give the value and phase relationship of the applied voltage necessary to send an alternating current i through a resistance R and an inductance L in series. Trigonometrically, the necessary voltage is given by

$$\begin{aligned}
 v &= iR \sin \omega t + \omega L i \sin \left(\omega t + \frac{\pi}{2} \right) \\
 &= iR \sin \omega t + \omega L i \cos \omega t \\
 &= i \sqrt{R^2 + \omega^2 L^2} \sin \left(\omega t + \tan^{-1} \frac{\omega L}{R} \right)
 \end{aligned}$$

(using para. 276), and is therefore a voltage which is sinusoidal, whose amplitude is $i\sqrt{R^2 + \omega^2 L^2}$, and which leads the current by an angle whose tangent is $\frac{\omega L}{R}$.

Vectorially, the same result is obtained by considering Fig. 127 (a), which represents the vectorial addition of the two vectors iR and ωLi .

The vector iR , in phase with and therefore drawn in the same direction as the vector representing the current, is shown by OP .

The vector ωLi , leading the current by 90° and therefore drawn as shown at right angles to the current vector, is shown on the same scale by OS .

The summation is carried out by completing the rectangle $OPQS$ and drawing the diagonal OQ . OQ is then the vectorial representation of the applied voltage which is equivalent to the separate voltages iR and ωLi . Its length is given by

$$\sqrt{OP^2 + PQ^2} = \sqrt{(iR)^2 + (\omega Li)^2} = i\sqrt{R^2 + \omega^2 L^2}.$$

This is the amplitude \mathcal{V} of the voltage required.

Hence $\mathcal{V} = \mathcal{I} \sqrt{R^2 + \omega^2 L^2},$

or $\mathcal{I} = \frac{\mathcal{V}}{\sqrt{R^2 + \omega^2 L^2}}.$

In R.M.S. values, $V = I \sqrt{R^2 + \omega^2 L^2}$, or $I = \frac{V}{\sqrt{R^2 + \omega^2 L^2}}.$

Also from the figure, if ϕ = the angle QOP,

$$\tan \phi = \frac{PQ}{OP} = \frac{\omega L \mathcal{I}}{\mathcal{I} R} = \frac{\omega L}{R}.$$

Hence the current lags on the voltage by an angle whose tangent is $\frac{\omega L}{R}$ or $\frac{\text{Reactance}}{\text{Resistance}}.$

Also $\omega L \mathcal{I} = \mathcal{V} \sin \phi$, and $\mathcal{I} R = \mathcal{V} \cos \phi.$

289. Impedance.—The expression $\sqrt{R^2 + (\omega L)^2}$ is called the **Impedance** of the circuit, and is denoted by the letter **Z (ohms)**. It may be defined as the ratio of the maximum value of the voltage to the maximum value of the current (irrespective of the fact that these may not occur at the same instant). Being a ratio of voltage to current, it is measured in ohms, though not necessarily a true ohmic resistance (*cf.* para. 284).

Example 30.

Let an alternating voltage $\mathcal{V} = 100$ volts at a frequency of 25 cycles per second be applied to a circuit of resistance 1.5 ohms and of inductance .01 henry. Find (a) the current flowing, and (b) the angle of lag :—

(a) $\omega = 2\pi f = 2\pi \times 25 = 50\pi = 157$ radians per sec.

Reactance $= X_L = \omega L = 157 \times .01 = 1.57$ ohms.

Resistance $= 1.5$ ohms.

Impedance $Z = \sqrt{R^2 + X_L^2} = \sqrt{1.5^2 + 1.57^2}$
 $= \sqrt{2.25 + 2.46} = \sqrt{4.71} = 2.17$ ohms.

$\mathcal{I} = \frac{\mathcal{V}}{Z} = \frac{100}{2.17} = 46$ amperes.

(b) $\tan \phi = \frac{X_L}{R} = \frac{1.57}{1.5} = 1.047.$

From a table of Tangents, ϕ is seen to be an angle of $46^\circ 19'.$

So we have a current of 46 amps. lagging 46° , or .13 of a cycle, behind the E.M.F. of 100 volts.

Plotting these as vectors and as curves, we get Fig. 128 (a) and (b). The voltage drop in the resistance

$$= IR = 46 \times 1.5 = 69 \text{ volts.}$$

The voltage balancing the counter E.M.F. of the inductance

$$= \omega L I = 1.57 \times 46 = 72.2 \text{ volts.}$$

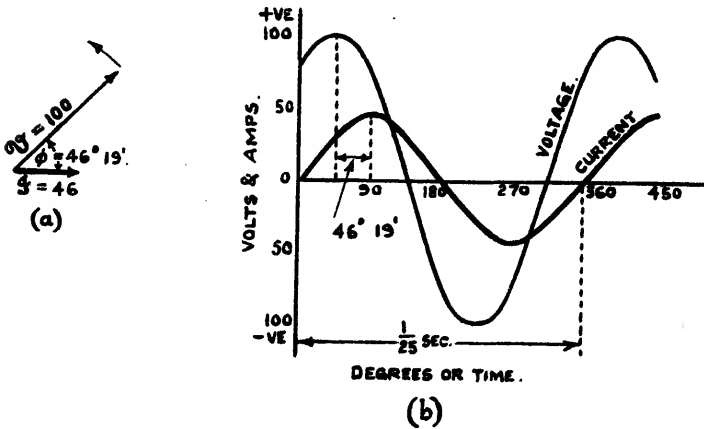


FIG. 128.

It seems impossible for the 100 applied volts to supply both these values, but the 69 volts and the 72.2 volts are not supplied at the same instant owing to the phase difference between the resistance and reactance voltages.

Let us take a radio frequency example.

Example 31.

Let an alternating P.D., $\mathcal{V} = 100$ volts, at a frequency of 10,000 cycles per second, be applied to a circuit of resistance 20 ohms and inductance 300 mics. Find (a) the current flowing, and (b) the angle of lag.

$$\omega = 2\pi \times 10,000 = 6.28 \times 10^4.$$

$$X_L = \omega L = 6.28 \times 10^4 \times \frac{300}{10^6} = 6.28 \times 3 = 18.84 \text{ ohms.}$$

$$R = 20 \text{ ohms.}$$

$$Z = \sqrt{20^2 + 18.84^2} = \sqrt{400 + 355} = \sqrt{755} \\ = 27.48 \text{ ohms.}$$

$$I = \frac{\mathcal{V}}{Z} = \frac{100}{27.48} = 3.64 \text{ amps.}$$

$$IR = 3.64 \times 20 = 72.8 \text{ volts.}$$

$$\omega L I = 3.64 \times 18.84 = 68.6 \text{ volts.}$$

$$(b) \tan \phi = \frac{18.84}{20} = .942. \quad \phi = 43^\circ 15'.$$

Let us try the effect of applying the radio frequency voltage of Example 31 to the circuit given in Example 30.

Example 32.

As before, $\omega = 2\pi \times 10^4$.

$$X_L = \omega L = 2\pi \times 10^4 \times .01 = 6.28 \times 10^2 = 628 \text{ ohms.}$$

$$R = 1.5 \text{ ohms.}$$

$$Z = \sqrt{1.5^2 + 628^2} = 628 \text{ ohms practically.}$$

$$I = \frac{V}{Z} = \frac{100}{628} = .163 \text{ amps.}$$

$$\tan \phi = \frac{628}{1.5} = 418. \quad \phi = 89^\circ 52' = 90^\circ \text{ practically.}$$

From this example we may deduce that with a radio frequency voltage applied to a circuit where the inductance is large compared with the resistance, the resulting current depends almost entirely upon the value of the inductance and is practically 90° out of phase with the applied voltage.

290. Capacity in an Alternating Current Circuit.—The effect of a condenser joined alone in an A.C. circuit is exactly opposite to that of an inductance; the current leads in phase on the voltage, as will now be shown.

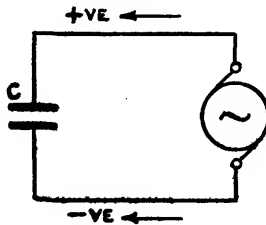


FIG. 129.

Suppose an alternating voltage, whose wave form is sinusoidal, is applied to a condenser C as in Fig. 129. The condenser is supposed to be perfect, *i.e.*, there are no losses due to leakage through the dielectric or across the surface of the dielectric at the edges, no resistances in the leads or plates, and no dielectric absorption. Such conditions cannot be achieved in practice, but losses can be allowed for by assuming resistances in series and parallel with the capacity, and using the theory applicable to such combinations, which is done later. At present, we are only to consider the theoretically perfect condenser.

In such a condenser the law $Q = CV$ is true; that is, the charge on the condenser at any instant in coulombs is given by the product of the capacity in farads and the voltage applied to it in volts.

With no losses, a change of voltage applied to the condenser is instantaneously accompanied by a proportional change in the charge on the condenser. Also the voltage across the condenser due to the charge on it is at every instant exactly equal and opposite to the applied voltage. This condenser voltage may be called the counter E.M.F. of the condenser.

Now, under the conditions that hold when an alternating voltage is applied to the condenser, *i.e.*, a voltage which is always changing, the consequent changes in charge on the condenser must constitute currents flowing into the condenser when the applied voltage is increasing and currents flowing out of the condenser when the voltage is decreasing. It is very important to distinguish clearly between these two things—quantity, or **charge** on the condenser, measured in coulombs, and **current**, or rate at which this charge is changing, increasing or decreasing, measured in coulombs per second, or amperes.

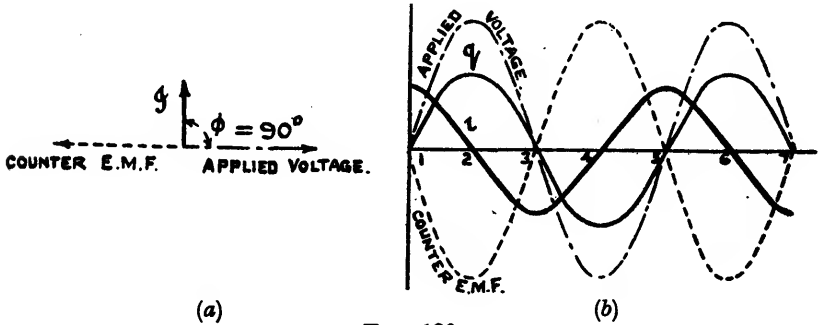


FIG. 130.

Fig. 130 (b) shows the conditions when an alternating voltage is applied to the condenser C as in Fig. 129. The curve marked applied voltage is a sine curve.

As this voltage changes, exactly similar changes occur in the charge of electricity introduced into the condenser, and this charge is shown by the curve marked q .

Due to this charge, a counter E.M.F. is set up across the condenser, which is exactly equal to the applied voltage at each instant. This E.M.F. is indicated by the curve marked Counter E.M.F. It is in antiphase with the applied voltage. Between moments 1 and 2 the applied voltage may be said to force an increasing charge into the condenser against the counter E.M.F.; between moments 2 and 3, as the applied voltage decreases, the counter E.M.F. forces this charge out of the condenser, until at moment 3 the charge is zero.

The same argument holds for the other half of the cycle, during which the condenser is charged in the opposite sense.

From the curve of charge q the current flow may be determined.

At the moment 1, the curve of charge is increasing very rapidly in a positive direction, and the rate of flow of electricity, *i.e.*, the current, has a maximum positive value there.

At the moment 2, the charge is a maximum but is not changing, and at that time, therefore, the current is zero; similarly for other specified points in the cycle.

The result is, that a curve of current is obtained which is 90° out of phase with either the curve of applied voltage or the curve of charge on the condenser, and in this case the current **leads** on the voltage. The **vectorial representation** is given in Fig. 130 (a).

The counter E.M.F. is in antiphase, or 180° out of phase with the applied voltage.

The current, which is the rate of change of the charge, the latter being in phase with the applied voltage, leads on the applied voltage by 90° .

291. The results found in para. **290** as regards phase relationship of current and voltage in a purely capacitive circuit can also be deduced by mathematical treatment, and at the same time a formula which gives the relationship between the amplitudes of current and voltage can be obtained.

Given an applied voltage $v = \mathcal{V} \sin \omega t$, the charge q on the condenser is given by

$$q = Cv = C\mathcal{V} \sin \omega t.$$

The current i = rate of change of charge

$$= \frac{dq}{dt} = \omega C\mathcal{V} \cos \omega t = \omega C\mathcal{V} \sin \left(\omega t + \frac{\pi}{2} \right).$$

So that for an applied voltage $\mathcal{V} \sin \omega t$, a current flows, which is also sinusoidal, with the same frequency as the voltage, and **leading** by 90° on the voltage. The amplitude of the current is $\mathcal{I} = \omega C\mathcal{V}$, or $\mathcal{I} = 2\pi f C\mathcal{V}$, \mathcal{I} being given in amperes, if C is in farads and \mathcal{V} in volts. This may be written

$$\mathcal{V} = \frac{\mathcal{I}}{\omega C} = \frac{\mathcal{I}}{2\pi f C}.$$

As with inductance, the R.M.S. values of **both** current and voltage can be substituted for the maximum values in the above equation, giving $I = \omega CV$, or $V = \frac{I}{\omega C}$.

If the current is $i = \mathcal{I} \sin \omega t$, the expression for the instantaneous applied voltage is

$$v = \frac{\mathcal{I}}{\omega C} \sin \left(\omega t - \frac{\pi}{2} \right).$$

292. Capacitive Reactance.—The relationship between current and voltage again resembles Ohm's Law.

In the expression $\mathcal{V} = \frac{I}{\omega C}$, the term $\frac{1}{\omega C}$ takes the place of R in Ohm's Law, and this term $\frac{1}{\omega C}$ is called **Capacitive Reactance**.

It is denoted by the symbol X_C , X being the general term for reactance, inductive or capacitive, and the suffixes L or C indicating which.

The unit in which it is measured must be the same as the unit of resistance, since it expresses the ratio of voltage to current and is therefore the **ohm**. It does not represent a true ohmic resistance, but it is convenient in calculations to give it the name "ohm."

The vector diagram given in Fig. 130 (a) can now be completed from the quantitative point of view by giving to the vectors concerned their correct values.

Since, however, we drew the vector diagram for inductance with the current horizontal, the figure may be redrawn as below in a similar manner.

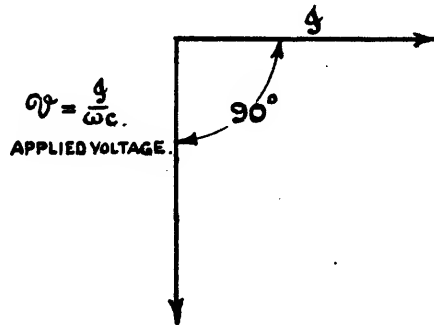


FIG. 131.

The applied voltage lags behind the current by 90° , and as regards amplitude is given by $\frac{I}{\omega C}$.

Example 33.

Find the current flowing through* a condenser of 45 jars if an alternating voltage $\mathcal{V} = 100$ volts is applied at a frequency of 300,000 cycles per second.

$$\omega = 2\pi f = 2\pi \times 300,000 = 1.884 \times 10^6.$$

* It is the custom to speak loosely of a current flowing "through" a condenser, when what is meant is a current charging a condenser alternately on either side, or a *displacement current* flowing in the condenser.

No conduction current actually flows through, but the electrons in the dielectric are strained alternately in each direction.

$$\begin{aligned}
 X_c &= \frac{1}{\omega C} = \frac{1}{1.884 \times 10^6} \times \frac{9 \times 10^8}{45} \text{ (converting jars to farads*)} \\
 &= \frac{20}{1.884} = 10.61 \text{ ohms.} \\
 I &= \frac{100}{10.61} = 9.4 \text{ amperes.}
 \end{aligned}$$

293. Condensers in Series and Parallel (*cf.* paras. 177 and 178).

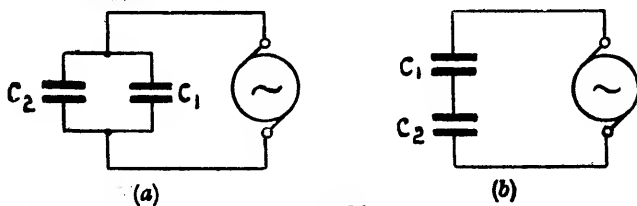


FIG. 132.

Condensers in Parallel.—If two condensers, C_1 and C_2 , are joined in parallel, as in Fig. 132 (a), and an alternating voltage V (using R.M.S. values throughout) is applied to both, currents I_1 and I_2 will flow through C_1 and C_2 such that

$$I_1 = \omega C_1 V \text{ and } I_2 = \omega C_2 V.$$

These currents will both lead by 90° on the applied voltage and will therefore be in phase. The total current flowing from the supply will be $I = \omega (C_1 + C_2) V$.

The same current would flow if the separate condensers were replaced by a single condenser with capacity $(C_1 + C_2)$, *i.e.*, the equivalent capacity of two (or more) condensers in **parallel** is the sum of their separate capacities.

Condensers in Series.—If two condensers, C_1 and C_2 , are joined in series, as in Fig. 132 (b), the same charging current I will flow round the circuit, or "through" each condenser.

The requisite applied voltage to overcome the counter E.M.F.s. of the condensers will be given by $\frac{I}{\omega C_1} + \frac{I}{\omega C_2}$, the components being additive because the individual voltages are both lagging 90° on the current, and so are in phase.

$$\therefore V = \frac{I}{\omega C_1} + \frac{I}{\omega C_2} = \frac{I}{\omega} \left(\frac{1}{C_1} + \frac{1}{C_2} \right).$$

The same voltage would be necessary to send a current I round a circuit with a single condenser C if $V = \frac{I}{\omega C}$.

* Readers who are not accustomed to the unit of a "jar" used throughout this book should remember that 1 jar

$= 1/900\text{th microfarad,}$
 $= 1,000 \text{ centimetres,}$
 $= 10/9\text{ths milli-microfarad.}$

Hence the equivalent value of the two capacities in series is C , where

$$\frac{I}{\omega C} = \frac{I}{\omega} \left(\frac{1}{C_1} + \frac{1}{C_2} \right) \quad \text{or} \quad \frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2}.$$

So for a number of condensers in series.

Hence capacities in series have an equivalent value given by the reciprocal law: the reciprocal of the equivalent capacity is the sum of the reciprocals of the separate capacities.

$$\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \&c.$$

For two capacities only $C = \frac{C_1 C_2}{C_1 + C_2}$.

Note that the rules for finding the equivalent value of capacities in series and parallel are exactly the opposite of those in the case of resistances or inductances.

294. Resistance and Capacity in Series.—The question of finding the voltage necessary to send an alternating current \mathcal{I} , or I , using the R.M.S. notation, round a circuit containing resistance and capacity in series, can be tackled by the graphical methods of paragraph 287, or by the shorter and more accurate methods, trigonometrical and vectorial, of para. 288.

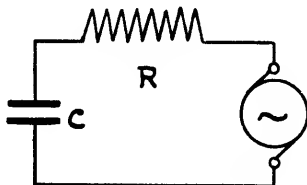


FIG. 133.

It is difficult to deduce accurate generalised results from the graphical method, and so attention will be confined to the other methods.

From previous results, the total voltage necessary to maintain an alternating current $i = \mathcal{I} \sin \omega t$ in the circuit above, with resistance R and capacity C , will be given by

$$\begin{aligned} v &= \mathcal{I}R \sin \omega t + \frac{\mathcal{I}}{\omega C} \sin \left(\omega t - \frac{\pi}{2} \right) \\ &= \mathcal{I}R \sin \omega t - \frac{\mathcal{I}}{\omega C} \cos \omega t. \\ &= \mathcal{I} \left(R \sin \omega t - \frac{1}{\omega C} \cos \omega t \right) \end{aligned}$$

$$\begin{aligned}
 &= \mathcal{I} \sqrt{R^2 + \frac{1}{\omega^2 C^2}} \left(\frac{R}{\sqrt{R^2 + \frac{1}{\omega^2 C^2}}} \sin \omega t - \frac{\frac{1}{\omega C}}{\sqrt{R^2 + \frac{1}{\omega^2 C^2}}} \cos \omega t \right) \\
 &= \mathcal{I} \sqrt{R^2 + \frac{1}{\omega^2 C^2}} \sin (\omega t - \phi),
 \end{aligned}$$

where $\tan \phi = \frac{1}{\omega CR}$.

The required voltage is therefore sinusoidal, with an amplitude of $\mathcal{I} \sqrt{R^2 + \frac{1}{\omega^2 C^2}}$, and lags behind the current by an angle whose tangent is $\frac{1}{\omega CR}$.

Vectorially, the same result is obtained by considering Fig. 134, which represents the vectorial addition of the two vectors $\mathcal{I}R$ and $\frac{\mathcal{I}}{\omega C}$.

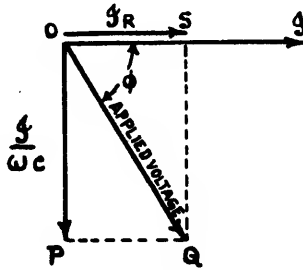


FIG. 134.

The vector $\mathcal{I}R$ in phase with the current vector \mathcal{I} and drawn along the same direction, is shown by OS.

The vector $\frac{\mathcal{I}}{\omega C}$ representing the voltage necessary to maintain the current through the capacity, and therefore lagging 90° on the current, is shown by OP.

The applied voltage is found by completing the rectangle OPQS and drawing the diagonal OQ.

$$OQ^2 = OP^2 + OS^2.$$

Thus the amplitude \mathcal{V} of the voltage required is given by

$$\mathcal{V} = \mathcal{I} \sqrt{R^2 + \frac{1}{\omega^2 C^2}}.$$

Also

$$\mathcal{I} = \frac{\mathcal{V}}{\sqrt{R^2 + \frac{1}{\omega^2 C^2}}}$$

A corresponding result holds for R.M.S. values.

Again, from the figure, if $\phi =$ angle SOQ ,

$$\tan \phi = \frac{SQ}{OS} = \frac{OP}{OS} = \frac{1}{\omega CR}.$$

Hence the current **leads** the voltage by an angle whose tangent

$$\frac{1}{\omega C R} = \frac{\text{Reactance}}{\text{Resistance}}.$$

The expression $\sqrt{R^2 + \frac{1}{\omega^2 C^2}}$ is called the **impedance** of the circuit and is denoted by the letter Z . It is measured in ohms.

As in para. 289, it may be defined as the ratio of the maximum value of voltage to the maximum value of current.

295. Circuits containing Inductance, Capacity and Resistance in Series.—We now come to the most important problem of all, and

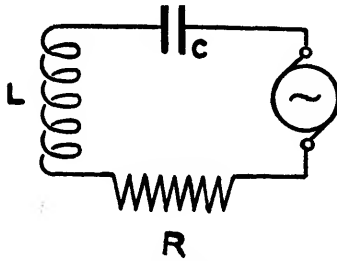


FIG. 135.

one with which we are frequently concerned in wireless telegraphy—namely, the case of a circuit containing **inductance, capacity and resistance in series** (as in Fig. 135).

In this case the alternator has three duties to perform ; it has to supply :—

- (a) A voltage $E_1 (= IR)$ to drive the current through the resistance R .
- (b) A voltage $E_2 (= \omega LI)$ to overcome the counter E.M.F. of the inductance L .
- (c) A voltage $E_3 \left(= \frac{I}{\omega C} \right)$ to overcome the counter E.M.F. of the condenser C .

Let us draw a vector diagram of these three voltages, taking due account of their relative phases (Fig. 136).

The vector $E_1 (= IR)$ is in phase with the current ;
the vector $E_2 (= \omega LI)$ is 90° in advance of the current ; and
the vector $E_3 \left(= \frac{I}{\omega C} \right)$ is 90° behind the current.

As E_2 and E_3 are exactly opposite in phase, we can obtain their resultant by subtracting the smaller from the greater (in the case illustrated by subtracting E_3 from E_2).

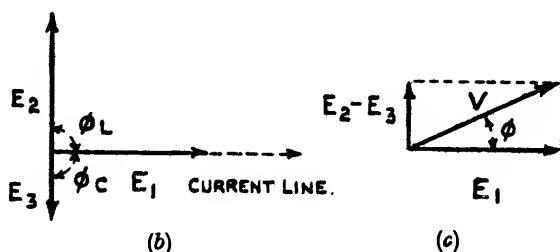


FIG. 136.

The problem then becomes the much simpler one of finding the resultant of two vectors—

$$E_1 = IR, \text{ and } (E_2 - E_3) = \left(\omega LI - \frac{I}{\omega C} \right) = I \left(\omega L - \frac{1}{\omega C} \right)$$

This is obtained as shown in Fig. 136 (b), the resultant applied voltage (V) obviously being such that—

$$V^2 = E_1^2 + (E_2 - E_3)^2 = I^2 R^2 + I^2 \left(\omega L - \frac{1}{\omega C} \right)^2$$

Therefore

$$V = I \sqrt{R^2 + \left(\omega L - \frac{1}{\omega C} \right)^2}$$

$$\text{or } I = \frac{V}{\sqrt{R^2 + \left(\omega L - \frac{1}{\omega C} \right)^2}}$$

This is the law for A.C. circuits containing inductance, capacity and resistance in series.

The expression $\sqrt{R^2 + \left(\omega L - \frac{1}{\omega C} \right)^2}$ is termed the **impedance** of the circuit and is denoted by the symbol Z (ohms).

The tangent of the angle (ϕ) of lag or lead is such that

$$\tan \phi = \frac{I \left(\omega L - \frac{1}{\omega C} \right)}{IR} = \frac{X_L - X_C}{R} = \frac{\text{Reactance}}{\text{Resistance}}$$

As the tendency for the current to lag depends on the value of the counter E.M.F. of the inductance (ωLI volts), and the tendency for the current to lead depends on the counter E.M.F. of the condenser ($\frac{I}{\omega C}$ volts), it can be seen that if ωLI is greater than $\frac{I}{\omega C}$

(or ωL is greater than $\frac{1}{\omega C}$) the resultant current will lag behind the applied E.M.F. ; if ωL is less than $\frac{1}{\omega C}$, the current leads the applied E.M.F.

The fact that the counter E.M.F.s of the inductance and capacity partially cancel one other means that the counter E.M.F. of the inductance helps to charge up the condenser, and the counter E.M.F. of the condenser helps to create the magnetic field round the inductance by the current it produces ; the assistance of the applied voltage is only required to balance the difference between these two counter E.M.F.s.

A voltmeter, if joined across the inductance, would read ωLI volts, and if joined across the condenser would read $\frac{I}{\omega C}$ volts, but if joined across the two would read the difference between the two voltages, *i.e.*, $\left(\omega LI - \frac{I}{\omega C} \right)$ volts.

Example 34.

An alternating voltage of 50 volts at a frequency of 158,000 cycles per second is applied to a circuit consisting of an inductance of 900 mics, a condenser of 1 jar, and a resistance of 10 ohms.

- Find (a) the current flowing ;
 (b) the phase angle of lag or lead ;
 (c) the P.D. across the inductance ;
 (d) the P.D. across the condenser.

$$(a) \omega = 6.28 \times 158,000 = 9.924 \times 10^5.$$

$$X_L = \omega L = 9.924 \times 10^5 \times \frac{900}{10^6} = 893.16 \text{ ohms.}$$

$$X_C = \frac{1}{\omega C} = \frac{1}{9.924 \times 10^5} \times \frac{9 \times 10^8}{1} = \frac{9,000}{9.924} = 906.7 \text{ ohms.}$$

$$X_C - X_L = 906.7 - 893.16 = 13.54 \text{ ohms.}$$

$$Z = \sqrt{R^2 + (X_C - X_L)^2} = \sqrt{10^2 + 13.54^2} \\ = \sqrt{283.4} = 16.83 \text{ ohms.}$$

$$I = \frac{V}{Z} = \frac{50}{16.83} = 2.97 \text{ amperes.}$$

$$(b) \tan \phi = \frac{X_C - X_L}{R} = \frac{13.54}{10} = 1.354. \quad \phi = 53^\circ 35'.$$

Since X_C is greater than X_L , ϕ will be an angle of lead.

$$(c) \omega LI = X_L I = 893.16 \times 2.97 = 2,655 \text{ volts.}$$

$$(d) \frac{I}{\omega C} = X_C I = 906.7 \times 2.97 = 2,694 \text{ volts.}$$

296. Resonance.—A question that naturally arises is whether we can arrange matters so that the counter E.M.F. of the inductance exactly balances the counter E.M.F. of the condenser.

This condition is attained if we arrange that—

$$\omega LI = \frac{I}{\omega C},$$

By dividing both sides of this equation by I we obtain

$$\omega L = \frac{1}{\omega C},$$

$$\text{or } \omega^2 = \frac{1}{LC},$$

$$\text{or } 2\pi f = \frac{1}{\sqrt{LC}},$$

$$\text{or } f = \frac{1}{2\pi\sqrt{LC}}, \text{ where } \begin{cases} L \text{ is measured in henries.} \\ C \text{ is measured in farads.} \end{cases}$$

This equation is frequently used in wireless telegraphy.

If we arrange to satisfy it, either by altering the applied frequency so that it is equal to $\frac{1}{2\pi\sqrt{LC}}$, or by altering the value of the inductance or the capacity of the circuit so that the expression $\frac{1}{2\pi\sqrt{LC}}$ is equal to the applied frequency f , then the counter E.M.Fs. of inductance and capacity “cancel out,” and none of the applied voltage is required to make good their difference.

That is to say, since $\omega L = \frac{1}{\omega C}$, the expression

$$I = \frac{V}{\sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}}$$

becomes $I = \frac{V}{\sqrt{R^2}} = \frac{V}{R}$, and, so far as the applied voltage is concerned, the circuit behaves as if it comprised resistance only.

The current neither lags nor leads, but is in phase with the applied voltage, because $\tan \phi = \frac{X_L - X_C}{R} = \frac{0}{R} = 0$, and therefore $\phi = 0^\circ$.

297. The condition described above is known as “**Electrical Resonance**,” and the frequency that satisfies the equation is known as the “**Resonant Frequency**.”

In series circuits the definition of resonance is that it is the condition under which, for a given applied voltage, the maximum

current flows in the circuit ; and of Resonant Frequency is, that it is the frequency at which a given applied alternating voltage will give a maximum value of current.

At such a frequency the impedance equals the resistance and has a minimum value, and the reactance is zero.

298. Acceptor Circuit.—A circuit comprising inductance and capacity in series, which is in resonance with the frequency applied to it, is said to be an “**Acceptor Circuit**,” for that frequency.

All circuits of the same LC value are obviously acceptor circuits for the same frequency.

For example, if in a certain circuit we double the inductance and halve the capacity the counter E.M.F.s of each will both be doubled, and they will still “cancel out.”

299. As in W/T we are generally working in units of mics and jars, and not of henries and farads, it is convenient to turn the above formula into terms of mics and jars, as follows :—

$$f = \frac{1}{2\pi\sqrt{LC}} \quad \begin{cases} L \text{ in henries.} \\ C \text{ in farads.} \end{cases}$$

$$= \frac{1}{2\pi\sqrt{\frac{L}{10^6} \times \frac{C}{9 \times 10^8}}} = \frac{\sqrt{9 \times 10^{14}}}{2\pi\sqrt{LC}} = \frac{3 \times 10^7}{2\pi\sqrt{LC}}$$

where $\begin{cases} L \text{ is measured in mics.} \\ C \text{ is measured in jars.} \end{cases}$

$$\text{or } \omega = \frac{3 \times 10^7}{\sqrt{LC}}.$$

If an approximate value of 6.28 be taken for 2π , then the above formula becomes

$$f = \frac{4.774 \times 10^6}{\sqrt{LC}} = \frac{4.8 \times 10^6}{\sqrt{LC}} \text{ *approximately.}$$

f in cycles per second,
L in mics,
C in jars.

Example 35.

(i) Find the correct resonant frequency for the circuit given in Example 34.

(ii) If an alternating voltage $V = 50$ volts at this frequency be applied to the circuit, find the answer to (a), (b), (c) and (d) as before.

* It must be remembered that the expression $f = \frac{4.8 \times 10^6}{\sqrt{LC}}$ is only a very rough approximation, and should not be used if an accurate answer is required.

$$(i) f = \frac{3 \times 10^7}{2\pi\sqrt{LC}} = \frac{3 \times 10^7}{2\pi\sqrt{900}} = \frac{3 \times 10^7}{2\pi \times 30} = \frac{10^6}{2\pi}$$

= 159,200 cycles per second.

$$(ii) (a) \omega = \frac{3 \times 10^7}{\sqrt{LC}} = \frac{3 \times 10^7}{\sqrt{900}} = \frac{3 \times 10^7}{30} = 10^6.$$

$$\begin{cases} X_L = \omega L = 10^6 \times \frac{900}{10^6} = 900 \text{ ohms.} \\ X_C = \frac{1}{\omega C} = \frac{1}{10^6} \times \frac{9 \times 10^8}{1} = 900 \text{ ohms.} \end{cases}$$

$$X_L - X_C = 0. \quad I = \frac{V}{R} = \frac{50}{10} = 5 \text{ amperes.}$$

$$(b) \tan \phi = \frac{X_L - X_C}{R} = \frac{0}{10} = 0, \text{ hence } \phi = 0^\circ.$$

$$(c) \omega LI = X_L I = 900 \times 5 = 4,500 \text{ volts.}$$

$$(d) \frac{I}{\omega C} = X_C I = 900 \times 5 = 4,500 \text{ volts.}$$

Notice particularly that the voltages across the inductance and capacity are very much greater than the applied voltage.

This occurs when the reactance of the inductance or of the capacity is great compared with the resistance of the circuit; advantage is continually taken of this fact in W/T circuits.

Example 36.

Find how much inductance is required in series with a condenser of 2 jars capacity to make the circuit an acceptor for a frequency of 500 kilocycles per second.

$$f = \frac{3 \times 10^7}{2\pi\sqrt{LC}} \quad (L \text{ in mics, } C \text{ in jars.})$$

$$500 \times 10^3 = \frac{3 \times 10^7}{2\pi\sqrt{LC}} = \frac{3 \times 10^7}{2\pi\sqrt{2L}}.$$

$$10\pi\sqrt{2L} = 300, \quad \pi\sqrt{2L} = 30.$$

$$2L = \left(\frac{30}{\pi}\right)^2 = \frac{900}{\pi^2} = \frac{900}{9.870} = 91.16.$$

$$\therefore L = 45.58 \text{ mics.}$$

300. Resonance in Daily Life.—The fact that very big voltages build up across the inductance and capacity of a circuit if, **and only if**, it is tuned to be in resonance, or nearly so, with the frequency of the current applied to it, is continually made use of in W/T. In fact, W/T would be quite impossible, if the phenomenon of resonance did not exist.

Resonance may be defined as the transference of energy from one system to another in a series of periodic impulses or waves timed exactly to coincide with the natural rate of vibration of the second system (but see para. 391).

Numerous cases of this phenomenon occur in daily life. Watch, for instance, one child swinging another in a swing; how carefully it gives each little push exactly as the swing has reached its limit and begins to go forward, and how the energy of each push is added to that of the moving swing so that very soon it would knock over a man who stepped in its way.

Walk across a room carrying a cup of tea, and note how quickly it slops over if your step is in time with its natural swing, and how, by walking with short irregular steps, it is much easier to avoid spilling it.

A good demonstration of resonance can also be given with a sponge in a bath. Just swing it to and fro in time with the wave it produces, and in a very few moments you will find the water is also swinging from end to end of the bath with rapidly increasing energy; here also you will find that exact timing of the impulses is the sole condition of success.

301. Currents at Resonance and Non-resonance.—In any circuit containing R , L and C in series, the current is given by the quotient of the applied voltage by the impedance.

At resonance, as we saw above, the impedance consists of resistance only, and $I = \frac{V}{R}$.

If, however, the frequency of the applied voltage is increased above or decreased below the resonant frequency, the equality of ωL and $\frac{1}{\omega C}$ does not hold, and the impedance increases, reducing the current flowing.

Various results follow from these statements:—

(1) **Phasing.**—If the frequency is **increased** above the resonant frequency, ωL becomes greater and $\frac{1}{\omega C}$ becomes smaller, and the resultant reactance will be inductive, so that the current will lag behind the applied voltage.

Conversely, if the frequency is **decreased** below the resonant frequency, ωL becomes smaller and $\frac{1}{\omega C}$ greater, so that the resultant reactance is capacitive, and the current will lead on the applied voltage.

(2) **Effect of amount of departure from resonance.**—The more the frequency of the applied voltage differs from the resonant value, the greater will be the value of the expression $\left(\omega L \sim \frac{1}{\omega C} \right)$

which is the resultant reactance, and hence the greater will be the impedance, and the less the current.

Take the case of an increase of frequency above the resonant frequency. The greater the divergence from resonance, the greater ωL becomes, and the less $\frac{1}{\omega C}$ becomes, hence the greater is their difference.

For a decrease below resonant frequency, the greater the divergence from resonance, the greater $\frac{1}{\omega C}$ becomes and the less ωL becomes, hence the greater is their difference.

(3) **Effect of the proportion of inductance to capacity.**—If the LC value and the resistance R of a circuit are kept constant, but the ratio $\frac{L}{C}$ is altered, the current at resonance is constant, but the current at non-resonance is decreased as this ratio increases.

In the expression for impedance $\sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}$, for a given ω , which does not correspond to resonance, an increase in the ratio $\frac{L}{C}$ means that ωL and $\frac{1}{\omega C}$ are both increased.

If L is increased n times, n being greater than 1, C must be divided by n to keep the product LC the same.

ωL then becomes $n \times \omega L$,

$$\frac{1}{\omega C} \text{ then becomes } \frac{1}{\omega \times \frac{C}{n}} = \frac{n}{\omega C},$$

both results for the different reactances having n times their previous value.

Hence the difference between them is n times what it was before, the impedance is increased, and the current is less.

(4) **Effect of resistance.**—As the resistance R of the circuit is increased, the current at the resonant frequency $\left(= \frac{V}{R}\right)$ is decreased. This effect is independent of the ratio of inductance to capacity in the circuit, except in so far as alteration of this ratio also alters the resistance.

The ratio of the current at non-resonant frequencies to the current at resonance is also affected by the resistance. In a circuit of constant L/C ratio, the current at non-resonant frequencies is increased relatively to the current at resonance when the resistance is increased. The ratio of the current at resonance to the current for a value of ω corresponding to a non-resonant frequency is given by

$$\frac{Z}{R} = \sqrt{1 + \frac{1}{R^2} \left(\omega L - \frac{1}{\omega C}\right)^2}.$$

For given values of ω , L and C , this expression decreases as R is increased.

These points will be illustrated by examples.

Example 37.

(a) In a series circuit consisting of an inductance of 10 mics, a capacity of 10 jars, and a resistance of 10 ohms, let an R.M.S. voltage of 100 volts be applied.

Find the R.M.S. current

(1) if the frequency of the applied voltage is the resonant frequency ;

(2) if it is 90 per cent. of the resonant frequency ;

(3) if it is 70 per cent. of the resonant frequency.

(b) Find the same three values of current if the resistance of the circuit remains 10 ohms, while the inductance is increased to 20 mics and the capacity reduced to 5 jars (same LC value = 100 as before).

(c) Find the same three values of current when the inductance is 20 mics, the capacity is 5 jars, and the resistance is increased to 20 ohms.

(a) The resonant frequency is f such that $2\pi f = \omega = \frac{3 \times 10^7}{\sqrt{LC}}$

$$\therefore \omega = \frac{3 \times 10^7}{\sqrt{100}} = 3 \times 10^6$$

(1) At resonance, current $I = \frac{V}{R} = \frac{100}{10} = 10$ amperes.

X_L and X_C are equal and cancel out, their value being

$$\omega L = 3 \times 10^6 \times \frac{10}{10^6} = 30 \text{ ohms.}$$

(2) In this case $\omega' = 90$ per cent. of the resonant ω
 $= 2.7 \times 10^6$.

$$X_L = \omega' L = 2.7 \times 10^6 \times \frac{10}{10^6} = 27 \text{ ohms.}$$

$$X_C = \frac{1}{\omega' C} = \frac{9 \times 10^8}{2.7 \times 10^6 \times 10} = \frac{90}{2.7} = \frac{30}{0.9} = \frac{10}{0.3} = 33\frac{1}{3} \text{ ohms.}$$

$$X_C - X_L = 6\frac{1}{3} \text{ ohms.}$$

$$Z = \sqrt{R^2 + (X_C - X_L)^2} = \sqrt{100 + (6\frac{1}{3})^2}$$

$$= \sqrt{140.11} = 11.83 \text{ ohms.}$$

$$\therefore \text{Current } I = \frac{100}{11.83} = 8.45 \text{ amperes.}$$

It will be noticed, as mentioned in para. 301 (1), that with the frequency decreased below resonance the capacitive reactance X_C is **greater** than the inductive reactance X_L and so the current **leads** on the voltage.

- (3) In this case $\omega' = 70$ per cent. of the resonant ω
 $= 2.1 \times 10^6$.

$$X_L = \omega' L = 2.1 \times 10^6 \times \frac{10}{10^8} = 21 \text{ ohms.}$$

$$X_C = \frac{1}{\omega' C} = \frac{9 \times 10^8}{2.1 \times 10^6 \times 10} = \frac{90}{2.1} = \frac{30}{0.7} = 42.86 \text{ ohms.}$$

$$X_C - X_L = 21.86 \text{ ohms.}$$

$$Z = \sqrt{10^2 + (21.86)^2} = \sqrt{577.9} \\ = 24.04 \text{ ohms.}$$

$$\text{Current } I = \frac{100}{24.04} = 4.16 \text{ amperes.}$$

This shows that the current is cut down more, the further from resonance is the frequency of the applied voltage (para. 301 (2)).

- (b)—(1) At resonance, with the same LC value and the same R, the current is still **10 amperes**.

- (2) With $\omega' = 2.7 \times 10^6$, $X_L = \omega' L = 2.7 \times 10^6 \times \frac{20}{10^8} = 54 \text{ ohms.}$

$$X_C = \frac{1}{\omega' C} = \frac{1}{2.7 \times 10^6} \times \frac{9 \times 10^8}{5} = 66.67 \text{ ohms.}$$

$$Z = \sqrt{100 + (12.67)^2} = \sqrt{260.4} = 16.14 \text{ ohms.}$$

$$\text{Current } I = \frac{100}{16.14} = 6.2 \text{ amperes.}$$

which is less than the value in case (a) (2) above, justifying the result of para. 301 (3), that the current at non-resonance is decreased as the ratio L/C increases.

- (3) With $\omega' = 2.1 \times 10^6$, $X_L = \omega' L = 2.1 \times 10^6 \times \frac{20}{10^8} = 42 \text{ ohms.}$

$$X_C = \frac{1}{\omega' C} = \frac{1}{2.1 \times 10^6} \times \frac{9 \times 10^8}{5} = 85.71 \text{ ohms.}$$

$$X_C - X_L = 43.71 \text{ ohms.}$$

$$Z = \sqrt{R^2 + (X_C - X_L)^2} = \sqrt{100 + (43.71)^2} \\ = \sqrt{2011} \\ = 44.84 \text{ ohms.}$$

$I = \frac{100}{44.84} = 2.23 \text{ amperes}$, again less than the corresponding result in (a) (3).

- (c)—(1) At resonance, $I = \frac{V}{R} = \frac{100}{20} = 5 \text{ amperes.}$

- (2) With $\omega' = 2.7 \times 10^6$, $X_C - X_L = 12.67 \text{ ohms}$, as in (b) (2).

$$Z = \sqrt{20^2 + 12.67^2} = \sqrt{560.4} = 23.67 \text{ ohms.}$$

$$I = \frac{100}{23.67} = 4.23 \text{ amperes.}$$

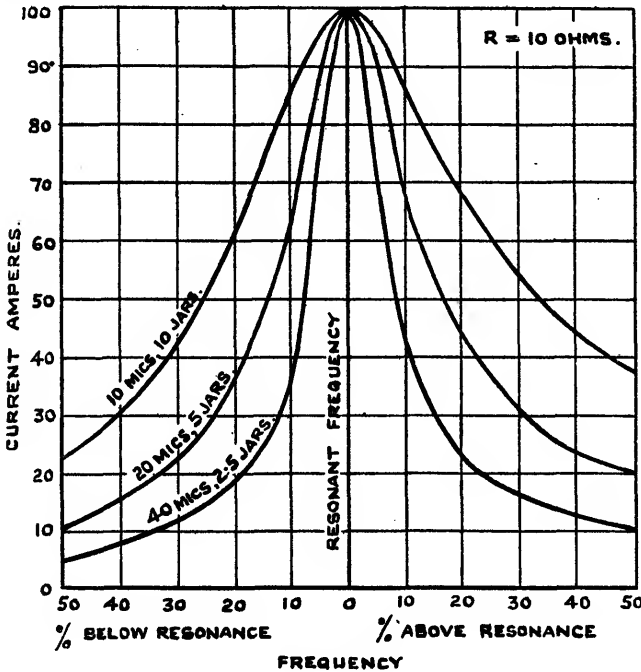
(3) With $\omega' = 2.1 \times 10^6$, $X_C - X_L = 43.71$ ohms, as in (b) (3).

$$Z = \sqrt{20^2 + 43.71^2} = \sqrt{2311} = 48.07 \text{ ohms.}$$

$$I = \frac{100}{48.07} = 2.08 \text{ amperes.}$$

It will be seen that in each of these cases the current is exactly half of the corresponding current in (a). In other words, the ratio of resonant to non-resonant current for the same departure from resonance, which was increased by doubling the L/C ratio, has been restored by doubling the resistance.

302. Resonance Curves and Selectivity.—The currents flowing when an E.M.F. of constant amplitude is applied to a circuit consisting of inductance, capacity and resistance in series, at the



R CONSTANT; L/C VARYING.

FIG. 137.

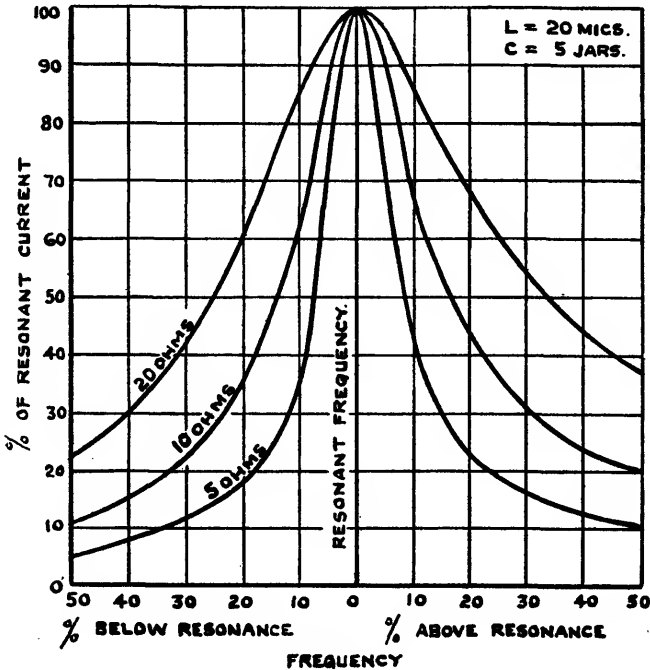
resonant frequency, and at various percentage frequency departures from resonance, may be graphed against percentage frequency above and below resonance. The resulting curve is known as the **resonance** or **response curve** of the circuit.

In Fig. 137 are shown three resonance curves for circuits of the same resistance and LC value, but with different ratios of inductance

to capacity; the results of Example 37 (a) and (b) correspond to points on two of these curves.

Each curve has a peak value at the resonant frequency, and the greater the ratio of inductance to capacity, the more sharply does the curve fall on either side of the peak. The curves are not symmetrical about the resonant frequency, and fall more sharply in the direction of decreasing frequency.

Resonance curves for circuits of the same LC value and L/C ratio, but of varying resistance, are shown in Fig. 138.



L/C CONSTANT; VARYING R.

FIG. 138.

In this case the current at resonance varies inversely as the resistance, and so a better basis of comparison is obtained by plotting the percentage of the resonant current flowing at various percentage departures from the resonant frequency. The curves are similar to those of Fig. 137; the smaller the resistance, the sharper is the resonance peak. Thus decreasing the resistance has the same effect as increasing the L/C ratio.

In practice the coil constituting the inductance in a series resonant circuit must possess some resistance, so that it is not possible to alter the L/C ratio without at the same time altering the

resistance. It is therefore important to consider the effects of joint variation of L/C ratio and resistance on the relation between the currents at the resonant and other frequencies. The effect at a given percentage departure from the resonant frequency may be obtained as follows :

Let ω_0 correspond to the resonant frequency, ω to the non-resonant frequency, and the percentage departure from resonance be 100α , *i.e.*, $\omega = \alpha\omega_0$.

The impedance at resonance, (ω_0), is $Z = R$.

The impedance at the frequency $\frac{\omega}{2\pi}$ is

$$\begin{aligned} Z' &= \sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2} \\ &= \sqrt{R^2 + \left(\alpha\omega_0 L - \frac{1}{\alpha\omega_0 C}\right)^2} = \sqrt{R^2 + \omega_0^2 L^2 \left(\alpha - \frac{1}{\alpha\omega_0^2 LC}\right)^2} \\ &= \sqrt{R^2 + \frac{L}{C} \left(\alpha - \frac{1}{\alpha}\right)^2}, \text{ since } \omega_0^2 = \frac{1}{LC} \end{aligned}$$

$$\therefore \frac{\text{Current at resonant frequency}}{\text{Current at non-resonant frequency}} = \frac{Z'}{Z} = \sqrt{1 + \frac{L}{CR^2} \left(\alpha - \frac{1}{\alpha}\right)^2}.$$

Hence, for a given departure from resonance, the current at the non-resonant frequency becomes a smaller proportion of the resonant current as $\frac{L}{CR^2}$ is increased.

It can easily be seen from this result, and was shown for a particular case in Example 37, that with any given LC value, if the resistance of an inductive coil increases proportionately to its inductance, the value of $\frac{L}{CR^2}$ is unaltered ; if the resistance increases

more slowly than the inductance, $\frac{L}{CR^2}$ is increased ; and if the

resistance increases faster than the inductance, $\frac{L}{CR^2}$ is diminished.

It is usually the case in practice that the ratio of the inductance to resistance in a coil increases as the inductance is increased, and so

$\frac{L}{CR^2}$ is increased by increasing the L/C ratio. Care must, however, be exercised in applying this result in practical W/T circuits where resistances other than that of the coil itself will normally be present.

The square root of $\frac{L}{CR^2}$, viz., $\frac{1}{R} \sqrt{\frac{L}{C}}$ is taken as a criterion of the **selectivity** (para. 492) of a series resonant circuit, *i.e.*, its response at frequencies away from resonance compared with its response at the resonant frequency.

It will be seen later (para. 397) that $\frac{1}{R} \sqrt{\frac{L}{C}}$ is in inverse proportion to the expression derived for the logarithmic decrement of an oscillatory circuit, in which a free oscillation has been set up.

The characteristics of a series resonant circuit may be summed up as follows :—

- (a) At the resonant frequency, the current is a maximum, is independent of the inductance and capacity, and depends solely on the resistance of the circuit.
- (b) At any other frequency, the current depends on all three quantities, and is smaller the further the given frequency departs from resonance.
- (c) Below the resonant frequency, the net reactance of the circuit is capacitive, and the current leads the voltage; above the resonant frequency, the net reactance is inductive and the current lags on the voltage.
- (d) For any given LC value, the ratio of the resonant current to the current at a given percentage departure from resonance depends on the value of $\frac{1}{R} \sqrt{\frac{L}{C}}$. The greater this value becomes, the sharper is the peak of the resonance curve.

303. Acceptor Circuits in Series.—It is often desirable to join two or more acceptor circuits in series.

The tuning of the whole circuit is not altered, but the ratio L/C is increased.

Suppose two circuits $L_1 C_1$ and $L_2 C_2$ are joined in series, and that each is tuned to the same LC value, *i.e.*, that $L_1 C_1 = L_2 C_2$.

$$\text{If } L_1 C_1 = L_2 C_2,$$

$$\text{then } L_1 = \frac{L_2 C_2}{C_1}, \text{ and } L_2 = \frac{L_1 C_1}{C_2}.$$

$$\text{Hence } L_1 + L_2 = \frac{L_2 C_2}{C_1} + \frac{L_1 C_1}{C_2}$$

$$= L_1 C_1 \left(\frac{1}{C_1} + \frac{1}{C_2} \right), \text{ since } L_1 C_1 = L_2 C_2$$

$$= L_1 C_1 \left(\frac{C_1 + C_2}{C_1 \times C_2} \right) \dots \dots \dots (a)$$

The LC value of the two inductances and the two capacities in series will be :—

$$(L_1 + L_2) \left(\frac{C_1 \times C_2}{C_1 + C_2} \right),$$

$$\begin{aligned} \text{which equals } L_1 C_1 \left(\frac{C_1 + C_2}{C_1 \times C_2} \right) \left(\frac{C_1 \times C_2}{C_1 + C_2} \right), \text{ from (a),} \\ = L_1 C_1 = L_2 C_2. \end{aligned}$$

Hence the LC value of the whole circuit is equal to that of either of the two combinations.

The ratio L/C is increased, because the equivalent inductance is increased and the equivalent capacity is decreased; hence their quotient, or ratio, is increased.

Actually the new value of the ratio $\frac{L}{C}$ is

$$\frac{\frac{L_1 + L_2}{C_1 C_2}}{C_1 + C_2} = \frac{(L_1 + L_2)(C_1 + C_2)}{C_1 C_2}$$

$$= \frac{L_1}{C_1} + \frac{L_2}{C_2} + \frac{L_1}{C_2} + \frac{L_2}{C_1},$$

so that it is greater than either of the separate ratios $\frac{L_1}{C_1}$ or $\frac{L_2}{C_2}$, and, in fact, exceeds their sum. This is further explained by an example:

Example 38.

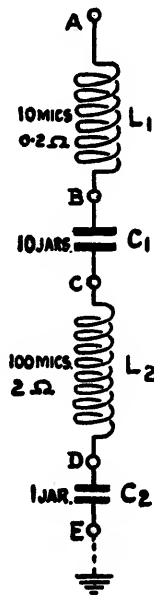


FIG. 139.

The circuits A to C and C to E are each tuned separately to 100 mic-jars.

The total inductance = $10 + 100 = 110$ mics.

The total capacity = $\frac{10 \times 1}{10 + 1} = \frac{10}{11}$ jars.

The LC value of the circuit A to E is therefore

$$110 \times \frac{10}{11} = 100 \text{ mic-jars,}$$

but notice that the ratio L/C is now $110 : \frac{10}{11}$, or $121 : 1$, instead of $1 : 1$ as in the case of the circuit A to C, or $100 : 1$ as in the case of the circuit C to E.

Hence, if two or more acceptor circuits are joined in series, the tuning of the whole circuit is not altered, but the ratio L/C is increased.

304. Voltage across the Inductance or Capacity at Resonance.—

The voltage across the inductance or condenser in a resonant circuit where the current is known may be found more conveniently in the following manner, without working out the frequency :—

The voltage across the inductance $= \omega LI$, which is equal to $\frac{I}{\omega C}$, the voltage across the condenser.

Hence $E_L = E_C = \omega LI$; but $\omega = \frac{1}{\sqrt{LC}}$.

Therefore $E_L = \frac{LI}{\sqrt{LC}} = I \sqrt{\frac{L}{C}}$, where $\begin{cases} L \text{ is in henries,} \\ C \text{ is in farads} \end{cases}$. . . (1)

Bringing henries to mics and farads to jars, we have

$$\begin{aligned} E_L = E_C &= I \sqrt{\frac{L}{10^6} \times \frac{9 \times 10^8}{C}} = I \sqrt{\frac{900L}{C}} \\ &= 30 I \sqrt{\frac{L}{C}}, \text{ where } \begin{cases} L \text{ is in mics,} \\ C \text{ is in jars.} \end{cases} \end{aligned} \quad (2)$$

This formula is useful in cases where the inductance and capacity of a circuit are both "concentrated," and not composed of several inductances and capacities in series or parallel.

Example 39.

In a circuit consisting of an inductance of 700 mics, a condenser of 1 jar, and a resistance of 30 ohms in series, find the voltage (a) across the condenser, (b) across the supply terminals, for a current $I = 10$ amperes at resonant frequency through the circuit.

(a) The voltage across the condenser

$$\begin{aligned} V_C &= 30 I \sqrt{\frac{L}{C}} \text{ volts} = 30 \times 10 \sqrt{\frac{700}{1}} = 300 \sqrt{700} \\ &= 300 \times 26.5 \\ &= 7,950 \text{ volts.} \end{aligned}$$

The maximum value of this voltage will be

$$V_0 = 7,950 \times \sqrt{2} = 7,950 \times 1.414 = 11,240 \text{ volts.}$$

(b) The voltage required across the circuit to force the current of $I = 10$ amperes through the resistance of the circuit at resonant frequency will be

$$V = IR = 10 \times 30 = 300 \text{ volts.}$$

The maximum value of this voltage will be

$$V = 300 \times \sqrt{2} = 300 \times 1.414 = 424.2 \text{ volts.}$$

305. It is interesting to note the distribution of voltage in the circuit of Example 38 above.

Assume that the resistance of L_1 is 0.2 ohm, that of L_2 is 2 ohms, and that a current of 1 ampere is flowing at the resonant frequency. Find the P.D.'s across the various portions of the circuit.

The voltage required to drive the current through the resistance of the acceptor circuit A to C $= IR = 1 \times 0.2 = 0.2$ volt.

Similarly, the voltage across the acceptor circuit C to E $= IR = 1 \times 2 = 2$ volts.

The total voltage required across the two acceptor circuits in series $= 2.2$ volts. The voltages across intermediate points in the circuits are, however, much greater. Thus the voltage across AB is given by

$$I \sqrt{R^2 + \omega^2 L^2}.$$

Since the LC value of both acceptor circuits is 100 mic-jars, the reactance ωL_1 of L_1 is given by the product of

$$\omega = \frac{3 \times 10^7}{\sqrt{100}} = 3 \times 10^6, \text{ and } L_1 = 10 \text{ mics} = \frac{10}{10^6} \text{ henries;}$$

$$\text{i.e., } \omega L_1 = 3 \times 10^6 \times \frac{10}{10^6} = 30 \text{ ohms.}$$

\therefore Voltage across AB $= 1 \sqrt{(0.2)^2 + (30)^2} = 30$ volts approximately.

The resistance is really so small compared with the reactance that it can be neglected, and the formula of the last paragraph would give the same result.

$$\text{Voltage A to B} = 30 I \sqrt{\frac{L}{C}} = 30 \times 1 \times \sqrt{\frac{10}{10}} = 30 \text{ volts.}$$

This is also the voltage from B to C, across the condenser C_1 .

Similarly, voltage C to D $=$ voltage D to E $= 300$ volts. The voltages across the various portions of the circuit are therefore as follows:—

A—B = 30 volts	C—D = 300 volts
(approximately).	(approximately).
B—C = 30 volts.	D—E = 300 volts.
A—C = 0.2 volt.	C—E = 2 volts.
A to E = 2.2 volts.	

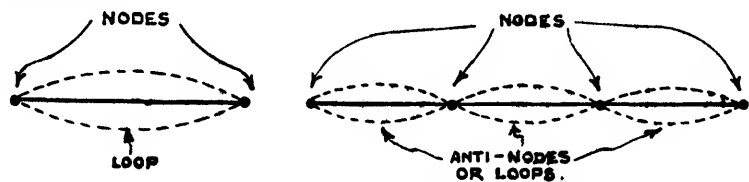
The voltage A to C $= 0.2$ volt is the vector sum of the voltages A to B and B to C; A to B being just a little greater than B to C,

and not in exact antiphase to it. The power required to maintain the current $= I^2 R = 2.2$ watts.

This example will be found useful when dealing with receiving circuits later.

306. Potential Nodes and Loops.—In W/T great variations of potential are encountered between various portions of a circuit, an important point when considering the value of the insulation that must be provided.

The portions of a circuit where the potential does not vary much are often referred to as "nodes of potential," and the points undergoing large potential changes are termed "anti-nodes" or "loops" of potential.



A vibrating violin string.

FIG. 140.

Take a taut violin string and pluck it; it will vibrate as shown by the dotted lines, Fig. 140 (a). The two ends which are secured cannot move; these two points are called "nodes" of vibration. The centre of the string will have a maximum of movement; this point is called a "loop" or "anti-node" of vibration.

Now press the string lightly at a point one-third of its length from one end, Fig. 140 (b). This point and the two ends are held fixed. The remaining two-thirds will automatically divide itself into two equal vibrating portions, with the mid-point stationary. The string will vibrate and emit a note of three times the frequency of the fundamental note as given by the string in Fig. 140 (a).

This note is called the **third harmonic**.

As can be seen from the figure, we have in this second case four nodes and three anti-nodes of vibration.

Various interesting experiments of this nature may be tried by applying regularly-timed jerks to a taut signal halyard or stay.

Similar results apply to the wireless circuit we have just been discussing. Suppose we earth the point E. Then the point D will be oscillating at 300 volts above and below earth potential.

The potential of the point C will only vary 2 volts either way.

The point B will vary 30 volts, and A will vary 0.2 volt with respect to C.

Thus A, C and E may be spoken of as nodes of potential, and B and D as anti-nodes, or loops of potential.

D must have insulation from earth adequate to stand 300 volts, C to stand 2 volts, B to stand 30 volts, and A to stand 2.2 volts.

PARALLEL COMBINATIONS OF RESISTANCE, INDUCTANCE AND CAPACITY IN A.C. CIRCUITS.

307. Resistance and Inductance in Parallel.—In the treatment of various types of parallel circuit, vector diagram methods will be used.

If an inductance L and a resistance R are joined in parallel (as in Fig. 141 (a)), and an alternating voltage V is applied to the combination, currents of different value and different phase relationship will flow through the two parallel paths, and the current from the supply will be their vector sum.

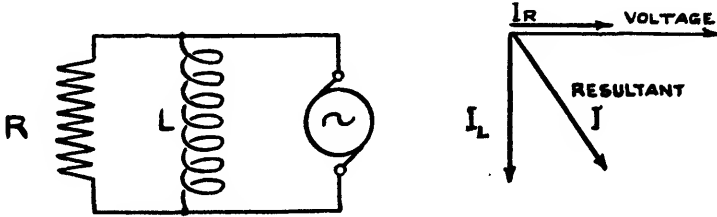


FIG. 141.

In this case,

(a) a current I_L will flow through the inductance, equal to $\frac{V}{\omega L}$ and lagging by 90° on the applied voltage;

(b) a current I_R will flow through the resistance, equal to $\frac{V}{R}$, in phase with the applied voltage.

The resultant current from the alternating supply is seen to be, from the vector diagram (Fig. 141 (b)),

$$\begin{aligned} I &= \sqrt{I_R^2 + I_L^2} = \sqrt{\left(\frac{V}{R}\right)^2 + \left(\frac{V}{\omega L}\right)^2} \\ &= V \sqrt{\frac{1}{R^2} + \frac{1}{\omega^2 L^2}} \end{aligned}$$

The angle of lag of current on voltage is given by ϕ , where

$$\tan \phi = \frac{\frac{1}{\omega L}}{\frac{1}{R}} = \frac{R}{\omega L}.$$

As before, the **impedance** Z of the parallel circuit is given by the ratio of the maximum (or R.M.S.) value of the voltage to the maximum (or R.M.S.) value of the current.

$$\text{In this case } Z = \frac{V}{I} = \frac{1}{\sqrt{\left(\frac{1}{R}\right)^2 + \left(\frac{1}{\omega L}\right)^2}} = \frac{\omega LR}{\sqrt{R^2 + \omega^2 L^2}}$$

The ratio of current to voltage, $\sqrt{\left(\frac{1}{R}\right)^2 + \left(\frac{1}{\omega L}\right)^2}$,

i.e. the reciprocal of the impedance is called the **admittance** of the circuit. It is denoted by the letter Y .

Example 40.

An alternating voltage of 100 volts at a frequency of 10,000 cycles per second is applied to a circuit consisting of a resistance of 20 ohms in parallel with an inductance of 300 mics. Find (a) the current and (b) the angle of lag.

$$\omega = 2\pi \times 10,000 = 6.28 \times 10^4.$$

$$X_L = \omega L = 6.28 \times 10^4 \times \frac{300}{10^6} = 6.28 \times 3 = 18.84 \text{ ohms.}$$

$$I_L = \frac{V}{\omega L} = \frac{100}{18.84} = 5.31 \text{ amps.}$$

$$I_R = \frac{V}{R} = \frac{100}{20} = 5 \text{ amps.}$$

$$\begin{aligned} \text{Resultant current} &= \sqrt{I_R^2 + I_L^2} = \sqrt{5^2 + 5.31^2} \\ &= \sqrt{25 + 28.17} = \sqrt{53.17} = 7.3 \text{ amps.} \end{aligned}$$

$$\tan \phi = \frac{I_L}{I_R} = \frac{5.31}{5} = 1.06. \quad \phi = 46^\circ 42'.$$

Hence the resultant current is one of 7.3 amperes lagging behind the applied voltage by an angle of $46^\circ 42'$.

308. From the above, the following general deduction may be made concerning the vectorial treatment of series and parallel circuits.

In series circuits—inductance, resistance, &c., in series—the current through each will be the same, and the voltage drop across each will be different. **Start by drawing the current line.** Lay off along it the voltage drop due to resistance, and at right angles to it the voltage drop due to reactance—inductive or capacitive, or both. Completion of the vector diagram gives the necessary supply voltage for the given current.

In parallel circuits—inductance, resistance, &c., in parallel—the voltage applied to each will be the same and the current through each will be different. **Start by drawing the voltage line.** Lay off along it the current through the resistance, and at right angles to it the current through the inductance or capacity. Completion of the vector diagram gives the resultant current for the given supply voltage.

In parallel circuits, either or both arms of the circuit may contain combinations of resistance and inductance or capacity; in which case the corresponding current vector will, as shown in paras. **307** and **309**, be inclined to the voltage vector at an angle between 0° and 90° .

309. Resistance and Capacity in Parallel.—Suppose that a resistance R and a capacity C are in parallel, as in Fig. 142 (a).

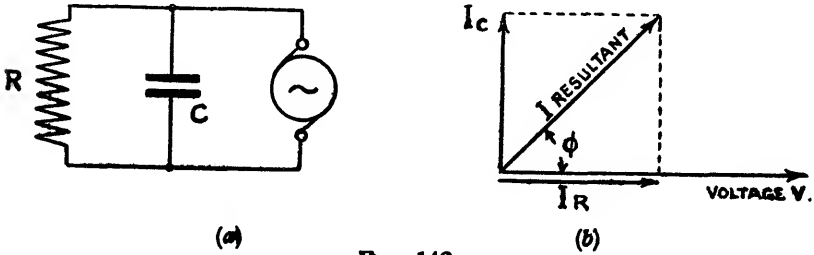


FIG. 142.

An alternating voltage V is applied to the combination. In this case,

- (a) a current I_R flows through the resistance $= \frac{V}{R}$, in phase with the applied voltage ;
- (b) a current I_C flows "through" (*i.e.*, charging up) the condenser, which leads the applied voltage by 90° , and is equal to $V\omega C$.

The resultant current from the alternating supply is seen to be, from the vector diagram, Fig. 142 (b),

$$I = \sqrt{I_R^2 + I_C^2} = V \sqrt{\frac{1}{R^2} + \omega^2 C^2}.$$

The angle ϕ , by which the current leads the voltage, is given by

$$\tan \phi = \frac{\omega C}{\frac{1}{R}} = \omega CR.$$

As in para. 307, $\sqrt{\frac{1}{R^2} + \omega^2 C^2}$, being the ratio of R.M.S. current to R.M.S. voltage, is termed the **admittance** of the parallel circuit, and its reciprocal,

$$\frac{1}{\sqrt{\frac{1}{R^2} + \omega^2 C^2}} = \frac{R}{\sqrt{1 + \omega^2 C^2 R^2}}$$

is the impedance of the circuit.

310. Equivalent Capacity and Series Resistance.—The case of a capacity in parallel with a resistance is important in practice, as it may represent a condenser with dielectric leakage, or leakage to earth from a high potential point in an aerial, or simply a condenser with an artificial resistance across it.

We propose to show that, for such a circuit, an equivalent series circuit comprising capacity and resistance can be substituted, giving the same impedance and the same phase angle.

Let us assume that a circuit containing C_1 and R_1 in series is equivalent to C and R in parallel.

By equality of impedances, $\sqrt{R_1^2 + \frac{1}{\omega^2 C_1^2}} = \frac{R}{\sqrt{1 + \omega^2 C^2 R^2}}$.

By equality of phase angles, $\frac{1}{\omega C_1 R_1} = \omega C R$.

$$\therefore \frac{R}{\sqrt{1 + \omega^2 C^2 R^2}} = \sqrt{R_1^2 + \frac{1}{\omega^2 C_1^2}} = R_1 \sqrt{1 + \frac{1}{\omega^2 C_1^2 R_1^2}}$$

$$= R_1 \sqrt{1 + \omega^2 C^2 R^2}$$

$$\therefore R_1 = \frac{R}{1 + \omega^2 C^2 R^2}$$

$$\text{Also } C_1 = \frac{1}{\omega^2 C R R_1} = \frac{1 + \omega^2 C^2 R^2}{\omega^2 C R^2} = C \left(1 + \frac{1}{\omega^2 C^2 R^2} \right).$$

In those practical cases where the resistance shunted across the condenser, e.g., the insulation resistance, is high compared with the

capacitive reactance, $\frac{R^2}{\omega^2 C^2}$ or $\omega^2 C^2 R^2$ is large compared to unity.

Thus a good approximation to R_1 is $\frac{R}{\omega^2 C^2 R^2} = \frac{1}{\omega^2 C^2 R}$, and C_1 very nearly equals C .

Hence a parallel resistance R can be replaced by a series resistance $\frac{1}{\omega^2 C^2 R}$, a result which is very useful when considering power losses.

The higher the leak resistance, the less power will be wasted.

It was shown, while dealing with condenser theory, that the various losses can all be represented by either resistances in series or parallel; this paragraph shows that they can all be shown simply by an equivalent resistance in series.

311. Inductance and Capacity in Parallel (neglecting Resistance).—The investigation of a parallel circuit, in which one side of the circuit contains inductance only, and the other capacity only, will now be undertaken. It must be remembered that such conditions are impossible in practice, but the results obtained from the theoretical case lead up to those in the general case where resistance is included.

In this paragraph, and succeeding paragraphs up to and including para. 315, the existence of resistance is entirely neglected.

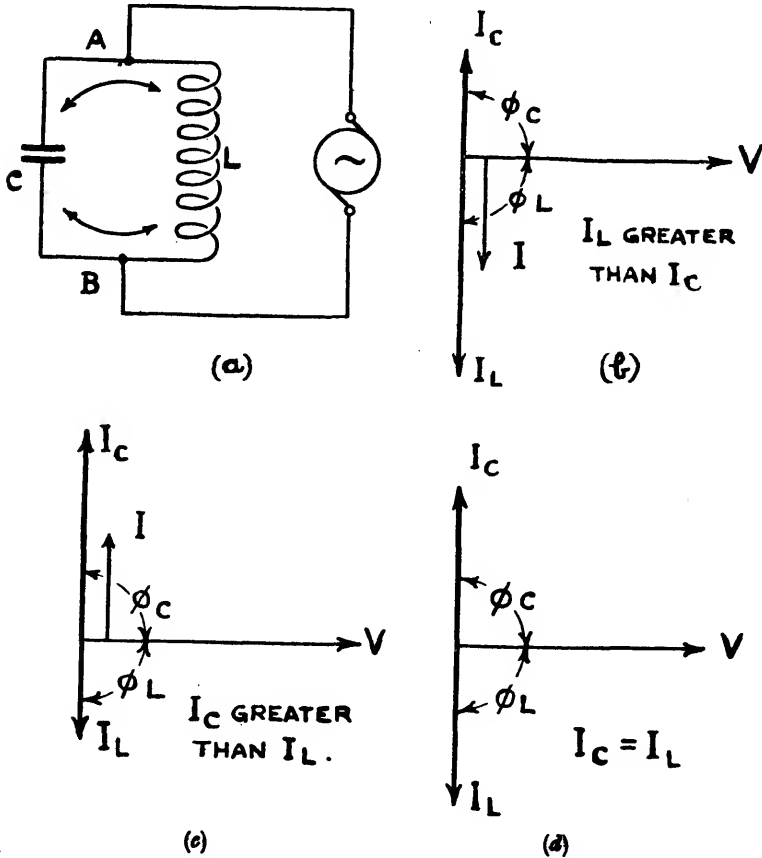


FIG. 143.

If an alternating voltage of V volts at a frequency of f cycles per second be applied to a circuit, as shown in Fig. 143 (a), the current flowing through the circuit from the supply can be arrived at as follows :—

- (a) A current I_L will flow through the inductance L such that
$$I_L = \frac{V}{\omega L},$$
 lagging behind the applied voltage V by an angle ϕ_L of 90° .
- (b) A current I_C will flow through the condenser C such that
$$I_C = V\omega C,$$
 leading on the applied voltage by an angle ϕ_C of 90° .

The supply current from the A.C. source of supply will be the vector sum of these separate currents.

In Fig. 143 (b), (c) and (d) are represented the different conditions that may obtain according to the relative magnitudes of I_L and I_C .

If, as in Fig. 143 (b), I_L is greater in magnitude than I_C , the vector sum of the two currents is actually their difference $I_L - I_C$. In this case $I = I_L - I_C = V \left(\frac{1}{\omega L} - \omega C \right)$ and lags 90° on the voltage.

If, as in Fig. 143 (c), I_L is less than I_C , the resultant of the two currents is $I_C - I_L = V \left(\omega C - \frac{1}{\omega L} \right)$, and leads 90° on the voltage.

312. Resonant case, neglecting Resistance.—If, as in Fig. 143 (d), $I_L = I_C$, the resultant current flowing from the A.C. source is $I_L - I_C = 0$.

In this case

$$\begin{aligned} V \times \omega C &= \frac{V}{\omega L} \\ \omega C &= \frac{1}{\omega L} \\ \omega^2 &= \frac{1}{LC}, \quad \omega = \frac{1}{\sqrt{LC}}, \quad f = \frac{1}{2\pi\sqrt{LC}} \end{aligned}$$

This is known as the **Resonant Frequency** for the circuit given.

The **Resonant Frequency** for the simple parallel circuit dealt with is defined to be the frequency at which an applied alternating voltage will send **no** current through the circuit.

The **impedance** of a simple parallel circuit at resonance is **infinite**; the **admittance** is zero.

313. Rejector Circuit.—A resonant circuit composed of inductance and capacity in parallel is termed a "**Rejector Circuit**," being exactly opposite in its effects to an acceptor circuit.

This opposition of effects applies to the various aspects considered in para. 301. These results will be developed in due course.

314. Circulating Current.—Although no current passes through the circuit, large currents may flow in the respective sides of the circuit. These currents are I_L and I_C respectively. The explanation is based on the fact that these currents are exactly equal and 180° out of phase; so that at the instant when the current from A to B in the inductive side of the circuit is a maximum, say, the current in the capacitive branch is also a maximum, but flowing from B to A; i.e., at that particular instant there is a current flowing "round" the circuit in a clockwise direction and having a maximum value.

From this point of view, the name "**circulating**" current is applied to the current flowing round the circuit. The current from the supply may be termed the make-up current. Here it is zero.

The value of the circulating current is given by

$$I_L = I_C = \frac{V}{\omega L} = V \omega C.$$

$$\omega = \frac{1}{\sqrt{LC}}, \text{ L and C being in henries and farads.}$$

\therefore Circulating current $= \frac{VC}{\sqrt{LC}} = V \sqrt{\frac{C}{L}}$ amps, when L is in henries and C in farads,

$$= V \sqrt{\frac{C}{9 \times 10^8 \times \frac{10^6}{L}}}$$

$$= \frac{V}{30} \sqrt{\frac{C}{L}} \text{ amps, when L is in mics}$$

and C in jars.

In the purely theoretical case we are considering, with no resistances, there is no expenditure of power to keep this circulating current flowing; in the practical case, with resistance, the small resultant of I_L and I_C , which is the make-up current, and cannot then be zero, provides the power necessary to maintain the circulating current.

315. Non-Resonant Case, neglecting Resistance.—Let us assume a different frequency of supply, which may be greater or less than the resonant one.

The make-up current at resonance is zero, the impedance being infinite. As, however, the frequency is increased above or decreased below the resonant frequency, the equality of $\frac{1}{\omega L}$ and ωC no longer holds, and the current through the rejector circuit, being $V \left(\frac{1}{\omega L} \sim \omega C \right)^*$, has a value other than zero.

As in para. 301, various results can be derived.

(1) **Phasing.**—If the frequency is increased above the resonant frequency, ωC becomes greater and $\frac{1}{\omega L}$ becomes smaller, so that the make-up current $V \left(\omega C - \frac{1}{\omega L} \right)$ is capacitive, and leads 90° on the voltage.

If the frequency is decreased below resonance, $\frac{1}{\omega L}$ is greater

* The sign \sim means "take the difference between" the two symbols it links.

than ωC and the resultant current $V \left(\frac{1}{\omega L} - \omega C \right)$ lags 90° on the voltage. These results were also derived in para. 311, and are seen to be in opposition to those of para. 301.

(2) **Effect of amount of departure from resonance.**—The further the frequency of the applied voltage departs from the resonant value, the greater will be the value of the expression $\left(\frac{1}{\omega L} - \omega C \right)$, and hence the greater will be the current flowing through the circuit. This result is again opposite to that of para. 301.

(3) **Effect of the proportion of inductance to capacity.**—If the LC value of the circuit is kept constant, but the ratio $\frac{C}{L}$ is increased, the current through the rejector at resonance is still zero, but the current at non-resonance increases as this ratio increases.

If C is increased n times, n being greater than 1, and L is divided by n to keep the product the same, then ωC becomes $n\omega C$ and $\frac{1}{\omega L}$ becomes $\frac{n}{\omega L}$, and the difference between the two becomes n times what it was before.

Hence the current $V \left(\frac{1}{\omega L} - \omega C \right)$ is increased n times. This result is the **same** as that in para. 301. In both cases, acceptor and rejector (taking the rejector circuit with **no** resistance), an increase in the ratio $\frac{L}{C}$ cuts down the current at a given non-resonant frequency, or increases the impedance of the circuit.

The rejector circuit is used to stop the flow of current at the resonant frequency through the circuit, and, by increasing C and diminishing L correspondingly, it can be made to allow large currents to pass through it at frequencies only slightly different from the resonant one.

In continuation of the results in (1) and (2) above, it is obvious that for frequencies very much greater or less than the resonant frequency, the rejector circuit is practically equivalent to a simple capacity C or a simple inductance L respectively.

Example 41.

If an alternating voltage of 1 volt be applied to a circuit consisting of an inductance of 4 mics in parallel with a condenser of 25 jars (a) at the resonant frequency, (b) at a frequency of 500,000 cycles per second, find the circulating current in the first case, and the current from the supply in the second case.

If the inductance is decreased to 1 mic and the capacity increased to 100 jars, find (c) the current from the supply corresponding to case (b) above.

$$(a) \text{ Circulating current} = \frac{V}{30} \sqrt{\frac{C}{L}} = \frac{1}{30} \sqrt{\frac{25}{4}} = \frac{1}{30} \times \frac{5}{2} \\ = \frac{1}{12} \text{ amp.} = 83.3 \text{ milliamps.}$$

$$(b) \omega = 2\pi \times 5 \times 10^5 = \pi \times 10^6 \text{ radians per sec.}$$

$$I_C = \omega CV = \frac{3.14 \times 10^6 \times 25 \times 1,000}{9 \times 10^8} \text{ mA} = 87.2 \text{ mA.}$$

$$I_L = \frac{V}{\omega L} = \frac{1}{3.14 \times 10^6} \times \frac{10^6 \times 1,000}{4} \text{ mA} = \frac{1,000}{12.56} \\ = 79.6 \text{ mA.}$$

$$I_C - I_L = 7.6 \text{ milliamps.}$$

The supply current is therefore one of 7.6 milliamps, leading by 90° , since the condenser current is greater than the inductance current.

$$(c) \omega = \pi \times 10^6 \text{ radians per sec., as before.}$$

$$I_C = \omega CV = \frac{3.14 \times 10^6 \times 100 \times 1,000}{9 \times 10^8} \text{ mA} = 349 \text{ mA.}$$

$$I_L = \frac{V}{\omega L} = \frac{10^6 \times 1,000}{3.14 \times 10^6 \times 1} \text{ mA} = 318.3 \text{ mA.}$$

$$I_C - I_L = 30.7 \text{ milliamps.}$$

The increase in the ratio $\frac{C}{L}$ has resulted in increasing the current at a given non-resonant frequency four times.

316. General case, including Resistance.—So far, we have been dealing with a circuit containing no resistance whatever.

In practice, such a state of affairs is not possible, although in an efficient circuit the resistances are small, especially in the condenser branch.

The general case, with resistance in each branch, will be considered first.

Owing to the presence of resistance, the current through the inductance branch will not now be equal to $\frac{V}{\omega L}$, but will be

$$I_L = \frac{V}{\sqrt{R_L^2 + (\omega L)^2}}$$

lagging on the applied voltage by an angle ϕ_L which is not 90° , but $\tan^{-1} \frac{\omega L}{R_L}$.

Similarly, the current through the condenser branch will be

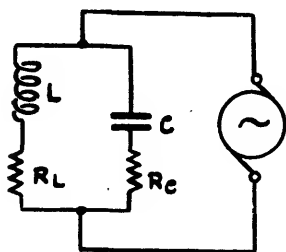
$$I_C = \frac{V}{\sqrt{R_C^2 + \left(\frac{1}{\omega C}\right)^2}},$$

leading by an angle ϕ_C , whose tangent is $\frac{1}{\omega C R_C}$.

The resultant supply current I is the vector sum of these two currents I_L and I_C .

Let us resolve I_L and I_C into components in phase with the applied voltage, and components 90° out of phase with it.

I_L is equivalent to $I_L \sin \phi_L$ lagging 90° behind the applied voltage, and $I_L \cos \phi_L$ in phase with it. Similarly, I_C is equivalent to $I_C \sin \phi_C$ leading 90° on the applied voltage, and $I_C \cos \phi_C$ in phase with it.



(a)

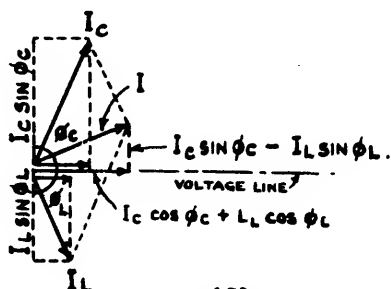
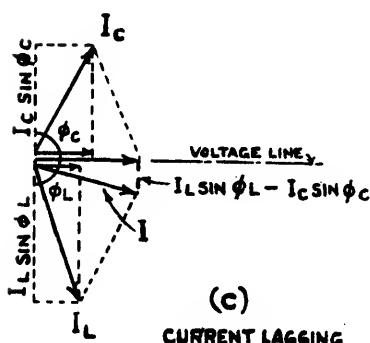
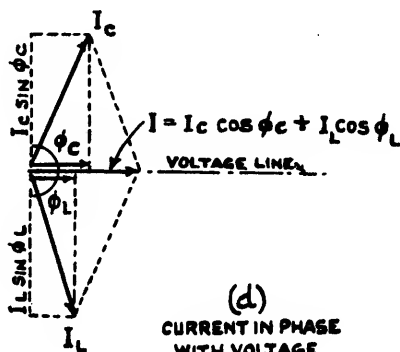
(b)
CURRENT LEADING(c)
CURRENT LAGGING(d)
CURRENT IN PHASE
WITH VOLTAGE

FIG. 144.

The supply current is then the vector sum of :—

- (a) $I_L \sin \phi_L \sim I_C \sin \phi_C$, 90° out of phase with the applied voltage; and
- (b) $I_L \cos \phi_L + I_C \cos \phi_C$, in phase with the applied voltage (see Fig. 144 (b), (c), (d)).

317. Resonance.—Resonance occurs in a parallel circuit, in which resistance is taken into account, when the supply current is in phase with the applied voltage.

For this to be the case, the vertical components of I_L and I_C in the vector diagram must be equal and opposite, i.e.,

$$I_L \sin \phi_L = I_C \sin \phi_C.$$

There is then no reactance offered by the circuit, and its impedance is a pure resistance only.

At the resonant frequency the supply current flowing through the circuit is not necessarily a minimum, so that, while in the acceptor circuit resonance may be looked upon as meaning maximum current (minimum impedance), or zero reactance, in this case zero reactance is the only correct definition of resonance.

***318. Resonant Frequency, and Effective Resistance of a Resonant Parallel Circuit.—**

At resonance $I_L \sin \phi_L = I_C \sin \phi_C$.

ϕ_L is an angle whose tangent is $\frac{\omega L}{R_L}$,

and so $\sin \phi_L = \frac{\omega L}{\sqrt{R_L^2 + (\omega L)^2}}$.

$$\begin{aligned} \therefore I_L \sin \phi_L &= \frac{V}{\sqrt{R_L^2 + (\omega L)^2}} \times \frac{\omega L}{\sqrt{R_L^2 + (\omega L)^2}} \\ &= \frac{V \cdot \omega L}{R_L^2 + (\omega L)^2} \end{aligned}$$

$$\text{Similarly, } I_C \sin \phi_C = \frac{V \cdot \frac{1}{\omega C}}{R_C^2 + \left(\frac{1}{\omega C}\right)^2}$$

Equating these two results:—

$$\begin{aligned} \frac{\omega L}{R_L^2 + (\omega L)^2} &= \frac{\frac{1}{\omega C}}{R_C^2 + \left(\frac{1}{\omega C}\right)^2} \\ \omega^2 L R_C^2 + \frac{L}{C^2} &= \frac{1}{C} R_L^2 + \frac{L^2}{C} \omega^2 \\ \omega^2 \left(L R_C^2 - \frac{L^2}{C} \right) &= \frac{R_L^2}{C} - \frac{L}{C^2} \\ \omega^2 &= \frac{R_L^2 C - L}{L R_C^2 C^2 - C L^2} = \frac{L - R_L^2 C}{L^2 C - L R_C^2 C^2} = \frac{1}{LC} \left(\frac{L - C R_L^2}{L - C R_C^2} \right), \end{aligned}$$

which gives the value of ω for resonance.

Since $f = \frac{\omega}{2\pi}$, the resonant frequency may then be found.

The current at resonance may now be derived by making use of the value of ω just found. The current is in phase with the voltage and so is given by

$$I = I_L \cos \phi_L + I_C \cos \phi_C$$

$$= \frac{V \cdot R_L}{R_L^2 + \omega^2 L^2} + \frac{V \cdot R_C}{R_C^2 + \frac{1}{\omega^2 C^2}}$$

by substitution for I_L , I_C , ϕ_L and ϕ_C .

$$\therefore \frac{I}{V} = \frac{R_L}{R_L^2 + \omega^2 L^2} + \frac{R_C}{R_C^2 + \frac{1}{\omega^2 C^2}}$$

But $\omega^2 L^2 = \frac{L}{C} \left(\frac{L - CR_L^2}{L - CR_C^2} \right)$, and $\frac{1}{\omega^2 C^2} = \frac{L}{C} \left(\frac{L - CR_C^2}{L - CR_L^2} \right)$.

$$\therefore \frac{I}{V} = \frac{CR_L (L - CR_L^2)}{L^2 - C^2 R_C^2 R_L^2} + \frac{CR_C (L - CR_L^2)}{L^2 - C^2 R_C^2 R_L^2}$$

$$= \frac{C (R_L + R_C) (L - CR_L R_C)}{L^2 - C^2 R_C^2 R_L^2}$$

$$= \frac{C (R_L + R_C)}{L + CR_C R_L}$$

Writing the total resistance $R_L + R_C$, as R , this gives

$$\frac{I}{V} = \frac{CR}{L \left(1 + \frac{C}{L} R_C R_L \right)}$$

$\frac{I}{V}$ is the admittance, Y , of the parallel circuit.

The impedance (or effective resistance, since current and voltage are in phase) is given by

$$Z = \frac{1}{Y} = \frac{L \left(1 + \frac{C}{L} R_C R_L \right)}{CR}$$

When R_L and R_C are small, as is generally the case in practice, the approximate value of the effective resistance of the circuit is $\frac{L}{CR}$, this result being in ohms, if L is in henries, C in farads, and R in ohms.

The supply current is $\frac{VRC}{L}$ amperes, V being in volts and the other quantities as above. The curious result must be noted that the supply current is **greater** and the impedance **less**, the **greater** the actual ohmic resistance included in the circuit.

If all the resistance of the circuit can be taken to be in the inductance branch, *i.e.* $R_C = 0$,

$$\omega^2 = \frac{1}{LC} - \frac{R_L^2}{L^2},$$

and the effective resistance of the circuit is given accurately by $\frac{L}{CR}$.

If we make the further approximation that R_L can be neglected,

$$\omega = \frac{1}{\sqrt{LC}}$$

The statement made in para. 317 that resonance does not give a minimum value of supply current can be easily proved by finding the value of the supply current for the frequency corresponding to

$\omega = \frac{1}{\sqrt{LC}}$ in the case where $R_C = 0$, but R_L is not taken equal to zero; though not in phase with the voltage, it will be found to be less than the supply current for the correct resonant frequency given by

$$\omega = \sqrt{\frac{1}{LC} - \frac{R_L^2}{L^2}}$$

The difference is very small, however, and it is always assumed in practice that the impedance of a rejector circuit is $\frac{L}{CR}$ for an applied frequency $= \frac{1}{2\pi\sqrt{LC}}$, and that this impedance can be regarded as a pure resistance, the supply current being practically in phase with the voltage.

319. The last approximate result can also be derived as follows:—

If R_L and R_C are considered to be very small, the currents I_L and I_C may be assumed equal and are given by $\frac{V}{\omega L}$ or $V\omega C$; either of

which expressions, with $\omega = \frac{1}{\sqrt{LC}}$, equals $V\sqrt{\frac{C}{L}}$.

The power expended by these circulating currents is given by

$$\begin{aligned} I_L^2 R_L + I_C^2 R_C &= V^2 \frac{C}{L} (R_L + R_C) \\ &= \frac{V^2 CR}{L}, \text{ where } R = R_L + R_C. \end{aligned}$$

This power is supplied by the source of alternating voltage.

If V is the supply voltage and I the supply current,

$$VI = \frac{V^2 CR}{L}$$

$$\therefore I = V \frac{CR}{L}$$

and the effective resistance of the rejector circuit

$$= \frac{V}{I} = \frac{L}{CR}$$

If C is in jars, and L in mics, the supply current

$$= VR \frac{C}{9 \times 10^8} \times \frac{10^6}{L} = \frac{VRC}{900L}$$

Example 42.

If an alternating voltage of 10 volts be applied to a circuit consisting of 100 jars in parallel with an inductance of 4 mics at the correct resonant frequency, find (a) the supply current required, (b) the circulating current, if the resistance of the circuit is equal to 0.1 ohm.

$$\begin{aligned}
 (a) \text{ Supply current} &= V \frac{RC}{900L} \quad (\text{para. 319}) \\
 &= 10 \times \frac{0.1 \times 100}{900 \times 4} = \frac{1}{36} = 0.028 \text{ ampere.} \\
 (b) \text{ Circulating current} &= \dot{I} = \frac{V}{30} \sqrt{\frac{C}{L}} \quad (\text{para. 314}) \\
 &= \frac{10}{30} \sqrt{\frac{100}{4}} = \frac{5}{3} = 1.67 \text{ amperes.}
 \end{aligned}$$

320. Non-Resonant Case, including Resistance.—When the vertical components of I_L and I_C are not equal, the supply current is the vector sum of ($I_L \sin \phi_L \sim I_C \sin \phi_C$) and ($I_L \cos \phi_L + I_C \cos \phi_C$), these being at right angles to each other vectorially (para. 316).

(1) **Phasing.**—If the applied frequency is greater than the resonant one, the current I_C through the condenser branch will be

$$\text{increased, and its vertical component } I_C \sin \phi_C = \frac{V \cdot \frac{1}{\omega C}}{R_0^2 + \left(\frac{1}{\omega C}\right)^2}$$

will be increased. (This result is strictly true only if $\frac{1}{\omega C} > R_0$, but this would hold in practice.) In the same way, for an increase in frequency above resonant frequency, the current I_L through the inductive branch is decreased, and its vertical component $I_L \sin \phi_L = \frac{V \omega L}{R_L^2 + (\omega L)^2}$ is decreased. The resultant of the out-of-phase components of current, being ($I_L \sin \phi_L \sim I_C \sin \phi_C$), is therefore capacitive, and the supply current will lead on the applied voltage.

If the applied frequency is less than the resonant one, the supply current lags on the applied voltage (*cf.* results of para. 315 for the case of no resistance).

(2) **Effect of departure from resonance.**—For frequencies sensibly different from resonance, the greater the departure from resonant frequency the greater will the current become through one branch of the circuit, and the less the current through the other, so that their vector sum, which is the supply current, increases.

It is difficult to give a strict proof of this statement. It is, of course, proved for the case of zero resistance in para. 315.

(3) **Effect of alteration of ratio of capacity to inductance.**—

An increase in the ratio $\frac{C}{L}$ not only affects the results at non-resonance, but, from the formula $I = \frac{VRC}{L}$, it increases the supply current at resonance. In the acceptor circuit the ratio $\frac{L}{C}$ or $\frac{C}{L}$ does not affect current at resonance. At non-resonance a strict proof on the lines of para. 315 cannot be given, owing to the complicated form of the vector diagram, but it may be assumed that the inclusion of resistance into the problem does not alter the results of para. 315, and that an increase in the ratio $\frac{C}{L}$ increases the current at non-resonance in the case where the circuit includes resistance. An example follows to illustrate these points.

Example 43.

(1) In a circuit consisting of 10 mics in parallel with 10 jars, the inductive branch having a resistance of 1 ohm and the capacitive branch no resistance, find (a) the current through the circuit for an applied voltage of 100 volts at the resonant frequency, (b) the correct resonant frequency, (c) the current at a frequency which is 70 per cent. of the resonant frequency.

(2) Repeat the calculation for a circuit of 5 mics and 20 jars, the resistance being as before.

In this example resistance has been introduced as being wholly in the inductance branch—the numerical calculation being much longer if it is included in both branches.

$$(1) (a) I = \frac{VRC}{L} = 100 \times \frac{1 \times 10}{9 \times 10^8} \times \frac{10^6}{10} \text{ ampere} = 0.11 \text{ ampere.}$$

$$(b) \omega^2 = \frac{1}{LC} - \frac{R_L^2}{L^2} = \frac{9 \times 10^8 \times 10^6}{10 \times 10} - \frac{1^2 \times 10^{18}}{10^8}$$

$$= 10^{12} \left(9 - \frac{1}{100} \right) = 10^{12} \times 8.99.$$

$$\therefore \omega = 10^6 \times \sqrt{8.99} = 2.9983 \times 10^6.$$

$$f = \frac{\omega}{2\pi} = 477,200 \text{ cycles per second.}$$

It is obvious that the value for ω is practically 3×10^6 , the result which would be obtained if R_L were neglected.

In other words, even with resistance considered, $\omega = \frac{1}{\sqrt{LC}}$ is a very close approximation to the " ω " corresponding to the resonant frequency.

For simplicity of calculation, $\omega = 3 \times 10^6$ will be used in (c).

(c) For a frequency which is 70 per cent. of the resonant frequency, $\omega = 2.1 \times 10^6$ approximately.

$$\text{In this case, } I_L = \frac{V}{\sqrt{R_L^2 + (\omega L)^2}}.$$

The out-of-phase component of I_L is $I_L \sin \phi_L$

$$= \frac{V \omega L}{R_L^2 + (\omega L)^2} \quad (\text{para. 320}).$$

$$\text{Now } \omega L = 2.1 \times 10^6 \times \frac{10}{10^6} = 21 \text{ ohms.}$$

$$\therefore I_L \sin \phi_L = \frac{100 \times 21}{1^2 + 21^2} = \frac{2,100}{442} = 4.75 \text{ amperes.}$$

The in-phase component of I_L is

$$I_L \cos \phi_L = \frac{V R_L}{R_L^2 + (\omega L)^2} = \frac{100 \times 1}{1^2 + 21^2} = \frac{100}{442} = 0.226 \text{ ampere.}$$

The current through the capacity is 90° out of phase and is given by

$$V \omega C = \frac{100 \times 2.1 \times 10^6 \times 10}{9 \times 10^8} = \frac{21}{9} = 2.33 \text{ amperes.}$$

The current I from the supply is the vector sum of the in-phase current and the resultant out-of-phase current, and is therefore given by

$$\begin{aligned} I^2 &= (4.75 - 2.33)^2 + (.226)^2 \\ &= (2.42)^2 + (.226)^2 \\ &= 5.86 + .05 = 5.91. \end{aligned}$$

$$\therefore I = 2.43 \text{ amperes}$$

which is very much greater than the current at resonance given by (a).

$$(2)-(a) \quad I = \frac{VRC}{L} = 100 \times \frac{1 \times 20}{9 \times 10^8} \times \frac{10^6}{5} = 0.44 \text{ ampere.}$$

$$\begin{aligned} (b) \quad \omega^2 &= \frac{1}{LC} - \frac{R_L^2}{L^2} = (9 \times 10^{12}) - \frac{1^2 \times 10^{12}}{5^2} \\ &= 10^{12} (9 - .04) = 10^{12} \times 8.96 \end{aligned}$$

$$\omega = 10^6 \times \sqrt{8.96} = 2.9933 \times 10^6,$$

still a very close approximation to 3×10^6 .

$$f = \frac{\omega}{2\pi} = 476,500 \text{ cycles per second.}$$

(c) Using $\omega = 3 \times 10^6$ as the resonant frequency, as before,

$$\omega' = 2.1 \times 10^6. \quad \omega' L = 2.1 \times 10^6 \times \frac{5}{10^6} = 10.5 \text{ ohms.}$$

$$I_L \sin \phi_L = \frac{V \omega' L}{R_L^2 + (\omega' L)^2} = \frac{100 \times 10.5}{1^2 + (10.5)^2} = \frac{1,050}{111.25} = 9.44 \text{ amperes.}$$

$I_L \cos \phi_L$ (in-phase component)

$$= \frac{V R_L}{R_L^2 + (\omega' L)^2} = \frac{100 \times 1}{111.25} = 0.90 \text{ ampere.}$$

The current I_0 , which is 90° out of phase, is

$$V\omega'C = \frac{100 \times 2.1 \times 10^8 \times 20}{9 \times 10^8} = 4.67 \text{ amperes.}$$

$$\begin{aligned} I \text{ (as before)} &= \sqrt{(9.44 - 4.67)^2 + (.90)^2} \\ &= \sqrt{(4.77)^2 + (.90)^2} \\ &= \sqrt{22.75 + .81} = \sqrt{23.56} = 4.85 \text{ amperes.} \end{aligned}$$

Thus the current **at resonance** and the current **at non-resonance** given by (a) and (c) have both been increased by an increase in the ratio of C to L.

It may also be observed, that an increase of R to say, double its amount, would double the current at resonance, but would have little effect on its value at frequencies differing much from resonance.

321. Resonance Curves.—The values of supply current flowing at different frequencies may be graphed, as in the case of the acceptor circuit (para. 302) and the result is known as a resonance curve.

Owing to the laborious calculation involved, exact curves have not been drawn, but from the theory and worked examples it is obvious that such curves would be as illustrated in the following figure:—

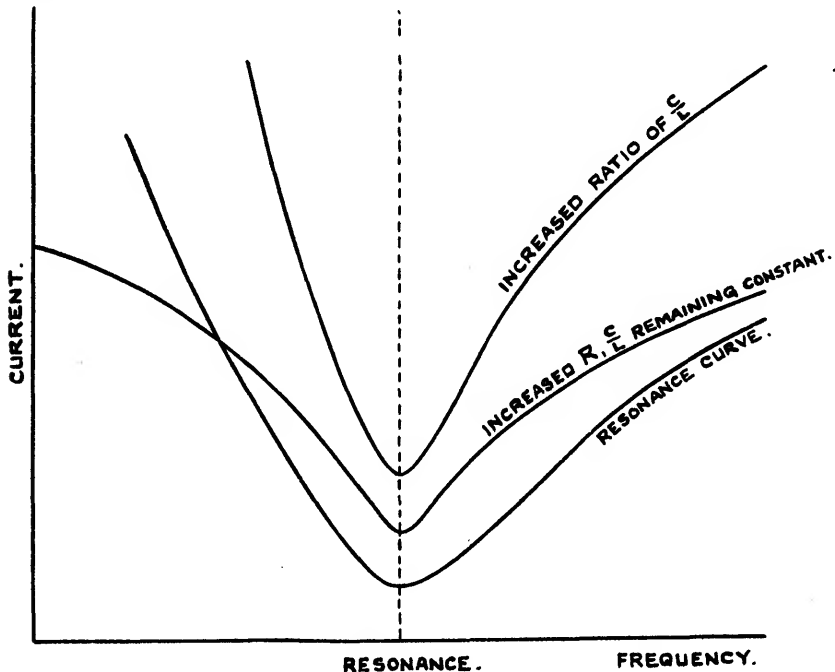


FIG. 145.

The properties of a rejector circuit may now be summed up :—

(a) Resonant Frequency.

- (1) If the circuit has no resistance losses at all, it allows no current to flow through it from the source of supply, although a large circulating current is set up in the circuit.
- (2) If it has any resistance losses, a supply current can flow through it, which is greater :—
 - (i) the greater the resistance losses ;
 - (ii) the greater the ratio C/L .
- (3) If the circuit contains resistance, the resonant frequency (at which the supply current is in phase with the voltage) is not entirely independent of the resistance, as is the case with the acceptor circuit.

(b) Non-Resonant Frequencies.

- (1) For non-resonant frequencies, the supply current increases—
 - (i) the farther the non-resonant frequency is from resonance ;
 - (ii) the greater the ratio C/L ;
 - (iii) the greater the resistance in the circuit.
- (2) If the non-resonant frequency is greater than the resonant frequency, the current leads on the applied voltage ; if less, the current lags on the applied voltage.

322. Comparison between Acceptor and Rejector Circuits.—

The essential difference is that the acceptor circuit is an easy path for currents at the resonant frequency, and a more difficult one for all others, whereas the rejector is a difficult path for currents at the resonant frequency and an easier path for all others.

At resonance, current through the acceptor is inversely proportional to its resistance, while current through the rejector is directly proportional to the resistance.

At resonance, current in the acceptor circuit is independent of the ratio of inductance to capacity ; in the rejector this is not so.

The resonant frequency for the acceptor circuit is independent of the resistance ; in the rejector circuit this is not so, but for small resistances in the circuit, the divergence from the value

$$f = \frac{1}{2\pi\sqrt{LC}},$$

which is correct for the acceptor, is not very great.

POWER IN ALTERNATING CURRENT CIRCUITS.

323. Comparison with D.C. Circuits.—In any **direct** current circuit, the expenditure of power is easily obtainable in any one of three ways :—

- (a) If the current and applied voltage are known, in amps and volts respectively, then the power expenditure (in watts) is equal to $V \times I$.
- (b) If the resistance of the circuit is known in ohms, and the current flowing through the resistance is known in amps., then the power expenditure (in watts) $= VI = IR \times I = I^2R$.
- (c) If the resistance of the circuit and the applied voltage are known, then the power expenditure $= VI = V \times \frac{V}{R} = \frac{V^2}{R}$.

In alternating-current circuits these simple relations do not all hold ; owing to the fact that the value of the applied voltage, and the value of current flowing, are continuously changing, and that they are generally out of phase with each other, the power supplied to the circuit is itself a variable quantity, and formulæ have to be derived which will give the **mean** power expenditure over a complete cycle.

324. A.C. Circuits containing Resistance only.—In an alternating current circuit in which the reactance is zero—for instance, a circuit which contains only a resistance R —the values of current and applied voltage are related to one another by the formula $I = \frac{V}{R}$ or, using R.M.S. values, $I = \frac{V}{R}$.

From the definition of the R.M.S. value of an alternating current, given in para. 274, which states that it is the equivalent value of D.C. current which would give the same power expenditure in a resistance R as an alternating current amplitude \mathcal{I} , the power expenditure in this case is I^2R or VI or $\frac{V^2}{R}$ when I and V are the R.M.S. values of current and voltage respectively.

The strict mathematical proof has already been given in para. 275 (a), which shows that the **mean** power expenditure over a cycle is $\frac{\mathcal{I}^2R}{2}$ which, **by definition**, is I^2R .

In an A.C. circuit containing resistance only, therefore, the power expenditure (in watts) is the product of V (volts) and I (amperes), as read by alternating current voltmeter and ammeter, which register R.M.S. values.

325. General case of A.C. Circuits.—We now consider A.C. circuits which have reactance as well as resistance, and in which the current and the voltage are therefore not in phase. Let us take the simple circuit illustrated in Fig. 146, consisting of a resistance R and an inductance L .

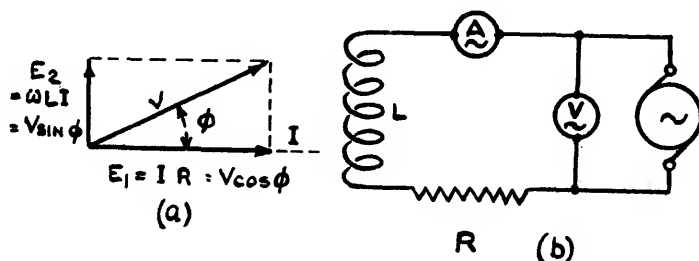


FIG. 146.

An ammeter (A) reads the R.M.S. value of the current flowing, and a voltmeter (V) reads the R.M.S. value of the applied voltage.

Then we know that the current will be lagging on the voltage by an angle ϕ , whose tangent is $\frac{\omega L}{R}$. The voltage V is performing two duties :—

- (a) It is supplying a component $E_1 = IR$ to force the current through the resistance of the circuit. This voltage is in phase with the current, and is known as the **in-phase** component of the total voltage.
- (b) It is supplying a component $E_2 = \omega LI$, to overcome the counter E.M.F. of the inductance. This component is 90° ahead of the current, and is known as the **wattless** component of the total voltage.

In the case of the inductance, the energy expended in creating a magnetic field round the inductance during part of the cycle is completely restored to the circuit during the remainder of the cycle. In other words, the mean expenditure of energy, when the current and voltage are 90° out of phase with each other, is zero. Hence the component of the voltage E_2 , or ωLI , does not, in conjunction with the current flowing, involve any power expenditure, and so arises the name "wattless" applied to it.

The only power required is that necessary to force the current through the resistance R . The voltage required to do this is E_1 , and from para. 324 the power expenditure is $E_1 I$.

(a) $E_1 = IR$.

\therefore The power required $= E_1 I = I^2 R$.

(b) $\frac{E_1}{V} = \cos \phi$.

Hence $E_1 = V \cos \phi$, and the power required $= VI \cos \phi$.

In terms of maximum values it is $\frac{\mathcal{V}}{\sqrt{2}} \frac{\mathcal{I}}{\sqrt{2}} \cos \phi$

$$= \frac{\mathcal{V}\mathcal{I} \cos \phi}{2}.$$

Similar considerations would apply to the case of a resistance in series with a condenser, or a resistance in series with both an inductance and a condenser. For the condenser the component of voltage 90° out of phase with the current is also "wattless," since the energy associated with it and the current is alternately taken from and returned to the circuit with no losses, and the mean power expenditure over a whole cycle is zero.

***326. Mathematical Proof of the General Case.**—We are assuming the general A.C. circuit, in which the current is not in phase with the voltage. The voltage can therefore be represented by $v = \mathcal{V} \sin \omega t$, and the current by $i = \mathcal{I} \sin (\omega t \pm \phi)$, according to whether it leads or lags.

At any moment the instantaneous power is the product of the instantaneous current and the instantaneous voltage, and is therefore

$$\begin{aligned} iv &= \mathcal{I}\mathcal{V} \sin \omega t \sin (\omega t \pm \phi) \\ &= \frac{\mathcal{I}\mathcal{V}}{2} \{ \cos (\pm \phi) - \cos (2\omega t \pm \phi) \} \\ &= \frac{\mathcal{I}\mathcal{V}}{2} \cos \phi - \frac{\mathcal{I}\mathcal{V}}{2} \cos (2\omega t \pm \phi) \end{aligned}$$

Since $(2\omega t \pm \phi)$ can take all values from 0 to 360° for the different values of t in a complete cycle, the second term in this expression has a **mean** value of zero.

The mean value of iv over a complete cycle is therefore

$$\frac{\mathcal{I}\mathcal{V}}{2} \cos \phi, \text{ or } IV \cos \phi,$$

which agrees with the result of para. 325.

$$I = \frac{V}{Z} \text{ and } \cos \phi = \frac{R}{Z}, \text{ hence } IV \cos \phi = I \times IZ \times \frac{R}{Z} = I^2 R.$$

327. Power Factor.—For the general A.C. circuit, with reactance and resistance, an A.C. voltmeter across the supply would read V , the R.M.S. voltage, and an A.C. ammeter in the circuit would read I , the R.M.S. current. The product of these readings VI is termed the "**apparent watts**," since this is the power that is **apparently** being expended on the circuit.

We have seen above that the **true** mean power expenditure is $VI \cos \phi$, and this is known as the "**true watts**."

The ratio of the mean power supplied, $VI \cos \phi$, to the product of R.M.S. voltage and current, VI , is given by $\frac{VI \cos \phi}{VI}$, or $\cos \phi$, and this is defined as the **Power Factor**.

Hence, **True watts = Power Factor \times R.M.S. voltage \times R.M.S. current.**

$$\cos \phi = \frac{\text{True watts}}{\text{Apparent watts}} = \frac{\text{Resistance } R}{\text{Impedance } Z}$$

True watts can be measured by a wattmeter.

$$\text{Power factor} = \cos \phi = \frac{\text{Wattmeter reading}}{\text{Ammeter reading} \times \text{Voltmeter reading}}$$

The expression for Power, I^2R (true watts), may be looked on as a method of determining the resistance of the circuit, which may be different under A.C. conditions from its D.C. value, due to hysteresis, eddy currents, &c.

We may thus define the effective resistance of an A.C. circuit as being

$$\frac{\text{True watts}}{(\text{R.M.S. current})^2}$$

the true watts expenditure being measured by a wattmeter and the current by an A.C. ammeter.

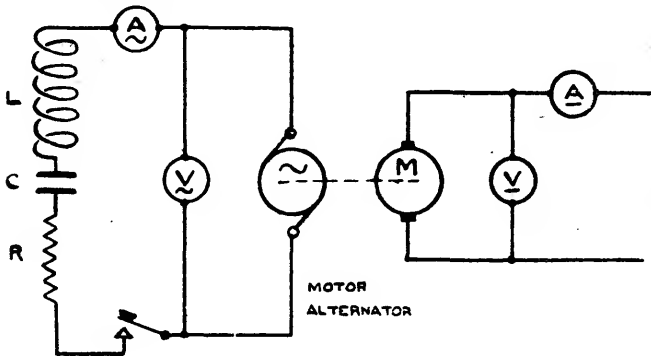


FIG. 147.

Example 44.

Find the Power Factor, true watts, and apparent watts for the circuit given in Example 34 above.

$$\text{True watts} = I^2 R = 2.97^2 \times 10 = 88.21 \text{ watts.}$$

$$\text{Apparent watts} = VI = 50 \times 2.97 = 148.5 \text{ watts.}$$

$$\text{Power Factor} = \cos \phi = \frac{R}{Z} = \frac{10}{16.83} = 0.5941,$$

$$\text{or, True watts} = VI \cos \phi = 50 \times 2.97 \times 0.5941 = 88.21 \text{ (again),}$$

$$\text{or, Power Factor} = \frac{\text{True watts}}{\text{Apparent watts}} = \frac{88.21}{148.5} = 0.5941.$$

323. If we have measuring instruments joined up to a motor alternator, as illustrated in Fig. 147, then the ammeter and the voltmeter on the A.C. side will read the apparent watts, while the

ammeter and the voltmeter on the D.C. side will give a measure of the true power being expended on the circuit, together with the power required to overcome the frictional losses of the machine.

If a reading of the D.C. ammeter be taken with the machine running, but with the A.C. circuit broken, and another reading when the A.C. current is completed, the power factor can be calculated.

329. If the circuit contains nothing but resistance, or if it is in resonance with the applied frequency, then current and voltage are in phase, $\phi = 0$, $\cos \phi = 1$, and apparent watts = true watts.

On the other hand, if the circuit contains inductance and/or capacity only, and no resistance, current and voltage are 90° out of phase, $\phi = 90^\circ$, $\cos \phi = 0$, and the true watts expenditure is zero.

330. Power Factor of a Condenser.—In the section on condensers in Chapter III, it was mentioned that various losses occur in practical condensers, which may be regarded as equivalent to resistances in series or parallel with the condenser. Again, it was shown in para. 310 that a parallel resistance can be replaced by an equivalent series resistance.

Let the total equivalent series resistance be R . For such a condenser, C , the current will not lead the voltage by 90° , but by an angle ϕ less than 90° , whose tangent is $\frac{1}{\omega CR}$. The power absorbed by the resistance will be $VI \cos \phi$, and $\cos \phi$ is known as the Power Factor of the condenser.

In general, if the losses are small, the phase angle of the condenser will not differ much from 90° , and $\cos \phi$ will be a small quantity.

$$\text{Now } \cos \phi = \frac{R}{Z} = \frac{R}{\sqrt{R^2 + \left(\frac{1}{\omega C}\right)^2}}.$$

When R is very small compared with $\frac{1}{\omega C}$, R^2 may be neglected in the denominator, giving

$$\text{Power Factor} = \cos \phi = \omega CR.$$

The expression $VI \cos \phi$ for the power absorbed by the equivalent series resistance is, of course, the power actually absorbed by the condenser owing to its various losses.

Also $\cos \phi = \sin (90^\circ - \phi) = \left(\frac{\pi}{2} - \phi\right)$ in circular measure, as the angle $(90^\circ - \phi)$ is small, so that the Power Factor may be taken as the angle which is the difference between 90° and the actual angle of lead of the current on the voltage (expressed in radians).

This angle is sometimes referred to as the "phase difference" of the condenser.

Example 45.

A condenser has a capacity of 1 jar and a phase difference of 1° . Find its equivalent series resistance, and the power absorbed by the condenser when an alternating voltage of 100 volts is applied to it at a frequency for which $\omega = 10^6$.

$$\omega C = \frac{10^6 \times 1}{9 \times 10^8} = \frac{1}{900} \therefore \frac{1}{\omega C} = 900 \text{ ohms.}$$

Since the phase difference is small, it is approximately equal to the power factor, and each may be taken equal to ωCR .

$$1^\circ \text{ (in circular measure) } = \omega CR.$$

$$\frac{1 \times \pi}{180} = \omega CR.$$

$$\therefore R = \frac{\pi}{180} \times \frac{1}{\omega C} = \frac{\pi}{180} \times \frac{900}{1} = 5\pi = 15.7 \text{ ohms.}$$

$$I = \frac{V}{Z} = V \omega C \text{ approximately (since } R \text{ is small compared to } \frac{1}{\omega C} \text{).}$$

$$= \frac{100}{900} = \frac{1}{9} \text{ ampere} = 0.11 \text{ ampere.}$$

$$\therefore \text{Power consumed} = I^2 R = \left(\frac{1}{9}\right)^2 \times 15.7 = \frac{15.7}{81} = 0.19 \text{ watts.}$$

$$\text{or Power} = VI \cos \phi = VI \times \text{phase difference} = 100 \times \frac{1}{9} \times \frac{\pi}{180} \\ = \frac{\pi}{16.2} = 0.19 \text{ watts.}$$

331. Rating of Alternators.—Makers of alternating current machines rate their machines as being capable of delivering so many kilo-volt amperes (k.V.A.) and not as capable of delivering so many kilo-watts (kW.).

That is to say, they guarantee that the machine will generate a certain voltage if kept revolving at the correct speed, and that it will stand a certain current without overheating.

They cannot guarantee it as being capable of producing a certain power under all conditions, because they do not know the nature of the load the user is going to put on it.

For example, if a machine guaranteed to deliver 5 kW. at 200 volts were put on to a circuit having a power factor of .7, it would then have to supply an apparent power $\frac{5,000}{.7} = 7,143$ watts, so that the true watts (5,000) should be equal to the apparent watts (7,143) multiplied by $\cos \phi$ (.7).

This would necessitate a current of $\frac{7,143}{200} = 35.7$ amps. instead of $\frac{5,000}{200} = 25$ amps.

The increased heating effect would damage the machine.

COUPLED CIRCUITS.

332. Methods of Coupling.—When two circuits are so arranged that energy can be transferred from one to the other, they are said to be coupled. Methods of coupling may be classified into **direct** and **mutual** coupling according to the nature of the path connecting the one circuit to the other.

Mutual coupling has already been encountered when discussing mutual induction, where, owing to the proximity of two inductances, the changing magnetic field due to a changing current in one sets up voltages across the other. The two circuits are entirely disconnected.

Direct coupling may be of two types :—

- (a) In one type, the two circuits may contain a common impedance, the actual portion of the circuit which is common being a resistance, an inductance, or a capacity.
- (b) In the other type, the two circuits may be connected together through an impedance, which does not form a part of either individual circuit.

These different types are illustrated below :—

Mutual Coupling.

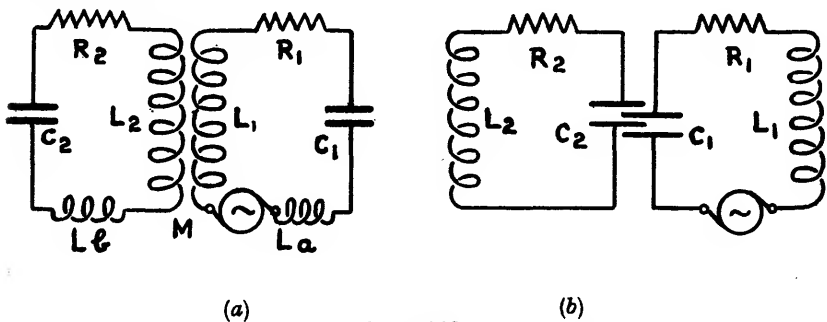


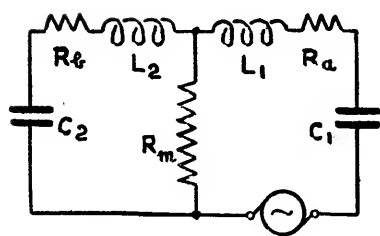
FIG. 148.

There are two types of mutual coupling, of which (a) above is by far the most common. In both there is a field common to both circuits, in (a) magnetic, in (b) electric; (a) is called **mutual inductive** coupling, or **mutual magnetic** coupling; (b) is called **mutual capacitive** or **mutual electrostatic** coupling.

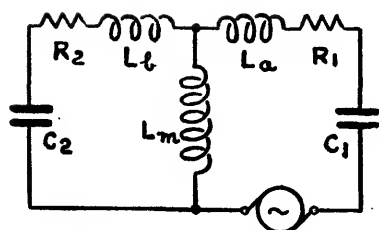
Direct Coupling of the first type.

With this type of coupling, the common impedance may be a resistance, as in (a), an inductance as in (b), or a capacity as in (c).

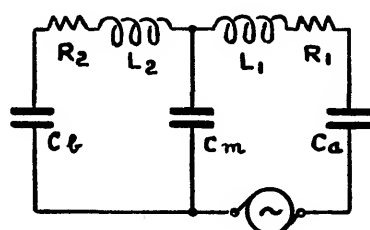
- (a) is called **resistive** coupling. The voltage drop due to the current in one circuit acts as an applied E.M.F. in the other.



(a)

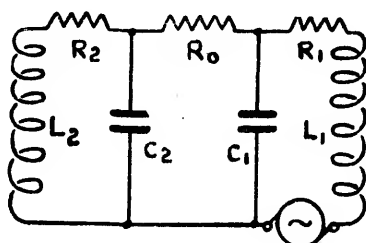


(b)

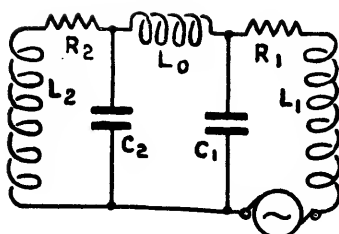


(c)

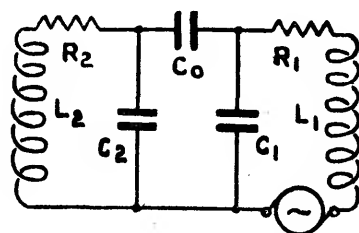
FIG. 149.



(a)



(b)



(c)

FIG. 150.

- (b) is called **inductive**, or **auto-inductive** coupling, to distinguish it from the previous case of Fig. 148 (a). The E.M.F. set up across the inductance by a changing current in one circuit acts as an applied E.M.F. in the other.
- (c) is called **capacitive** coupling. A changing current in one circuit sets up voltage variations across the condenser, which act as an applied E.M.F. in the other.

Direct Coupling of the second type.

Figs. 150 (a), (b) and (c) respectively show types of coupling in which the two circuits $L_1 R_1 C_1$ and $L_2 R_2 C_2$ are connected by a resistance R_0 , an inductance L_0 , and a capacity C_0 , these impedances not forming part of the individual circuits.

The general division of coupling could have been made otherwise, of course, into resistance coupling, electromagnetic coupling and electrostatic coupling, these three subdivisions incorporating all types in which the coupling between the circuits is due to resistance, inductance, or capacity, whether direct or mutual.

The important types of coupling are those referred to above as :—

- (1) Mutual Inductive. Fig. 148 (a).
- (2) Resistive. Fig. 149 (a).
- (3) Auto-Inductive. Fig. 149 (b).
- (4) Direct Capacitive. Fig. 149 (c) and Fig. 150 (c).

333. Coupling Factor.—As a measure of the extent to which one circuit affects the other, it is necessary to have a definition of the degree of coupling between the two circuits. This is done by means of a **coupling coefficient**, or **coupling factor**, denoted by the symbol **K**.

The coupling factor is defined as the ratio of the mutual or common reactance of the two circuits to the square root of the product of the total **similar** reactances in the separate circuits. Thus, for the case of **mutual inductive** coupling, Fig. 148 (a), the coupling factor is

$$\frac{\omega M}{\sqrt{\omega (L_1 + L_a) \times \omega (L_2 + L_b)}} = \frac{M}{\sqrt{(L_1 + L_a) (L_2 + L_b)}}$$

which agrees with the value of **K** already given in Chapter III, where L_a and L_b were zero. For the case of **auto-inductive** coupling, Fig. 149 (b), the common reactance is ωL_m ; the separate total similar reactances of the two circuits are

$$\omega (L_a + L_m) \text{ and } \omega (L_b + L_m)$$

$$\text{so that } K = \frac{L_m}{\sqrt{(L_a + L_m) (L_b + L_m)}}.$$

For the case of **capacitive** coupling, Fig. 149 (c), the common capacitive reactance is $\frac{1}{\omega C_m}$.

The capacitive reactance of the circuit $C_a C_m R_1 L_1$ is given by $\frac{1}{\omega C_1} = \frac{1}{\omega C_a} + \frac{1}{\omega C_m} = \frac{1}{\omega} \frac{C_a + C_m}{C_a C_m}$, and a similar expression holds for the capacitive reactance of the other circuit.

$$K = \frac{\frac{1}{\omega C_m}}{\sqrt{\frac{1}{\omega} \times \frac{C_a + C_m}{C_a C_m} \times \frac{1}{\omega} \frac{C_b + C_m}{C_b C_m}}} = \sqrt{\frac{C_a C_b}{(C_a + C_m)(C_b + C_m)}}.$$

For the cases illustrated in Figs. 150 (b) and (c), where the direct coupling is by means of an inductance or capacity not included in either circuit, formulæ may be obtained as follows :—

$$\text{Fig. 150 (b) : } K = \sqrt{\frac{L_1 L_2}{(L_1 + L_0)(L_2 + L_0)}}$$

$$\text{Fig. 150 (c) : } K = \frac{C_0}{\sqrt{(C_1 + C_0)(C_2 + C_0)}}.$$

In these cases the "mutual reactance" can only be obtained by finding the voltage generated in the second circuit when a current of 1 ampere flows in the first. It is not proposed to carry out this investigation here, but the results are quoted for reference.

Circuits are said to be "tightly" or "loosely" coupled together, according as the energy transferred from one to the other is large or small; or, in terms of the coupling factor, according as K is large or small.

For the most common type of coupling, viz. mutual inductive, the coupling factor K may be increased in one of two ways :—

(a) By moving the coupled coils closer together.

(b) By increasing the inductances of the two coils, L_1 and L_2 , which take part in the mutual inductive action, and decreasing the inductances of the two coils L_a and L_b , which do not so take part, in order to maintain resonance in both circuits. For auto-inductive coupling, K may be increased by increasing the common inductance L_m and decreasing L_a and L_b to maintain resonance. For capacitive coupling, K may be increased by decreasing the value of the common capacity C_m and increasing C_a and C_b to maintain resonance. For capacitive coupling of the type illustrated in Fig. 150 (c), K is increased by increasing C_0 .

334. Free and Forced Oscillations in coupled circuits.—Two types of oscillatory action occurring in coupled circuits are of importance in W/T circuits:—

- (1) **Free Oscillations.**—In this case two circuits are coupled together and one is set in oscillation, the energy being transferred to the other by means of the coupling. There is **no** continuously applied E.M.F. Since the theory of free oscillatory action in one circuit alone is not considered until Chapter VII, the more complicated theory of the frequencies at which two coupled circuits oscillate freely will be postponed till then.
- (2) **Forced Oscillations.**—In this case a source of alternating E.M.F. is included in one of the two circuits which are coupled together. It is found that the degree of coupling has an effect on the frequency to which the circuits are **resonant**. If the coupling is small, and the two circuits are separately resonant to the same frequency, they will continue to be resonant to that frequency when coupled together; but, if the coupling is increased, they will be resonant to **two frequencies, one higher and one lower** than the frequency to which they are resonant separately.

The presence of the second circuit also affects the resistance and the reactance of the first circuit to an extent depending on the degree of coupling.

These effects will now be considered in more detail.

335. Forced Oscillations, Equivalent Resistance and Reactance.—Let us take the case of **mutual inductive** coupling, and let the constants of the circuit be as shown in the following figure:—

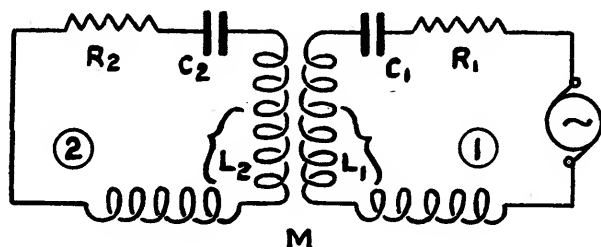


FIG. 151.

The currents flowing in the circuits (1) and (2) are I_1 and I_2 . Let the frequency of the applied voltage be f , and let $2\pi f = \omega$.

The voltage induced in circuit (2) by a current I_1 in circuit (1) is $E_2 = \omega M I_1$.

The secondary current is $I_2 = \frac{\omega M I_1}{Z_2}$ which lags or leads on E_1 by an angle ϕ whose tangent is $\frac{X_2}{R_2}$.

This current may therefore be split up into two components, $I_2 \cos \phi$ in phase with E_2 , and $I_2 \sin \phi$, which is 90° out of phase with E_2 .

$$\text{Now } I_2 \cos \phi = I_2 \frac{R_2}{Z_2} \text{ and } I_2 \sin \phi = I_2 \frac{X_2}{Z_2}.$$

Equivalent Resistance.—The component of current $I_2 \cos \phi$ gives an induced voltage in circuit (1) equal to $\omega M I_2 \cos \phi$ or $\omega M I_2 \frac{R_2}{Z_2}$, and lagging 90° on $I_2 \cos \phi$.

Since the voltage E_2 , with which $I_2 \cos \phi$ is in phase, lags 90° on the current I_1 , and the induced voltage in circuit (1) lags 90° on $I_2 \cos \phi$, this induced voltage in circuit (1) is 180° out of phase with the current I_1 in circuit (1).

It is therefore equivalent to an E.M.F. in circuit (1) acting in direct opposition to the component of the applied voltage E which is driving the current I_1 through the ohmic resistance of circuit (1), and so we can say that the in-phase component of the applied voltage less the induced E.M.F., $\omega M I_2 \frac{R_2}{Z_2}$, is equal to $I_1 R_1$.

For simplicity, write the in-phase component of E as E_R and the 90° out-of-phase component as E_X .

i.e. E_R is in phase with I_1 , and

E_X is 90° out of phase with I_1 .

$$\text{Then } E_R - \omega M I_2 \frac{R_2}{Z_2} = I_1 R_1.$$

$$\text{or } E_R = I_1 R_1 + \omega M I_2 \frac{R_2}{Z_2}.$$

$$\text{But } I_2 = \frac{\omega M I_1}{Z_2}.$$

$$\therefore E_R = I_1 R_1 + \omega^2 M^2 I_1 \frac{R_2}{Z_2^2} = I_1 \left(R_1 + \frac{\omega^2 M^2 R_2}{Z_2^2} \right).$$

This is equivalent to saying that the apparent resistance of circuit (1) is increased from R_1 to $\left(R_1 + \frac{\omega^2 M^2 R_2}{Z_2^2} \right)$ because it is coupled to circuit (2).

Equivalent Reactance.—The reactance in both circuits will be assumed inductive, so that the currents lag on the voltages.

The component of the secondary current I_2 lagging 90° on the secondary E.M.F. E_2 is $I_2 \sin \phi$. This component gives rise to an

induced E.M.F. in the primary circuit of value $\omega MI_2 \sin \phi$, or $\omega MI_2 \frac{X_2}{Z_2}$, lagging 90° on $I_2 \sin \phi$, and therefore lagging 180° on E_2 . E_2 lags by 90° on the primary current I_1 , and so $\omega MI_2 \frac{X_2}{Z_2}$ lags by 270° (i.e. leads by 90°) on I_1 .

Hence $\omega MI_2 \frac{X_2}{Z_2}$ is in phase with E_x .

Hence $E_x + \omega MI_2 \frac{X_2}{Z_2} = I_1 X_1$

$$E_x = I_1 \left(X_1 - \frac{\omega^2 M^2}{Z_2^2} X_2 \right).$$

This is equivalent to saying that the apparent reactance of circuit (1) is decreased from X_1 to $\left(X_1 - \frac{\omega^2 M^2}{Z_2^2} X_2 \right)$ because it is coupled to circuit (2).

As might be anticipated, the equivalent resistance and reactance are obtained in the case of auto-inductive coupling by substituting ωL_m , and in the case of capacitive coupling by substituting $\frac{1}{\omega C_m}$, for ωM in the above results.

The complete theory of resistive coupling, as in Fig. 149 (a), leads to a slightly different result, viz. that the equivalent resistance is decreased from R_1 to $R_1 - \frac{R_m^2}{Z_2^2} R_2$, while the equivalent reactance is increased from X_1 to $X_1 + \frac{R_m^2}{Z_2^2} X_2$.

336. Forced Oscillations. Resonant Frequencies.—From the results of the last paragraph the complete formulæ for I_1 and I_2 in the general case are:—

$$I_1 = \frac{E}{Z_1'} = \frac{E}{\sqrt{\left[R_1 + \left(\frac{\omega M}{Z_2} \right)^2 R_2 \right]^2 + \left[X_1 - \left(\frac{\omega M}{Z_2} \right)^2 X_2 \right]^2}}$$

$$I_2 = \frac{\omega MI_1}{Z_2} = \frac{\omega ME}{Z_1' Z_2} = \frac{\omega ME}{Z_2 \sqrt{\left[R_1 + \left(\frac{\omega M}{Z_2} \right)^2 R_2 \right]^2 + \left[X_1 - \left(\frac{\omega M}{Z_2} \right)^2 X_2 \right]^2}}$$

If the reactance term is put equal to zero in the expression for the impedance given in the denominator above, I_1 will be in phase with the applied voltage and the solution of the equation

$$X_1 - \frac{\omega^2 M^2}{Z_2^2} X_2 = 0.$$

will give the resonant frequency.

We shall assume the resistance R_2 to be so small compared with X_2 , that it can be neglected, and therefore $Z_2 = X_2$.

$$\text{Hence } X_1 = \frac{\omega^2 M^2}{X_2^2} X_2 = \frac{\omega^2 M^2}{X_2},$$

$$\text{or } \omega^2 M^2 = X_1 X_2$$

$$\begin{aligned} \omega^2 M^2 &= \left(\omega L_1 - \frac{1}{\omega C_1} \right) \left(\omega L_2 - \frac{1}{\omega C_2} \right) \\ &= \omega^2 L_1 L_2 \left(1 - \frac{1}{\omega^2 L_1 C_1} \right) \left(1 - \frac{1}{\omega^2 L_2 C_2} \right). \end{aligned}$$

Let the two circuits be separately tuned to the same frequency

$$\frac{\omega_0}{2\pi}.$$

$$\text{Then } (\omega_0)^2 = \frac{1}{L_1 C_1} = \frac{1}{L_2 C_2}$$

$$\therefore \omega^2 M^2 = \omega^2 L_1 L_2 \left(1 - \frac{(\omega_0)^2}{\omega^2} \right) \left(1 - \frac{(\omega_0)^2}{\omega^2} \right)$$

$$\frac{M^2}{L_1 L_2} = K^2 = \left(1 - \frac{(\omega_0)^2}{\omega^2} \right)^2$$

$$\therefore \pm K = 1 - \frac{(\omega_0)^2}{\omega^2}$$

$$\text{or } (\omega_0)^2 = \omega^2 (1 \pm K)$$

$$\omega_0 = \omega \sqrt{1 \pm K}$$

$$\omega = \frac{\omega_0}{\sqrt{1 \pm K}}.$$

Thus, there are two frequencies of resonance, one higher and one lower than the individual equal resonant frequencies.

For all the other types of coupled circuits which have been considered a similar result is obtained, **given certain conditions** as regards the resonant frequencies of the separate circuits. Thus, for auto-inductive coupling, the resonant frequencies are given by

$$\frac{\omega_0}{\sqrt{1 \pm K}}, \text{ provided } (L_a + L_m) C_1 = (L_b + L_m) C_2 = \frac{1}{(\omega_0)^2}.$$

Fig. 149 (b).

For capacitive coupling, the resonant frequencies are $\frac{\omega_0}{\sqrt{1 \pm K}}$,

provided $L_1 \times \frac{C_a C_m}{C_a + C_m} = L_2 \times \frac{C_b C_m}{C_b + C_m} = \frac{1}{(\omega_0)^2}$; in other words, provided the products of the total inductance and the total capacity in both circuits are equal.

The coupling factors K appropriate to each case are given in para. 333.

It can be proved that, for all other types of coupled circuits, **except** mutually coupled circuits, **different conditions from those just quoted** will give different results as regards resonant frequencies.

In the case of mutually coupled circuits, the circuits are entirely disconnected, and the resonance as regards the individual circuits can only have the meaning we have assigned to it, that

$$L_1 C_1 = L_2 C_2 = \frac{1}{(\omega_0)^2}.$$

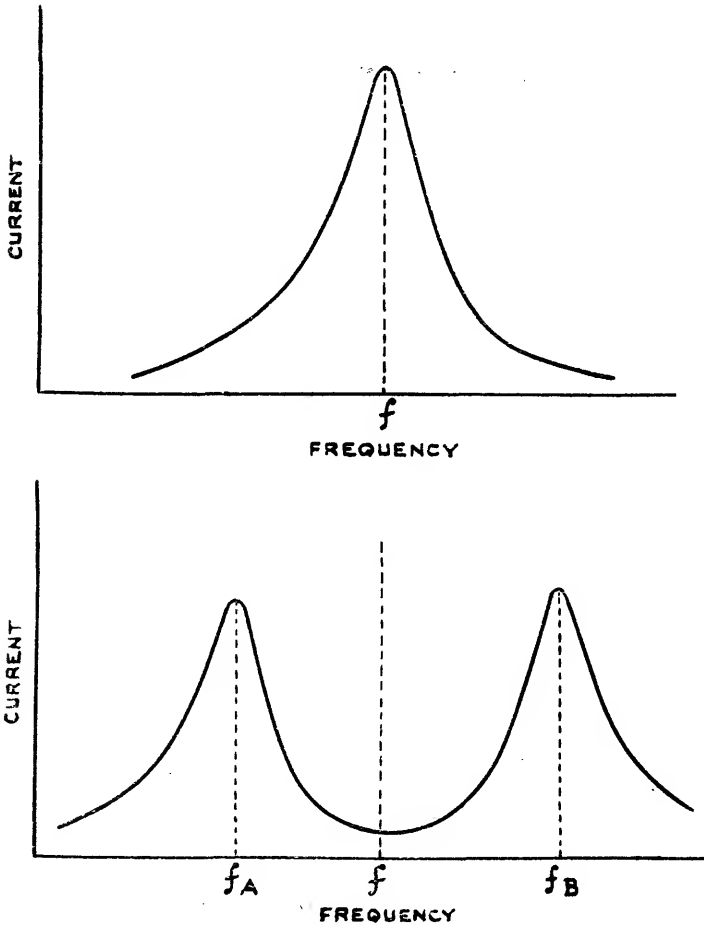


FIG. 152.

In the other cases, however, different conditions may be applied to the separate circuits.

Thus, for **auto-inductive** coupling, if we choose the condition that $L_a C_1 = L_b C_2 = \frac{1}{(\omega_0)^2} = LC$, the combination of circuits will

be resonant to the frequencies

$$\frac{1}{2\pi\sqrt{LC}} \text{ and } \frac{1}{2\pi\sqrt{LC\left(1 + \frac{L_m}{L_a} + \frac{L_m}{L_b}\right)}}$$

one of which is seen to be fixed and independent of the coupling.

Also, for **capacitive** coupling, if we choose the condition that $L_1C_a = L_2C_b = \frac{1}{(\omega_0)^2} = LC$, the combination of circuits will be resonant to the frequencies

$$\frac{1}{2\pi\sqrt{LC}} \text{ and } \frac{1}{2\pi\sqrt{LC\left(1 - \frac{C_a + C_b}{C_m + C_a + C_b}\right)}}$$

one of which is fixed and independent of the coupling.

To illustrate the theory of mutual coupling given above, Fig. 152 is appended showing a comparison between the resonance curves of a series resonant circuit by itself, and of the same circuit tightly coupled to another similar circuit.

Example 46.

Two circuits $L_1R_1C_1$ and $L_2R_2C_2$ are coupled mutually.

$L_1 = 200$ mics, $R_1 = 15$ ohms, $C_1 = 5$ jars.

$L_2 = 96$ mics, $R_2 = 8$ ohms, $C_2 = 10$ jars.

$M = 15$ mics.

An alternating voltage of 100 volts at a frequency $\frac{10^6}{2\pi}$ is applied to $L_1R_1C_1$. Find the equivalent resistance R_1' , the equivalent reactance X_1' and the currents I_1 and I_2 .

$$\omega = 2\pi \times \frac{10^6}{2\pi} = 10^6 \text{ radians per second.}$$

$$\omega L_1 = \frac{200}{10^6} \times 10^6 = 200 \text{ ohms.}$$

$$\frac{1}{\omega C_1} = \frac{9 \times 10^8}{5 \times 10^6} = 180 \text{ ohms.}$$

$$X_1 = 20 \text{ ohms, } R_1 = 15 \text{ ohms, } \therefore Z_1 = 25 \text{ ohms.}$$

$$\omega L_2 = \frac{96}{10^6} \times 10^6 = 96 \text{ ohms.}$$

$$\frac{1}{\omega C_2} = \frac{9 \times 10^8}{10 \times 10^6} = 90 \text{ ohms.}$$

$$X_2 = 6 \text{ ohms. } R_2 = 8 \text{ ohms. } \therefore Z_2 = 10 \text{ ohms.}$$

$$\omega M = 10^6 \times \frac{15}{10^6} = 15 \text{ ohms.}$$

$$\begin{aligned}\text{Equivalent resistance } R_1' &= R_1 + \left(\frac{\omega M}{Z_2}\right)^2 R_2 \\ &= 15 + \left(\frac{15}{10}\right)^2 \times 8 \\ &= 33 \text{ ohms.}\end{aligned}$$

$$\begin{aligned}\text{Equivalent reactance } X_1' &= X_1 - \left(\frac{\omega M}{Z_2}\right)^2 X_2 \\ &= 20 - \left(\frac{15}{10}\right)^2 \times 6 \\ &= 6.5 \text{ ohms.}\end{aligned}$$

$$\text{Current } I_1 = \frac{E}{Z_1} = \frac{100}{\sqrt{33^2 + 6.5^2}} = \frac{100}{\sqrt{1089 + 42.3}} = 2.97 \text{ amps.}$$

$$\text{Current } I_2 = \frac{\omega M I_1}{Z_2} = \frac{\omega M E}{Z_1' Z_2} = \frac{15 \times 100}{33.63 \times 10} = 4.46 \text{ amps.}$$

Since the total reactance in circuit (1) is inductive, the circuit may be said to be equivalent to a resistance $R_1 = 15$ ohms and a self-inductance L given by $\left(L_1 - \frac{1}{\omega^2 C_1}\right)$

$$\begin{aligned}&= \left(\frac{200}{10^6} - \frac{9 \times 10^8}{5 \times 10^{12}}\right) \text{ henries,} \\ &= \left(\frac{200}{10^6} - \frac{900}{5 \times 10^6}\right) \text{ henries} = (200 - 180) \text{ mics} \\ &= 20 \text{ mics.}\end{aligned}$$

This result could also have been obtained by dividing the inductive reactance $\omega L = 20$ ohms by $\omega = 10^6$. When circuit (1) is coupled to circuit (2), however, its inductive reactance is only 6.5 ohms, so that its self-inductance is 6.5 mics. Therefore, coupling the circuit to another circuit has increased its resistance from 15 to 33 ohms and at the same time decreased its inductance from 20 to 6.5 mics.

An example is given below in which the primary current is known and it is desired to find the current in the secondary circuit. This corresponds to practical cases that occur in W/T, for example, the reading of an ammeter coupled to an oscillatory circuit by means of an ammeter transformer.

Example 47. [Fig. 153.]

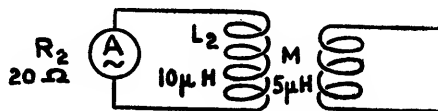


FIG. 153.

Let the primary current I_1 be 10 amps, the mutual inductance M be 5 mics, the secondary inductance L_2 be 10 mics, the resistance

of the ammeter R_2 be 20 ohms, and let ω be equal to 10^6 radians per second.

$$\text{Then } E_2 = \omega M I_1 = 10^6 \times \frac{5}{10^6} \times 10 = 50 \text{ volts.}$$

$$X_2 = \omega L_2 = 10^6 \times \frac{10}{10^6} = 10 \text{ ohms.}$$

$$Z_2 = \sqrt{R_2^2 + X_2^2} = \sqrt{20^2 + 10^2} = \sqrt{500} = 22.37 \text{ ohms.}$$

$$I_2 = \frac{E_2}{Z_2} = \frac{50}{22.37} = 2.237 \text{ amps.}$$

Hence the ammeter reading will be 2.237 amps, when the primary current is 10 amps.

337. Transient Conditions.—It must be remembered that the relationships between current and voltage developed throughout this chapter are those which pertain during "steady" conditions. When a switch is suddenly made in a circuit containing inductance or capacity, there is a period of transience, during which the current flowing is a combination of the forced oscillations we have considered and of "free" oscillations at the "natural" frequency of the circuit. The latter ultimately die away, leaving only the forced oscillations which continue until the switch is broken. The theory of free oscillations is considered in Chapter VII.

THE OPERATOR "J."

338. A short account of the use of the mathematical operator " j ". ($= \sqrt{-1}$), in solving alternating current problems will be given in the following paragraphs. This method is to be recommended for the simplification it introduces in writing down the equations necessary for the solution of any problem. It does not shorten the computation required to obtain a result in cases which can be treated as a combination of impedances in series and parallel, but its use, or that of some equivalent operator, is essential in analysing more complicated circuits such as alternating current bridge networks.

339. Number.—The idea of number was originally confined to the positive whole numbers or positive integers, and its first extension was to positive fractions or ratios of one positive integer to another. The next discovery was that it was not always possible to represent exactly the ratio of one length to another as a fraction whose numerator and denominator were integers. Two simple examples are the ratio of a diagonal of a square to its side ($\sqrt{2}$) and the ratio of the circumference of a circle to its diameter (π). Such numbers as $\sqrt{2}$ and π are called incommensurable. The introduction of the Arabic notation for numbers, in which the value of a digit is indicated by its position in a number, led to the use of zero

as the number to which small numbers tend as a limit. For instance, in 123, 132, 312, the digit 3 represents successively three, thirty and three hundred. To represent thirty it is necessary to have another digit (0) following the digit 3, to indicate that 3 stands for 3 tens, and yet this other digit must not increase the size of the number as would be the case if 1, 2, &c., were used. Hence, arose the necessity for the symbol 0. A logical extension of the idea of large numbers to indefinite limits gives infinity (∞) as the other end from zero of the list of positive numbers.

The operation of subtraction led to the idea of negative numbers. The answer to the problem 7 minus 3 can be given purely by the use of positive numbers, the answer being 4. But if the converse problem 3 minus 7 is considered, no positive number can be found which represents the result. The answer is obviously exactly the opposite of the answer to 7 minus 3, and so we are led to the idea of the number -4 , which represents the exact opposite of the number $+4$. The ideas of negative fractions, incommensurable numbers and $-\infty$ are now an easy extension.

All the numbers between $-\infty$ and $+\infty$ so far considered are called "**real**" numbers.

340. Operators.—Consider the problem $7 + 3$. The plus sign really stands for two separate things :—

- (1) It indicates the positive number $+3$.
- (2) It tells us that this number, $+3$, is to be added to 7. In other words, it indicates the **operation** to be performed on $+3$.

Similarly, in the problem $7 - 3$, the minus sign can either be looked on as indicating the number -3 , in which case the problem tells us to perform the **operation of addition** of -3 to 7 [$7 + (-3)$], or it may be taken to mean the **operation of subtraction** of $+3$ from 7 [$7 - (+3)$].

Thus, plus and minus signs, as well as being marks distinguishing numbers of opposite kinds, may also be considered as marks representing the **operations** of addition and subtraction.

341. Imaginary Numbers.—Even in quite elementary problems it is found that the answer cannot always be expressed as a real number. Consider the square root of a negative number, *e.g.*, $\sqrt{-4}$. No real number can be found such that when it is multiplied by itself, the answer is -4 . Suppose we proceed as far as possible in the solution :—

$$\sqrt{-4} = \sqrt{4 \times -1} = \sqrt{4} \times \sqrt{-1} = \pm 2\sqrt{-1}$$

since $\sqrt{4}$ is ± 2 .

The problem of giving a meaning to the square root of **any** negative number thus reduces to that of finding a meaning for $\sqrt{-1}$. For convenience, $\sqrt{-1}$ is usually written as j .

Thus $\sqrt{-4} = \pm 2j$.

A number such as $2j$ is called an **imaginary** number. In other words, an imaginary number is any real multiple, positive or negative, of j . The name "imaginary number" should be looked on as a technical mathematical term, in the same way as is "real number." As regards their mathematical behaviour, it can be easily shown that all the ordinary mathematical operations such as addition, multiplication, &c., may be performed in exactly the same way for both real and imaginary numbers. The proof of this cannot be given here, but may be found in any textbook on Algebra.

Since $j = \sqrt{-1}$, it follows that $j^2 = -1$.

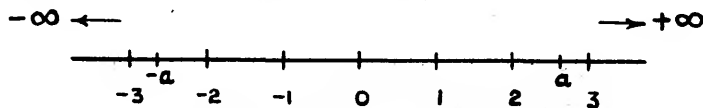
Similarly, $j^3 = j^2 \times j = -1 \times j = -j$,

$j^4 = j^2 \times j^2 = -1 \times -1 = +1$,

$\frac{1}{j} = \frac{j}{j^2} = \frac{j}{-1} = -j$, and so on.

342. Complex Numbers.—These are numbers which contain both a real and an imaginary part, *e.g.*, $2 + 3j$, $5 - 4j$, &c.

Any complex number may thus be taken to be of the form $a + bj$, where a and b are real numbers.



Representation of Real Numbers.

FIG. 154.

343. Graphical Representation of Numbers.—All real numbers may be represented as points on an infinitely long straight line, as indicated in Fig. 154. Zero is fixed at some point 0 in the line and the other real numbers filled in on any convenient scale. The line extends an infinite distance in both directions from the zero point, positive numbers being represented by points to the right of 0, negative numbers by points to the left of 0. It can further be shown that every point on the line corresponds to a real number. If we could magnify the line indefinitely after every real number had been given a place on it, no gaps between its points could be observed. It follows, therefore, that there is no possibility of representing the imaginary numbers on this line.

In obtaining a graphical representation for imaginary numbers, we turn to the interpretation of j as an **operator**. In deriving an imaginary number from a real number, the real number is multiplied by j .

Consider the simplest real number, unity.

$$1 \times j \times j = 1 \times j^2 = 1 \times -1 = -1.$$

The performance of the operation j twice over, (j^2), on $+1$ converts it into -1 . Reference to the line representing the real numbers shows that we may arrive at the point -1 , from the point $+1$, by turning the line 01 through 180° . This turning movement may be performed clockwise or counter-clockwise. We may thus interpret the operation of multiplying $+1$ by j^2 as equivalent to turning the line 01 through 180° . Multiplying by j^2 is the same as multiplying by j twice in succession. Thus multiplication by j may be interpreted as the operation which, performed twice in succession, gives a rotation of 180° . It follows that multiplication by j may be given the meaning of a rotation through $\frac{180^\circ}{2} = 90^\circ$.

Just as a positive number has two real square roots, e.g., $\sqrt{4} = \pm 2$, so we saw that the operation $\sqrt{-4}$ gave formally two roots $+2j$ and $-2j$. The fact that the operation of turning through 180° above may be accomplished either in a clockwise or counter-clockwise direction enables us to give a meaning to this formal result. Thus, $\sqrt{-4}$ may be represented by turning the line 02 through 90° either counter-clockwise or clockwise. **Counter-clockwise rotation through 90° is taken as the operation of multiplying a number by $+j$: clockwise rotation through 90° as the operation of multiplying a number by $-j$.**

As $\frac{1}{j} = -j$, the operation of dividing by j may also be interpreted as a clockwise rotation through 90° , and so appears as the reverse of multiplying by j , as we should expect.

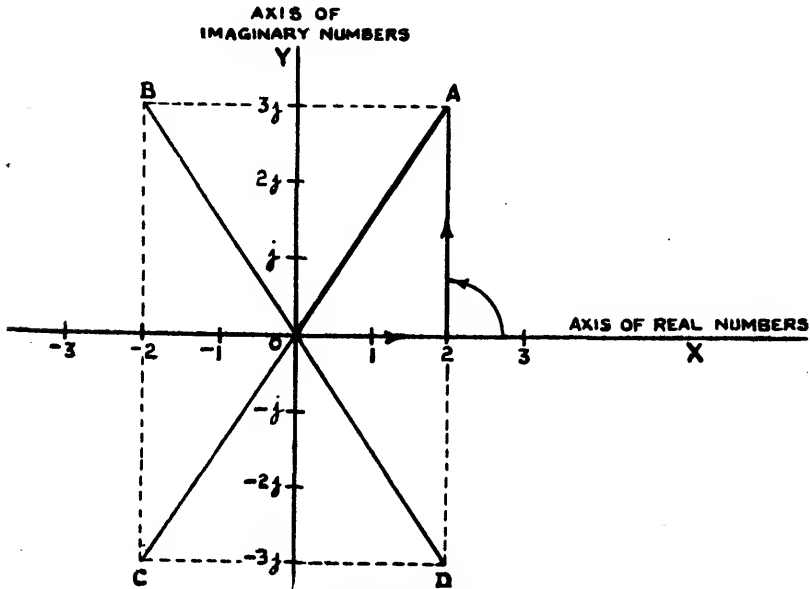
Imaginary numbers may thus be represented graphically by drawing a line through the point 0 at right angles to the line representing the real numbers. All real multiples of j are represented by points on this line, which extends an infinite distance in both directions. The graphical representation of real and imaginary numbers thus assumes the form of two axes at right angles as shown in Fig. 155. It is called the **Argand Diagram**.

A complex number may be represented on the Argand Diagram by a combination of the operations which give the representation of real and imaginary numbers. Consider, for example, the number $2 + 3j$. The operations to be performed are :—

- (1) To traverse a distance of 2 units to the right of 0 along the axis of real numbers. This brings us to the point 2.
- (2) Having arrived at point 2, to turn through 90° counter-clockwise and traverse three units in this new direction. We thus arrive at a point (A) in the plane of the diagram which represents the number $2 + 3j$.

The complex number $2 + 3j$ may be considered to be represented either by the point A or by the line OA , just as 2 is represented by the point 2 or the line 02 .

It will be seen that to specify OA completely, not only its **length** must be given, but also its **direction** with respect to one of the axes.



Argand Diagram.

FIG. 155.

The line OB , for instance, has the same length as OA , but has a different direction. It represents the number $-2 + 3j$. Similarly OC represents $-2 - 3j$ and OD represents $2 - 3j$.

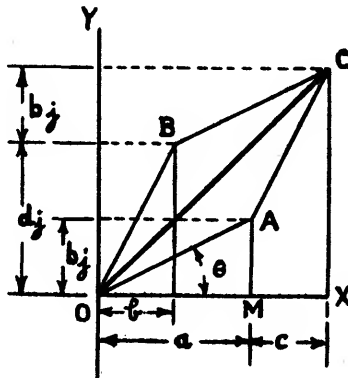


FIG. 156.

The representation of complex numbers is thus a **vectorial** representation. Complex numbers are equivalent to vectors on the Argand Diagram. It is this result which furnishes their utility in alternating current problems.

The vector addition law, *i.e.*, the parallelogram law, can easily be seen to apply to complex numbers. We may represent two complex numbers by $a + jb$ and $c + jd$, where a , b , c and d are real numbers.

Addition by the ordinary rules of algebra gives their sum as

$$a + jb + c + jd = (a + c) + j(b + d),$$

i.e., their sum is another complex number.

$a + jb$ is represented by OA in Fig. 156, and $c + jd$ by OB. The parallelogram OACB is completed. It is easily seen from the figure that OC, the diagonal of the parallelogram, represents the number $(a + c) + j(b + d)$.

The complex number $a + jb$ may also be expressed in a form which shows more obviously its vectorial nature.

Let the length of OA in Fig. 156 be taken as r and the angle XOA, which it makes with the axis of real numbers, as θ .

Then in triangle OAM,

$$a = r \cos \theta,$$

$$b = r \sin \theta.$$

(The length of AM is obviously b units; it is written bj merely to indicate its direction on the Argand Diagram.)

It follows that $a + jb = r \cos \theta + jr \sin \theta = r (\cos \theta + j \sin \theta)$.

The relations connecting r , θ , a and b may also be written

$$r = \sqrt{a^2 + b^2}.$$

$$\theta = \text{angle whose tangent is } \frac{b}{a}, \left(\tan^{-1} \frac{b}{a} \right).$$

r or $\sqrt{a^2 + b^2}$, which gives the length of OA, is called the **modulus**, and θ or $\tan^{-1} \frac{b}{a}$, which gives the direction that OA makes with the positive axis of real numbers (OX), is called the **argument** of the complex number $a + jb$.

344. Multiplication and Division of Complex Numbers.—The object in each case is to represent the answer as a complex number.

This presents no difficulty in the case of multiplication. If the two numbers are $a + jb$ and $c + jd$, ordinary algebraic multiplication and the substitution of -1 for j^2 gives the answer as

$$ac - bd + j(bc + ad),$$

which is in the required form.

Division requires a little more manipulation. The problem is to express $\frac{a + jb}{c + jd}$ as an ordinary complex number. The method of doing this is to multiply both numerator and denominator by the complex number $c - jd$, which is called the conjugate of $c + jd$. The fraction is unchanged in value by this operation, which is equivalent to multiplying it by unity, but the denominator now becomes a real number, *viz.*,

$$(c + jd)(c - jd) = c^2 + d^2.$$

The whole operation is as follows :—

$$\begin{aligned}\frac{a + jb}{c + jd} &= \frac{a + jb}{c + jd} \times \frac{c - jd}{c - jd} = \frac{ac + bd + j(bc - ad)}{c^2 + d^2} \\ &= \frac{ac + bd}{c^2 + d^2} + j \frac{bc - ad}{c^2 + d^2}\end{aligned}$$

which is in the form of an ordinary complex number.

The method rather than the results of these operations should be noted.

Example 48.

Express $\frac{1 + 2j}{2 + 3j}$ in the form $a + jb$

$$\begin{aligned}\frac{1 + 2j}{2 + 3j} &= \frac{(1 + 2j)(2 - 3j)}{(2 + 3j)(2 - 3j)} = \frac{2 - 6j^2 + 4j - 3j}{4 - 9j^2} = \frac{8 + j}{13} \\ &= \frac{8}{13} + \frac{1}{13}j.\end{aligned}$$

345. The significance of these operations is better realised if the two complex numbers are taken as

$$r(\cos \theta + j \sin \theta) \text{ and } r'(\cos \theta' + j \sin \theta').$$

Their product is

$$\begin{aligned}rr'(\cos \theta + j \sin \theta)(\cos \theta' + j \sin \theta') \\ = rr'[(\cos \theta \cos \theta' - \sin \theta \sin \theta') + j(\sin \theta \cos \theta' + \cos \theta \sin \theta')] \\ = rr'[\cos(\theta + \theta') + j \sin(\theta + \theta')].\end{aligned}$$

In words, the **modulus** of the product of two complex numbers is the **product** of their moduli, and the **argument** of their product is the **sum** of their arguments.

The quotient is

$$\begin{aligned}\frac{r(\cos \theta + j \sin \theta)}{r'(\cos \theta' + j \sin \theta')} &= \frac{r(\cos \theta + j \sin \theta)(\cos \theta' - j \sin \theta')}{r'(\cos \theta' + j \sin \theta')(\cos \theta' - j \sin \theta')} \\ &= \frac{r(\cos \theta \cos \theta' + \sin \theta \sin \theta') + j(\sin \theta \cos \theta' - \cos \theta \sin \theta')}{r'(\cos^2 \theta' + \sin^2 \theta')} \\ &= \frac{r}{r'}[\cos(\theta - \theta') + j \sin(\theta - \theta')].\end{aligned}$$

In words, the **modulus** of the quotient of two complex numbers is the **quotient** of their moduli, and the **argument** of the quotient is the **difference** of their arguments.

These results correspond to the ordinary rules for vector multiplication and division.

346. Application to A.C. Problems.—The vector representation of alternating quantities has already been fully considered, and as complex numbers are equivalent to vectors they may also be used to exhibit the relations in amplitude (or R.M.S. value) and

phase of alternating quantities. The method will be seen most easily by considering some simple examples.

- (1) In a purely resistive circuit, the current I and E.M.F. E are in phase and connected by the relation $E = RI$.

The vector diagram in this case consists of two lines of the magnitude of E and I in the same direction. On the Argand Diagram they are thus most simply represented by two distances along the positive axis of real numbers.

- (2) In a purely inductive circuit, the applied E.M.F. leads the current by 90° and their relative magnitudes are given by $E = \omega LI$.

Thus, if I is represented by a distance from O along the positive real axis, E is represented by a distance from O along the positive imaginary axis of magnitude ωLI .

E is thus fully described with respect to I by the equation $E = j\omega LI$.

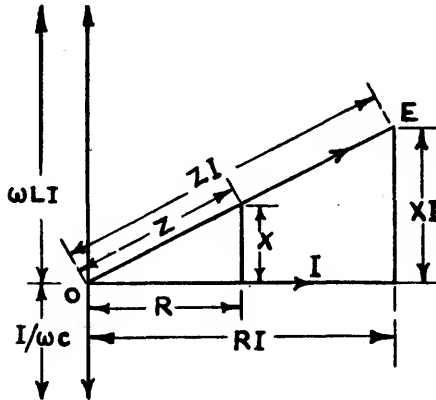


FIG. 157.

- (3) In a purely capacitive circuit, $E = \frac{I}{\omega C}$ and lags on I by 90° . Under the same conditions as in (2) above, it may therefore be fully represented by

$$E = \frac{-jI}{\omega C} \text{ or since } -j = \frac{1}{j}, \text{ by } E = \frac{I}{j\omega C}.$$

If we now consider any alternating current circuit, the relation between the magnitude of the current and the applied E.M.F. is given by $E = ZI = (\sqrt{R^2 + X^2}) I$, and, if we take X to be inductive, E leads I by an angle ϕ whose tangent is $\frac{X}{R}$, ($\phi = \tan^{-1} \frac{X}{R}$).

Now the complex number $R + jX$ is such that its length on the Argand Diagram (modulus), is $\sqrt{R^2 + X^2}$, and the angle it makes with the positive direction of the real axis (argument), is $\tan^{-1} \frac{X}{R}$, so that E is fully represented by the equation

$$E = (R + jX) I,$$

and the impedance Z may be represented as

$$Z = R + jX.$$

This case is shown in Fig. 157, which represents the vector diagram of a circuit with R , L and C in series.

$$Z = R + j\left(\omega L - \frac{1}{\omega C}\right),$$

$$\text{and } E = ZI = \left[R + j\left(\omega L - \frac{1}{\omega C}\right) \right] I.$$

It is obvious that the complex number representing Z may also be written

$$Z = R + j\omega L + \frac{1}{j\omega C}.$$

347. Resonance Condition.—If I is taken along the real axis, then whenever E and I are in phase, the coefficient of j in the complex expression of E must be zero. In other words, to obtain the condition for resonance, the coefficient of j in the representation of Z as a complex number should be equated to zero. For example, in the series oscillatory circuit the coefficient of j in the expression for Z is $\omega L - \frac{1}{\omega C}$, and $\omega L - \frac{1}{\omega C} = 0$, or $\omega^2 = \frac{1}{LC}$ is the resonant condition.

348. To illustrate the above remarks, two examples which have been worked from first principles earlier in this chapter will now be worked through by the use of the operator j .

(1) **Parallel circuit with resistance in each branch.** One branch consists of inductance L and resistance R_1 in series. The complex number representing its impedance is thus $R_1 + j\omega L$ and the current I_1 , flowing through it is

$$I_1 = \frac{E}{R_1 + j\omega L}$$

The impedance of the other branch (capacity C and resistance R_2) is

$$R_2 - \frac{j}{\omega C}$$

and the current I_2 , flowing through it is given by

$$I_2 = \frac{E}{R_2 - \frac{j}{\omega C}}$$

The total current flowing through the circuit (the make-up current) is $I = I_1 + I_2$

$$\begin{aligned}\therefore I &= \left[\frac{1}{R_1 + j\omega L} + \frac{1}{R_2 - \frac{j}{\omega C}} \right] E \\ &= \left[\frac{R_1 - j\omega L}{R_1^2 + \omega^2 L^2} + \frac{R_2 + \frac{j}{\omega C}}{R_2^2 + \frac{1}{\omega^2 C^2}} \right] E \\ \therefore \frac{1}{Z} = \frac{I}{E} &= \frac{R_1}{R_1^2 + \omega^2 L^2} + \frac{R_2}{R_2^2 + \frac{1}{\omega^2 C^2}} \\ &\quad + j \left(\frac{\frac{1}{\omega C}}{R_2^2 + \frac{1}{\omega^2 C^2}} - \frac{\omega L}{R_1^2 + \omega^2 L^2} \right)\end{aligned}$$

The resonant frequency is given by the condition that the co-efficient of j is zero.

$$\text{i.e. } \frac{\frac{1}{\omega C}}{R_2^2 + \frac{1}{\omega^2 C^2}} = \frac{\omega L}{R_1^2 + \omega^2 L^2}$$

$$\therefore R_1^2 + \omega^2 L^2 = \omega C \times \omega L \left(R_2^2 + \frac{1}{\omega^2 C^2} \right) = \omega^2 R_2^2 LC + \frac{L}{C}$$

$$\therefore \omega^2 L (L - CR_2^2) = \frac{1}{C} (L - CR_1^2)$$

$$\therefore \omega^2 = \frac{1}{LC} \times \frac{L - CR_1^2}{L - CR_2^2}$$

and the resonant frequency f is

$$f = \frac{1}{2\pi\sqrt{LC}} \cdot \sqrt{\frac{L - CR_1^2}{L - CR_2^2}}$$

If now R_1 and R_2 are so small that CR_1^2 and CR_2^2 may be neglected in comparison with L , f may be taken approximately at resonance

as $f = \frac{1}{2\pi\sqrt{LC}}$ The impedance at resonance may be found from the relation

$$\frac{1}{Z} = \frac{R_1}{R_1^2 + \omega^2 L^2} + \frac{R_2}{R_2^2 + \frac{1}{\omega^2 C^2}}$$

Substituting $\frac{1}{LC}$ for ω^2 , and neglecting R_1^2 and R_2^2 in comparison with $\omega^2 L^2$ and $\frac{1}{\omega^2 C^2}$, this gives

$$\frac{1}{Z} = \frac{R_1}{L/C} + \frac{R_2}{L/C} = \frac{C(R_1 + R_2)}{L},$$

$$\therefore Z = \frac{L}{C(R_1 + R_2)}.$$

(2) **Two circuits coupled by mutual inductance.** (Fig. 151).—The impedance of the primary circuit is taken as $R_1 + jX_1$ and of the secondary as $R_2 + jX_2$, and the currents flowing are taken as I_1 and I_2 .

The R.M.S. value of the E.M.F. induced in the secondary circuit by the primary current is ωMI_1 . If M is positive it lags 90° on the primary current; if M is negative it leads the primary current by 90° . Both cases are covered by writing the induced E.M.F. as $-j\omega MI_1$. Similarly the E.M.F. induced in the primary circuit by the secondary current is $-j\omega MI_2$.

Kirchoff's Law applied to the two circuits then gives:—

$$(a) \text{ Primary } (R_1 + jX_1) I_1 = E - j\omega MI_2$$

$$(b) \text{ Secondary } (R_2 + jX_2) I_2 = -j\omega MI_1$$

$$\text{Hence } I_2 = \frac{-j\omega MI_1}{R_2 + jX_2}$$

Substituting this in the primary circuit equation gives

$$(R_1 + jX_1) I_1 = E + \frac{j^2 \omega^2 M^2 I_1}{R_2 + jX_2}$$

$$\therefore E = \left[R_1 + jX_1 + \frac{\omega^2 M^2}{R_2 + jX_2} \right] I_1$$

The equivalent impedance of the two circuits is thus given by

$$Z = \frac{E}{I_1} = R_1 + jX_1 + \frac{\omega^2 M^2}{R_2 + jX_2}$$

$$= R_1 + jX_1 + \frac{\omega^2 M^2 (R_2 - jX_2)}{R_2^2 + X_2^2}$$

$$= R_1 + \frac{\omega^2 M^2 R_2}{R_2^2 + X_2^2} + j \left(X_1 - \frac{\omega^2 M^2 X_2}{R_2^2 + X_2^2} \right).$$

The equivalent resistance is thus $R = R_1 + \frac{\omega^2 M^2 R_2}{R_2^2 + X_2^2}$,

and the equivalent reactance is $X = X_1 - \frac{\omega^2 M^2 X_2}{R_2^2 + X_2^2}$.

These results should be compared with those derived in para. 335.

CHAPTER VI.

THE TRANSFORMER, MEASURING INSTRUMENTS, R.F. EFFECTS.

349. One of the most important advantages of alternating currents over continuous currents is the extreme ease with which the transformation from a low to a high voltage, or *vice versa*, may be accomplished. This process is effected by means of **Transformers**.

A transformer consists essentially of two insulated coils, known as the **primary** and **secondary** windings, wound over a closed iron magnetic circuit. Alternating current at one voltage is supplied to the primary; from the secondary an alternating current is taken at a higher or lower voltage than that supplied to the primary. If the voltage is increased, the transformer is said to "step-up" the voltage; if decreased, the transformer is said to "step-down" the voltage.

Both types are used in wireless work; step-up transformers, for instance, to give voltages suitable for applying to the anode of a thermionic valve; step-down transformers, to give voltages suitable for the filament supply.

350. **Construction of Typical Transformer.**—Two iron cores made of thin sheets or laminations of iron, averaging about

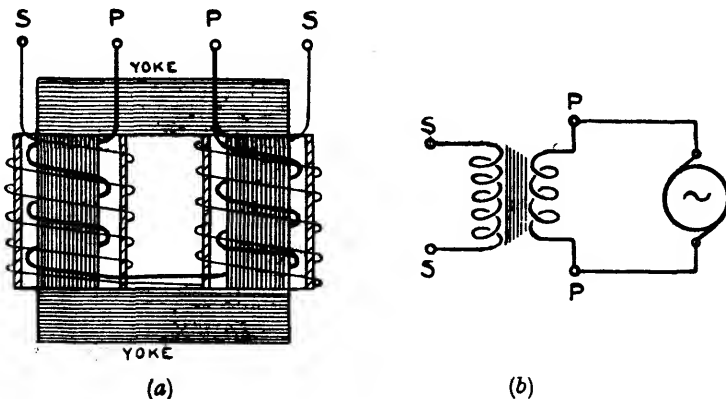


FIG. 158.

0.012-inch thick, are built up, each sheet being slightly jappeded or oxidised. The ends of the iron cores are connected by two iron yokes constructed in the same manner.

On each core is wound a coil of insulated copper wire, called the **Primary**; these two coils are connected in series, as illustrated in Fig. 158 (a).

The primary windings are covered with insulating sleeves, made of good insulating material—micanite, mica, or presspahn.

Over these again are wound the **Secondary** Coils, which have more turns than the Primary if the alternating voltage is to be increased.

The two secondary windings are joined in series, and their ends are brought to two terminals heavily insulated with ebonite or porcelain.

351. Primary E.M.F.—If an alternator is connected up to the primary terminals, and the secondary terminals are left disconnected or on “open circuit,” as illustrated diagrammatically in Fig. 158 (b), then an alternating current will pass through the primary winding, and an alternating flux will be set up in the core.

Since the primary winding has a great many turns, and the magnetic path for the lines of force is a very good one, the inductance of the primary with the secondary on open circuit will have a very large value, of the order of several henries.

The applied voltage of the alternator has to perform two functions:—

- (1) Cause the current to flow through the resistance of the primary winding.
- (2) Balance the induced E.M.F. in the primary winding due to its self inductance.

It was seen in the preceding chapter that, in any circuit comprising inductance and resistance only,

$$I = \frac{V}{\sqrt{R^2 + (\omega L)^2}};$$

this formula applies here, where I is the magnetising current flowing from the alternator through the primary, V the voltage of the alternator, L the inductance, and R the resistance of the primary winding.

The inductance of the primary is, however, so great compared with its resistance that the latter may be neglected. Let L_1 denote the inductance of the primary winding, L_2 the inductance of the secondary winding, and I_m the magnetising current. The current I_m is then equal to $\frac{V}{\omega L_1}$, and the alternator voltage V is exactly balanced by the induced E.M.F. of the primary, i.e., $V = \omega L_1 I_m$ volts.

I_m , the magnetising current, will be lagging by 90° on the alternator E.M.F.

These facts are expressed in the following vector diagram :—

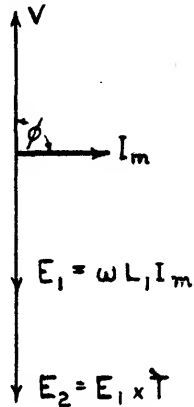


FIG. 159.

The magnetising current will be an exceedingly small one, since the impedance of the transformer is so great.

352. Secondary E.M.F. Secondary on open circuit.— In a well-designed transformer, practically all the flux due to the current flowing through the primary will cut every turn of the secondary winding also, as it expands and collapses round the primary winding.

From this it follows that the E.M.F. induced in each turn of the secondary is equal to that induced in each turn of the primary, so that the ratio of the total primary to the total secondary E.M.F. is equal to the **ratio of the number of primary turns to the number of secondary turns.**

Thus

$$\frac{\text{Primary E.M.F.}}{\text{Secondary E.M.F.}} = \frac{\text{Primary Turns}}{\text{Secondary Turns}}$$

The ratio of Secondary to Primary Turns is spoken of as the **Transformation Ratio**, and is denoted by the letter T .

If the transformation ratio is greater than 1, the transformer is a "step-up" one; if less than 1, the transformer is a "step-down" one.

Since the secondary voltage of the transformer is the **induced** E.M.F. due to the alternating flux associated with the primary current I_m , the secondary voltage ($E_2 = E_1 \times T$) is in phase with the induced E.M.F. in the primary, and hence is 180° out of phase with the voltage applied to the transformer by the alternator, as shown in Fig. 159.

As an example, let us take a transformer with 100 primary turns, supplied by an alternator giving an alternating supply of 100 volts.

Leave the secondary on open circuit. Then the counter E.M.F. produced in the primary coil will be nearly as much as 100 volts—for the small magnetising current required to give this E.M.F. across the large primary inductance gives a negligible voltage drop across the primary resistance.

Since the primary winding consists of 100 turns, the counter E.M.F. in each turn will be nearly 1 volt.

Again, each turn of the secondary winding will have the same voltage induced in it as each turn of the primary winding, for both are interlinked with the same magnetic flux. The voltage produced in any turn of the secondary winding will, therefore, be practically 1 volt.

If, for example, the secondary winding consists of 10 turns its voltage will be about 10 volts, if of 100 turns its voltage will be about 100 volts, if of 1,000 turns its voltage will be about 1,000 volts, &c.

353. Total Primary No-load Current.—In the first vector diagram, Fig. 159, we assumed the current in the primary to be lagging exactly 90° behind the applied voltage. If losses, however, are considered, the primary current will lag by some angle less than 90° , and the current may be resolved into two components, one lagging

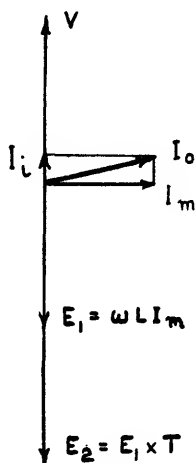


FIG. 160.

by 90° on, and the other in phase with the applied voltage. The component in phase with the applied voltage is necessary to account for the power lost, partly in the primary resistance, but mainly in hysteresis and eddy current losses in the core (these will be referred to more fully later).

This component may be termed the **iron-loss current**, and denoted by the symbol I_0 .

The component which lags by 90° on the applied voltage is I_m , the magnetising current.

The total primary no-load current I_0 is the resultant of these two components, and hence the vector diagram given in Fig. 159 should be amended as in Fig. 160.

It must be pointed out that a transformer should never be used with the same supply voltage at a frequency lower than that for which it was designed, *e.g.*, a 100-volt, 500-cycle transformer should not be used on a 100-volt, 50-cycle supply. The lower the frequency, the greater must be the magnetising current flowing in the primary winding to make the induced E.M.F. equal to the applied E.M.F. (since $V = \omega L_1 I$). In addition, this larger current will bring the core nearer to saturation, thus decreasing the effective value of L_1 , and a still larger current will flow. The winding, not being designed to carry such large currents, will be burnt out immediately. The same supply voltage at a higher frequency can, however, be used, *e.g.*, a 100-volt 50-cycle transformer can be used on a 100-volt 500-cycle supply, for in this case the current will be reduced.

354. Effect of Load Applied to Secondary.—So far the secondary has been taken to be on open circuit, *i.e.*, no current has been flowing in it. Closing the secondary by an external circuit allows the secondary E.M.F. to give rise to a secondary current. This secondary current I_2 will set up an alternating magnetic flux proportional to itself and to the number of turns through which it flows, *i.e.*, proportional to the secondary ampere-turns.

By Lenz's Law this secondary flux is in opposition to the cause which produces it, and hence in opposition to the primary flux already existing in the core. Thus part of the primary flux is cancelled, and the flux-linkage with the primary winding for a given primary current is decreased.

This is equivalent to saying that the primary inductance is decreased, and in consequence an increased current will flow through the primary to restore the state of equilibrium present under no-load conditions; in other words, to restore the original value of the primary flux, and the approximate equality of applied voltage and counter E.M.F. of self-induction in the primary.

This additional current flowing through the primary must set up a flux, therefore, exactly equal and opposite to the demagnetising effect produced by the secondary ampere-turns; hence the primary ampere-turns due to the additional primary current must be equal to the secondary ampere-turns.

This additional primary current is known as the **load** component of total primary current, and from the equality of ampere-turns just stated,

$$\frac{\text{Primary load current}}{\text{Secondary current}} = \frac{\text{secondary turns}}{\text{primary turns}} = T.$$

If we call the load component of primary current I_1 , then

$$I_1 = I_2 \times T.$$

The total current flowing in the primary with the secondary on load is, therefore, the vector sum of the original no-load current and the additional, or load, current.

The greater the load on the secondary, the greater will be the primary load current, since $I_1 = I_2 \times T$. Therefore, with a very large current being taken from the secondary, the load current in the primary will be so much greater than the no-load current that it may be regarded as equal to the total primary current I_1 . In this case, if we neglect the no-load current,

$$I_1 = I_2 \times T.$$

Also, neglecting losses,

$$\begin{aligned} E_2 &= E_1 \times T \\ &= V \times T. \end{aligned}$$

$$\therefore I_1 \times E_1 = E_2 \times I_2.$$

Therefore the energy input equals the energy output.

This is to be expected since losses have been neglected. In the practical case there are various losses which result in the transformer being less than 100 per cent. efficient. The complete theory can best be explained by the use of vector diagrams, and in the following paragraphs the vector diagrams will be drawn for purely resistive, inductive and capacitive loads, and also for the more practical case of a circuit possessing both resistance and reactance.

In drawing vector diagrams for voltage step-down transformers it is usual, in order to keep both sides of the diagram approximately the same size, to draw the voltages on the secondary side to a scale T times that of the voltage on the primary side, and the currents on the primary side to a scale T times that on the secondary side. For the same reason the scales are adjusted in exactly the opposite manner in the vector diagrams for voltage step-up transformers. This convention is employed in the vector diagrams in this chapter.

355. Purely Resistive Load.

Fig. 161 shows the vector diagram for a purely resistive load.

Neglecting the resistance and the leakage reactance of the two windings, the magnetising current I_m lags 90° on the applied voltage V . The flux Φ is in phase with the magnetising current. The total primary no-load current is given by I_0 , the resultant of I_m and I_w , the iron-loss current.

The alternating flux Φ cutting the secondary winding sets up the alternating voltage E_2 across the terminals of the secondary, E_2 being equal to $E_1 \times T$, or $V \times T$.

The secondary current I_2 is in phase with E_2 , because the load is a pure resistance, and is equal to $\frac{E_2}{R_2}$.

This current I_2 has a demagnetising effect on the core, and since E_1 must remain equal to V whether the transformer is on load or not, and E_1 depends on the value of the flux, a primary load current I_1 must flow in antiphase to I_2 and of such magnitude that the ampere-turns due to it exactly balance the secondary ampere-turns. Hence $I_1 = I_2 \times T$, and with our convention as regards scale it is shown as being the same length as the vector representing I_2 .

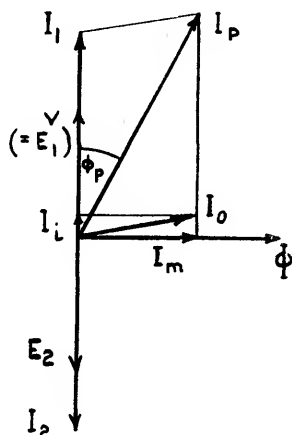


FIG. 161.

The total alternator current, I_p , is the resultant of I_0 and I_1 , lagging on the applied voltage by an angle ϕ_p . The greater the secondary current I_2 , the greater is I_1 , and the more nearly are the primary current I_p and the primary voltage V in phase.

In other words, the greater the load on the secondary the nearer to unity is the power factor in the primary circuit, while the secondary power factor is exactly unity. The transformer on full-load behaves as if almost non-inductive, voltages and currents on both sides being practically in phase.

356. Purely Inductive and Purely Capacitive Loads.—The corresponding vector diagrams for purely inductive and purely capacitive secondary loads are shown in Fig. 162 (a) and (b).

In the case of the inductive load the resistance R_s of last paragraph is replaced by an inductance L_s . The secondary current I_2 , therefore, lags 90° on the secondary voltage E_s , and is equal to

$$\frac{E_s}{\omega L_s}.$$

To keep the value of flux the same as under no-load conditions, and maintain the equivalence of V and E_1 , the alternator will have to supply a current I_1 in antiphase to I_2 and equal to $I_2 \times T$. I_1 , with our scale convention, is shown equal in length to I_2 . The total primary current I_p is, as before, the vector sum of I_0 and I_1 .

The secondary current lags by 90° on the secondary voltage, and the primary current I_p lags by nearly 90° on the applied voltage. The greater the load, the greater is I_1 , and the more nearly does the angle of lag of the primary current I_p approach 90° .

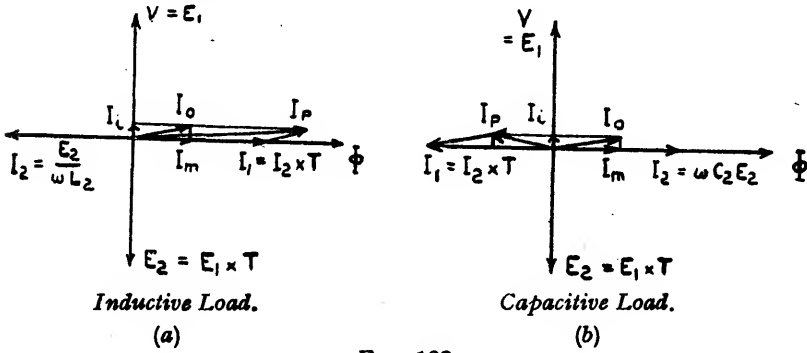


FIG. 162.

The capacitive case is simply the reverse of the inductive case, and the diagram need not be explained.

357. General Case. Secondary Load both Resistive and Reactive.—The reactance in the secondary circuit will be taken

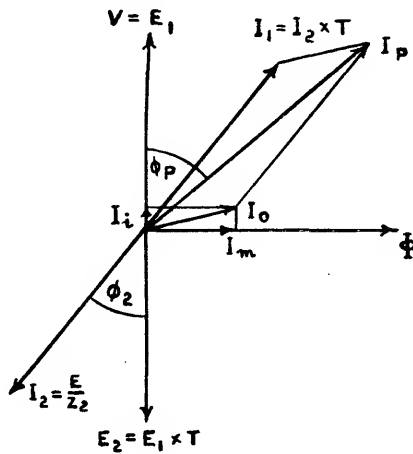


FIG. 163.

to be inductive, so that the secondary current I_2 lags on the secondary voltage E_2 by an angle ϕ_2 , whose cosine is the power factor of the secondary circuit.

As before, the demagnetising effect of this secondary current must be counterbalanced by a current I_1 in the primary circuit, in antiphase to it and equal to $I_2 \times T$.

The total primary current is, as before, given by the resultant of I_0 and I_1 , and is I_p in the figure.

It will be seen from the figure that the angle of lag of the primary current I_p on the applied voltage, the angle ϕ_p , is greater than the angle of lag ϕ_s of the secondary current on the secondary voltage, provided the secondary circuit is not so highly inductive that ϕ_s exceeds the angle of lag of the primary no-load current I_0 .

Also, as before, the greater the load on the secondary and in consequence the greater I_1 , the more nearly equal do the angles of lag, ϕ_p and ϕ_s , become.

If a leading current were taken from the secondary, the angle of lead ϕ_p of the primary current would be slightly less than the angle of lead of the secondary current.

358. Transformer Losses.—The losses in a transformer may be considered under the following headings :—

1. Magnetic Leakage.
2. Copper Losses.
3. Eddy Current Losses.
4. Hysteresis Losses.

The last two are losses which occur in the iron core, and in the simple vector diagrams already given they have been allowed for by introducing the current I_i (iron-loss current) as a component of the total primary no-load current. These losses will be explained more fully in paras. 363 and 364.

The first two types of loss will now be considered, and a complete vector diagram drawn allowing for their effect.

359. Magnetic Leakage.—It is very nearly, but not quite, true that the whole of the magnetic flux passes through the iron core of the transformer and links with all the turns on both windings.

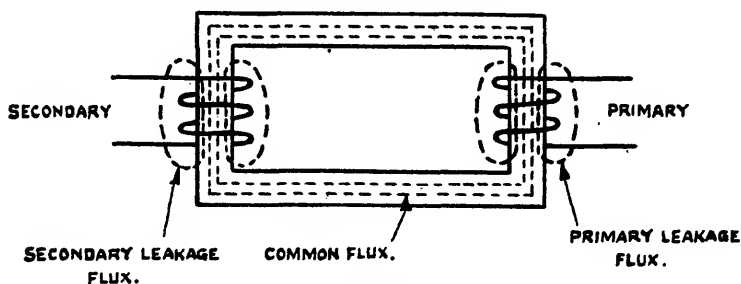


FIG. 164.

Some of the lines of flux produced by the primary current, instead of passing round the main iron magnetic circuit, will take shorter paths as illustrated in the figure above, mainly through air. Since they do not cut the secondary winding, they serve no useful purpose as regards generation of secondary voltage, and are known as the **primary leakage flux**.

Similarly, the current flowing in the secondary will produce some lines of flux which do not link with the primary. These constitute the **secondary leakage flux**.

The leakage flux from each winding is produced by the current in that particular winding, and is unaffected by the current in the other winding. This leakage flux is therefore in both cases in phase with, and proportional in amount to, the current in the particular winding which is producing it. Thus, both leakage fluxes increase with load, while the common flux remains practically constant.

The effect of the primary leakage flux is to add a certain amount of reactance to the primary winding, which uses up part of the primary applied voltage. Less of the applied voltage, therefore, has to be balanced by the counter E.M.F. due to the common alternating flux, and hence there is less common flux and less secondary voltage induced. The inductance in series with the primary circuit which would give the same reactance as that due to the leakage field in the primary winding is known as the **primary leakage inductance**.

In the secondary circuit, some of the secondary voltage will be used in overcoming the reactance due to the leakage flux, so that the terminal voltage will be reduced. The inductance which, in series with the secondary circuit, would give the same effect is known as the **secondary leakage inductance**.

The effect of magnetic leakage upon the transformation ratio is to reduce the secondary terminal voltage for a given primary applied voltage.

It is to diminish this voltage drop that the transformer coils are wound one upon the other, as previously described. By this construction the amount of leakage flux is reduced to a minimum.

If the coils are arranged in any other way, as, for instance, in Fig. 164, with the whole primary on one leg of the core and the secondary on the other, much larger paths would be available for the leakage flux and the falling-off of the secondary terminal voltage would be considerably increased.

A vector representation of the effect of magnetic leakage is given in Fig. 165.

360. Copper Losses.—The resistances of the primary and secondary windings must be taken into consideration. When the secondary is on open circuit, the primary no-load current I_0 is very small, and the voltage drop due to this current flowing through the resistance of the primary is negligible.

With the secondary on closed circuit, however, a very much larger current I_1 flows in the primary and a considerable current I_2 in the secondary, and the losses due to these currents must be taken into account. The primary ohmic drop reduces the proportion of the applied voltage balanced by the counter E.M.F. due to the

flux; the consequent decrease in flux decreases the secondary voltage.

In addition, there is an $I_2 R_2$ voltage drop in the secondary circuit, so that the secondary **terminal** voltage is further decreased below the value obtained by assuming the windings to have no resistance.

A vector diagram to illustrate this is given in Fig. 165.

The copper losses are minimised by using large section copper wire or flat copper strip for the primary and secondary windings.

In order to keep down eddy current losses in the copper conductors they are often subdivided and made up of several insulated strands joined in parallel.

The total copper losses are $I_1^2 R_1 + I_2^2 R_2$ watts.

361. General Vector Diagram.—The vector diagram shown in Fig. 165 is applicable to the general case of a secondary load, in

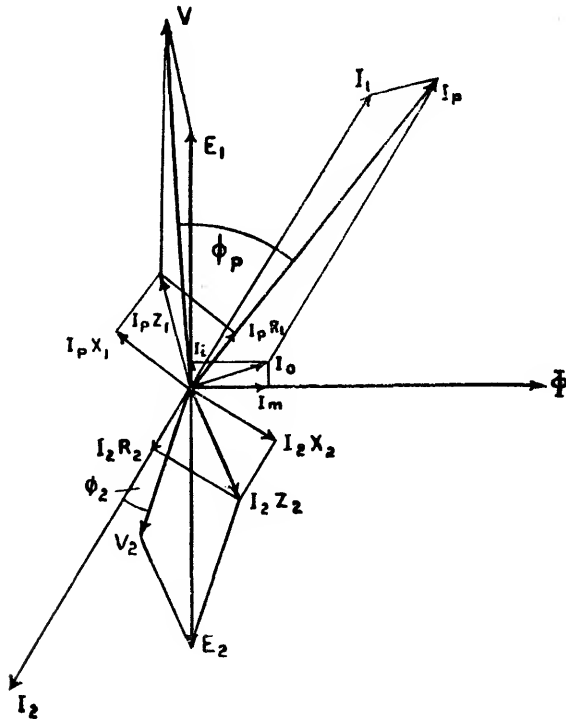


FIG. 165.

which the current lags behind the voltage (*cf.* Fig. 163, para. 357), taking into account the resistances of the windings, and the reactances equivalent in effect to the magnetic leakage.

Superimposed on Fig. 163, para. 357, we have on the primary side the voltages $I_1 R_1$ and $I_1 X_1$, giving a resultant $I_1 Z_1$.

These are respectively the resistance drop and the leakage reactance drop in the primary circuit, and hence the primary applied voltage is given by V , the resultant of E_1 and $I_1 Z_1$. Actually this shows that E_1 , the residual part of the primary applied voltage that is balanced by the counter E.M.F. of the flux, is less than V .

On the secondary side the resistance and leakage reactance drops combine to give $I_2 Z_2$, and hence V_2 , the secondary terminal voltage, is less than the secondary induced voltage E_2 , being given by the vector difference of E_2 and $I_2 Z_2$.

It is obvious from the diagram, that as the load is increased, both I_1 and I_2 increase, and for a given applied voltage V , which is the condition under which the transformer works, V_2 falls for an increase of load.

It has been convenient to use the diagram as shown, in continuation of previous diagrams, but it must be remembered that V is the constant factor, being the applied voltage, and so Φ , the flux, is smaller than when resistance and leakage reactance are neglected.

It may be noted that in this case, where the secondary load is inductive, the power factor in the primary is less than that in the secondary.

If the diagram is drawn for a capacitive load, it will be found that the power factor in the primary is greater than that in the secondary; and also that it is possible under certain conditions for V_2 to be greater than E_2 ; that is, for the secondary terminal voltage to increase above its no-load value when load is applied.

362. Regulation of a Transformer.—The Regulation of a transformer is defined as the change in the terminal secondary voltage from no-load conditions to full-load conditions for a constant applied primary voltage, or $(E_2 - V_2)$.

It may also be defined as a percentage of the terminal secondary voltage on full load, thus :—

$$\text{Percentage regulation} = \frac{100 (E_2 - V_2)}{V_2}.$$

The regulation depends on the size of the transformer and the conditions of service. From the result quoted at the end of para. 361, with a capacitive load and a leading secondary current, the regulation may be a negative quantity.

363. Core Losses.—The two other types of loss mentioned in para. 358 will now be considered. They may be referred to by the common name of iron-losses or core-losses.

Eddy Current Losses.—If the core were solid throughout, large circulating currents would be set up in it in the same plane as the direction of the windings. This is indicated in Fig. 166 (a) which shows a cross section of one leg.

These currents are termed "**Eddy Currents.**"

They represent an expenditure of energy which would heat up the core unduly, and damage the insulation of the winding.

They are kept down to a very small value by "laminating" the cores and yokes as indicated in Fig. 166 (b), *i.e.*, by making them up of a number of thin sheets of iron laid together and insulated from each other by varnish, shellac, or tissue paper.

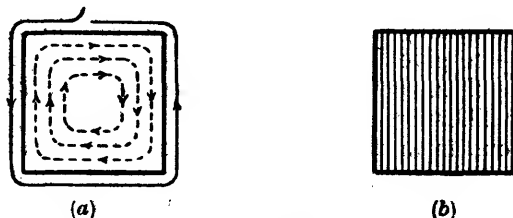


FIG. 166.

In this manner the eddy currents are forced to travel in very narrow high-resistance paths, and are kept down to such a small value that their effect is not serious.

The laminations are frequently "L"-shaped, being pushed inside the coils from alternate ends.

364. Hysteresis Losses.—During each alternating current cycle, the ferro-magnetic core is taken through a cycle of magnetisation. Hence energy is lost due to hysteresis, (para. 89), and appears as heat in the core.

The higher the frequency of the alternating current and the greater the flux density in the core the greater will be this loss.

Hard steel has a greater hysteresis loss than soft iron. Hence transformers have their cores constructed of soft iron either pure or alloyed with a small percentage of silicon. The alloyed iron—known as "Stalloy"—is more expensive, but is found to cause considerably less hysteresis and eddy current loss. Other alloys used for cores are permalloy and μ -metal.

It will be remembered that a further effect of hysteresis is that flux changes lag behind the current changes producing them.

365. Efficiency of Transformers.—In a well-designed transformer, the expenditure of energy due to these various causes is not very great.

The efficiency of a transformer is expressed as a percentage.

The percentage efficiency is $\left(\frac{\text{output}}{\text{input}} \times 100 \right)$ per cent.

The input, however, is equal to the output plus the losses.

Therefore, percentage efficiency may be written:—

Percentage efficiency

$$\begin{aligned}
 &= \frac{\text{output}}{\text{output} + \text{iron losses} + \text{copper losses}} \times 100 \\
 &= \frac{I_2 V_2 \cos \phi_2}{I_1 V_1 \cos \phi_1 + \text{iron losses} + \text{copper losses}} \times 100
 \end{aligned}$$

The iron losses remain practically constant at all loads when the primary supply voltage is constant.

The copper losses, $I_1^2 R_1 + I_2^2 R_2$ watts, vary as the square of the currents flowing.

It can be proved that the efficiency is greatest when the load is such that the copper losses are equal to the (constant) iron losses.

This need not necessarily be the full load for which the transformer is designed, but in practice it is arranged that the transformer is most efficient in the neighbourhood of full load.

The efficiency only falls off slowly as the inequality between iron and copper losses increases.

The same transformer can be built with a large section core and few turns (an "iron" transformer) or a small section core and many turns (a "copper" transformer). Copper is expensive and economy can be exercised in the amount used without a serious decrease in efficiency.

The following table gives the efficiencies which might reasonably be expected in modern transformers:—

Output in kW.	1	5	10	20	50	100
Efficiency	94%	95%	95.5%	96%	96.5%	97%

It is evident that a transformer is an extremely efficient piece of apparatus.

366. Connections of Windings.—In transformers where the primary and secondary windings are arranged in two coils on the two legs of the core, the step-up varies according to whether the two primary and two secondary coils are joined in series or parallel.

For example, in a transformer with 100 secondary turns for every primary turn, with:—

- (a) Primaries in series. Secondaries in parallel,
Step-up = 50 : 1.
- (b) Primaries in series. Secondaries in series,
Step-up = 100 : 1.
- (c) Primaries in parallel. Secondaries in parallel,
Step-up = 100 : 1.
- (d) Primaries in parallel. Secondaries in series,
Step-up = 200 : 1.

In certain Service spark sets, arrangements (a) and (b) are used for convenience in charging the condenser. The condenser in the oscillatory circuit, which is charged up from the transformer, may have several different values, and, due to energy considerations (which will be referred to in Chapter VIII), it is necessary to have alternative values for the voltage obtainable from the secondary of the transformer. The primary windings are connected permanently in series, and the secondaries are changed over from series to parallel by means of a switch, as illustrated in Fig. 168.

367. Earth in Centre of Transformer.—The centre point of the two secondary windings is always earthed in large transformers, the reason being as follows:—

Suppose that a maximum voltage of 14,000 volts is being produced across the secondary of the transformer; then each terminal will alternately reach a potential of 7,000 volts above and below "earth" potential or "zero."

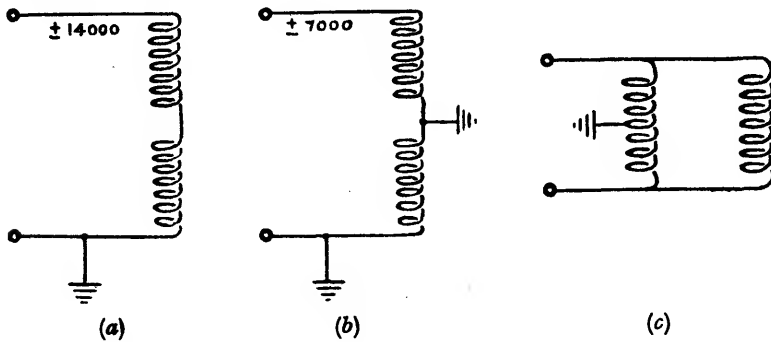
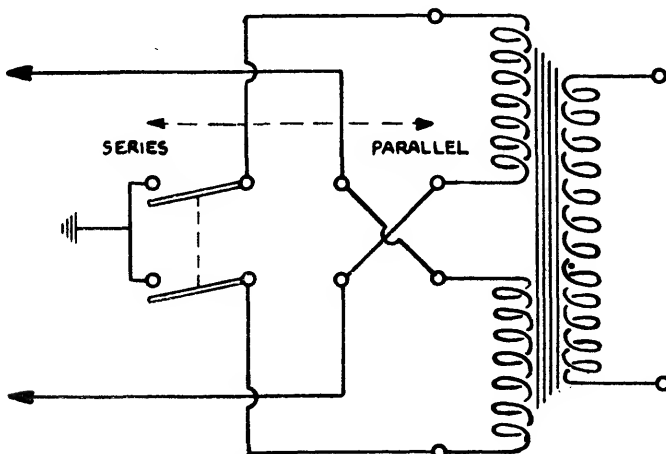


FIG. 167.

The thickness of the insulation on the secondary winding of the transformer is calculated so as to be sufficient, with a fair margin of safety, to stand this P.D.



Series-Parallel Switch.

FIG. 168.

If now an earth leak were to develop on one side of the transformer, the terminal on this side would automatically be fixed at earth potential. The same flux is still cutting the secondary.

Hence as shown in Fig. 167 (a) the potential of the other end of the winding would be alternating between 14,000 volts above and below earth instead of 7,000 volts.

Since the casing and core of the transformer are connected to earth, it is evident that the insulation of the winding of the transformer secondary will be excessively strained at this point.

To obviate this, the centre point of the secondary winding is permanently connected to earth, as in Fig. 167 (b); or, if the secondaries are in parallel, as in Fig. 167 (c); the outer ends are thereby prevented from reaching a greater potential above earth than their normal 7,000 volts.

Under these conditions, should an earth develop at one terminal of the winding, that half will be put on short circuit, and the secondary voltage will be only half its proper value; the short-circuited half of the secondary will call for a large primary current, and the A.C. cut-outs in the supply mains should blow.

Fig. 168 illustrates a Series-Parallel switch for earthing the centre point of the two windings when in the series position corresponding to Fig. 167 (b).

368. Cooling of Transformers.—It is necessary to make arrangements for radiating away the heat generated in the core and windings by the various losses referred to in paras. 360, 363 and 364, as otherwise the insulation of the windings will be impaired.

This is effected by means of air or oil cooling.

In small transformers, the windings are merely enclosed in a well ventilated iron case, and the heat is radiated away by convection through the air.

Air cooling is cheap and clean, but the transformer must not be allowed to get damp, or the insulation of the high tension winding will suffer, nor must it be put away in a corner or covered up where no air will reach it or circulate round it.

Sometimes the case is filled with an insulating compound, in order to enable the clearance between the windings and casing to be decreased without fear of sparking over.

Air blast cooling may also be employed. Cold air is forced into the transformer by means of fans.

For oil cooling, the tank is filled with good insulating oil. The heat is then conveyed through the oil to the sides of the tank and radiated away from there.

The oil protects the windings against damp, and the insulation of the windings is also materially assisted by its presence.

It is necessary to make the lid of the tank perfectly air-tight; otherwise the oil will absorb moisture from the atmosphere and lose its insulating property.

It is also necessary to arrange for the expansion of the oil when it becomes heated. This is done by providing an "Expansion Tank" connected to the transformer case by a short length of pipe.

Oil cooling may also be made more efficient by having an air blast on the tank, or a circulation of cold water in pipes at the top of the tank to cool the oil.

369. Equivalent Circuits.—In considering circuits in which a transformer is included, it is often convenient in calculations to reduce the circuit to an equivalent one in which the transformer is eliminated, that is, to find what values of R , L and C , when connected on the primary side of the transformer, would give the same relationship between primary voltage and current in the simple circuit as that which holds when R_2 , L_2 and C_2 are actually included in the secondary circuit.

Let the secondary be connected to a circuit of resistance R_2 ohms, inductance L_2 henries, and capacity C_2 farads.

Let E_2 be the secondary induced voltage and I_2 the secondary current, V the primary applied voltage and I_p the primary current.

Then, as in para. 354, neglecting losses,

$$V \times I_p = E_2 \times I_2.$$

The relationship between E_2 and I_2 is given by

$$E_2 = I_2 \sqrt{R_2^2 + \left(\omega L_2 \sim \frac{1}{\omega C_2} \right)^2}$$

Also $E_2 = V \times T$, and $I_p = I_2 \times T$ (para. 354).

$$\text{Therefore } V \times T = \frac{I_p}{T} \sqrt{R_2^2 + \left(\omega L_2 \sim \frac{1}{\omega C_2} \right)^2}$$

$$\begin{aligned} V &= \frac{I_p}{T^2} \sqrt{R_2^2 + \left(\omega L_2 \sim \frac{1}{\omega C_2} \right)^2} \\ &= I_p \sqrt{\left(\frac{R_2}{T^2} \right)^2 + \left(\frac{\omega L_2}{T^2} \sim \frac{1}{\omega C_2 T^2} \right)^2} \\ &= I_p \sqrt{R_1^2 + \left(\omega L_1 \sim \frac{1}{\omega C_1} \right)^2}, \end{aligned}$$

where $R_1 = \frac{R_2}{T^2}$; $L_1 = \frac{L_2}{T^2}$; $C_1 = C_2 T^2$.

But this is exactly the relationship found between V and I_p in a simple series circuit whose constants are R_1 , L_1 and C_1 .

Hence, for purposes of calculation, the combined circuits with a transformer included can be replaced by a single circuit, the equivalent value of the resistance R_2 in the secondary being $\frac{R_2}{T^2}$ when it is transferred to the primary side; the equivalent primary inductance corresponding to L_2 being $\frac{L_2}{T^2}$; and the equivalent primary capacity corresponding to C_2 being $C_2 \times T^2$.

The equivalent impedance Z_1 corresponding to the secondary impedance Z_2 is $\frac{Z_2}{T^2}$.

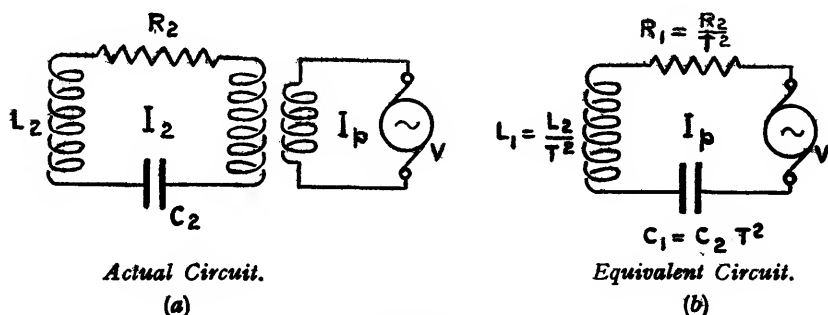


FIG. 169.

370. Auto-Transformer.—This is a special type of transformer used for a small step-up or step-down of voltage.

It is a transformer with only one winding, as shown in Fig. 170 (a) and (b), and, having an iron-cored coil, possesses a large inductance.

To step-up the alternator voltage, tapings are taken as shown in Fig. 170 (a), $P_1 P_2$ being the primary terminals, and $S_1 S_2$ the secondary terminals.

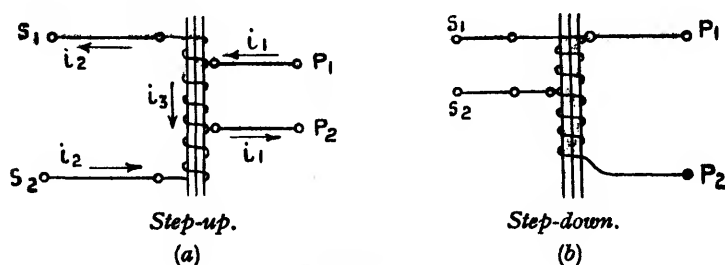


FIG. 170.

If an alternator is joined to $P_1 P_2$ its voltage will be balanced by the counter E.M.F. of the turns N_1 between the terminals $P_1 P_2$. The flux due to the primary current will thread the whole coil of N_2 turns, and consequently the P.D. between the secondary terminals $S_1 S_2$ will be greater than the alternator voltage in the proportion of $\frac{N_2}{N_1}$, or the transformation ratio T . If a load is joined across the secondary terminals $S_1 S_2$, a current I_2 will flow in the secondary circuit.

As before, it must be counterbalanced by a current I_1 in the primary, and $I_1 = I_2 \times T$. Neglecting losses, I_1 can be taken to be the whole primary current.

Also, I_1 and I_2 are in antiphase, so that, as shown in the figure, the current in the common portion of the winding is given by $I_1 - I_2 = I_3$.

To step the voltage down, the transformer would be used as shown in Fig. 170 (b).

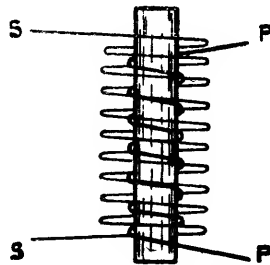
It can be shown that there is a considerable saving in the amount of copper and in the copper losses with an auto-transformer as compared with a two-coil transformer, but this saving diminishes rapidly as the transformation ratio T increases. Hence, an auto-transformer is only suitable for a small step-up or step-down of voltage.

Its great disadvantage is that there is direct electrical connection between primary and secondary. If a break occurs in the winding between S_1 S_2 in a step-down transformer, for instance, the higher primary voltage is applied to the low tension apparatus and leads. This is another reason for limiting the utility of the auto-transformer to cases where T is small.

In addition, part of the winding has to be thick enough to stand the primary current, and the whole well enough insulated to stand the secondary voltage.

For these reasons an iron-cored auto-transformer is not used in wireless telegraphy for alternating currents, but the principle is an important one to understand, as it often comes in when considering, for example, how to join up aerial coils in oscillating circuits, and in many other cases.

371. Open-core Transformers.—By an "open-core" transformer is meant one where the iron magnetic circuit is not complete, as in the induction coil.



Open Core Transformer.

FIG. 171.

Fig. 171 illustrates the primary and secondary windings of an induction coil.

Here the windings are wound over an iron core, but the return path for the magnetic lines of force consist of air, and magnetic leakage (and consequently the effective inductance on full load) is much greater.

372. Air-Core Transformers.—By this term are meant the various arrangements for transferring energy from one oscillatory circuit to another by mutual inductive coupling, as described later in connection with transmitting and receiving circuits.

The primary and secondary windings have no iron core, and so their inductance is only of the order of microhenries, but the principles which govern the behaviour of the closed core transformer apply to them with certain limitations.

A.C. MEASURING INSTRUMENTS.

373. Ammeters and Voltmeters.—It was pointed out in para. 274 that any instrument whose pointer deflection is proportional to the first power of the current is useless for A.C. measurements, as the mean deflection of the pointer is zero. If, however, the pointer movement depends on the square of the current a mean deflection of a definite amount is produced, and an instrument operating on this principle, and calibrated for direct current, reads R.M.S. values of alternating current. Of the instruments described in Chapter III, this eliminates the moving coil ammeter or voltmeter, leaving as measuring instruments suitable for A.C.,

- (a) Hot-wire ammeter and voltmeter.
- (b) Moving-iron ammeter and voltmeter.
- (c) Electrostatic voltmeter.

To these may be added :—

- (d) Thermo-couple ammeter and voltmeter, which are used very seldom in D.C. work, but largely used for radio-frequency measurements.

374.—(a) Hot Wire Instruments.—These have been almost entirely superseded for low-frequency measurements, *e.g.*, in power supply circuits, by the more accurate moving-iron instruments, but are still largely used for radio-frequency measurements as their readings are independent of frequency over a considerable range. Up to about 50 kc/s. shunts of ordinary type can be used to extend the range of these instruments, but above this frequency “skin effect” becomes important in altering the effective resistances and the current paths must be made absolutely symmetrical with respect to the hot wire. This is sometimes effected by making the hot wire one of a number of conductors in parallel, arranged in the form of a squirrel cage, but a better method is to step-down the current to a suitable value by a radio-frequency transformer, the primary winding carrying the current to be measured and the hot-wire being connected across the secondary. The iron core of such a transformer must be very thoroughly laminated to prevent excessive power losses. With such a transformer, currents of 1,000 amperes can be measured at any frequency up to at least 1,000 kc/s

(b) **Moving Iron Instruments.**—As mentioned above, these instruments are used for low-frequency as opposed to radio-frequency measurements. Within this range their readings are moderately independent of frequency, particularly in the case of the ammeter. The voltmeter, owing to its relatively high self-inductance, may be seriously affected, so that if calibrated at 50 cycles/sec. an error of

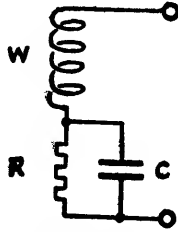


FIG. 172.

as much as 40 per cent. may occur at 500 cycles/sec. The arrangement shown in Fig. 172 illustrates one method of compensating for this defect. *W* is the winding and *R* is the non-inductive series resistance. To compensate for frequency errors, *R* is shunted by a condenser *C*. The reactance of this condenser decreases as the frequency increases and so the impedance of the *C*-*R* parallel circuit decreases sufficiently to compensate for the increase in reactance of the winding.

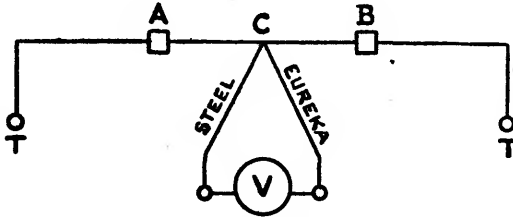
A well-designed attracted-iron instrument will give almost as accurate readings as a moving-coil instrument and is often employed in A.C. circuits where accuracy is necessary, *e.g.*, in measuring the filament voltage of valves.

(c) **Electrostatic Voltmeter.**—It was shown in para. 189 that the pointer movement of this instrument was proportional to the square of the P.D. between the fixed and moving plates, so that when calibrated for D.C. voltage it reads R.M.S. alternating voltages. As it is unaffected by temperature changes, stray magnetic fields, changes of frequency or wave-form, and wastes no power, it is a very suitable instrument for A.C. measurements.

(d) **Thermo-Couple Instruments.**—Thermo-electric effects are fully described in the chapter on wavemeters. It is sufficient to state here that if two wires of different metals are connected to form a closed circuit, and one junction of the two wires is raised in temperature relatively to the other, an E.M.F. is set up between the hotter and colder junctions, proportional to their temperature difference. The E.M.F. is not affected by inserting a wire of a third material in some intermediate part of the circuit.

Fig. 173 shows the arrangement of an ammeter based on this effect. The thermo-junction *C* is attached to a heater wire *AB*, which carries the current to be measured. The heat developed in

AB is proportional to the square of the current, and so the rise in temperature, and consequently the thermo-electric E.M.F. produced, is roughly proportional to the square of the current. The moving-coil milli-voltmeter V measures this E.M.F., so that its deflection is also roughly proportional to the square of the current through the heater, and it can be calibrated in R.M.S. values.



Thermo-couple Ammeter.

FIG. 173.

This instrument is largely used for radio-frequency measurements. For large currents (above 10 amps.), or high frequencies (above 1,000 kc/s.), errors due to "skin effect" are appreciable and special calibration is necessary. The radio-frequency current may also find its way into the thermo-couple circuit with consequent losses due to stray capacity. To prevent this, the heater is sometimes electrically insulated from the thermo-couple, while maintaining close thermal contact with it. This precaution is only effective up to medium radio-frequencies.

375. Frequency Meters.—The measurement of frequency is an important part of alternating current practice which has no parallel in direct-current work. Measurement of radio-frequencies is accomplished by wavemeters, which are described in another chapter. Here we are simply concerned with the instruments which measure frequencies from 50 to 1,400 cycles/sec., such as are encountered in the circuits used for H.T. and filament supplies in transmitters.

376. The three types of frequency meter used in the Service are :—

- (a) Reed type.
- (b) Inductor type.
- (c) Magneto-generator type.

(a) **Reed Type.**—It has been seen that in an electrical oscillatory circuit there is a definite frequency (the resonant frequency) at which an applied E.M.F. gives a maximum current. A mechanical oscillatory system behaves in a similar manner. The amplitude of vibration produced by an applied periodic force depends on the period and is a maximum when the frequency of the applied force is the same as the resonant frequency of the oscillatory system.

This is the principle of the reed type of frequency meter, a diagram of which is shown in Fig. 174.

A small piece of soft iron *X* is attached to one end of a thin steel blade *R* (the "reed"), whose other end is firmly held to the supporting structure. This structure carries a laminated core of soft iron, and wound on this is a coil *W*, through which flows the current whose frequency is to be ascertained. The alternating flux due to the core magnetises *X*, and the resultant force between *X* and the core is proportional to the square of the current, as in a moving-iron

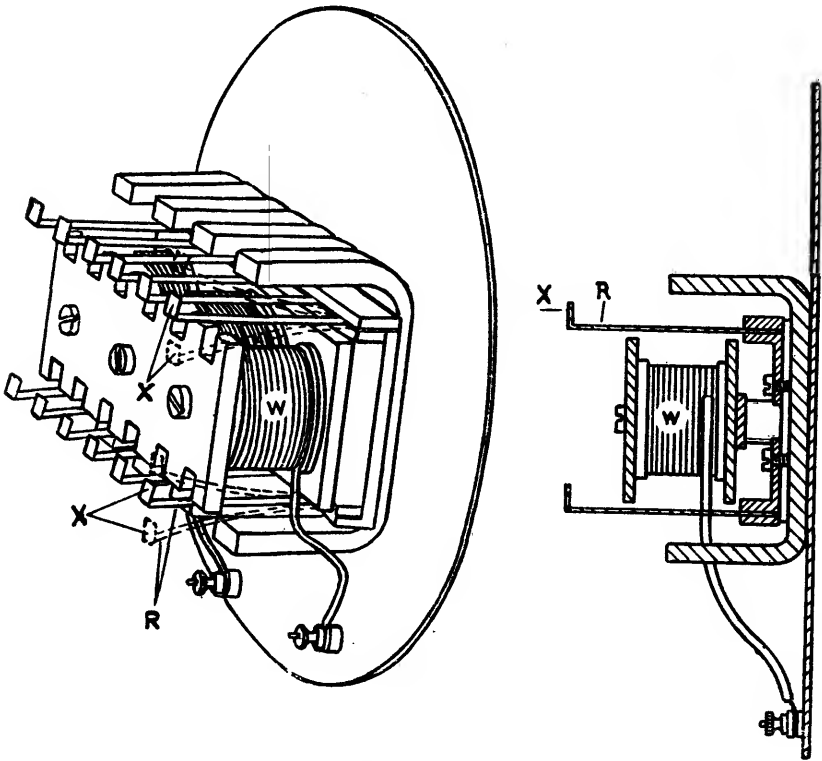


FIG. 174.

ammeter. It is therefore always in the same direction and has two maxima per cycle of alternating current. The reed is thus acted on by a periodic force of twice the frequency of the alternator. If this force has the same frequency as the resonant frequency of the reed, large vibrations are set up in the reed, but as the reed is a lightly-damped system, its amplitude of vibration is very small if the applied frequency has any other value.

A number of reeds of differing known frequencies are mounted in this fashion and only the one whose resonant frequency is twice the alternator frequency gives a notable response. The top of each

reed is whitened or reddened and is visible through a slot in the cover of the instrument. When its amplitude of vibration is large

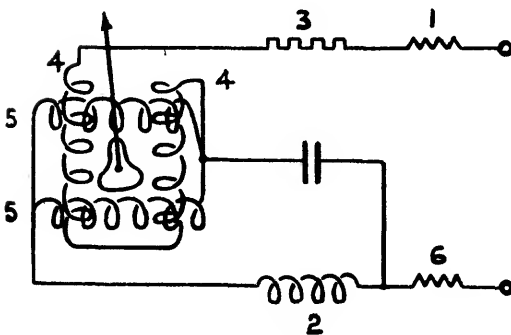
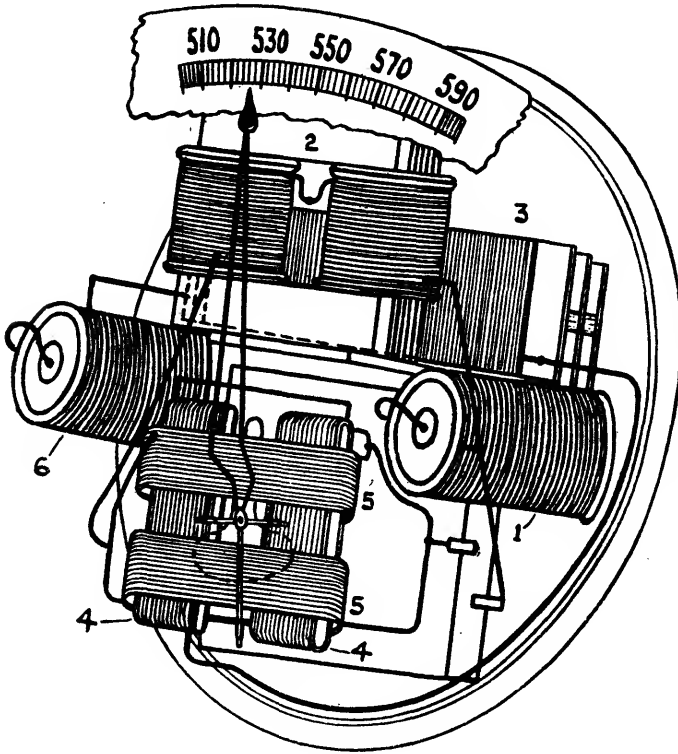


FIG. 175.

it appears to the eye as a broad white or red band because of the persistency of the impression on the retina. Reeds of neighbouring frequencies show a slight broadening and the remainder are not

appreciably affected. This is a very accurate and sensitive type of frequency meter.

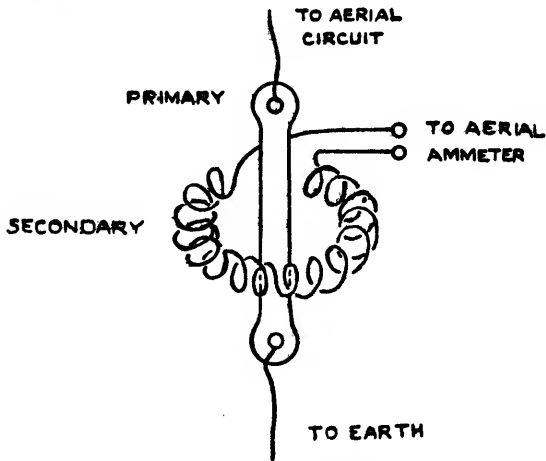
(b) **Inductor Type.**—This is a simpler instrument designed to cover only a small range of frequencies around some standard frequency. The principle is shown in Fig. 175. Two field coils whose axes are at right angles act on a light freely-pivoted iron vane carrying a pointer. The direction in which the pointer sets itself depends on the direction of the resultant field of the two windings. One end of each winding is common and the other ends are connected to an inductance and resistance respectively. The current through the resistive path is nearly independent of frequency, while that through the inductive path is inversely proportional to the frequency ($I = V/\omega L$). The ratio of the currents and therefore the direction of the resultant field thus alters with the frequency, giving a different equilibrium position of the pointer at each frequency.

(c) **Magneto-Generator Type.**—In this instrument a small magneto-generator is driven off the shaft of the alternator whose frequency is required. The E.M.F. of the generator is directly proportional to the speed of rotation of the armature and is read on a moving-coil voltmeter. The frequency of an alternator in cycles/sec. is given by the product of the speed in revs./sec. and the number of pairs of poles (para. 196). It is thus directly proportional to the reading of the voltmeter, which can be calibrated for frequencies.

377. Aerial Ammeter Transformer.—It was seen in para. 185 that, in order to increase the range of ammeters for measuring direct currents, they might be shunted. This procedure cannot be applied to ammeters for A.C. because, in general, the inductances of both the ammeter itself and the shunt are appreciable and their mutual inductance may also be large enough to matter. The distribution of current between ammeter and shunt is determined, of course, by the ratio of their total impedances, which may be very different from the ratio of their resistances and moreover varies with frequency. This effect will be particularly pronounced at radio-frequencies, when the ratio of the inductances may often give a better criterion of the distribution of current than the resistance ratio.

A common Service method of measuring aerial current, when the ammeter has too small a maximum reading to be inserted directly in the aerial circuit, is to employ a "toroidal transformer," as illustrated in Fig. 176, the primary being the aerial wire and the secondary the toroidal coil whose terminals are connected to the ammeter. The concentration of the flux in a toroid inside the coil itself has already been referred to on several occasions (*cf.* para. 152). Thus the mutual flux-linkage of the primary and secondary is small

and the current flowing in the coil is considerably stepped down compared with the aerial current. Further, the back reaction of the current flowing in the coil on the aerial and other neighbouring circuits is minimised.



Toroidal Transformer.

FIG. 176.

VARIATION OF ELECTRICAL QUANTITIES WITH FREQUENCY.

378. The assumption of constant resistance, inductance and capacity at various frequencies is by no means justified in practice; for instance, the ohmic resistance of a circuit is much larger to a radio-frequency current than to a direct current, and again a coil which at first sight would seem an inductance, may actually have capacitive reactance. A short account of some of the more outstanding points in the behaviour of electrical circuits will now be given.

379. Resistance.—The definition of the resistance of a circuit by means of Ohm's Law is really equivalent to the statement that all the electrical energy supplied to the circuit is directly converted to heat. Now, though this is the case when a direct current is flowing, it ceases to hold for alternating currents. It has already been seen that part of the energy supply to an A.C. circuit may be transferred to neighbouring circuits if there is mutual induction. Likewise, condenser losses also account for some of the energy. Eddy currents and hysteresis cause energy loss, and in high-frequency circuits, energy radiated as "wireless waves" may be a considerable fraction of the energy input. The idea of the resistance of a circuit as being merely the "ohmic" resistance of its conductors has,

therefore, to be generalised. Resistance is directly connected with power dissipation by the formula $RI^2 = P$ (power), which has been seen to hold both for direct and alternating currents. It is therefore convenient, (in fact, it is forced on us by circumstances), to define the resistance of a circuit to alternating current as

$$R = \frac{P}{I^2}.$$

This amounts to finding an equivalent "ohmic" resistance for each source of energy dissipation, such that when the said resistance is multiplied by the square of the current it gives the energy dissipated per second. These resistances, however, no longer obey Ohm's Law and so the resistance as above defined will vary with current and frequency.

380. Skin Effect.—It is found that even the true "ohmic resistance" of a circuit to alternating current, *e.g.*, the ratio of P.D. to current in a straight conductor, is not constant but increases with frequency. This is due to the fact that when an alternating current flows in a conductor, the distribution of the current over the cross section of the conductor is not uniform.

If a direct current of 2 amps., for instance, were flowing through a conductor whose cross section is 0.02 sq. cms., we might split the conductor lengthwise into two conductors each of cross section 0.01 sq. cm., and it would be found that under the same conditions 1 amp. flowed in each part. The conclusion that the resistance of a conductor is inversely as its area was derived from such reasoning (para. 61). If the total resistance of the original conductor was 1 ohm., the resistance of each part would be 2 ohms, giving their equivalent resistance in parallel as 1 ohm. The total power dissipation RI^2 is 4 watts, whether it is calculated as $1 \Omega \times (2 \text{ amps})^2$ or $2 \times 2 \Omega \times (1 \text{ amp})^2$.

Suppose now that instead of the current distributing itself equally between the two parts, it could be arranged that 1.5 amp. flowed in one part and 0.5 amp. in the other. The power dissipation would then be given by

$$2 \Omega \times (1.5 \text{ amp})^2 + 2 \Omega \times (0.5 \text{ amp})^2 = 4.50 + 0.50 = 5 \text{ watts.}$$

and the equivalent resistance, as calculated from the power dissipation would be

$$R = \frac{P}{I^2} = \frac{5 \text{ watts}}{(2 \text{ amps})^2} = 1.25 \Omega$$

i.e., the equivalent resistance has increased by 0.25Ω owing to the non-uniform current distribution.

This uneven distribution of current across the cross section of a conductor actually occurs when an alternating current is flowing, owing to the E.M.F.s induced in the conductor. Consider the case of a straight conductor. The lines of flux are circles round the

conductor. During one cycle, they start with zero number, rise to a maximum, fall again through zero to a minimum (or maximum value in the opposite direction) and rise to a zero value again. The number interlinking with the material of the conductor is thus continually altering and E.M.F.s are induced in the conductor. In the central part of the wire all the flux lines, in being produced or in disappearing, must cut the material of the conductor; but towards the outer surface of the conductor this is not the case. The flux lines which were established in the central part of the conductor itself obviously do not cut the outer parts of the conductor in disappearing, and so make no contribution to the E.M.F. induced in these outer parts. The induced E.M.F. is thus not constant over the cross section of the wire, but increases in value from the surface to the centre. The result is that the current finds an easier path along the outer parts of the conductor, and a greater part of it proportionately flows there than in the central parts. This uneven distribution of current leads, as shown above, to the power dissipation in the conductor being increased, and so to an increase in its resistance.

This tendency of alternating currents to flow in the outer parts or "skin" of a conductor is called "skin effect."

We should expect it to increase with frequency (f) since the higher the frequency, the greater is the rate of change of flux; and with increase in the area of cross-section of the conductor, which increases the disproportion between the flux cutting its outer and central parts. At radio frequencies it is very pronounced and practically no current flows except in the skin of a conductor. For this reason transmitting inductances are often made of copper tubing instead of solid copper. The effect also depends in amount on the permeability μ and resistivity ρ of the conductor. Expressed as a formula, this may be written

$$\frac{R_{A.C.}}{R_{D.C.}} \propto d \cdot \sqrt{\frac{\mu f}{\rho}}$$

where d is the diameter of the conductor.

It is important to keep the resistance of leads, coils, &c., used in W/T circuits as low as possible to prevent power losses as heat, and this is generally achieved by the use of stranded wire. If the strands are insulated from each other, and interwoven so that each strand has the same part of its total length on the surface of the resulting cable, and therefore the same proportion in the interior of the cable, as the others, a much more uniform distribution of current over the cable may be obtained and a corresponding decrease in skin effect. The skin effect in the individual strands themselves, however, is still pronounced at high frequencies. Another obvious precaution is never to use iron wire because of its high permeability. If, in aerials, for example, iron wire must be

employed because copper wire is not strong enough, it should be galvanised. The galvanised outer skin will then carry all the high frequency current.

For use as aerial wire on board ship, insulated wire is inefficient as it collects a film of soot, &c., from funnel smoke, which gives it a high resistance skin. On the other hand, bare wire is corroded by sulphur fumes for the same reason, which also increases its skin resistance. Bare wire coated with anti-sulphuric enamel is therefore generally used.

When the conductor is wound as a coil or solenoid, the non-uniformity of distribution of current is still more marked than when it is straight. Owing to the arrangement of the flux lines in a coil (Fig. 18), the side of the wire further from the former is cut by more flux lines than the nearer side and so the current tends to flow on the inner side of the wire. Thus the increase of resistance with frequency is greater in a conductor wound as a solenoid than in a straight conductor. The greater the number of layers in the winding the more pronounced is the skin effect, as would be expected. Another factor tending to increase the apparent resistance of a coil is its self-capacity, which is considered below.

381. Inductance.—In winding a coil, the straightforward method is to wind the wire along the former in the bottom layer, as shown in various previous figures representing solenoids, then to wind another layer above the first one in the reverse direction until the starting point of the winding is again arrived at, and so on. The result is that adjacent turns in the winding may be at very different potentials. In a two-layer winding, for example, the first and last turns wound on would be adjacent, and their difference of potential would be the total P.D. across the winding. They are separated from each other by the insulation of the wire and so form in effect a condenser. It is obvious that the condenser is in parallel with the total winding. In a similar manner, every turn of the winding has a certain capacity to every other turn, and the combined effect of these capacities may be taken as a capacity in parallel with the inductance of the coil. The coil is thus really a parallel circuit of the type discussed in Chapter V, and instead of its reactance increasing steadily with frequency, it rises to a peak value at the frequency at which the self-capacity of the coil tunes with its inductance. At higher frequencies the reactance is capacitive. This is further discussed in Chapter XIII.

It has already been explained that at its resonant frequency the "rejector circuit" behaves as a very large resistance and it will be obvious that even at frequencies removed from resonance, this tendency will be to some extent in evidence, *i.e.*, the effect of the self-capacity is to increase the component of the applied E.M.F. in phase with the current and so to increase the power loss in the coil. In other words, the A.C. resistance of the coil is increased.

The effect of self-capacity in a single layer coil is not nearly so large, as adjacent turns do not differ so much in potential. It can be decreased in a multilayer coil by winding the turns on top of each other in the same plane as in the ordinary frame coil winding, Fig. 177 (a), or by arranging the layers in a banked winding, as in

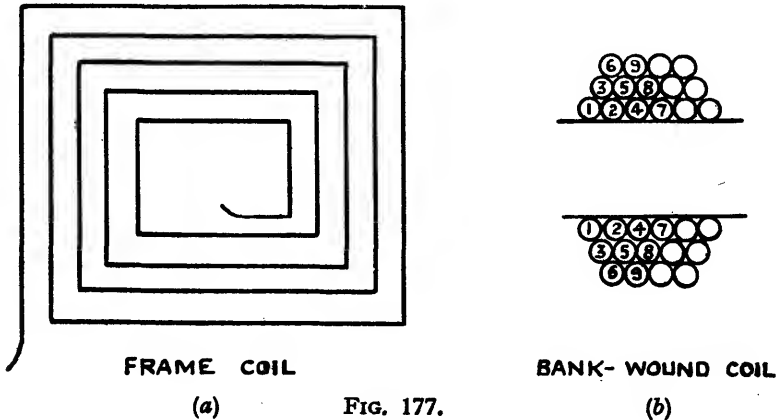
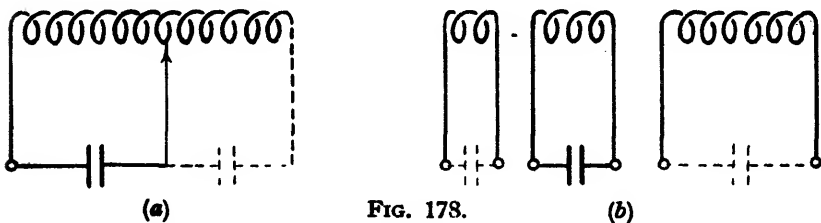


Fig. 177 (b), where the order in which the turns are wound on is shown by the numbers. Another method is to adjust the thickness of the air spacing between turns and layers. A spacing approximately equal to the diameter of the wire gives minimum self-capacity effect. The object in each case is to reduce the P.D. between adjacent turns.



362. Tapped Inductances.—It often happens in wireless sets that tuning adjustments are obtained by varying the amount of inductance in a circuit. This may be effected by having a variable contact on a coil, which according to its position alters the number of turns in circuit; alternatively, several coils of different inductance may be separately wound on the same former and the inductance altered by varying the number of such windings included in the circuit. These methods are illustrated in Fig. 178 (a) and (b). The unused turns or windings are supposed to be "dead," *i.e.*, to have no effect on the rest of the circuit, but actually, owing to their self-capacity, it will be seen that they are in reality closed oscillatory circuits,

which are closely coupled by auto-inductive or mutual coupling to the circuit aimed at. If these unwanted circuits happen to tune at a frequency not greatly different from that being employed, they may absorb a large amount of energy from the main circuit, and increase its apparent resistance considerably (para. 335). In addition, they alter its resonant frequency and may even produce a noticeable double frequency effect. It may often be found advisable to short-circuit the unused turns, as the impedance of their inductance by itself may be much greater than the impedance of the turns in conjunction with their self-capacity: a much smaller current will then flow in the unused turns. The question of the best procedure depends on the number of unused turns. A large number should be short-circuited; a small number should be left open.

333. Cores for Inductances.—The advantages of ferromagnetic cores in increasing and concentrating the flux-linkage with a coil for a given current have already been discussed. The difficulties found in their use with radio frequency currents arise principally from the loss of energy due to eddy currents in the core; comparatively speaking, the hysteresis loss can be neglected. Even with well-laminated iron cores the losses at radio-frequencies due to eddy currents render their use prohibitive. Two effects are produced:—

- (1) Owing to the power losses, the apparent resistance of the coil is greatly increased.
- (2) The flux produced by the eddy currents is in the opposite direction to that produced by the magnetising current in the coil. The result is that the resultant flux in the core is much smaller than with a direct current of the same strength, *i.e.*, the permeability of the core appears to be considerably lower.

The eddy current losses increase with frequency and so the inductance of the coil decreases as the frequency increases. This may be so pronounced that the inductive reactance of an iron-cored coil remains nearly constant over a large range of frequencies or even falls as the frequency increases. In general iron-cored coils are used only in the audio-frequency stages of wireless sets.

The copper losses in such coils are generally negligible compared with the iron losses, and little advantage is gained by using wire of large diameter.

Many endeavours have been made to reduce the iron losses at high frequencies, and special iron and steel alloys, such as permalloy (an alloy of steel and nickel), have been developed, possessing especially large permeabilities for small magnetising currents. Another method is to reduce the size of the laminations by using iron dust for the core, the dust particles being coated with insulating cement. The eddy current loss is thus considerably reduced, but in the earlier cores of this type it was found that the permeability

was also very much less. It was later discovered that, by subjecting the coated dust particles to high pressures in moulds, they could be made to bind together, and a material of low eddy current loss and yet of high permeability could be obtained.

384. It is usually the case that when coils are being used for wireless purposes, a direct current is flowing through them, and the high-frequency alternating current, often of smaller amount, is superimposed on this. The result is an iron-cored coil is that the iron is already magnetised to a degree corresponding to the point on the permeability curve for the direct current flowing. Care must therefore be taken when the alternating current is superimposed that the maximum current values do not bring the iron to a saturated condition. The effect of this is shown in Fig. 179 (a). It produces

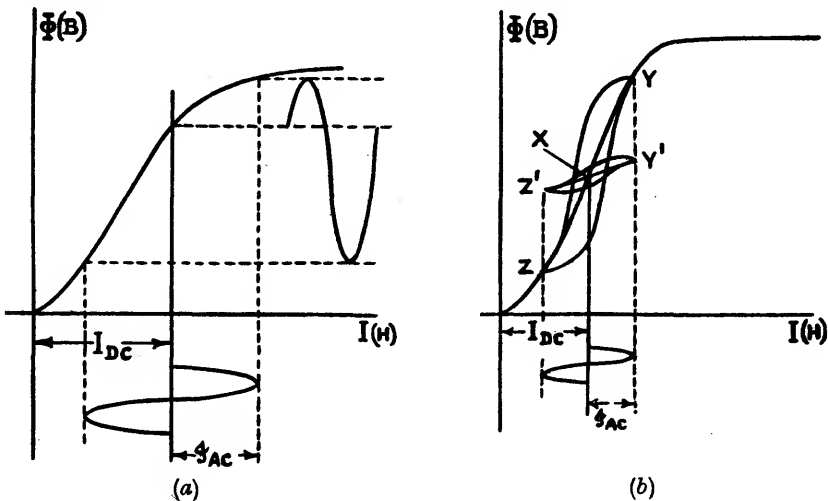


FIG. 179.

a distortion of the alternating flux wave from the sinusoidal form and therefore a similar distortion in the wave form of the induced E.M.F. The permeability is also less the nearer the direct flux is to saturation point, and so the value of the induced E.M.F. is decreased. It may quite often occur that an air gap in the iron core will produce an increase in the A.C. inductance of the coil, by bringing down the working point on the permeability curve, owing to the increased reluctance of the core.

Even on the steep part of the permeability curve the apparent permeability for alternating current is less than would be found for direct current. This is shown in Fig. 179 (b). The loop described in a cycle of magnetisation corresponding to a cycle of the alternating current is not the YZ loop, as might be expected, but the loop $Y'Z'$. The average slope of $Y'Z'$ is less than that of YZ , i.e., the A.C. permeability is decreased.

385. Transformers.—The remarks above on ferromagnetic cores also apply to transformers. The limitations which prevent high transformation ratios will be better appreciated when the conditions under which transformers operate in wireless circuits have been considered. It may be pointed out at this stage, however, that the self-capacity of the windings means that at high frequencies the transformer secondary is closed through a circuit of comparatively low reactance. In addition, the coupling between the primary and secondary coils at radio-frequencies may be as much capacitive as inductive, owing to the self-capacity between the windings, and caution must be exercised in applying the results derived earlier in this chapter for power transformers.

A simple example of this is to be found in the variometer. If the magnetic coupling between the fixed and moving coils is alone considered, the mutual inductance is zero when the coils are at right angles, and no E.M.F. should be produced in either coil by changes of the current flowing in the other. At high frequencies it is found that the capacitive coupling produces an E.M.F. in this position and zero E.M.F. is actually obtained at some other angle where the mutual inductive and capacitive couplings are equal and opposite.

386. Capacity.—The effect of dielectric losses, etc., in increasing the resistance of a capacitive circuit have already been discussed. Perhaps the most important point to grasp about capacities at high frequencies is the low reactance of even minute capacities and the easy shunt paths they provide, with consequent loss of H.F. current where it is required. This may best be realised by giving a comparative table of the reactances of a capacity of 1 jar and an inductance of 100 microhenries at various frequencies.

$f.$	1 jar $\left(X_c = \frac{9 \times 10^9}{2\pi f} \right)$	100 mics ($X_L = 2\pi f \times 10^{-4}$)
100 cycles/sec.	14.3 megohms.	0.0063 ohm.
1 kc./s.	143,000 ohms.	0.63 ohm.
1 Mc./s.	143 ohms.	630 ohms.
100 Mc./s.	1.43 ohms.	63,000 ohms.

In wireless sets for high frequencies it is often found that the inductive reactance of a proposed earth lead is high enough to provide an appreciable obstacle to H.F. currents, and a large condenser is inserted in the lead to reduce the reactance to a negligible amount (H.F. earth). By-pass condensers are also frequently used to conduct H.F. currents to earth by the shortest possible path in order to prevent unwanted coupling between the various stages of a wireless receiving circuit. Examples of this will be found in later chapters.

CHAPTER VII.

THE OSCILLATORY CIRCUIT : DAMPED OSCILLATIONS.

387. Wireless Radiation.—Communication by wireless has been rendered possible within the last fifty years because methods have been devised by which disturbances of an electro-magnetic nature can be propagated through the aether from transmitting apparatus and received on suitable receiving apparatus at a distance from the transmitter.

The investigation of the subject falls therefore into three sections :—

- (1) The methods by which the electro-magnetic disturbances are produced.
- (2) Conditions affecting their passage from transmitter to receiver through the intervening medium—the aether.
- (3) The methods by which the electro-magnetic disturbances are detected and rendered apparent to the senses.

The first of these sections is to be considered in this chapter and subsequent chapters on the Spark Transmitter, the Arc Transmitter and the Valve Transmitter.

The first essential in wireless transmission is the production of a high-frequency alternating current in a suitable circuit, from which, as we shall see, it is possible to detach a certain amount of energy which creates the electro-magnetic disturbance. Such a high-frequency alternating current may be of two different types, its wave form being either "**Damped**" or "**Undamped**," also referred to as "**Continuous**." (See Chapter I.)

The wave form of the resultant electro-magnetic disturbance is the same as that of the alternating current which produces it.

The spark system of wireless telegraphy gives damped waves ; the arc system continuous waves ; and the valve system damped or continuous waves at will.

There is another method, the high frequency alternator system, which produces continuous waves, but it is not used in the Navy. For efficient radiation, the frequency of the alternating current in the transmitting circuit must be high, so that the design of an alternator which would produce the requisite frequencies is difficult, and the alternative methods given above are preferred.

Historically, the first type of transmitter to be rendered effective was the spark transmitter, producing damped waves. We shall therefore start our investigation of the subject by considering the production of damped waves, or, what amounts to the same thing, damped high frequency alternating current.

There are two definite divisions in this investigation :—

- (1) The theory of the oscillatory circuit. A circuit containing inductance and capacity, with small resistance, has damped high-frequency alternating currents produced in it if a certain amount of energy is isolated in the circuit, and allowed to discharge through the circuit. This subject is dealt with in the present chapter.
- (2) The charging circuit arrangements. These are the methods by which the oscillatory circuit is supplied at frequent intervals with the necessary amount of energy for oscillatory action to take place, and rendered efficient as a transmitter of damped waves. They are dealt with in Chapter VIII.

The distinction between these two headings must be clearly understood.

388. The Oscillatory Circuit.—The oscillatory circuit used in a spark transmitter is made up of a condenser in series with an inductance and a spark gap. The first thing about this circuit which must be clearly grasped is that the spark gap is included purely and simply as a means by which energy can be introduced into and isolated in the circuit. The obvious way in which to introduce energy into the circuit is to charge up the condenser to a potential V , when the energy stored in the condenser will be $\frac{1}{2} CV^2$ joules. If the circuit from one plate of the condenser to the other were continuous it would be impossible to charge it up to a definite potential difference, and so the spark gap is introduced.

During the oscillatory action, however, which is what we are investigating, the insulation of the spark gap breaks down, and the circuit becomes effectively one containing R , L and C in series.

The Mechanical Case.—Before going on to the detailed investigation of the electrical circuit, it is useful to consider a mechanical analogy.

Let us take a mechanical oscillator, consisting of a weight and a spring. (Fig. 180.)

The spring is fixed in a horizontal position to a wall and carries the weight on its free end. Imagine the weight to be pulled and held to one side, as in Fig. 180 (*b*) (1). The spring is bent, thereby storing up energy in itself, and has a strong tendency to spring back again. The energy stored in the spring is known as potential energy. When it is released, it will spring straight, setting the weight in motion. The moving weight, owing to its inertia, will not stop dead when the spring has straightened (*b*) (2), but will continue past the middle position and swing to the right until it comes momentarily to rest in position (*b*) (3).

When the weight is in position (*b*) (2), there is no energy locked up in the spring, which is not deformed, but the weight possesses kinetic energy by virtue of its motion. In position (*b*) (3), when

the weight is just at rest, the energy is again stored up in the spring as potential energy. It will then swing back again. So we have a transference of energy from the potential to the kinetic form occurring every quarter cycle, if we look on the sequence of events from position (1) until the weight returns there as being a complete cycle.

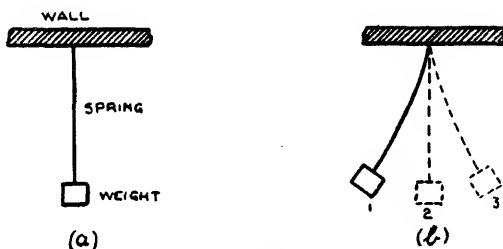


FIG. 180.

The oscillation continues until the energy originally stored in the spring has been expended in various frictional losses.

The rate at which the arrangement will oscillate will depend on the weight and on the length and elasticity of the spring. A short stiff spring and a small weight will oscillate much more rapidly than a long weak spring and a heavy weight.

The above is an exact mechanical analogy for the electrical oscillations we are about to consider—the mass representing the inductance, the spring the condenser, the velocity of the weight the current and the compression of the spring the voltage to which the condenser is charged.

The Electrical Case.—Let the condenser be C , the inductance L , and the spark gap G , as in Fig. 181.

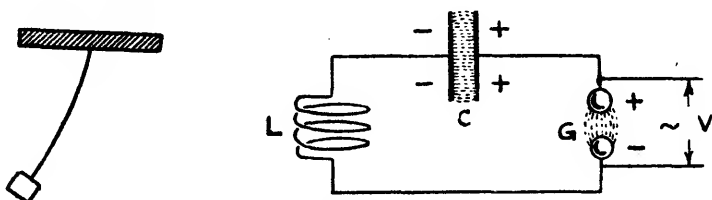


FIG. 181.

The source of high voltage supply V is assumed capable of charging the condenser C up to a high voltage at regular intervals.

(a) Assume the condenser to be given a charge of Q coulombs (as in Fig. 181). Then the right-hand plate of the condenser and the upper spark plug will have a positive charge, and an equal and opposite negative charge will be distributed over the left-hand plate, the surface of the inductance L , and the

lower spark plug. Between the condenser plates the electrons in the dielectric will be strained to one side, and similarly for the electrons in the spark gap. The potential difference across the condenser will be

$$V = \frac{Q}{C} \text{ volts.}$$

The energy stored in the condenser will be

$$\frac{1}{2} CV^2 \text{ joules.}$$

(b) Assume that the spark gap G has been set to such a width that, when the charge put into the condenser has risen to its maximum value, the voltage across the gap is sufficient to break down the insulation of the air. The spark gap now becomes effectively a resistance included in the circuit. There will be a drift of electrons in a counter-clockwise direction, or an electric current from the right-hand plate across the gap, through L , to the left-hand plate.

A conductive bridge is set up between the spark plugs, composed of positive and negative ions of air, and small particles of copper driven off from the spark plugs. This constitutes a convection current.

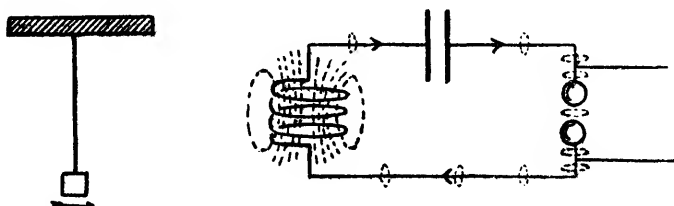


FIG. 182.

As the current gradually rises from zero it builds up a magnetic field, and when it has reached its maximum value I , the energy stored in the magnetic field is $\frac{1}{2}LI^2$. At the same time, the potential difference across the condenser is zero, so that all the energy in the system is now in the magnetic field, instead of being in the electric field, as at the beginning of the current flow. Assuming no resistance losses, $\frac{1}{2}LI^2 = \frac{1}{2}CV^2$.

In the mechanical analogy, at the corresponding point in the cycle, the energy is all kinetic and situated in the moving weight, while at the beginning it was all potential and stored in the spring.

(c) When the P.D. across the condenser is zero, there is no voltage left to keep the current flowing.

It will not, however, stop instantaneously, but will continue flowing for some time while the magnetic field dies down. By Lenz's law, the collapsing magnetic field sets up an E.M.F. in the inductance opposing the cause of its collapse, *i.e.*, the reduction of the P.D. and the current to zero, and hence it tends to

maintain the current flowing in the **same** direction. When the magnetic field has entirely died away, the condenser is charged up again, but now positively on the left-hand plate, and negatively on the right-hand plate.

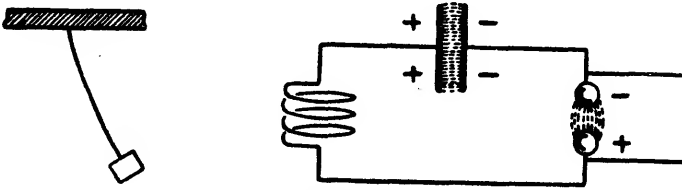


FIG. 183.

At this stage of the operation, the energy in the system is again in the form $\frac{1}{2} CV^2$.

In the mechanical case, the energy was at this stage concentrated once again in the spring, the latter being in position (3) in Fig. 180 (b).

The conductive bridge between the spark plugs is not broken down at this moment, because the P.D. across the condenser immediately sets up an increasing current in the opposite direction.

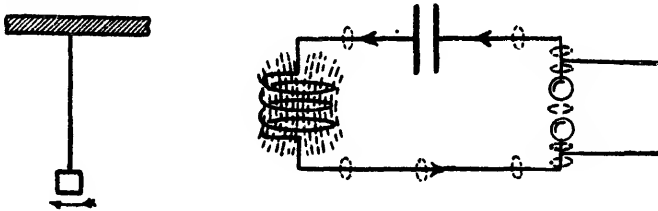


FIG. 184.

(d) As before, after an interval of time, the current rises to a maximum value I and the P.D. across the condenser falls to zero, the whole of the energy being again in the form $\frac{1}{2} LI^2$.

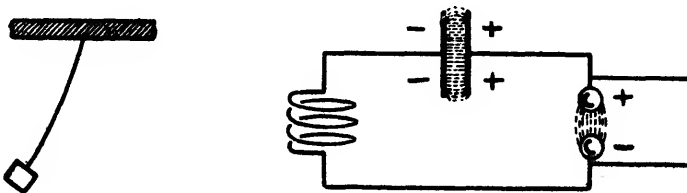


FIG. 185.

(e) Once again the magnetic field round the inductance dies away, and the current continues to flow in a clockwise direction, until it charges up the condenser in the same manner as that in

which it was originally charged. We have thus traced out one complete cycle of the oscillatory action, corresponding to a complete movement of the weight from position 1 back to position 1 in the mechanical analogy.

389. Relative Phases of Current and Voltage.—In the foregoing, nothing has been said about the ohmic and other losses, which are responsible in practice for a steady diminution in the energy stored in the circuit.

If these are neglected for the moment, graphs can be drawn showing the relationship between the values of instantaneous current flowing and P.D. across the condenser at different points in the cycle.

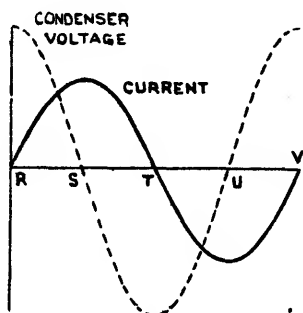


FIG. 186.

The moment "R" in Fig. 186 corresponds with Fig. 181. The condenser is fully charged and no current is flowing out of it. Between "R" and "S" the P.D. across the condenser is falling and the current round the circuit is rising, energy being present during this stage both in the electric and in the magnetic field. At moment "S" (see Fig. 182) the condenser is completely discharged, and the current has risen to its maximum value. Between "S" and "T" the condenser is becoming charged in the opposite direction, and the current into it gradually decreases because the increasing charge in the condenser sets up a stronger back E.M.F.

Moment "T" corresponds to Fig. 183, moment "U" to Fig. 184, moment "V" to Fig. 185, and so on. From an inspection of Fig. 186, we therefore see that in the oscillatory circuit, condenser voltage and current flowing are 90° out of phase. This is only true if resistance is neglected, under which conditions, of course, the voltage and current curves will have the same amplitudes during succeeding cycles as during the first.

If resistance losses are taken into account, the successive amplitudes of both the voltage and the current curves decrease in value, and the oscillatory action is said to be **damped**. The action will go on until the energy left in the circuit is insufficient to maintain the ionisation of the spark gap. The complete series of cycles of

current or voltage variation which occurs during the oscillatory discharge of a condenser is termed a "Train of Waves" or a "Wave Train." The correct representation of the voltage to which the condenser is charged during a wave train is given in Fig. 187, and a similar curve for the current flowing is given in Fig. 188.

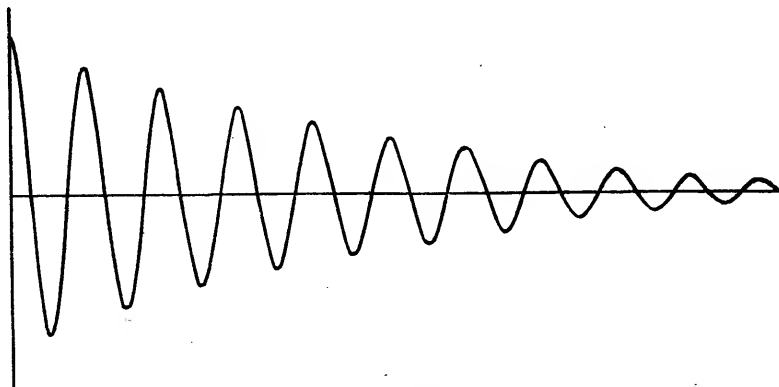


FIG. 187.

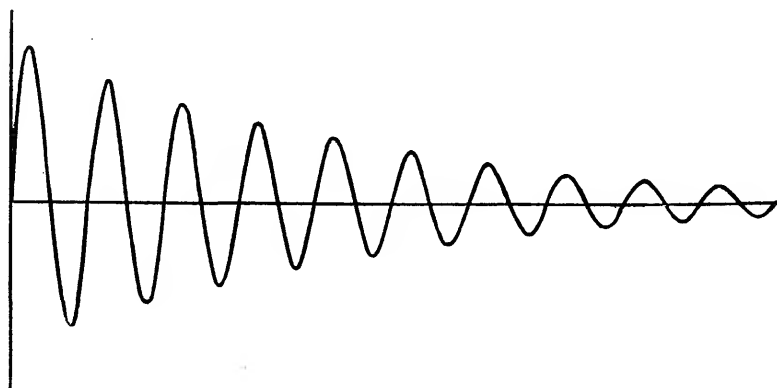


FIG. 188.

It may be remarked that the current and voltage curves in this case are not exactly 90° out of phase with each other.

***390. Mathematical Treatment.**—The complete theory of the wave forms of current and voltage in the oscillatory circuit, and the frequency at which the action occurs, can only be investigated by a mathematical treatment on the lines of that already given in paras. 158 and 174, which deal respectively with the case of resistance and inductance, and resistance and capacity, in the same circuit.

As regards mathematical treatment, the circuit can be taken as consisting of L, C and R, the resistance R representing not only

the ohmic losses in the leads and the conductive spark gap, but also energy radiated, etc. A detailed account of the various damping losses is given later.

We shall deal with the case already considered in para. 333, in which the condenser is fully charged and the current zero at time $t = 0$.

Let V be the voltage across the condenser when fully charged.

Q be the charge on the condenser at voltage V .

v be the voltage across the condenser at time t .

q be the charge on the condenser at time t .

i be the instantaneous value of current at time t after the condenser begins to discharge.

Then we have the following relationships :—

$$q = Cv. \quad Q = CV.$$

Also the current i is the rate at which the charge on the condenser is decreasing, so that

$$i = - \frac{dq}{dt} = - C \frac{dv}{dt}$$

At time t , the voltage v is maintaining a current i through the resistance R and also overcoming the counter E.M.F. of the inductance L .

The equation representing the relationship between voltage and current in the circuit is therefore

$$v - L \frac{di}{dt} = iR$$

$$\text{or } v - L \frac{di}{dt} - iR = 0.$$

Now $i = - C \frac{dv}{dt}$, so that $\frac{di}{dt} = - C \frac{d^2v}{dt^2}$.

$$\therefore v + LC \frac{d^2v}{dt^2} + RC \frac{dv}{dt} = 0.$$

$$\text{or } \frac{d^2v}{dt^2} + \frac{R}{L} \cdot \frac{dv}{dt} + \frac{v}{LC} = 0.$$

A similar differential equation can be derived in terms of i or q , with the same coefficients as in this case.

The solution of a differential equation of this nature is known to be of the form $v = Ae^{\lambda t}$. If we assume this answer, differentiate to find $\frac{dv}{dt}$ and $\frac{d^2v}{dt^2}$, and substitute in the differential equation, the latter becomes

$$Ae^{\lambda t} \left(\lambda^2 + \frac{R}{L} \lambda + \frac{1}{LC} \right) = 0.$$

Equating $\lambda^2 + \frac{R}{L}\lambda + \frac{1}{LC}$ to zero, it is found that there are two roots of the equation, given by $\lambda_1 = -\frac{R}{2L} + \sqrt{\frac{R^2}{4L^2} - \frac{1}{LC}}$, and $\lambda_2 = -\frac{R}{2L} - \sqrt{\frac{R^2}{4L^2} - \frac{1}{LC}}$ respectively.

There are therefore two independent solutions of the form $e^{\lambda t}$, and the general solution is obtained by multiplying them respectively by different constants, and adding the results.

Hence the complete solution of the differential equation is given by $v = Ae^{\lambda_1 t} + Be^{\lambda_2 t}$.

Before discussing the significance of the arbitrary constants, the values of λ_1 and λ_2 will be inspected. These values may be real and unequal, real and equal, or imaginary and unequal, according as the quantity under the square root $\left(\frac{R^2}{4L^2} - \frac{1}{LC}\right)$, is positive, zero or negative. *i.e.*, according as R is greater than, equal to, or less than $2\sqrt{\frac{L}{C}}$.

The first two cases are examples of non-oscillatory, or **unidirectional discharge**, in which the resistance is so great that the current, starting at zero, rises to a maximum and dies away again to zero without reversing its direction; in other words, the resistance losses are so high that all the energy in the circuit is wasted in these before the first half cycle is completed. The current graph is similar to that in para. 174, where no inductance was present in the circuit. The other case, $R < 2\sqrt{\frac{L}{C}}$, gives the case of oscillatory discharge, in which we are interested.

The two arbitrary constants, A and B , can be determined in any given case by considering the initial conditions at $t = 0$. In our case these are—

- (1) that $i = 0$ at $t = 0$.
- (2) that $v = V = \frac{Q}{C}$ at $t = 0$.

The complete solution will not be worked out, but the result will be quoted.

It is found that, with these initial conditions, the value v of the condenser voltage at any time t is given by

$$v = Ve^{-\alpha t} \sqrt{1 + \frac{\alpha^2}{\omega^2}} \cos(\omega t - \phi) \quad (A).$$

where $\phi = \tan^{-1}\left(\frac{\alpha}{\omega}\right)$.

" α " and " ω " are recognised symbols for the quantities $\frac{R}{2L}$ and $\sqrt{\frac{1}{LC} - \frac{R^2}{4L^2}}$ respectively.

The value i of current at time t is $-C \frac{dv}{dt}$, and so by differentiation of the expression for v above,

$$i = \frac{V}{\omega L} e^{-\alpha t} \sin \omega t \quad (\text{B}).$$

The last equation (B) represents an oscillatory current whose frequency is $\frac{\omega}{2\pi}$ and whose amplitude is $\frac{V}{\omega L} e^{-\alpha t}$ and therefore decreases as the time increases; in other words, the oscillations are damped.

The equation (A) represents the instantaneous value of the voltage across the condenser. It is periodic, with the same frequency as the current, and its amplitude $V e^{-\alpha t} \sqrt{\frac{1+\alpha^2}{\omega^2}}$ also decreases with time.

The frequency $\frac{\omega}{2\pi}$ at which the oscillatory action takes place is given (written in full) by $f = \frac{1}{2\pi} \sqrt{\frac{1}{LC} - \frac{R^2}{4L^2}}$ and is known as the **Natural Frequency** of the circuit. This frequency, and various results arising from it, will be discussed in the next few paragraphs, and after that the question of damping.

391. Natural Frequency.—This frequency $f_N = \frac{1}{2\pi} \sqrt{\frac{1}{LC} - \frac{R^2}{4L^2}}$, called the **Natural Frequency**, must be carefully distinguished from the frequency $f_R = \frac{1}{2\pi} \sqrt{\frac{1}{LC}}$, which was found in Chapter V to be the **resonant frequency** of a circuit comprising inductance capacity and resistance in series.

The natural frequency is the frequency of **free** oscillations, *i.e.*, the frequency at which the circuit will oscillate if left to do so; while the resonant frequency is the frequency at which **an applied voltage** will give **the biggest current**, *i.e.*, at which **forced** oscillations have a maximum amplitude.

The theoretical grounds from which these formulæ are derived are entirely different; but it so happens that they give practically the same result, and the simple formula $f = \frac{1}{2\pi} \sqrt{\frac{1}{LC}}$ is generally used for the natural frequency. This is quite legitimate as long as it is understood that it is a close approximation to the truth, and arises from different considerations from those dealt with under resonance.

If the resonant frequency is denoted by f_R and the natural frequency by f_N , the exact relationship between them is given by

$$(2\pi f_N)^2 = (2\pi f_R)^2 - \alpha^2$$

and so the natural frequency is always smaller than the resonant frequency.

Even if R is small enough to justify the approximations above, it must be remembered that in a circuit at its **resonant** frequency, the current is $\frac{V}{R}$, where V is the applied voltage, and that the resistance, however small, is the only quantity limiting the size of the current under resonant conditions.

In the usual wireless circuits, the resistance R is so small that the term $\frac{R^2}{4L^2}$ is negligible compared with $\frac{1}{LC}$; that is, R is very much less than $2\sqrt{\frac{L}{C}}$, and so the assumption that Natural Frequency

$$(f_N) = \frac{1}{2\pi\sqrt{LC}} \text{ is justified.}$$

In this formula L is measured in **henries**, and C in **farads**.

If L is in **mics**, and C in **jars**, the formula becomes

$$\begin{aligned} f &= \frac{3 \times 10^7}{2\pi\sqrt{LC}} \text{ cycles per second.} \\ &= \frac{3 \times 10^4}{2\pi\sqrt{LC}} \text{ kilocycles per second, or kc/s.} \end{aligned}$$

Example 49.

Find the correct natural frequency of free oscillations in a circuit in which $C = 50$ jars, $L = 2$ mics, and R is 1 ohm. Also find the approximation to the natural frequency, using the formula for resonant frequency, and the minimum value of R which would make the discharge of the condenser unidirectional.

$$\begin{aligned} \text{Natural frequency} &= \frac{1}{2\pi} \sqrt{\frac{1}{LC} - \frac{R^2}{4L^2}} \\ &= \frac{1}{2\pi} \sqrt{\frac{1}{\frac{2}{10^6} \times \frac{50}{9 \times 10^8}} - \frac{1}{4 \times \frac{2}{10^6} \times \frac{2}{10^6}}} \\ &= \frac{1}{2\pi} \sqrt{\frac{9 \times 10^{14}}{100} - \frac{10^{12}}{16}} \\ &= \frac{1}{2\pi} \sqrt{9 \times 10^{12} - \frac{10^{12}}{16}} = \frac{10^6}{2\pi} \sqrt{9 - \frac{1}{16}} \\ &= \frac{10^6}{2\pi} \sqrt{8.9375} = \frac{10^6 \times 2.9895}{2\pi} \\ &= 475,700 \text{ cycles/sec.} \\ &= 475.7 \text{ kc/s.} \end{aligned}$$

$$\text{Resonant frequency} = \frac{1}{2\pi} \sqrt{\frac{1}{LC}}$$

$$\begin{aligned} \text{or, using mics and jars,} &= \frac{3 \times 10^7}{2\pi\sqrt{LC}} = \frac{3 \times 10^7}{2\pi\sqrt{100}} = \frac{3 \times 10^6}{2\pi} \\ &= 477,400 \text{ cycles per second} = 477.4 \text{ kc./s.} \end{aligned}$$

These results are very nearly equal.

To make the discharge non-oscillatory, R must be at least equal to $2\sqrt{\frac{L}{C}}$.

$$\begin{aligned} R &= 2\sqrt{\frac{L}{C}} = 2\sqrt{\frac{\frac{2}{10^6}}{\frac{50}{9 \times 10^8}}} = 2\sqrt{\frac{900 \times 2}{50}} \\ &= 2\sqrt{36} = 12 \Omega \end{aligned}$$

so that with the resistance equal to 12 ohms or more the circuit would not oscillate freely. The term "aperiodic" is sometimes used to denote the type of discharge in this case.

392. Oscillation Constant.—The quantity LC is called the **oscillation constant** of the circuit. Provided this product remains the same, no matter what changes are made in the individual values of L and C , the resonant frequency, and, to all intents and purposes, the natural frequency, remain constant.

393. Period.—If the circuit oscillates at the rate of f cycles per second, then one cycle will last for a period of $1/f$ of a second.

$$\text{Natural period} = \frac{1}{\frac{1}{2\pi\sqrt{LC}}} = 2\pi\sqrt{LC} \quad \begin{array}{l} \text{seconds, where } L \\ \text{is in henries and} \\ C \text{ in farads.} \end{array}$$

$$\text{Natural period} = \frac{1}{\frac{3 \times 10^7}{2\pi\sqrt{LC}}} = \frac{2\pi\sqrt{LC}}{3 \times 10^7} \quad \begin{array}{l} \text{seconds, where } L \\ \text{is in mics and } C \\ \text{in jars.} \end{array}$$

Example 50.

Find the natural period of the circuit given in Example 49.

$$\begin{aligned} \text{Period} &= \frac{2\pi\sqrt{100}}{3 \times 10^7} = \frac{2\pi}{3 \times 10^6} = 2.1 \times 10^{-6} \text{ seconds.} \\ &= 2.1 \text{ micro-seconds.} \end{aligned}$$

394. Wavelength.—So far we have only discussed high-frequency oscillatory currents in a circuit. If the circuit in which these currents are flowing is an efficient radiator (para. 408), energy is sent out in the form of electro-magnetic waves. The wave form, frequency, and damping of the electro-magnetic wave are the same as those of the oscillatory current which produces it. It is a well-established fact that all electro-magnetic waves (light being a well-known example) travel at the same rate in empty space, which is very nearly equal to 3×10^8 metres per second. (See para. 13.)

The relation between frequency f and wavelength λ has been given in Chapter I.

If v is the speed of travel of a wave,

$$v = \lambda \times f.$$

In the case of electro-magnetic radiation in free space, therefore, λ (in metres) $= \frac{3 \times 10^8}{f}$

$$= 3 \times 10^8 \div \frac{1}{2\pi\sqrt{LC}} = 6\pi\sqrt{LC} \times 10^8 \text{ metres,}$$

where L is in henries and C in farads.

These units are too large for wireless circuits, so the formula is usually given in mics and jars.

$$\lambda \text{ in metres} = 3 \times 10^8 \div \frac{3 \times 10^7}{2\pi\sqrt{LC}} = 20\pi\sqrt{LC} \quad \begin{array}{l} \lambda \text{ in metres.} \\ L \text{ in mics.} \\ C \text{ in jars.} \end{array}$$

$$= 62.8\sqrt{LC}$$

(In units of micro-henries and micro-farads

$$\lambda = 600 \pi\sqrt{LC} = 1,885\sqrt{LC}.)$$

Three methods may thus be used to describe a wireless wave :—

- (a) Its **oscillation constant**, or **LC value**. This is inconvenient for commercial or inter-Service use, on account of the various units used for capacity.
- (b) Its **wavelength**, as determined above.
- (c) Its **frequency**, in cycles per second, or in **kilocycles per second**, written **kc/s**. (See para. 15.)

Method (c) is now the **standard** method of referring to wireless waves in the Service and in commercial practice ; it is convenient for determining readily the allocation of waves for different uses so that they will not interfere with one another. Further, the frequency, unlike the wavelength, is unaffected by the medium in which the wave is propagated.

395. Inter-relationship of Formulæ.—The formulæ given in paras. 391 and 394 can be arranged in several ways :—

(a) To find frequency from LC value :—

$$f = \frac{3 \times 10^4}{2\pi\sqrt{LC}}$$

(b) To find frequency from wavelength :—

$$f = \frac{3 \times 10^5}{\lambda}$$

(c) To find wavelength from LC value :—

$$\lambda = 62.8\sqrt{LC}.$$

(d) To find wavelength from frequency :—

$$\lambda = \frac{3 \times 10^5}{f}$$

(e) To find LC value from frequency :—

$$LC = \left(\frac{3 \times 10^4}{2\pi f} \right)^2$$

(f) To find LC value from wavelength :—

$$LC = \left(\frac{\lambda}{62.8} \right)^2$$

f in kc/s.
 λ in metres
 L in mics
 C in jars

396. Damping, Persistency, Decrement.—It has been shown in the preceding paragraphs that the resistance in an oscillatory circuit, if small, has a negligible effect on the natural frequency of free oscillation, which may be considered to be determined by the values of inductance and capacity present in the circuit.

We now proceed to investigate the meaning of the term e^{-at} , which occurs in the expressions both for voltage across the condenser and for current flowing in the circuit, in the results given in para. 390.

If we write I for $\frac{V}{\omega L}$ the expression for current becomes

$$i = Ie^{-at} \sin \omega t.$$

If this is drawn, with a horizontal axis representing time, the resultant graph is of the form shown below.

Instead of being a sine curve, with constant amplitude, which would be represented by $i = I \sin \omega t$, the amplitude is seen to decrease continually, because of the term e^{-at} , which becomes less and less with increase of time. Such a curve is known as a "damped sine wave." As t increases, the amplitude ultimately becomes very small, and theoretically, with t becoming infinite, the amplitude finally reaches zero, showing that the energy present in the circuit has been completely wasted in resistance losses. The greater the damping losses, the more rapidly does the amplitude diminish.

The amplitudes of the successive waves shown in Fig. 189 are the heights AB, CD, EF, &c.

It will be proved that the ratio of consecutive maxima in the same direction is a constant, so that if CD is 80 per cent. or 0·8 of AB, then EF is 80 per cent. or 0·8 of CD, and so on. The amplitudes thus decrease in "geometrical progression." The curved line

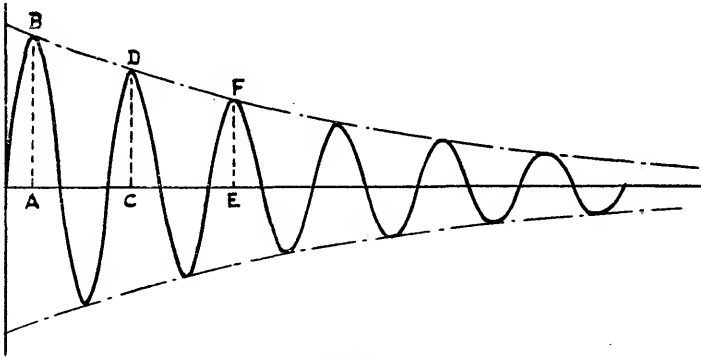


FIG. 189.

BDF drawn through the successive amplitudes in the same direction is a logarithmic curve. The constant ratio just referred to can be derived from the constants of the circuit as follows :—

We shall define "**Persistency**" to be the ratio of one peak value to the peak value immediately preceding it in the same direction ; and its reciprocal, "**Decrement**," to be the ratio of one peak value to that immediately succeeding it in the same direction.

The quantity " α " = $\frac{R}{2L}$ (see para. 390) is called the **Damping Factor**.

By differentiation of the expression for current, $i = I e^{-\alpha t} \sin \omega t$, it is easy to show that the current first attains a maximum value after a time given by t_1 , such that $\tan \omega t_1 = \frac{\omega}{\alpha}$ or $\omega t_1 = \left(\frac{\pi}{2} - \phi\right)$.

The actual time t_1 is immaterial from the point of view of decrement ; what is important is that successive maxima in the same direction occur one complete period, two complete periods, &c., after t_1 .

$$\text{Now a period} = \frac{1}{f} = \frac{2\pi}{\omega}.$$

$$\text{Therefore decrement} = \frac{e^{-\alpha t_1}}{e^{-\alpha(t_1 + \frac{2\pi}{\omega})}} = \frac{e^{-\alpha(t_1 + \frac{2\pi}{\omega})}}{e^{-\alpha(t_1 + \frac{4\pi}{\omega})}}, \text{ and so on.}$$

Each of these expressions equals $e^{+\frac{2\pi\alpha}{\omega}}$, which is independent of the place in the wave train where the ratio of one peak value to the succeeding one is taken.

The decrement is thus constant throughout the wave train, and is given by $e^{\frac{2\pi\alpha}{\omega}}$.

The persistency is, of course, $e^{-\frac{2\pi\alpha}{\omega}}$.

397. Logarithmic Decrement.—Instead of using the persistency, or the decrement, to measure the rate at which the amplitude decreases, the Naperian Logarithm of the decrement is generally used. This is simply the logarithm of the decrement to the base e (not base 10), and is, from the formula above, equal to $\frac{2\pi\alpha}{\omega}$.

Note.—Numerically the Naperian Logarithm of a quantity is derived from the Common Logarithm (to the base 10) by multiplying the Common Log by 2.3026.

The expression Logarithmic Decrement is generally referred to by the contraction "**Log. Dec.,**" and written δ (delta).

$$\text{Log. Dec.} = \frac{2\pi\alpha}{\omega} = \frac{\alpha}{f}.$$

$$\text{Now } \alpha = \frac{R}{2L} \text{ and } \omega = \sqrt{\frac{1}{LC} - \frac{R^2}{4L^2}},$$

which, if R is small, is very nearly $\frac{1}{\sqrt{LC}}$

$$\therefore \text{Log. Dec. } \delta = \frac{2\pi\alpha}{\omega} = \frac{\pi R}{\omega L} = \frac{R}{2fL} = \pi R \omega C = \pi R \sqrt{\frac{C}{L}}.$$

All these equivalent forms can be used to give the log. dec. of a circuit, according to the constants which are known.

R , L and C are, of course, in ohms, henries and farads respectively.

If L is in mics and C in jars,

$$\begin{aligned} \delta &= \pi R \sqrt{\frac{\frac{C}{9 \times 10^8}}{\frac{L}{10^6}}} = \pi R \sqrt{\frac{C}{900L}} \\ &= \frac{\pi R}{30} \sqrt{\frac{C}{L}} = \frac{R}{9.6} \sqrt{\frac{C}{L}} \end{aligned}$$

Similar formulæ in terms of mics and jars can be obtained for all the equivalent forms above.

398. Number of Oscillations in a Wave-train.—Although, in theory, the oscillations take an infinite time to reach zero value, they actually become of negligible size after a certain time.

It is customary to consider that the wave train is effectively ended when the amplitude of current has fallen to 1 per cent. of its initial value. This is, of course, quite an arbitrary value to choose.

Let the number of effective oscillations, on this assumption, be N .

Let the amplitude of the N^{th} oscillation be written I_N .

$$\text{Then } I_N = I_1 e^{-(N-1)\delta}$$

$$\text{or } \frac{I_1}{I_N} = e^{(N-1)\delta}.$$

But we are taking $\frac{I_1}{I_N}$ to be equal to 100.

$$\therefore 100 = e^{(N-1)\delta}$$

$$\log_e 100 = (N-1)\delta$$

$$N-1 = \frac{\log_e 100}{\delta}, \text{ or } N = 1 + \frac{\log_e 100}{\delta}.$$

The logarithm of 100 to the base e is 4.605.

$$\therefore N = 1 + \frac{4.605}{\delta}.$$

Thus, if the log. dec. of a circuit is 0.1, there will be $1 + \frac{4.605}{0.1} = 1 + 46 = 47$ oscillations, before the current amplitude is reduced to 1 per cent. of its initial value.

399. Duration of a Wave-train.—As we have now derived formulæ for the period of free oscillation in an oscillatory circuit, and for the number of oscillations which may be considered effective, a practical example can be worked out to give the duration of a wave-train.

Example 51.

Let R be 1 ohm, $C = 5$ jars, $L = 5$ mics.

$$\text{Period} = \frac{2\pi\sqrt{LC}}{3 \times 10^7} \text{ seconds (para. 393).}$$

$$= \frac{2\pi\sqrt{25}}{3 \times 10^7} \text{ seconds} = \frac{10\pi}{3 \times 10^7} \text{ seconds.}$$

$$= \frac{\pi}{3} \times 10^{-6} \text{ seconds} = 1.047 \text{ micro-seconds.}$$

$$\text{" } \delta \text{ ", for this circuit,} = \frac{R}{9.6} \sqrt{\frac{C}{L}} = \frac{1}{9.6} \sqrt{\frac{5}{5}} = \frac{1}{9.6} = 0.104.$$

Therefore the number of effective oscillations is

$$1 + \frac{4.6}{0.104} = 1 + 44 = 45,$$

and the duration of the wave-train is

$$(1.047 \times 45) \text{ micro-seconds} = 47 \text{ micro-seconds.} \\ = 0.000047 \text{ second.}$$

400. Damping Losses in the Practical Circuit.—The mathematical investigation in the paragraphs above has been concerned with the

purely theoretical circuit containing resistance, inductance, and capacity, and the damping, or dying away of the oscillations, has been determined in terms of these quantities. The practical circuit we started with at the beginning of the chapter will now be considered, and the various losses, which can all be covered by the term damping losses, investigated in turn. It will be found that ohmic resistance is by no means the only way in which energy is dissipated, but other forms of loss can be considered as adding a certain amount of equivalent resistance to the circuit, so that the theoretical results hold good if the resistance term in them is replaced by a total equivalent, or effective, resistance.

This subject will also be referred to in Chapter XVIII. The damping losses may be classed under the following headings:—

- (a) Damping due to ohmic resistance.
- (b) Damping due to spark gap resistance.
- (c) Damping due to eddy currents induced in neighbouring conductors.
- (d) Damping due to condenser losses.
- (e) Damping due to dielectric losses in surrounding dielectrics.
- (f) Damping due to energy radiated away into the surrounding ether.

401.—Ohmic Resistance.—Since $\delta = \frac{R}{9.6} \sqrt{\frac{C}{L}}$, the greater the ohmic resistance the more quickly will the wave be damped out. It is necessary to take account of the increase in resistance produced in the conductors by “skin effect” at these high frequencies.

As was explained in para. 380, in a cylindrical conductor a high frequency current tends to flow on the outer surface layers and there is practically no current in the centre of the conductor, whereas a direct current takes advantage of the whole cross sectional area of the conductor.

Similarly, in a strip conductor, the current is concentrated on the surface and its density increases towards the two edges.

In consequence, the resistance of any conductor to high frequency currents is much greater than its resistance to low frequency or direct currents. The variation of this effect with frequency, permeability, resistivity and area of cross section of the conductor has already been mentioned.

As a numerical illustration, a copper rod, 0.59 inches in diameter, offers to currents of a frequency of a million cycles per second a resistance which is about 51 times its steady current resistance.

Tables can be constructed to give the maximum size of conductor which can be used at different frequencies so that the H.F. resistance varies by less than some definite amount (say, 1 per cent.) from the D.C. resistance.

In all high frequency circuits it is of the utmost importance to provide conductors having the greatest possible area for the high

frequency currents to flow on, if the damping of the circuit is not to be excessive.

402. Spark Gap Resistance.—The resistance of the spark gap, once the latter is broken down and the oscillatory action is taking place, is still a fairly large one compared with the other resistances in the circuit, and so exercises a considerable effect on the damping. In addition, the theory is complicated by the fact that the resistance of the spark gap is not a constant quantity, but increases as the current decreases. The effect of this is to cause the decreases in successive amplitudes to be constant, instead of being in geometrical progression, so that if the spark gap resistance is the preponderating factor in the total resistance of an oscillatory circuit, the line joining the peak values in the resulting wave-train will be straight, instead of curved as in Fig. 189, and the wave-train will have a definite ending in a finite time.

403. Eddy Current Losses.—There will be currents induced in neighbouring circuits by mutual induction, and these cause a loss of energy from the circuit and hence an increased damping. The practical case of an oscillatory aerial circuit will be further considered in Chapter XVIII.

404. Condenser Losses and Losses in Neighbouring Dielectrics.—The various losses in a condenser have been already considered in the section on condensers, and it has also been shown that, whatever the causes of loss of energy, a condenser may be looked upon as a perfect condenser with a resistance in series with it (paras. 176 and 310). Application of this theory to an aerial circuit will be made in Chapter XVIII.

405. Energy Radiated.—In an oscillatory circuit such as we have been considering, a certain amount of energy is radiated away into the surrounding aether, in the form of electro-magnetic waves, which affect receiving apparatus at a distance. This also is a damping loss, but one that, with certain limitations, we want to be as great as possible.

The amount of energy radiated in this way is dependent upon the design of the circuit, and the subject will be considered further in the next few paragraphs which deal with the difference between closed and open oscillators.

These radiation losses can also be represented by an equivalent series resistance in the circuit. This resistance is known as the **Radiation Resistance** of the oscillatory circuit.

406. Effective Resistance of an Oscillatory Circuit.—All these damping losses result in energy being taken from the oscillatory circuit, and so can be represented by an equivalent total resistance included in the circuit, such that the energy lost in this equivalent resistance in I^2R losses would be equal to the total

damping losses. This equivalent total resistance is known as the **effective resistance** of the oscillatory circuit. If the effective resistance of the circuit is R ohms, the formulæ already found can be used to give the natural frequency of oscillation, the logarithmic decrement, the number of cycles in a wave-train, &c.

For example, we saw in para. 399 that, if the log. dec. of a circuit is 0.1, the wave will be reduced to less than 1 per cent. of its initial amplitude after 45 cycles.

If the inductance is 100 mics and the capacity is 1 jar, then this damping could have been caused by an effective resistance of R' ohms, determined as follows:—

$$\delta = \frac{R'}{9.6} \sqrt{\frac{C}{L}}.$$

$$\begin{aligned} \text{Therefore } R' &= 9.6 \delta \sqrt{\frac{L}{C}} = (9.6) (0.1) \sqrt{\frac{100}{1}} \\ &= 9.6 \text{ ohms.} \end{aligned}$$

In this case the effective high frequency resistance of the circuit would be said to be 9.6 ohms. The ohmic resistance present in the circuit might be considerably less.

407. The Closed Oscillator.—The oscillatory circuit so far described is known as a “closed oscillator.” The inductance and the capacity used have both been assumed to be artificial and compact in size. The condenser is of large capacity value, so that it is not necessary to charge it up to an excessive voltage V to isolate in it a given quantity of energy ($\frac{1}{2} CV^2$). The inductance is of the value of only a few micro-henries and is designed to have as small a resistance as possible to the high frequency currents passing through it.

With such a circuit two fields, one electric, and one magnetic, are set up. Both these fields are of an oscillatory nature, passing through cycles of positive and negative values, and are practically 90° out of phase with one another.

The **Magnetic Field** surrounds the wires and, more particularly, passes through the turns of the inductance. There is also a magnetic field between the plates of the condenser, due to the alternating displacement current in the dielectric. Theoretically, the magnetic field extends to an infinite distance; practically, however, it falls off to a very small value at a distance of a few feet from the oscillator. The **Electric Field** is established between the plates of the condenser. Theoretically, again, this field extends to an infinite distance; but, practically, if the plates are close together, it falls to a negligible value at a distance of a few inches from the edges of the plates.

It thus follows that the only space near the closed oscillator which is called upon to bear both a magnetic and an electric field,

is the space between the plates of the condenser, and, as will be seen in the next paragraph, this produces inefficiency in the radiation of electro-magnetic energy.

408. The Open Oscillator.—Let us gradually separate the plates of the condenser. Then, as the plates separate, a greater volume of the space near the oscillatory circuit is carrying both the fields.

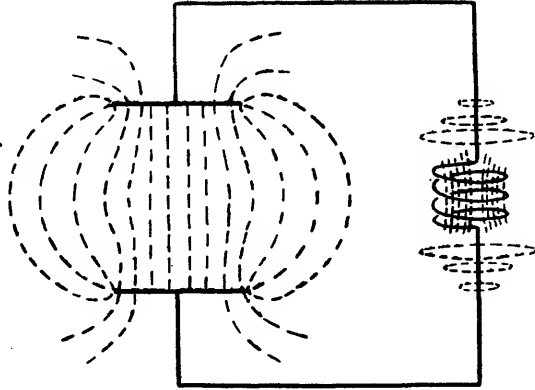


FIG. 190.

It can be shown mathematically that a varying magnetic field and a varying electric field at right angles to each other in the same space give rise to an electro-magnetic wave which is travelling at right angles to both; this electro-magnetic wave represents energy detached from the circuit, and it moves away from the source of its propagation with a velocity of 3×10^8 metres per second. The electro-magnetic wave itself consists of alternating electric and magnetic fields, which remain **at right angles to each other in space, but are in time phase**. Equal amounts of energy are stored in the electric and the magnetic components of the electro-magnetic wave. We can now bring together the ideas of induction and radiation and state that the total electric and magnetic fields with an open oscillator can be split up into two sets of components:—

(a) **Inductive.**—The component electric and magnetic fields are in space quadrature (at right angles to each other) and in time quadrature (90° out of phase with each other) and the effect is simply to cause a transference of energy from one to the other in the neighbourhood of the circuit without any energy being detached. The inductive field at any point varies inversely as the square of the distance from the oscillator to the point.

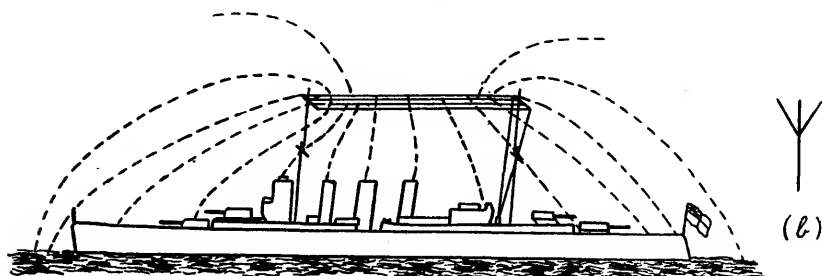
(b) **Radiative.**—The electric and magnetic fields of radiation are in space quadrature and time phase, and therefore represent a motion of energy at right angles to both in space. This radiated energy is, of course, equivalent to the energy which would be

dissipated in the fictitious equivalent radiation resistance. The greater the space occupied by both fields the more energy is radiated. A circuit specially designed to give good radiation, like the circuit of this paragraph, in which the condenser plates are widely separated, is known as an **open oscillator**. The stronger the total electric and magnetic fields the greater the energy radiated, hence high voltages and large currents are necessary to give good radiation. The radiation field varies inversely as the first power of the distance from the oscillator. It thus decreases much more slowly with increasing distance than the induction field, and, at the distances over which wireless communication is practised, only the radiation field is effective. Its magnitude is also directly proportional to the frequency, hence the use of radio-frequencies in wireless telegraphy.

A fuller account of wireless radiation is given in Chapter XVIII.

409. The Aerial Circuit.—The first type of open oscillator consisted of two metal plates joined by a wire, the inductance being in the wire and the capacity that between the plates.

It was found later that the earth could be used as one of the plates of the condenser, and the higher the other plate the more efficiently did the oscillator radiate. The spark gap was in series with the circuit.



(a)

Ship Aerial.

FIG. 191.

The normal transmitting antenna is of this type, though the use of the spark gap in the aerial circuit itself is now prohibited. In the well-known "Roof" type of aerial, as fitted in ships and ashore, the overhead wires form one plate of the condenser, and the earth itself forms the other plate. The dielectric consists of the intervening layer of air.

An aerial is conventionally represented as in Fig. 191 (b)

410. Natural Capacity of an Aerial.—The higher we make the overhead, or "roof" part of an aerial, the better it radiates energy.

Since the capacity of a condenser varies inversely with the thickness of the dielectric—in this case the distance between the aerial and earth—the capacity of the aerial will be small.

Again, the capacity varies directly as the area of dielectric charged, that is, of the opposed plates.

The earth is big, but the overhead portion of an aerial is very limited, especially in a ship. Consequently, this factor also goes to keep the condenser capacity small. In fact, the capacity of the aerial feeder wire to the trunk along which it is led from the transmitter and which is in parallel with the aerial capacity proper, forms an appreciable part of the total capacity.

We therefore expect the capacity of a ship aerial to be small, and in practice it varies from about 0.3 jar in a submarine to about 2 jars in a big ship. (Shore station aerals may have larger capacities than these.)

The natural capacity of an aerial is denoted in the Service by the letter " σ " (sigma) to distinguish it from other capacities.

411. Natural Inductance of an Aerial.—An aerial will likewise have a certain natural inductance, made up of the inductance of the wires composing the "roof," and of the "feeders" (or wires from the W/T Office to the roof).

As these wires are in parallel with one another, their total inductance will not be very great, and generally lies between 10 and 70 mics. for ship aerals.

412. Primary and Aerial Circuits.—The oscillation is generated in a closed circuit, generally termed the primary circuit. To give

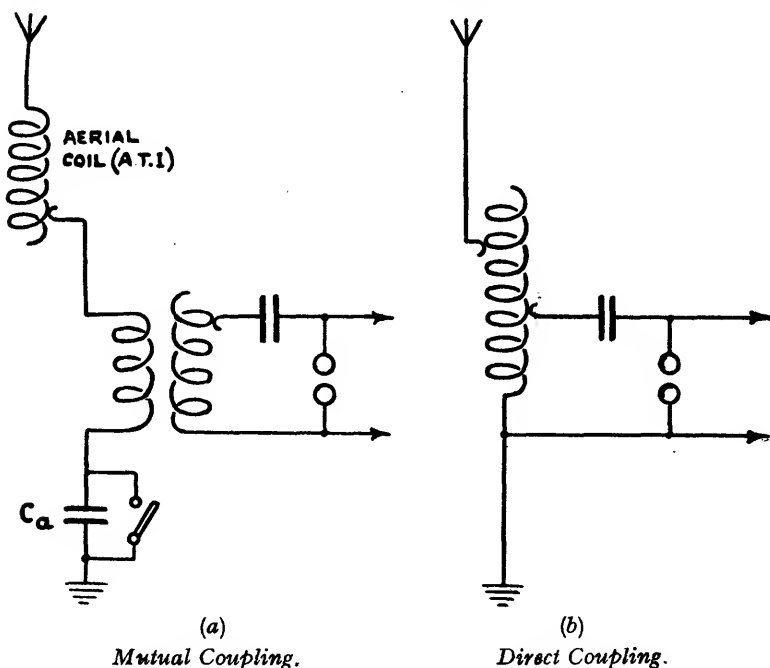


FIG. 192.

efficient radiation, we must transfer the oscillation to an open oscillator, in practice the aerial circuit.

The theory of the interaction of coupled circuits, when one of the circuits is set freely oscillating, will be given in para. 421.

The type of coupling employed is either mutual or direct as illustrated in Fig. 192 (a) and (b). The direct coupling in this case is auto-inductive.

413. Mutual Coupling (Fig. 192 (a)).—Here a coil of wire (known as the "Mutual Coil") in series with the aerial, is placed close to the primary inductance.

As the lines of magnetic flux rise and fall round the primary inductance, they also cut the mutual coil and induce an oscillatory E.M.F. across it, which will tend to set up an oscillatory current in the aerial.

414. Direct Coupling (Fig. 192 (b)).—Here a small portion of inductance is common to both circuits, with the same result—that an oscillatory E.M.F. is produced across a portion of the aerial circuit by the oscillatory discharge of the primary condenser. It should be noted that the circuit is that of the auto-transformer.

415. Tuning of Primary and Aerial Circuits.—We have already seen that the primary circuit has a certain natural frequency of its own, depending on its capacity and inductance. Similarly, the aerial circuit will have a certain natural frequency of its own at which it will oscillate.

This natural frequency will be equal to $\frac{3 \times 10^7}{2\pi\sqrt{LC}}$, where LC is the natural inductance of the aerial, L_{aa} , multiplied by its natural capacity, σ (sigma).

Note.—The frequency whose LC value is the same as the $L_{aa} \sigma$ value of the aerial is termed the "**Fundamental Frequency.**"

To radiate a wave of given frequency and, therefore, known LC value, we must first adjust the LC value of the primary oscillatory circuit to the correct value; the next step is to adjust the LC value of the aerial circuit to the same value. If the LC value required is different from that corresponding to the fundamental frequency of the aerial circuit, the LC value of the aerial circuit must be increased or decreased.

In practice, the LC value of an aerial is generally increased by adding inductance in series, and decreased by adding capacity in series, as in Fig. 192 (a).

416. Increasing the Wave frequency.—To increase the frequency of the aerial circuit, the LC value must be decreased. The inductance which is always in the circuit is the natural inductance of the aerial itself (L_{aa}) and that of the mutual coil. The total

inductance could only be decreased by adding inductance in parallel, and it is easier to decrease the capacity, which is done by adding capacity in series. A condenser C_s (see Fig. 192 (a)) is therefore placed in the aerial circuit; if not required, it is short-circuited by the switch shown.

Example 52.

An aerial circuit has the following constants :—

$$L_{ae} = 30 \text{ mics, } L_m = 10 \text{ mics, } \sigma = 1.2 \text{ jars.}$$

It is required to tune the circuit to a frequency of 1,200 kc./s. The LC value corresponding to the frequency given is, (para. 395),

$$\begin{aligned} LC &= \left(\frac{3 \times 10^4}{2\pi f} \right)^2 \\ &= \left(\frac{3 \times 10^4}{2\pi \times 1,200} \right)^2 = \left(\frac{300}{24\pi} \right)^2 = \left(\frac{25}{2\pi} \right)^2 \\ &= \frac{625}{4 \times 9.8696} = 15.8 \text{ mic-jars.} \end{aligned}$$

$L_{ae} \sigma = (40 \times 1.2) = 48 \text{ mic-jars}$, which is too large.

The capacity must, therefore, be reduced by adding a condenser in series with σ .

As a rule, the aerial condenser is not adjustable. The total capacity value is, at most, $\frac{15.8}{40} = 0.395 \text{ jars}$.

Let us try a condenser of 0.5 jars in series. The resultant capacity of this, in series with the aerial capacity, is given by :—

$$C = \frac{\sigma \times C_s}{\sigma + C_s} = \frac{(0.5) \times (1.2)}{1.7} = \frac{0.6}{1.7} = 0.353 \text{ jar.}$$

The total capacity has thus been reduced a little too much, and some inductance must be added to get exactly in tune.

The coil marked "Aerial Coil" in Fig. 192 can be used for adding inductance in series. It is also known as the "Aerial Tuning Inductance."

With a capacity of 0.353 jar, the total inductance required is $\frac{LC}{C} = \frac{15.8}{0.353} = 44.7 \text{ mics}$.

There is already 40 mics in the circuit. Therefore, by adding 4.7 mics on the aerial coil, the aerial is finally tuned exactly to 1,200 kc./s.

417. Decreasing the Wave Frequency.—In this case the LC value must be increased, and the inductance is therefore increased by adding inductance in the aerial coil.

Example 53.

With the same circuit constants as before (Example 52), it is required to tune the aerial to a frequency of 300 kc./s. The required

$$\begin{aligned} \text{LC value} &= \left(\frac{3 \times 10^4}{2\pi \times 300} \right)^2 = \left(\frac{50}{\pi} \right)^2 \\ &= \frac{2500}{9.8696} = 253.3 \text{ mic-jars.} \end{aligned}$$

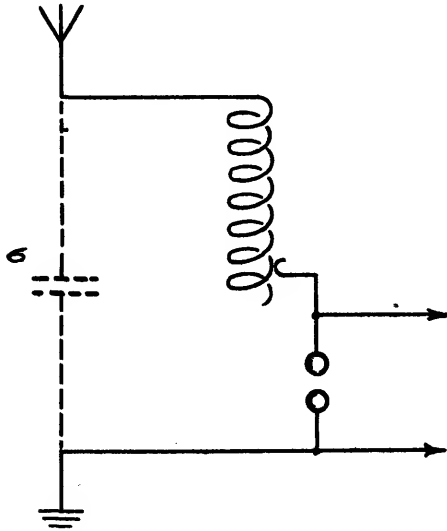
$$\text{Total inductance required} = \frac{\text{LC}}{\sigma} = \frac{253.3}{1.2} = 211 \text{ mics.}$$

Inductance already in circuit = 40 mics.

Therefore, the inductance required in the aerial coil is $(211 - 40) = 171$ mics, to tune the circuit exactly to 300 kc./s.

To sum up.—In order to transmit a wave of a given frequency, we must first adjust the primary circuit to the LC value which gives this frequency, and then tune the aerial circuit to the same LC value as that to which the primary is tuned. The actual method of making this tuning adjustment is given in Chapter XX.

418. The Plain Aerial System.—As already mentioned, the coupled circuit system was not used when wireless telegraphy by the



Plain Aerial.

FIG. 193.

spark system was first developed. The spark gap was joined directly in series with the aerial, with the source of high voltage supply across it, as shown in Fig. 193.

The capacity intermittently charged up to discharging voltage was thus the capacity of the aerial itself.

The result of this arrangement was a very heavily damped wave, because of the high resistance of the spark gap. The wave-train was of the nature shown in Fig. 194, *i.e.*, one which started with a big initial voltage, but which was damped out very quickly after only a few oscillations.

A wave of this nature was very objectionable, because it interfered considerably with all receiving aerials in the neighbourhood, whether tuned to it or not and—as will be understood after reading Chapter X—made selective working in a fleet impossible.

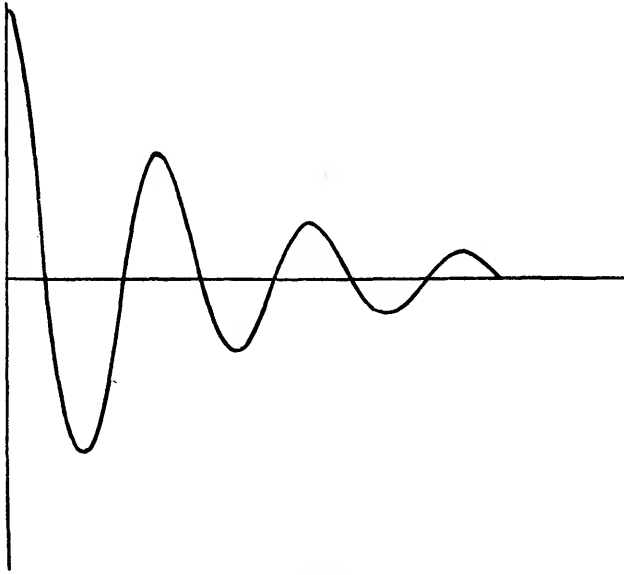


FIG. 194.

A second objection to the system is that, since the charging mains are directly connected to the aerial, a very bad shock would be taken from the latter by anyone touching it, and further, unless the insulation of the aerial is good, no spark can be obtained.

This system has long been prohibited by International Convention because of the unavoidable interference associated with it. The coupled circuit system has replaced it. In this system, the decrement is less, because the spark gap resistance is not present in the aerial circuit. The frequency transmitted is, however, dependent on the coupling factor between the two circuits, and we shall proceed in para. 421 to consider the physical and mathematical treatment of free oscillations in coupled circuits.

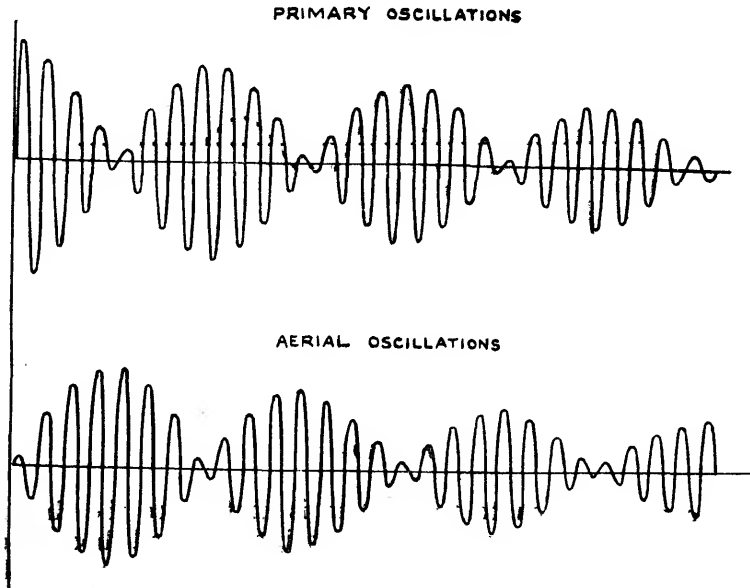
419. Tight and Loose Coupling.—It was stated in para. 413 that the oscillation set up in the primary circuit is transferred to the aerial circuit by mutual induction between the primary and mutual coils.

Since our object is to induce the maximum energy possible into the aerial, it might appear at first sight that the closer the mutual coil is to the primary the better.

This is not the case, for the following reasons :—

Consider the oscillation in the primary circuit ; each time the magnetic field rises and falls round the primary inductance a certain amount of energy is transferred to the aerial via the mutual coil, until eventually all the energy originally available in the primary circuit has been transferred to the aerial circuit.

Similarly, when the aerial circuit is oscillating, it will gradually transfer its energy back to the primary, until eventually, all the energy has been handed back to the primary circuit, ignoring I^2R losses and the amount radiated.



Tight Coupling.

FIG. 195.

This transfer and re-transfer of energy between primary and aerial continues until all the energy originally available has been expended in overcoming the various losses in the two circuits.

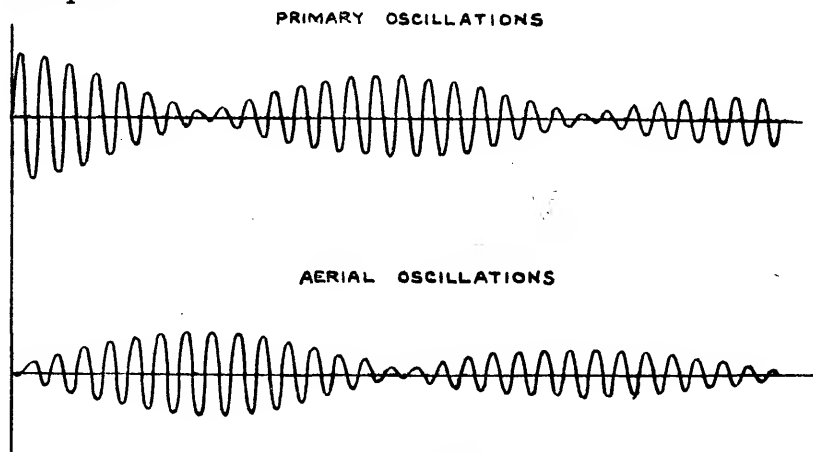
Every time the primary is set in oscillation, a great deal of energy will be expended in damping losses in the spark gap, while in the aerial the chief expenditure of energy is by radiation ; the latter expenditure is very desirable, but the former is not.

If the mutual coil is placed close up to the primary, the energy is transferred very rapidly backwards and forwards, as in Fig. 195.

This is known as "Tight Coupling." The effect will be as follows:—

It will be seen from the above that the oscillations in both primary and aerial die away and rise again several times in a series of "beats" before the wave train is entirely damped out.

Each time the oscillations are transferred to the primary, the damping effect of the spark gap is experienced, with consequent loss of energy, which might otherwise have been available for radiation into space.



Loose Coupling.

FIG. 196.

If the mutual coil be moved further away from the primary, the energy will be transferred more slowly between the two circuits, as in Fig. 196. The primary circuit is not set in oscillation so often before the wave train finally dies away and the primary damping losses are decreased. This is known as "Loose Coupling."

The advantages of loose coupling in lessening interference are discussed below. All damped wave transmission, however, is liable to cause "shock" excitation of receiving aerials whether tuned to it or not, and so to produce interference.

420. Brushing or "Corona Discharge."—If the aerial voltage is excessive the insulation of the air between the aerial and neighbouring earthed conductors breaks down, and a violet-blue discharge occurs. This represents a waste of energy, and also discloses the position of the ship at night. High aerial voltages are produced in sharply tuned circuits, *i.e.*, circuits with a large $\frac{L}{C}$ ratio.

On the other hand, if the $\frac{L}{C}$ ratio is decreased so as to flatten the tuning, the band of frequencies on which energy is radiated is widened and the possibility of interference is again increased.

421. Radiation of Two Wave-Frequencies.—The question of the resonant frequencies of two coupled circuits with forced oscillations due to a source of alternating E.M.F. in one has already been considered in Chapter V.

It is proposed to investigate here the other type of oscillations, viz., free oscillations (*see* para. 334). Free oscillations occur when one circuit is set oscillating, as in the case of the primary circuit of a spark transmitter, the energy being transferred and re-transferred by means of the coupling from one circuit to another until it is all consumed in damping losses. There is **no** continuously applied source of E.M.F. in either circuit.

It will be found that, although both circuits have the same LC value and therefore equal natural frequencies, there are two frequencies of free oscillation of the combination, both differing from the natural frequency of the independent circuits.

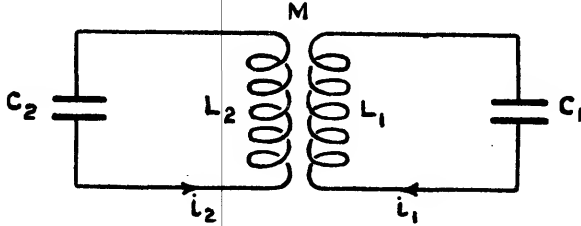


FIG. 197.

Let the two circuits be $L_1 C_1$ and $L_2 C_2$ as in the figure, where $L_1 C_1 = L_2 C_2 = L_0 C_0$.

Let the mutual inductance be M .

The resistances of the circuits will be neglected.

When the two circuits are both in oscillation and interacting one on the other, there will be currents flowing in both. Let these be i_1 and i_2 .

Let $\Omega = 2\pi \times$ natural frequency F of the individual circuits.

Let $\omega = 2\pi \times$ frequency f of free oscillations when coupled.

$$\text{Then } \Omega^2 = \frac{1}{L_0 C_0} = \frac{1}{L_1 C_1} = \frac{1}{L_2 C_2}$$

At any instant the voltage induced in the second circuit due to the current in the first is $\omega M i_1$.

This leads to the equation:—

$$\omega M i_1 = \left(\omega L_2 \sim \frac{1}{\omega C_2} \right) i_2.$$

$$\text{Similarly, } \omega M i_2 = \left(\omega L_1 \sim \frac{1}{\omega C_1} \right) i_1,$$

$\omega M i_2$ being the voltage induced in the first circuit from the changing current in the second.

Multiplying together these two equations, we obtain

$$\begin{aligned}\omega^2 M^2 i_1 i_2 &= \left(\omega L_1 \sim \frac{1}{\omega C_1} \right) \left(\omega L_2 \sim \frac{1}{\omega C_2} \right) i_1 i_2 \\ \text{or } \omega^2 M^2 &= \left(\omega L_1 \sim \frac{1}{\omega C_1} \right) \left(\omega L_2 \sim \frac{1}{\omega C_2} \right) \\ &= \omega^2 L_1 L_2 \left(1 - \frac{1}{\omega^2 L_1 C_1} \right) \left(1 - \frac{1}{\omega^2 L_2 C_2} \right) \\ &= \omega^2 L_1 L_2 \left(1 - \frac{\Omega^2}{\omega^2} \right)^2 \\ \therefore \frac{M^2}{L_1 L_2} &= K^2 = \left(1 - \frac{\Omega^2}{\omega^2} \right)^2 \\ \therefore 1 - \frac{\Omega^2}{\omega^2} &= \pm K \\ \therefore \frac{\Omega^2}{\omega^2} &= 1 \pm K \\ \therefore \omega^2 &= \frac{\Omega^2}{1 \pm K} \text{ or } \omega = \frac{\Omega}{\sqrt{1 \pm K}}.\end{aligned}$$

There are thus two values for ω , one greater and one less than Ω . Since $\Omega = 2\pi F$ and $\omega = 2\pi f$,

$$f = \frac{F}{\sqrt{1 \pm K}}.$$

Thus the frequency of oscillation of the two coupled circuits has two values, which may be written f_1 and f_2 ,

$$f_1 = \frac{F}{\sqrt{1 + K}} \text{ and } f_2 = \frac{F}{\sqrt{1 - K}}.$$

It will be noticed that these two frequencies are exactly the same as the two frequencies of resonance obtained in para. 336 for forced oscillations in coupled circuits. This result holds generally, provided the resistances of the circuits are small, so that any complex arrangement of circuits will oscillate freely at the same frequencies as those for which the reactance is zero when an alternating E.M.F. is introduced somewhere in the arrangement.

In the practical case of a spark primary and aerial circuit, the two frequencies of free oscillation and consequently of radiation, will correspond to two different wave lengths, λ_1 and λ_2 , which will be sent out by the transmitter.

Since wavelength is inversely proportional to frequency, $\lambda_1 = \lambda \sqrt{1 + K}$, and $\lambda_2 = \lambda \sqrt{1 - K}$, where λ is the wavelength corresponding to the natural frequency of each circuit separately. Also, since F is the frequency whose LC value is $L_0 C_0$, the frequencies f_1 and f_2 are those which correspond to $L_0 C_0 (1 + K)$ and $L_0 C_0 (1 - K)$.

Each of the circuits has therefore apparently two LC values, neither of which is equal to their individual LC value, L_0C_0 .

The difference between the wave frequencies radiated by the aerial depends on the degree of coupling K between the two circuits.

Example 54.

In a spark transmitting circuit tuned to 500 kc./s., the primary condenser is 40 jars, the aerial capacity is 1.5 jars, and the mutual inductance between primary and aerial circuits is 2 mics. Find the two wave frequencies radiated, and the percentage coupling.

The LC value of each circuit will be

$$\left(\frac{3 \times 10^4}{2\pi \times 500} \right) = \left(\frac{30}{\pi} \right)^2 = 91.2 \text{ mic-jars.}$$

$$L_1 = \frac{91.2}{40} = 2.28 \text{ mics ; } L_2 = \frac{91.2}{1.5} = 60.8 \text{ mics.}$$

$$K = \frac{M}{\sqrt{L_1 L_2}} = \frac{2}{\sqrt{2.28 \times 60.8}} = 0.17.$$

$$f_1 = \frac{500}{\sqrt{1+K}} \text{ kc./s.} = \frac{500}{\sqrt{1.17}} \text{ kc./s.} = \frac{500}{1.082} = 462.1 \text{ kc./s.}$$

$$f_2 = \frac{500}{\sqrt{1-K}} \text{ kc./s.} = \frac{500}{\sqrt{0.83}} \text{ kc./s.} = \frac{500}{0.911} = 548.9 \text{ kc./s.}$$

The percentage coupling = $K \times 100 = 17$ per cent.

422. We seem here to have two conflicting statements. It was stated in para. 419 that the energy in the aerial is in the form of beats, as shown in Figs. 195 and 196, and in para. 421 that two waves, differing in frequency, are set up in the aerial.

The truth of both these statements can be reconciled by drawing these two waves occurring simultaneously.

In Fig. 198, curve A shows f_2 , the higher frequency set up in the aerial, and curve B shows f_1 , the lower frequency.

As these frequencies are different, it follows that if they start in step they will work out of step, then in step, then out of step and so on, just as two men walking side by side and taking a different length of stride, get in and out of step in turn.

When the two waves are in step, it means that the currents due to the two waves are flowing in the same direction, and when out of step, in opposite directions.

The **total** current at any moment can be found by adding the heights of these curves together at each moment.

The result is shown in Curve C, which corresponds to the lower curve of Fig. 195 or 196.

From this we see that the current in the aerial rises and falls in a series of beats, being a maximum when the waves are exactly in phase, and a minimum when the two waves are exactly out of phase.

Similarly, the instantaneous voltage in the aerial can be found by drawing two voltage curves for the two frequencies and adding them together.

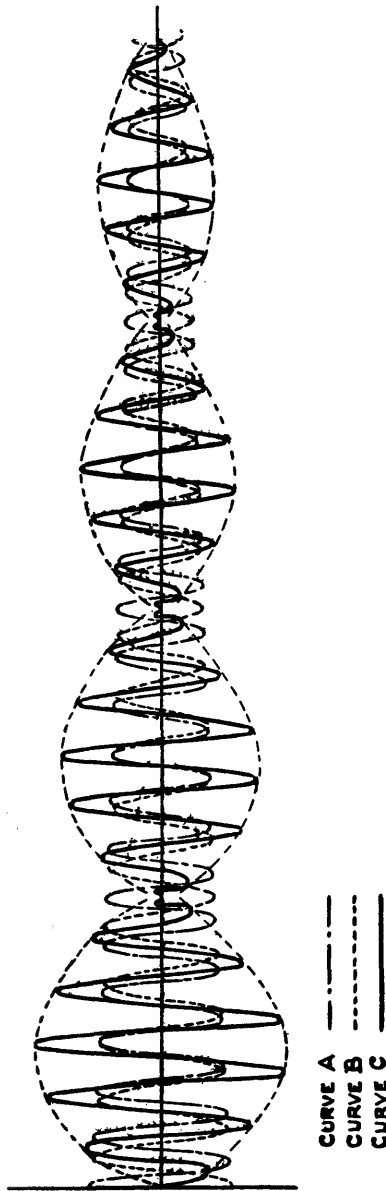


FIG. 198.

Hence, the total voltage and current in the aerial does rise and fall in a series of beats, and these beats are due to the two waves oscillating simultaneously at the two different frequencies, f_1 and f_2 .

The number of times per second at which the separate waves get into step is equal to the difference of the two frequencies, f_1 and f_2 .

$$\text{Now } f_1 = \frac{F}{\sqrt{1+K}} \text{ and } f_2 = \frac{F}{\sqrt{1-K}}.$$

$$\text{If } K \text{ is small, } f_2 - f_1 = F \{(1-K)^{-\frac{1}{2}} - (1+K)^{-\frac{1}{2}}\}$$

$$= F \left\{ 1 + \frac{K}{2} - 1 + \frac{K}{2} \right\} = KF,$$

approximately.

Therefore, the beat frequency, or the number of times per second at which the energy is transferred backwards and forwards from one circuit to another, is directly proportional to the coupling factor K , and the natural frequency F of the circuits taken individually.

This renders plausible the statement made in para. 419 with regard to tight and loose coupling. For tight coupling, higher powers of K would need to be considered in the above argument as the approximation, (K small compared with unity), is hardly justified.

423. Drawback of Radiating Two Waves.—In designing W/T circuits we have two objects always before us :—

- (a) To radiate as much energy as is required on the wave we wish to transmit.
- (b) To radiate as little energy as possible on any other wave.

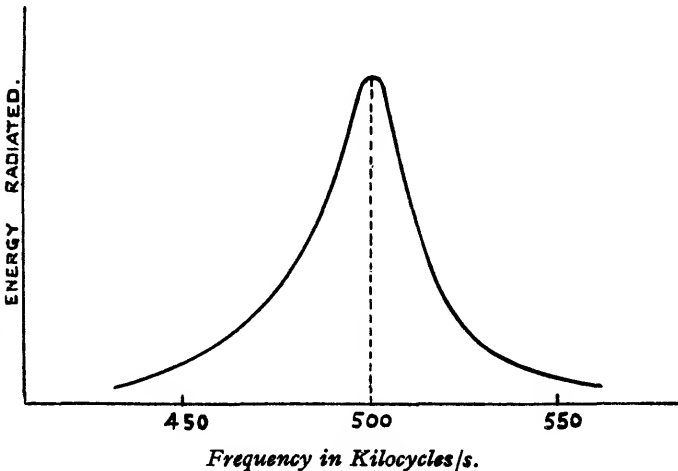


FIG. 199.

If this second object is fulfilled, then communication from a fleet to a number of shore stations or outlying ships is possible with a minimum amount of interference.

For example, the ideal condition is illustrated in Fig. 199.

This curve, known as an "Energy Distribution Curve," represents energy radiated by a transmitter at different frequencies. It corresponds to the case of the spark transmitter with very loose coupling, in which case the two component frequencies of the complex oscillation, f_1 and f_2 , are very close together, and their joint effect is to give a maximum of radiation at the natural frequency of the individual circuits. The curve can be also looked upon (and in practice can be plotted accordingly), as a graph of the energy received by a receiving circuit when adjusted to different frequencies. In Fig. 199, the transmitting ship, tuned to 500 kc./s., radiates a maximum of energy on this wave, but practically none on 450 and 550 kc./s., and therefore does not interfere with other ships trying to communicate on these waves.

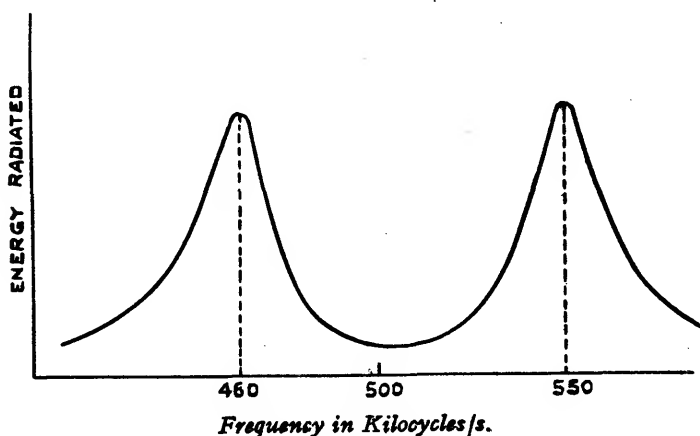


FIG. 200.

If, however, the coupling is tight, the current in the aerial circuit is made up of two component frequencies which differ considerably, and the energy distribution curve in this case will have two distinct maxima at these different frequencies, giving a curve as in Fig. 200.

- This means that the total energy available is divided, and also that interference with other lines of communication is caused.

For example, if the correct wave is 500 kc./s., and if, owing to the tightness of coupling, we radiate two waves, one 460 kc./s. and the other 550 kc./s. (a result which would be given approximately by a coupling of 17 per cent.), then we shall get an energy distribution curve as in Fig. 200, which gives bad interference on these wave frequencies. The tighter the coupling the further apart are the peaks in the Energy Distribution Curve, and the less is the energy actually radiated on the wave to which the circuits are tuned. It is found in practice that it is only for coupling of, say 5 per cent. or less, that the separate maxima merge into one and give a curve such as in Fig. 199.

In Chapter VIII, details will be given of a method by which the interaction between the circuits is stopped soon after the oscillatory action starts, so that the aerial circuit continues to oscillate after that at one frequency only, its natural frequency.

424. Factors affecting Coupling.—For the type of coupling normally employed, *viz.*, mutual coupling, the results of para. 333 can be applied to give methods of varying the amount of coupling, or the coupling factor K . K depends on two factors:—

- (a) How close the mutual coil is to the primary coil.
- (b) How many turns of mutual coil are used.

If the mutual coil is pushed closer to the primary coil, or if the inductance in the mutual coil is increased and that in the aerial coil decreased to maintain the same total value in the aerial circuit, the mutual inductance M between the primary and the mutual is increased, and K , which is equal to $\frac{M}{\sqrt{L_1 L_2}}$, is increased.

From the expression $\sqrt{L_1 L_2}$, which depends on the total inductance used in the primary and aerial circuits, we learn that, with a given primary and aerial capacity, the higher the wave frequency, the less must L_1 and L_2 be, and hence, for a given adjustment of the mutual coil, K is increased.

Also, if two ships, one with an aerial of large capacity and one with an aerial of small capacity, are both transmitting on the same wave and using the same value of M , then the one with the larger aerial capacity will have the tighter coupling, for she will have the lesser total inductance L_2 in the aerial circuit to tune to that wave.

Example 55.

Two ships—"A" and "B"—both have their primary and aerial circuits tuned to 500 LC. Each has a primary condenser of 50 jars and a mutual inductance M , between primary and aerial circuits, of 10 mics. The aerial capacity of ship A is 2 jars, and that of ship B is 1.2 jars. Find the percentage coupling in each case.

$$\text{In each case } L_1 = \frac{500}{C_1} = \frac{500}{50} = 10 \text{ mics.}$$

Ship A.

$$L_2 = \frac{500}{2} = 250 \text{ mics.}$$

$$K = \frac{M}{\sqrt{L_1 L_2}} = \frac{10}{\sqrt{10 \times 250}} = \frac{10}{50} = 0.2 = 20 \text{ per cent. coupling.}$$

Ship B.

$$L_2 = \frac{500}{1.2} = 416.6 \text{ mics.}$$

$$K = \frac{10}{\sqrt{10 \times 416.6}} = \frac{10}{64.5} = 0.155 = 15.5 \text{ per cent. coupling.}$$

425. To sum up, we may say that :—

- (a) Tight coupling results in a wave that starts with a big initial amplitude, because the energy is transferred to the aerial so quickly that very little energy is lost by the time the energy is all concentrated in the aerial for the first time. On the other hand, with loose coupling the energy transference is so slow that by the time the whole energy is in the secondary it will be considerably decreased in value by damping losses. Tight coupling therefore gives a shock effect on neighbouring aerals and a danger of brushing.
- (b) Tight coupling results in the energy being transferred more frequently to the primary circuit, with high losses each time due to the high resistance of the spark gap. With loose coupling, the primary is not set into oscillation so often.
- (c) With tight coupling a double frequency effect is produced, resulting in the radiation being most effective on wave frequencies widely differing from the frequency to which the aerial is tuned.

With looser coupling the double frequency effect becomes less and less marked, until with very loose coupling, the energy distribution curve may be said to have a single peak at the frequency to which the aerial is tuned. Interference will be considerably reduced.

Loose coupling is preferable therefore, but the coupling must not be too loose, or the energy will be mostly dissipated in the primary and not reach the aerial at all.

By using a special type of spark gap (see Chapter VIII), the double frequency effect may be almost entirely avoided and tighter coupling used.

426. Oscillatory Circuit Excited by Continuous and Alternating Voltages.—During the preceding part of this chapter we have been considering free oscillations set up by charging the condenser in an oscillatory circuit and allowing it to discharge, and have applied the theoretical results to the practical spark transmitter.

Two other cases may be considered in which free oscillations occur :—

- (1) When a circuit containing L , C and R in series is connected to a source of continuous voltage, and $R < 2\sqrt{\frac{L}{C}}$.

In this case, we shall see that in course of time, the voltage across the condenser will be the same as the voltage of supply, but that there will be an oscillatory action at the natural frequency of the circuit taking place during the transient period before the final state is reached.

- (2) When a circuit containing L , C and R in series is connected to a source of alternating voltage. In this case, we shall see that the final conditions are those of Chapter V, a forced oscillation at the applied frequency, but that during the period of transience, there will be free oscillations at the natural frequency of the circuit superimposed on the forced oscillations.

***427. Continuous Voltage.**—In this case the equation representing voltage relations in the circuit is only slightly different from the equation of para. 390. Instead of being $v - L \frac{di}{dt} - iR = 0$, it becomes $E - L \frac{di}{dt} - iR = v$, where E is the applied D.C. voltage, i is the instantaneous current flowing to charge up the condenser, and v is the counter E.M.F. of the condenser. The switch in Fig. 201 is closed at the start of the action.

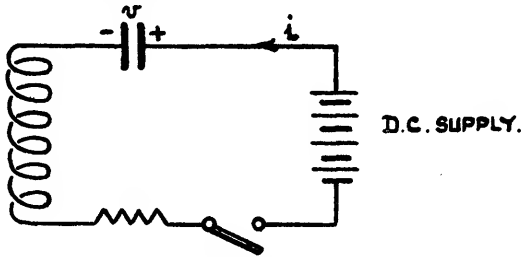


FIG. 201.

$$i = \frac{dq}{dt} = C \frac{dv}{dt}, \text{ and } \frac{di}{dt} = C \frac{d^2v}{dt^2},$$

so that the equation becomes

$$E = v + LC \frac{d^2v}{dt^2} + CR \frac{dv}{dt},$$

$$\text{or } \frac{d^2v}{dt^2} + \frac{R}{L} \frac{dv}{dt} + \frac{v}{LC} = \frac{E}{LC}.$$

This equation, apart from the constant term on the right, is the same as the differential equation of para. 390, and will give, therefore, the same type of solution. In particular, the same three cases arise as in para. 390, of which we need only consider one,

that for which R is less than $2\sqrt{\frac{L}{C}}$, which therefore gives oscillatory action. The conditions in this case which determine the arbitrary constants are that :—

(1) at $t = 0$, $v = 0$.

(2) at $t = 0$, $i = 0$, and therefore $\frac{dv}{dt} = 0$.

The following, if R is small, is the approximate solution of the equation :—

$$v = E (1 - e^{-\alpha t} \cos \omega t)$$

where $\alpha = \frac{R}{2L}$ and $\omega = \frac{1}{\sqrt{LC}}$ approximately, as before.

This equation shows that the voltage across the condenser varies at the frequency which is the natural frequency of the circuit, the amplitude of the variation about its final steady value E decreasing with increase of time at a rate depending on the damping factor $\frac{R}{2L}$.

The maximum voltage across the condenser (when $\omega t = \pi$) is very nearly double the supply voltage if the damping is small.

The full solution of the equation is

$$v = E \left\{ 1 - e^{-\alpha t} \left(\cos \omega t + \frac{\alpha}{\omega} \sin \omega t \right) \right\}$$

$$i = \frac{E}{\omega L} e^{-\alpha t} \sin \omega t.$$

The current in the circuit is therefore also oscillatory and decreases ultimately to zero due to damping losses. The form of the expression is the same as that in para. 390.

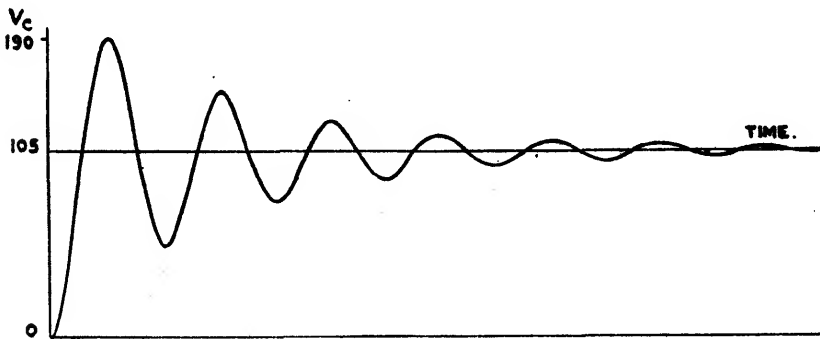


FIG. 202.

The figure above illustrates the varying voltage across the condenser in the case taken. The non-oscillatory case is similar to para. 174.

***423. Alternating Voltage.**—In this case the equation becomes

$$E \sin \omega' t = v + L \frac{di}{dt} + iR.$$

The frequency of the applied E.M.F. $\left(\frac{\omega'}{2\pi}\right)$ is taken to be different from the natural, or resonant, frequency (which are
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practically equal) of the circuit $\left(\frac{\omega}{2\pi}\right)$. In this case we shall derive the differential equation in a form which can be solved directly for current i .

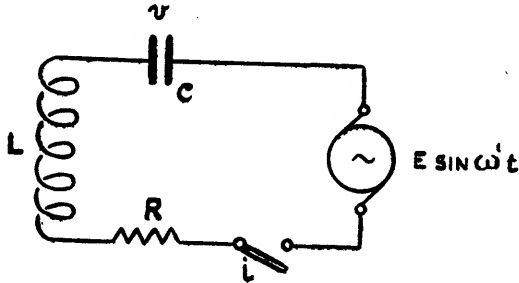


FIG. 208.

Differentiating the equation above as it stands :—

$$L \frac{d^2 i}{dt^2} + R \frac{di}{dt} + \frac{dv}{dt} = \omega' E \cos \omega' t.$$

Now $v = \frac{q}{C}$ and $\frac{dv}{dt} = \frac{1}{C} \frac{dq}{dt} = \frac{i}{C}$.

$$\therefore L \frac{d^2 i}{dt^2} + R \frac{di}{dt} + \frac{i}{C} = \omega' E \cos \omega' t.$$

The left-hand side of the equation is again of the same form as in the two preceding cases, and will give the same type of solution—in other words, there will be a free oscillation at the natural frequency

of the circuit, if R is less than $2\sqrt{\frac{L}{C}}$.

The particular solution obtained from the right-hand side of the equation represents the forced oscillation.

The full solution of the equation is quoted only, and is :—

$$i = I' \sin (\omega' t - \phi) + I e^{-\alpha t} \sin (\omega t - \theta)$$

where

$$I' = \frac{E}{\sqrt{R^2 + \left(\omega' L - \frac{1}{\omega' C}\right)^2}},$$

$$\tan \phi = \frac{\omega' L - \frac{1}{\omega' C}}{R},$$

$$\alpha = \frac{R}{2L} \text{ and } \omega = \sqrt{\frac{1}{LC} - \frac{R^2}{4L^2}}.$$

The first term in the equation for i represents the forced oscillation, at the frequency of the applied E.M.F. ; and the second

term represents the free oscillation, at the natural frequency of the circuit, which dies away at a rate determined by the damping factor $\frac{R}{2L}$.

The initial current wave form is therefore a complicated one, composed of the two wave forms given.

With the initial conditions that—

- (1) at $t = 0$, $i = 0$.
- (2) at $t = 0$, $v = 0$.

and an impressed resonant frequency, *i.e.*, such that $(\omega')^2 = \frac{1}{LC}$, a simple solution can be obtained if the damping of the circuit is small. This last condition means, of course, that R may be neglected in the expression for ω , making

$$\omega = \omega' = \frac{1}{\sqrt{LC}}.$$

In this case the solution reduces to the form

$$i = \frac{E}{R} \sin \omega t \left(1 - e^{-\frac{R}{2L}t} \right).$$

By integrating, v = voltage across condenser,

$$= \frac{E}{\omega CR} \left\{ \frac{R}{2\omega L} e^{-\frac{R}{2L}t} \sin \omega t - \left(1 - e^{-\frac{R}{2L}t} \right) \cos \omega t \right\}.$$

After the free oscillation dies away, only the forced oscillation, at the frequency of the applied E.M.F., remains, and the oscillatory action in the circuit may be said to settle down to its "steady state" in accordance with the theory of Chapter V.

We have now investigated the theory of free oscillations in an R, L, C circuit with no applied voltage, a constant applied voltage, and an alternating applied voltage, and in each case free oscillatory action takes place at the natural frequency of the circuit, and lasts for a time determined by the damping losses, provided that

$$R < 2\sqrt{\frac{L}{C}}.$$

CHAPTER VIII.

THE SPARK TRANSMITTING CIRCUIT.

429. Methods of Charging up the Condenser.—In Chapter VII we saw that a condenser in series with an inductance and a spark gap, if charged up to a voltage sufficient to break down the insulation of the spark gap, would give a high-frequency oscillatory current in the circuit. This current is damped out at a rate which depends on the losses in the circuit, and by using a suitable type of circuit it may be made to produce an electro-magnetic wave in the æther which carries energy to a distance.

The effective duration of the oscillatory current, and hence of the oscillation in the æther, is very short, and, to render the apparatus effective for communication, it is necessary to have methods by which the condenser may be charged up to discharging voltage at regular intervals. The action mentioned above will then be repeated every time the condenser discharges across the spark gap, and will give a series of wave-trains occurring regularly. It will be seen, when we consider the question of reception, that the most suitable **spark train frequencies** (*i.e.*, the frequencies of the condenser discharges) are those which correspond to the frequencies of easily audible sounds (say, 250 to 1,000 cycles per second), and the various methods of regularly charging and discharging the condenser are designed to work at these frequencies. The methods used, which will be described in this chapter, fall under two headings:—

- (1) Direct-current supply methods.
- (2) Alternating-current supply methods.

Direct-current methods are only suitable for low-power work, owing to the difficulty of obtaining high-voltage D.C. supplies. They all work on some sort of intermittent make-and-break principle, and will be described in detail under the headings of:—

- (a) Induction coil.
- (b) Attracted armature buzzer.
- (c) Motor-driven buzzer.

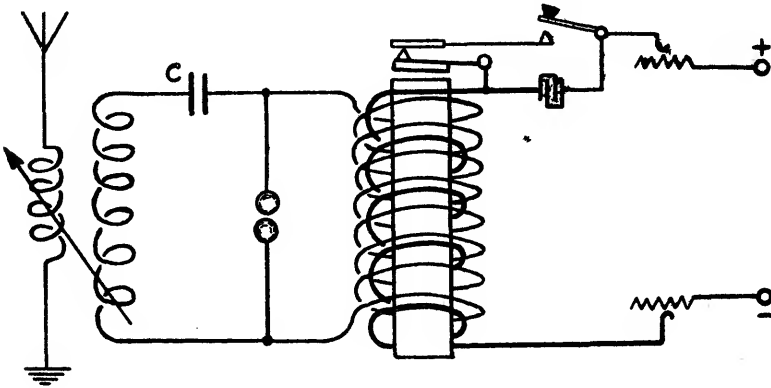
The second type, using alternating-current supply, is universally used for large power work. The voltage of the alternator is usually stepped-up by a transformer, so that the method is referred to as:—

- (d) Alternator and transformer method.

430. The Induction Coil.—If it is not convenient to run an alternator, owing to there not being a D.C. supply of high enough voltage or power available, or if it is required to work a small set

off a low-voltage D.C. supply or accumulator, the induction coil may be used.

The induction coil may be said to convert a D.C. supply into an intermittent current charging the transmitting condenser at a very much higher voltage than that of the supply.



Induction Coil.

FIG. 204.

The induction coil method is used in the Service in some spark transmitters fitted as attachments to continuous-wave transmitting sets for emergency purposes.

431. Construction.—An induction coil consists of a **primary coil** of thick wire wound with a number of turns on an iron core composed of a bundle of soft iron wires.

The primary coil is enclosed in an ebonite tube. Outside this again is the **secondary coil**, which consists of some miles of fine copper wire.

It is built up of a number of flat coils or sections, made by winding the wire between paper discs in a spiral form. The coils are joined in series, inner ends and outer ends of coils being joined together alternately, so that the current flows in the same direction in each coil.

By connecting them in this manner, there is no undue potential strain between the end of one section and the end of the next section to which it is joined.

In series with the primary winding is joined the "Interrupter" or "Make and Break."

This consists of a soft iron armature, secured to the top end of a flat steel spring whose tension can be adjusted by means of an ebonite wheel and adjusting screw.

This armature is close to one end of the iron core, the play between the two being about $\frac{1}{16}$ -inch.

The armature carries a small platinum contact. Close to the vibrating armature is a fixed standard carrying an adjustable contact, with a platinum tip. These contacts are normally held together by the tension of the spring.

Suitable terminals are provided to which the sending key should be joined.

A condenser of large capacity is joined across the key and interrupter contacts; its functions are explained below.

Two resistances are joined in series with the D.C. supply in order to limit the voltage to a value suitable for working the coil.

432. Action.—When the key is pressed, a current flows from the main positive lead, through the key, across the interrupter contacts, through the primary winding, through the resistance, and back to negative. The core of the coil is magnetised, and the armature is attracted to it. The contacts of the interrupter are therefore suddenly separated, and the current through the primary falls to zero very rapidly.

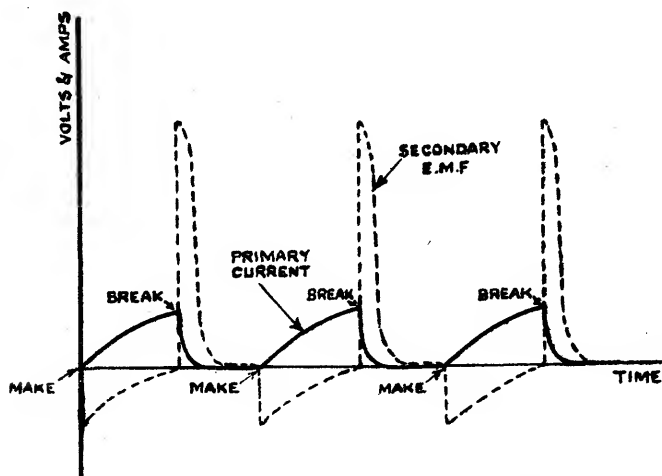
As soon as the primary current has died away, the armature will be released by the core, and will fly back; and so its contact will make connection again with the fixed contact. The primary current will again start to flow, and the cycle of events be repeated.

433. Secondary Voltage.—When the primary circuit is made, as indicated above, the primary current rises comparatively slowly owing to the big self-inductance of the primary coil. There are induced voltages set up in both the primary and secondary windings due to the charging current and changing flux. In the primary the induced voltage is simply the counter E.M.F. of self-induction, the maximum value of which is the value of the impressed voltage (at the beginning of the current flow, when the current is zero, and the applied voltage is entirely balanced by the induced voltage).

The induced voltage in the secondary while the primary current is growing can therefore have a maximum value which is N times the primary applied voltage, where N is the ratio of the number of secondary to primary turns. This secondary voltage is small compared with the voltage induced when the primary current is dying away, as we shall see.

When the contacts of the interrupter are separated, the primary current falls to zero. Again there are voltages induced in both the primary and secondary circuits due to changing current and changing flux, but the presence of the large-capacity condenser results in the rate of change of current and flux being much greater than during the period when the current is growing. The secondary E.M.F., being given by the product of the number of turns and the rate of change of flux, is therefore much greater than during the growth of current.

434. Condenser Action.—(1) The first function of the condenser across the interrupter is to minimise the sparking which tends to occur at the interrupter contacts. When the primary circuit is interrupted, the counter E.M.F. of self-induction due to the current decreasing would tend to keep the current flowing across the small gap that forms. Instead of this happening, the counter E.M.F. charges up the shunting condenser which had previously been short-circuited by the interrupter, and which, being of large capacity, does not rise to a high potential. The potential across the gap is therefore limited, and sparking is avoided. The break between the contacts is cleaner and a higher E.M.F. is induced in the secondary.



Primary Current and Secondary Voltage in Induction Coil.

FIG. 205.

(2) As the condenser discharges, its discharge current opposes the primary current and helps it to die away. The circuit in which the primary current decays is really a highly damped oscillatory circuit and if the resistance is not too large, a damped oscillation will be set up. In any case the rate of decay of current and flux is increased.

For these two reasons the E.M.F. induced in the secondary coil during the period of decay of the primary current is much greater than that induced during the period of growth of the primary current, the determining factor being the rate of change of current.

In both cases, of course, the secondary voltages depend on the amplitude of the primary current and its attendant flux, and the number of turns in the secondary.

A figure is appended, showing the variation of primary current and induced E.M.F. in the secondary circuit.

435. The Oscillatory Circuit.—Let us connect the secondary of an induction coil to an oscillatory circuit, consisting of a condenser, an inductance, and a spark gap. When the interrupter breaks the primary circuit, the very high induced E.M.F. in the secondary charges the condenser to such a high voltage that a spark takes place between the balls.

The high resistance of the gap having now been bridged by a spark, the condenser discharges through the inductance and the spark gap, and a high-frequency oscillatory current is set up in the circuit, and energy is radiated.

This action takes place at each break of the interrupter.

The frequency at which wave-trains are radiated is determined by the mechanical constants of the system—the tension of the spring and the position of the armature, &c.

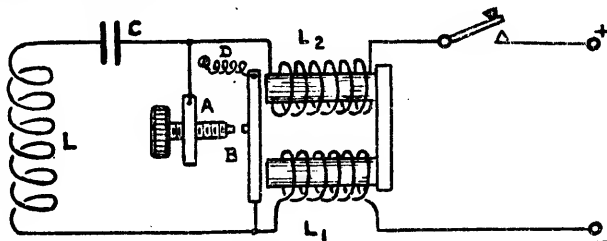
436. Limitation of an Induction Coil.—Although an induction coil may produce a momentary voltage of as much as 150,000 volts, yet it is not a very effective means of charging up a condenser for the reason that the **duration** of the high secondary voltage is very short.

The charging circuit consists of a condenser C being charged through a resistance R_s , the resistance of the secondary winding.

It was seen in para. 174 that, in such a case, the time taken to charge up the condenser is proportional to CR_s , the time constant of the circuit.

Since the duration of the high secondary voltage is short, CR_s should be small to enable the condenser to be charged to a high potential. C is fixed by H.F. oscillatory conditions, and therefore R_s should be small. The secondary winding must, however, have a large number of turns of fine wire, and so R_s in practice is large. Hence an induction coil is only suitable for charging up a comparatively small condenser; the energy isolated in the oscillatory circuit for each wave-train is small and the amount of energy radiated is correspondingly small.

437. The Attracted Armature Buzzer.—Another method of energising an oscillatory circuit is by means of a buzzer. This is illustrated in Fig. 206.



Typical Buzzer.

FIG. 206.

Here we have a fixed contact A, and a contact B carried on an armature which is free to vibrate. A and B are normally held together by a spiral spring D.

Joined to B and A are two coils of high inductance, L_1 and L_2 .

This arrangement constitutes the buzzer. A D.C. source of supply is joined to L_1 and L_2 , and is made and broken by a signalling key.

Across the make and break AB is joined a circuit, LC.

438. Action.—(a) When the key is pressed current flows through L_2 , across the contacts AB, and back through L_1 . The cores of L_1 and L_2 are magnetised and attract the armature B away from A.

(b) When the D.C. circuit is suddenly broken at AB, the current cannot suddenly cease owing to the high inductance of the coils L_1 and L_2 , so that the condenser C is charged up by the inductive kick from L_1 and L_2 , the right-hand plate being charged positively and the left-hand one negatively.

The energy that was stored in the magnetic fields round L_1 and L_2 is transferred to C when the current in L_1 and L_2 has been reduced to zero.

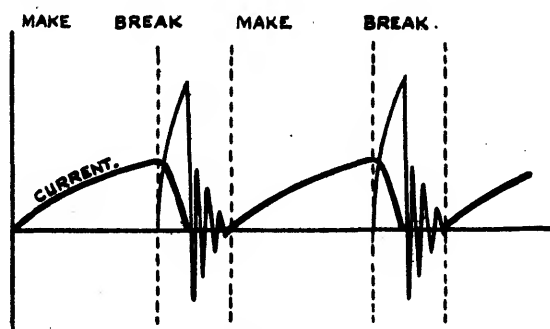


FIG. 207.

(c) The cores of L_1 and L_2 being demagnetised, the moving contact flies back towards the fixed contact.

When the two contacts are nearly together again, the P.D. between the condenser plates causes a spark to pass across the small gap so formed, and the discharge of the condenser C sets up a high frequency oscillation in the circuit formed by the condenser C, the inductance L and the small spark gap between A and B.

(d) When A and B touch once more, a direct current will flow through L_1 and L_2 , and the operation will continue.

This action may be summarised as in Fig. 207.

The thick line illustrates the slow rise of the direct current through L_1 and L_2 at the moment of "make," because of their self-inductance; and the sudden fall of the direct current at the moment

of "break," because of the P.D. across the condenser C at this moment. This is tending to send a current in the opposite direction to that flowing in the inductances and so the latter current decays more quickly than it rose. The condenser in the oscillatory circuit performs, from this point of view, the same function as the shunting condenser used with the induction coil.

The thin line shows the voltage applied to the condenser C at the moment of "break" and the high frequency oscillation set up in the circuit LC.

It is important to arrange the mechanical constants of the circuit correctly, so that, when the two contacts are nearly together and a spark is due to take place, the condenser will be charged up to its maximum voltage.

439. The above method of energising an oscillating circuit is used extensively :—

- (a) To energise a transmitting oscillator, when only a comparatively short range is required.
- (b) To energise receiving circuits for testing crystals, or for tuning purposes.

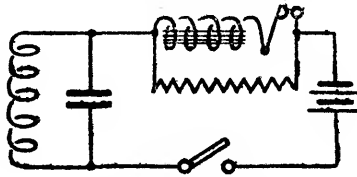
An advantage of this method of energising a transmitting oscillator is that, with reasonably loose couplings, only one wave is emitted from the aerial, and not two waves (due to interaction of primary and aerial circuits) as previously described.

The spark in the primary oscillatory circuit is extinguished in an exceedingly short time because the large mass of metal in its vicinity is very efficient in conducting away the heat which normally sustains the ionisation in the gap. Thus the primary is put on open circuit and no energy is transferred back to it from the aerial circuit. The high-frequency oscillation continues in the aerial circuit alone at the natural frequency of the latter. This type of action is known as "**quenching**," and will be considered in paras. **454** and **455** with respect to a special type of spark gap which effects it very efficiently.

440. Buzzer Tester for D/F.—The circuit of this buzzer, which is used to energise D/F aeriels for testing purposes, is shown in Fig. 208. When the switch is made, there is a closed circuit through the oscillatory inductance, buzzer coil and buzzer contacts. The buzzer coil is energised and separates the contacts.

The magnetic energy made available by the decreasing flux-linkage of the circuit as the current dies away is converted to electrostatic energy in the condenser and so a damped radio frequency oscillation takes place in the LC circuit. It will be seen that it is not necessary in this case for a spark to pass between the buzzer contacts. In fact, the passage of a spark is undesirable, for the gap is in parallel with the LC circuit and its low resistance when ionised would produce a large damping effect on the oscillations.

To prevent this, a resistance of 100 ohms is shunted across the spark gap. This is large enough to keep the oscillatory circuit sharply tuned and of low decrement, but it limits the P.D. across the contacts to a value which is too small to cause a spark.



Buzzer Tester for D/F.

FIG. 208.

This resistance is short circuited through the buzzer coil when the contacts are together and therefore does not reduce the charging current from the battery.

441. The Motor-driven Buzzer.—A motor buzzer is a more efficient method of energising an oscillating circuit than the attracted armature type of buzzer.

It may be illustrated diagrammatically as follows :—

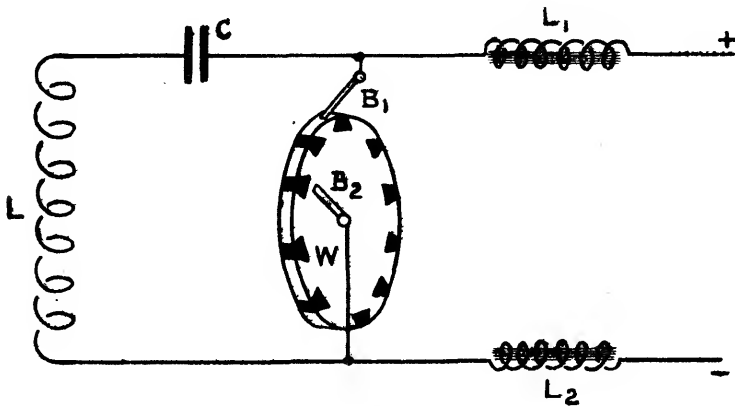


FIG. 209.

W represents a wheel driven round at a high speed by a small motor. In its edge are set a number of insulating segments of mica.

Bearing on its edge and on its side are two brushes, B_1 B_2 , across which is joined the oscillating circuit LC.

Joined to the two brushes are two coils of large inductance L_1 and L_2 .

A D.C. source of supply, interrupted by a hand key, is connected to the two inductance coils.

442. Action.—The action of this type of buzzer is very much the same as that of the attracted armature type previously described.

- (a) When brush B_1 is bearing on a **conducting** segment of the wheel a steady current will flow through L_1 and back through L_2 .
- (b) When the **insulating** segment on the wheel comes under brush B_1 , connection between the brushes is broken. The counter E.M.Fs. of L_1 and L_2 (due to the magnetic fields set up round them by the current) will charge up condenser C .
- (c) When the gap between brush B_1 and the next edge of the conducting segment is small enough, the condenser C will discharge in a high frequency oscillation, a small spark gap being formed across a portion of the surface of the insulating segment.

This sparking should occur on the under side of the brush. If it occurs above the brush it indicates that the wheel is dirty or that excessive power is being used.

Fig. 207 will serve to illustrate also the action of the motor buzzer. The period marked "make" is that during which the brush is bearing on a conducting segment, and the period marked "break" that during which the brush bears on an insulating segment.

This design of buzzer, like the previous type, produces a quenched wave.

With this type of buzzer, as with the attracted armature buzzer, it is important to arrange the circuit so that the current falls to zero, and the condenser C is therefore charged up to its maximum voltage, when the gap between the brush and the next conducting segment is small enough for a spark to take place.

During the period of break the circuit L_1, L, C, L_2 is effectively an oscillatory circuit with a steady source of supply; the current will fall to zero and the condenser charge up, in a period which is about a quarter of the **natural** period of oscillation of **this** circuit. L_1 and L_2 should be so arranged therefore that the duration of "break" should be just less than this quarter-period. The condenser will then be at its maximum voltage when the spark is due to take place.

Power may be reduced by inserting series resistances in the supply circuit.

443. The brushes should bear as firmly as possible on the surface of the wheel, and the standards on which they are mounted should have no play in them.

Excessive sparking at the brush, loose brush holders, or a pitted or dirty wheel may cause a bad note and loss of range.

The condenser is charged and discharged every time the brush bears on an insulated segment. Therefore the number of times per second the condenser is charged is equal to the speed of the wheel in revolutions per second multiplied by the number of insulating segments.

444. The Alternator and Transformer Method.—The methods hitherto described for charging up a transmitting condenser are only used under the special conditions referred to.

The almost universal practice for energising spark oscillatory circuits of $\frac{1}{2}$ kW. sets and upwards is to use an alternator or rotary converter and transformer.

It is essential to charge the condenser up to a very high voltage, for otherwise an excessively big condenser would be needed to store up a reasonable quantity of energy, (energy = $\frac{1}{2} CV^2$), which would mean that only large LC values could be obtained in the oscillating circuit. We might produce this voltage directly from some **high voltage alternator**, but such machines are very difficult and expensive to construct, and are not at all efficient; also the whole circuit would have to be very carefully enclosed to prevent fatal shocks being taken off it.

Fortunately, as explained in Chapter VI, it is a very easy matter to increase the voltage of an alternating current by utilising a step-up transformer.

Our simplest arrangement is then as shown in Fig. 210, namely, a low voltage alternator (or rotary converter) of a power suitable for the set we are using, delivering its alternating current at a frequency depending on the number of its poles and the speed at which it is run.

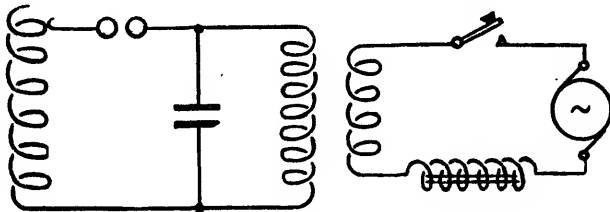


FIG. 210.

The circuit as shown may be divided into—

- (a) Low Tension circuit and High Tension circuit.
- (b) Charging or Low Frequency circuit and oscillatory or High Frequency circuit.

(a) **Low-tension Circuit.**—This includes everything to the right of the transformer in Fig. 210. From the terminals of the alternator current will flow through the primary of the transformer and across a signalling key of some sort. The impedance coil will be described later.

High-tension Circuit.—This includes the secondary of the transformer and everything to the left of it in Fig. 210. The secondary terminals are connected to the oscillatory circuit, which consists of a condenser, spark gap, and inductance. The current flowing from the secondary terminals of the transformer will be a **small** one at a **high voltage**, and at the same frequency as that of the alternator.

(b) **Charging Circuit.**—In Fig. 210 this means the condenser and everything to the right of it.

Oscillatory or H.F. Circuit.—This means the condenser, spark gap and inductance.

Note.—As regards the arrangement of the oscillatory circuit, it is a matter of indifference whether we take the transformer leads to each side of the condenser, or to each side of the spark gap, as in Fig. 211.

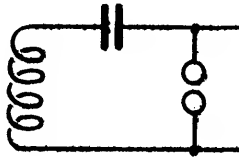


FIG. 211.

In the latter case the charging current flows through the transmitting inductance. Both arrangements may be found in practice.

445. The Charging Circuit.—It has been customary to consider the charging circuit from the point of view of resonance, and to show theoretically that the condenser will be charged up to the maximum voltage if the charging circuit is made resonant to the frequency of the alternator. On this assumption the impedance coil was regarded as a variable inductance which tuned the charging circuit, the latter being considered as a single circuit in accordance with the theory by which inductance or capacity on the secondary side of a transformer can be given an equivalent value and transferred to the primary side (*see para. 369*). In sets in use in the Service, however, the LC value of this equivalent single circuit, even with the minimum setting of the impedance coil, is always greater than the LC value which would correspond to the frequency of the supply, so that the theory of resonance of the charging circuit is not borne out in practice. It will be seen that the function of the impedance coil in the practical case is really to act as a regulator of the voltage impressed on the primary of the transformer, and in addition to assist in preventing what is known as "arcing."

446. Condenser Voltage.—Let us assume that the circuit is made at the beginning of a cycle of alternator voltage. A current will start to flow in the charging circuit and the voltage across the condenser will start to build up. From the theory at the end of Chapter VII (para. 428) the conditions at first will be "transient"; in other words, the graphs of current and voltage to begin with will not be sinusoidal, but will be the resultant of a sine wave (representing the final conditions of forced oscillations), and a damped wave at the natural frequency of the circuit (representing free oscillations).

The complete wave form will be of the type shown in Fig. 212.

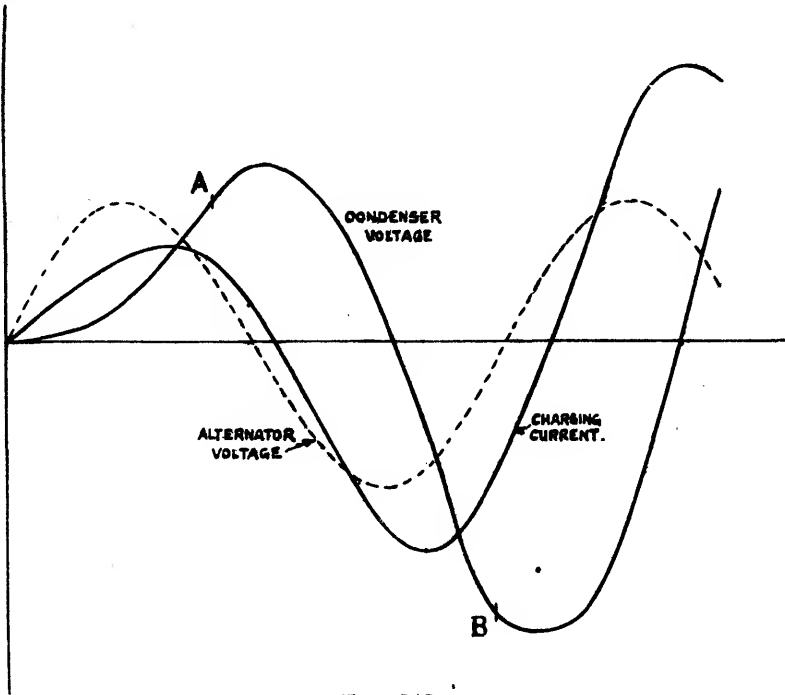


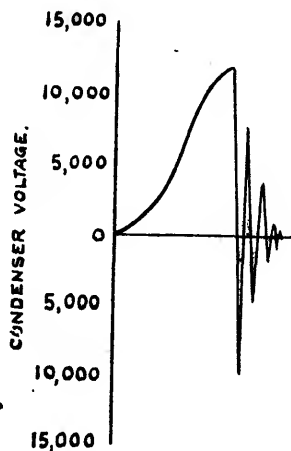
FIG. 212.

When the transient stage is over, the curve representing current will be a sine wave, lagging on the alternator voltage by a definite phase angle, while the voltage across the condenser will also be sinusoidal in form and lagging by 90° on the current.

These final conditions are not, however, of importance, because the spark gap is adjusted so that it breaks down at some voltage less than the final maximum value, for example, at point A in the figure above; under these circumstances, the transient conditions recur again after the spark gap breaks down, and the initial current and voltage variations are repeated.

447. Spark Train Frequency.—Let us consider in detail what happens when a spark occurs. We shall take first the simple case of a fixed spark gap. By this is meant a gap in which the plugs are fixed relatively to each other while sparking is taking place, but whose distance apart can be varied before the transmitting key is pressed. Let the spark gap be set at such a distance that the voltage across the condenser, and hence across the gap, corresponding to that at point A (Fig. 212), breaks down the insulation of the gap.

An oscillatory action takes place in the high-frequency circuit, the spark gap being now equivalent to a resistance. Energy is transferred by means of the mutual coupling to the aerial circuit, and a certain proportion is radiated into space in the form of electro-magnetic waves. The time during which the oscillatory action occurs is very small indeed compared with that of one cycle at the frequency of the alternator, so that the curve representing voltage across the condenser during the charging period and the consequent oscillatory discharge will be somewhat as shown.



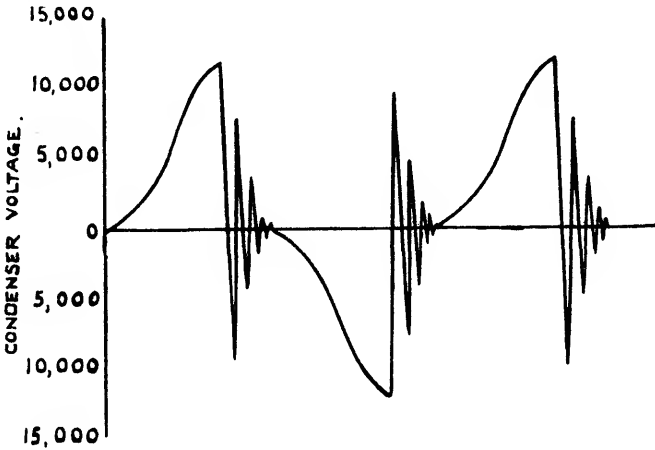
Condenser Voltage under Discharge Conditions.

FIG. 213.

The voltage across the condenser when the discharge is just completed is, of course, zero. If the break-down voltage is so arranged that, at the end of the ensuing discharge, a cycle of alternator voltage is just commencing again, exactly the same sequence of events will be repeated.

The case we have taken, of spark gap breakdown before the condenser voltage had reached its first maximum value, and the completion of the oscillatory action when the alternator voltage is again zero, as at the beginning, will obviously give **one spark per half-cycle** of the alternator voltage.

The sequence of events during succeeding half-cycles may be illustrated by the following graph of condenser voltage.



One Spark per Half-cycle.

FIG. 214.

If the spark gap were set at such a distance that the voltage at point B (Fig. 212) was just sufficient to break down its insulation (this voltage being greater than the first peak value), we should



One Spark per Cycle.

FIG. 215.

get **one spark per cycle** instead of one spark per half-cycle, and the curve representing condenser voltage would be of the form shown in Fig. 215.

With the type of gap in common use in the Service, the asynchronous rotary gap, these curves, which give a regular recurrence of events, have to be modified. This type of gap will be treated later.

448. Arcing and the Impedance Coil.—When the insulation of the spark gap is broken down, and the high frequency oscillatory current is flowing across the gap, the intense heat generated by the spark at the moment of discharge is sufficient to volatilise some of the metal of which the spark gap electrodes are made. This volatilised metal forms a conductive bridge from one electrode to the other. So long as a certain minimum current is passing across the gap, this conductive bridge is maintained. Now, when the spark gap becomes a conductor, the current from the alternator, which has hitherto been charging up the condenser, will tend to flow across the gap, maintaining its conductive property and preventing the condenser from being charged up again. It is therefore necessary to take every possible precaution for restoring the insulating properties of the spark gap as soon as the high-frequency discharge is completed.

Apart from mechanical devices for prevention of arcing, such as an air blast, which will be discussed later in dealing with the different types of gap, the inductance of the charging circuit, coupled with that of the impedance coil, tends to prevent charging current flowing across the gap. When the gap becomes conductive, the condenser is short-circuited and the charging circuit may be considered to consist of inductance and resistance only. The high inductive reactance cuts down the current to such an extent that it is insufficient to maintain the conductivity of the gap; the latter being restored, the condenser comes into the charging circuit again and the current charges up the condenser once more.

The **Impedance Coil** is therefore useful as an added inductance in the charging circuit towards the prevention of arcing.

A further use of the impedance coil is in controlling the power taken from the alternator. The larger the inductance inserted in the circuit by the impedance coil, the further from resonance are the conditions in the charging circuit; the more is the charging current cut down, and the less is the voltage built up across the condenser. The impedance coil may therefore be looked on as a voltage regulator, controlling the voltage in the condenser and hence the power radiated.

449. Power taken in Charging a Condenser.—When a condenser, C farads, is charged up to a discharging voltage of V volts, the amount of energy stored up in it is $\frac{1}{2} CV^2$ joules.

This energy is dissipated in damping losses in the primary and aerial circuits while the condenser is discharging in a high-frequency oscillation across the spark gap.

If we charge up the condenser to a voltage of V volts N times per second (*i.e.*, if N is our spark train frequency), then we shall be expending energy at the rate of $\frac{1}{2} NCV^2$ joules per second. But a joule per second is a watt, the unit of power.

Hence the power required to charge a condenser of C farads to a voltage of V volts N times per second is $\frac{1}{2} NCV^2$ watts.

Expressing power in kilowatts and capacity in jars, the equivalent formula is:—

$$\begin{aligned}\text{Power in kW.} &= \frac{1}{2} \times N \times \frac{C}{9 \times 10^8} \times V^2 \times \frac{1}{1000} \\ &= \frac{NCV^2}{1.8 \times 10^{12}}\end{aligned}$$

Example 56.

Find the power required to charge a condenser of 200 jars to a voltage of 15,000 volts, at a frequency of 300 cycles per second, with a spark train frequency of one spark train per cycle.

$$\text{kW.} = \frac{200 \times 15,000^2 \times 300}{1.8 \times 10^{12}} = 7.5.$$

From the above formula, we can deduce that the—

- (a) greater the size of the condenser charged,
- (b) higher the sparking voltage,
- (c) higher the spark train frequency,

the more is the power required to charge the condenser and the more is the power passed to the aerial and radiated away every second, the preponderating factor being V .

In Service sets the condenser in the oscillatory circuit is composed of separate elements, which may be joined in series or in parallel (*see also* para. 182). The parallel setting is used when a high LC value in the oscillatory circuit is necessary, *i.e.*, on long wavelengths or low wave frequencies. In order to keep as much equivalence as possible in the power radiated on different wave frequencies, the voltage to which the condenser is charged up should be smaller the greater the value of C . As the transformer secondary is in two halves, which may also be joined in series or in parallel, the voltage across the condenser can be reduced by using the parallel connection, or increased by using the series connection. This explains the rule employed in these sets:—

Condensers in parallel, transformer secondaries in parallel.

Condensers in series, transformer secondaries in series.

450. Design of Spark Gaps.—Charging circuits are very similar in their general arrangement, differing in detail only according to the power of the set and the range of waves required.

It is chiefly in design of spark gaps that sets differ.

The objects aimed at may be classified as follows :—

(a) **To Prevent Arcing.**—The effect of arcing has been already discussed.

The object is so to cool the conductive bridge that it is extinguished as soon as the oscillatory discharge is over.

Apart from the effect of the inductance in the charging circuit, and the additional inductance of the impedance coil, various methods of preventing arcing are included in the designs of the different types of spark gap.

(b) **To Effect Quenching.**—Quenching is effected by devising a very much more rapid method of cooling the spark gap and restoring its insulating properties than any method used for prevention of arcing. It effects this cooling so quickly that not only is arcing prevented, but the interaction and exchange of energy between the primary and aerial circuits is stopped as soon as the energy has been transferred for the first time to the aerial circuit. The theory of quenching will be considered in paras. 454 and 455, which deal with the special type of gap designed to effect this action.

(c) **To Produce a High Spark Train Frequency.**—The pitch of the note heard in the telephones at the receiving station depends on the spark train frequency of the transmitting station. If a fairly high frequency alternator is used, the conditions of one spark per cycle will give a high note.

If, however, the alternator or rotary converter is a low frequency one, we may increase the spark train frequency by using such a short spark gap as to give one spark per half cycle or even more.

The higher the spark train frequency the more difficult it is to prevent arcing, because the electrodes become hotter. With one type of gap, the asynchronous rotary gap, the spark train frequency is independent of the alternator frequency, and can be adjusted to any value desired.

The different types of gap employed are :—

- (a) The fixed gap.
- (b) The synchronous rotary gap.
- (c) The asynchronous rotary gap.
- (d) The quenched gap.

These will now be considered in detail. The last two are the types used in Service sets.

451. (a) The Fixed Spark Gap.—By this is meant a gap in which the plugs are fixed relatively to each other while sparking is taking place, but can be set to any required distance apart before the transmitting key is pressed.

Various shapes are illustrated in Fig. 216.

Such gaps are quite suitable for low power, or for low sparking frequencies, and are simple, requiring little attention beyond a periodical cleaning.

Two waves are emitted from the aerial circuit, the amount by which their frequencies differ depending on the coupling between the primary and aerial circuits.

The gap electrodes should be constructed of non-arcing material.

During the time the gap is conductive its resistance should be low, therefore the distance apart of the electrodes must not be too great. If, however, the distance apart is too short, the tendency towards arcing will be greater. The sparking surfaces should be parallel and clean, otherwise the spark will jump across at the same point, causing the surfaces to be burnt away.

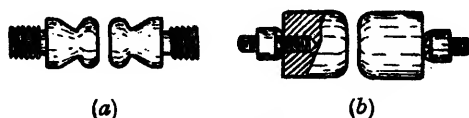


FIG. 216.

For higher powers or higher spark train frequencies a **Blower** is often provided to prevent arcing troubles. This is an arrangement for forcing a powerful blast of air between the electrodes ; it consists of a high-speed electrically-driven fan forcing air through a small porcelain nozzle, and is placed just below the gap.

452. (b) The Synchronous Rotary Gap or Discharger.—A **Synchronous Rotary Gap** denotes a form of design in which a metal wheel carrying a number of studs or spokes projecting from its edge rotates between two fixed electrodes, the rotating wheel being rigidly fastened to the alternator shaft. When the studs are opposite the fixed electrodes the spark jumps from one electrode to the wheel stud, through the wheel, and back through the second gap to the other electrode.

As the speed of the wheel and the alternating frequency both depend on the speed of revolution of the motor driving the alternator, the number of times per second at which the condenser voltage reaches a peak value and the number of opportunities it has of discharging can be made equal, and the position of the studs arranged so that these conditions occur simultaneously.

As the number of cycles per revolution of the alternator is equal to the number of pairs of its poles, and as in one revolution of the spark wheel the number of opportunities of sparking is equal to the number of studs on the wheel, it follows that to get **one spark train per half-cycle** the wheel should have as many studs as the alternator has poles.

The position of the fixed studs can be altered relatively to the moving studs by mounting them on a rocker which can be moved in one direction or the other. The condenser can thus attain its peak value of voltage when the studs are exactly opposite each

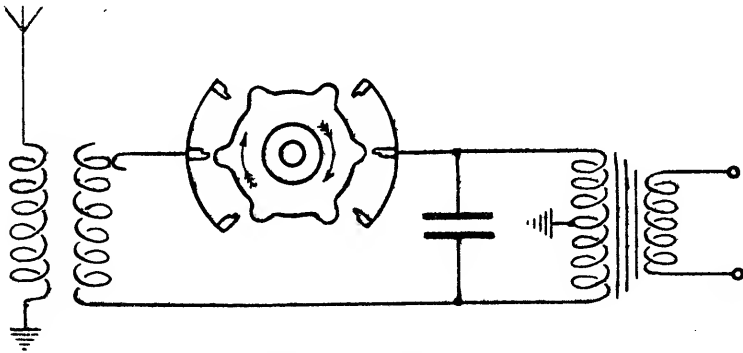
other, and the gap separation is adjusted for the breakdown to occur just before this happens.

Arcing is prevented by the increasing separation of the electrodes as the wave train takes place, and also by the fanning and cooling caused by their rapid motion.

In larger sets, air blasts are also used to cool the electrodes and prevent arcing.

It should be noted that the motor buzzer wheel, as previously described, is only a special variety of a synchronous gap. As the supply is a direct-current one, the spark train frequency is entirely dependent on the speed of revolution.

453. (c) The Asynchronous Rotary Gap.—This is the type of gap used in main spark sets in the Service.



Asynchronous Rotary Gap.

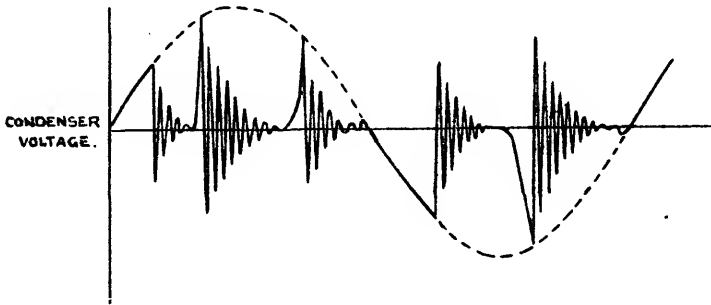
FIG. 217.

The principle of the rotating wheel with studs and the fixed electrodes between which it rotates is exactly the same as for the synchronous gap, the essential difference in the action being that the **speed of rotation of the wheel is entirely independent of the speed of the alternator.** The wheel is driven by a separate motor. The word "asynchronous," which means "out of time with," refers to this independence of speed of rotation.

The gap between the fixed and moving electrodes is adjusted to be as small as possible. A spark will occur whenever the gap between a fixed electrode and the point approaching it is small enough for the condenser voltage to force a spark across it. The great advantage of the asynchronous gap is that it is possible to produce a high spark train frequency from a low frequency supply. This being the case, it is obvious that there may be several sparks during one cycle of alternator supply, and consequently these occur at different points in the cycle. The conditions are not exactly repeated each time as in the case of the synchronous spark, because the charging current from the alternator is charging up the condenser

during different parts of its own cycle or variation, and hence neither the voltage to which the condenser is charged nor the voltage of breakdown is constant. Since the gap is adjusted to be as small as possible, there will be a discharge each time the studs come opposite each other unless the condenser voltage is very small. Sometimes, however, this condition holds and a spark is missed.

Not only is it possible to miss a spark altogether, but the interval between sparks is not absolutely constant. If the charging current after one breakdown is such that the condenser voltage before the next discharge is greater than usual the spark will take place over a longer gap, *i.e.*, when the studs are a little further apart than usual. If the condenser voltage is less than its average value, the spark will take place a little later than usual.



Condenser Voltage with Asynchronous Rotary Gap.

FIG. 218.

In addition, the energy stored in the condenser and the proportion radiated in the separate wave-trains is variable.

The disadvantages of this type of gap—the possibility of a spark being missed, irregular time-intervals between sparks, and irregular amplitudes of wave-trains—result in the note heard at a receiving station being impure.

The great advantage is the high spark train frequency which can be got from a low frequency supply, giving more energy radiation and a high and easily readable note at the receiver.

Arcing is prevented, as in the synchronous gap, by the mechanical separation of the electrodes and the draught of air. If necessary, an air blast may be fitted in addition.

A figure is given which shows the way in which the condenser voltage may be considered to vary during one cycle of low-frequency alternator supply, several spark discharges taking place during this period.

454. (d) The Quenched Gap—Theory of Quenching.—The **Quenched Gap** is designed to put out the spark much more quickly than any of the methods used for prevention of arcing. It was seen

in Chapter VII that when the primary oscillator has transferred its energy to the aerial, the aerial re-transfers it to the primary, and the energy is transferred and re-transferred backwards and forwards between the two circuits until it has been entirely expended in various damping losses, as illustrated thus :—

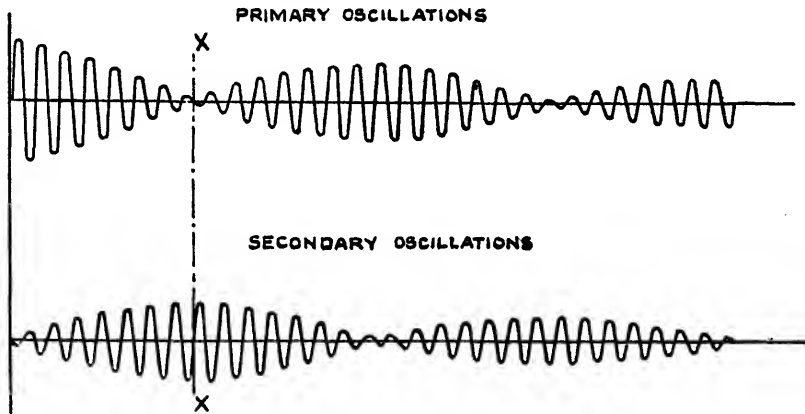


Diagram of Primary and Aerial Currents in Ordinary Coupled Circuit.

FIG. 219.

This transfer and re-transfer of energy produces two harmful results :—

- (a) The heavy damping in the spark gap is wasting the energy of the high-frequency oscillation during the whole time of transmission.
- (b) Two waves are emitted which interfere with ships working on other waves of different frequency.

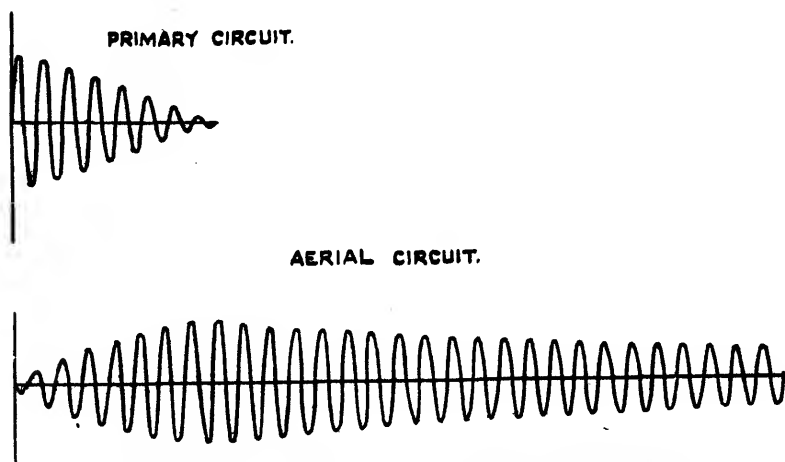
If, in some way, we can manage to put the spark out at the moment X in Fig. 219 (when the primary energy is at a minimum), then the conditions are entirely changed.

As soon as the whole of the energy has been transferred to the aerial the conductivity of the gap is destroyed and the energy which is now in the aerial cannot return to the primary, and will therefore continue to oscillate in the aerial circuit until the whole of it has been radiated in the form of electro-magnetic waves or lost unavoidably in resistance in the aerial circuit.

The coupling between the circuits can be increased, and more energy transferred from the closed to the open circuit. After the spark is "quenched," the aerial circuit oscillates at its own natural frequency, and it is only for a comparatively small period during the transfer of energy that two wave frequencies are being radiated. These are both considerably different from the natural frequency of the single circuit, especially with the tight coupling employed, and cause interference at the beginning of the wave-train. The tight

coupling causes the aerial oscillation to start off with a very big initial amplitude, causing a shock effect by setting up free oscillations in neighbouring aerals.

A diagram of the current oscillations in the two circuits is as shown in Fig. 220.



Primary and Aerial Currents in Quenched Gap Circuit.

FIG. 220.

The upper diagram represents the oscillations in the primary circuit until the spark is "quenched" at the moment X (see Fig. 219), and the lower diagram those in the aerial circuit. The advantages and disadvantages of quenching may be summarised as follows :—

Advantages.

- (a) Only one wave frequency radiated for the greater part of the wave-train.
- (b) No tendency to arc. The time involved in quenching is probably that corresponding to 3 or 4 cycles of high-frequency oscillation, which is very small compared with the time in which anti-arcing devices become efficient. One spark per half-cycle can therefore be obtained quite easily with the quenched gap.
- (c) The comparatively high resistance of the gap is in circuit for a very short time. As the unwanted power loss in the aerial circuit can be made considerably less than that in the primary circuit, a greater proportion of the energy input is converted to radiation energy.
- (d) Tight coupling (up to 20 per cent.) can be used, and therefore rapid transference of energy is ensured. The tighter the coupling the less is the time of transference and the less are the losses in the primary circuit.

Disadvantages.

- (a) The tight coupling causes the aerial oscillation to start with a big amplitude which shocks neighbouring aerials into oscillation.
- (b) During the period of interaction two wave frequencies are radiated, each differing from the frequency of free oscillation of the aerial.

Quenching is achieved by the use of the special design of gap called the **Quenched Gap**, which will now be considered in detail. The rapid mechanical rotation of the motor buzzer is sometimes considered to give a quenching action.

455. (a) The Quenched Gap—Construction.—The Quenched Gap, as now described, is used in spark attachments to main valve transmitting sets in the Service. Quenching is simply a matter of cooling the spark path rapidly enough.

A method of accomplishing this is to make use of the property metals have of conducting heat. In the quenched gap, the spark gap is broken up into a number of very short gaps in series with each other. The electrodes are made of copper, plated with silver, these metals being good heat conductors. In addition, large cooling or radiating fins are provided, so that there is a large mass of metal surrounding the gap and plenty of metal surface exposed to the surrounding air. In addition, it may be necessary in some cases to employ an air blast to cool the fins sufficiently. The gaps are so short that no part of the air dielectric is far from the metal of the electrodes. There are two methods of building up a quenched gap, shown in Fig. 221 (a) and (b).

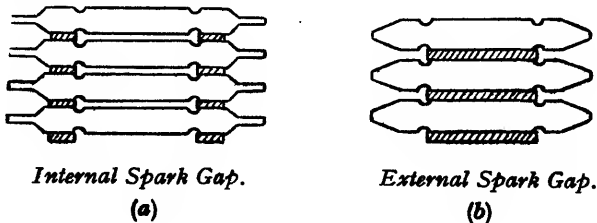
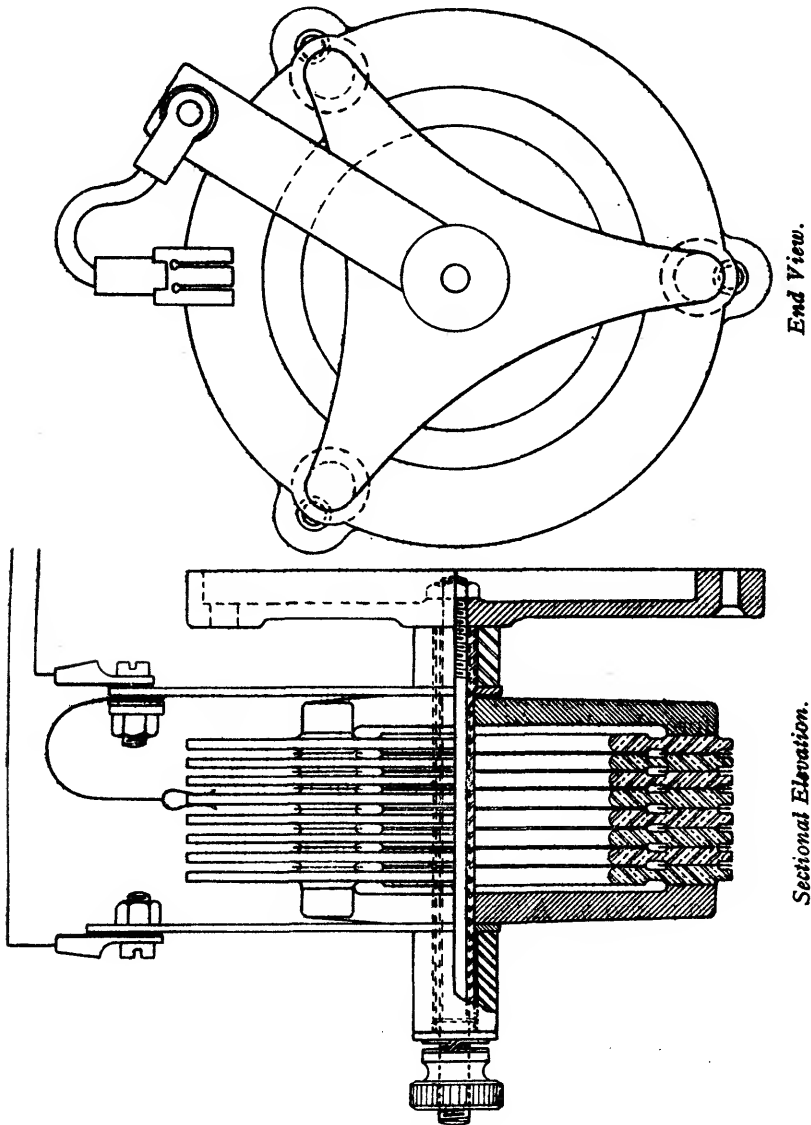


FIG. 221.

In Fig. 221 (a) the gap consists of a number of metal discs with circular grooves cut in them. The discs are separated by washers of mica, about 0.2 mm. in thickness, which are inserted between the discs at the outer edges and extend a little way across the circular grooves. The sparking surface in the centre of the discs is thus completely shut off from the surrounding air.

In Fig. 221 (b), the mica washers are placed between the electrodes inside, and just extending into, the space between the circular grooves. The sparking takes place between the edges of the electrodes. This is the type used in the Service.

In both cases the spark is rapidly extinguished because of the adequate cooling properties of the gap, and because the mutual



Quenched Gap.

FIG. 222.

repulsion of the ions in the ionised air between the electrodes forces the spark to the outer edges of the sparking surface, where it becomes lengthened across the circular groove. In the "internal" gap it

is probable that the removal of the oxygen in the gap by oxidation of the electrodes is an important factor in assisting its quenching properties.

Fig. 222 shows a complete quenched spark gap as used in the Service. The number of single gaps used in series can be varied by short-circuiting the others by a spring clip, which is attached to one terminal of the spark gap by a length of conducting wire and can be fastened to any one of the radiating fins. The number is varied according to the condenser break-down voltage, the voltage for each individual gap being about a thousand volts.

456. The Charging Circuit Complete.—We are now in a position to discuss broadly the general requirements of a Spark Transmitting installation (see Fig. 224).

Power Supply.—The alternating current will be supplied from either a motor alternator or a rotary converter.

Duplicate machines are generally supplied in H.M. ships, placed in separate watertight compartments, and fed through duplicate starters from either side of the ring main system.

Normally the machine in use should be fed from the section of the ring main on the disengaged side in action.

With both motor alternators and rotary converters a **Starter** will be necessary, with a no-volt-release coil, and some arrangement to prevent an excess current being taken from the mains. This may take the form of a fuse in the lead from one main, or of an overload release coil on the starter.

A **Motor Field Regulator** will be provided for varying the speed, and therefore the alternating frequency. (As previously explained, the voltage of the alternating current supplied by a **rotary converter** will always remain about 65 per cent. of the supply voltage, whatever the speed.)

An **A.C. Ammeter** and a **Frequency Meter** will also be provided: With a motor alternator a **D.C. Ammeter** and an **A.C. Voltmeter** will be supplied; also an **Alternator Field Regulator**, in order to vary the voltage of the alternating current.

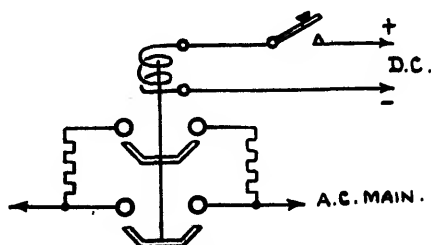
From the slip rings of the machine in use come the A.C. mains, in series with which we shall have the Signalling Key, an Impedance Coil, a safety arrangement of some sort, and, finally, the primary of the step-up transformer.

457. Signalling Key.—If the alternating current is small and its voltage is low, signalling may be effected by making and breaking the alternating current circuit by means of a hand key. (See also para. 468.)

If, however, the current is too great, or the voltage of a dangerously high value, a **Magnetic Key** will be used. This consists of a single-pole break between two contacts, which is short-circuited by a moving contact. The key is energised by direct current being

supplied to the bobbin of a solenoid when a hand key is pressed, as illustrated in Fig. 223.

When the low-tension current is too great to be interrupted conveniently, the magnetic key may be placed in the high-tension circuit, but special precautions must then be taken about its insulation.



Magnetic Key.

FIG. 223.

The drawback of this arrangement is that, even when the key is not pressed, the secondary terminals of the transformer are alive. It is used in certain high-power shore stations.

458. Safety Arrangements.—It is necessary to prevent the operator from accidentally receiving a fatal shock.

For a shock to prove fatal, about one-eighth to one-sixteenth of an ampere of direct or low-frequency alternating current will be necessary, one ampere being certainly enough to kill a man; far heavier currents at high frequency, however, can be safely withstood.

The resistance of the human body varies in different individuals and with the degree of moisture of the skin but, normally, the resistance of a man from one hand to the other is about 20,000 ohms; the injurious effects of the shock will also depend on the state of health. A very much worse shock is taken if the terminals of the mains are firmly grasped with the hands, or held in a pair of pliers, than if they are only touched with the finger-tips.

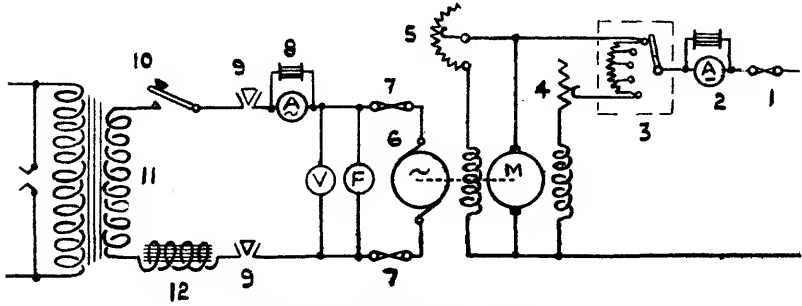
Any voltage above 1,250 volts direct or low-frequency supply will probably prove fatal, but voltages of 500 and above are dangerous.

If the alternator voltage is below 220 volts, the high-tension side of the transformer, the primary oscillating circuit, and the aerial circuit will be the only dangerous parts of the circuit.

These may be protected by enclosing the transformer, oscillating circuit and aerial coil in a cage, and arranging that the cage cannot be opened without breaking the low-tension alternating-current circuit at one point at least. It is better if both mains are broken, one by each of two cage doors. Thus, if the key is pressed when either cage door is open, nothing will happen.

If, however, the alternating voltage is high, it will be necessary to arrange in addition that it is not possible to receive a shock from the alternator mains.

This can be effected either by placing the various instruments included in the low-tension circuit inside the safety cage, or by enclosing them in boxes the lids of which cannot be opened when the primary circuit of the alternator is alive.



Typical Low-tension Circuit.

FIG. 224.

459. Low-tension Circuit.—Fig. 224 represents a typical arrangement of a Low-tension Circuit.

- 1 is a fuse in the positive D.C. main.
- 2 is a D.C. Ammeter with its shunt.
- 3 is a starter for the motor of the Motor Alternator (6).
- 4 is the Motor Field Regulator.
- 5 is the Alternator Field Regulator.
- 6 is the Motor Alternator. (If a Rotary Converter is supplied in lieu, then the Alternator Field Regulator (5) and field magnet winding will be omitted.)
- 7, 7 are A.C. fuses, one in each A.C. main.
- 8 is the A.C. Ammeter with its Shunt.
- V and F are respectively the A.C. Voltmeter and Frequency Meter.
- 9, 9 are the breaks in the low-tension A.C. circuit, completed when the cage doors are closed.
- 10 is the hand key for signalling. If the A.C. current is large, this is replaced by a magnetic key.
- 11 is the Transformer.
- 12 is the Impedance Coil.

If power is obtained from a Rotary Converter, then the A.C. Ammeter and Voltmeter may be omitted.

460. High-tension Circuit.—The high-tension circuit comprises the secondary of the transformer, the leads to the primary oscillating circuit, and the oscillating circuit itself, made up of primary condenser, primary inductance and spark gap.

In this circuit it is necessary to protect the transformer and condenser against excessive strains on their insulation, which may arise in several ways.

These risks, and the methods of guarding against them, are as follows :—

461. Safety Gaps.—The transformer is protected against any excessive rise in its own voltage by having safety gaps fitted across its terminals.

The gap is so arranged that if the terminal voltage of the secondary rises above its normal working limit, then an A.C. metallic arc will form across the gap.

Typical safety gaps are shown in Fig. 225.

Fig. 225 (a) consists of two bent pieces of copper wire. Fig. 225 (b) consists of two brass discs, mounted eccentrically so that the gap length can be adjusted by rotating them.



Safety Gaps.

FIG. 225.

The arc will rise, on account of its own heat, sliding along the wires. The higher it rises the longer becomes the gap which it must bridge, so that it automatically breaks down.

The gap must be kept set at the exact distance laid down in the Handbook for the set in question.

In the same way the transmitting condenser is fitted with safety gaps to prevent it being punctured by any abnormal rise of voltage.

It is commonly the practice, where several sections of a condenser are employed in series, to earth the centre point of the central sections.

This equalises the capacity to earth of each element of the condenser.

462. Back Oscillations.—When the condenser discharges across the spark gap it is quite possible that some of the high-frequency oscillatory current, instead of flowing across the spark gap, will try to flow back through the secondary of the transformer.

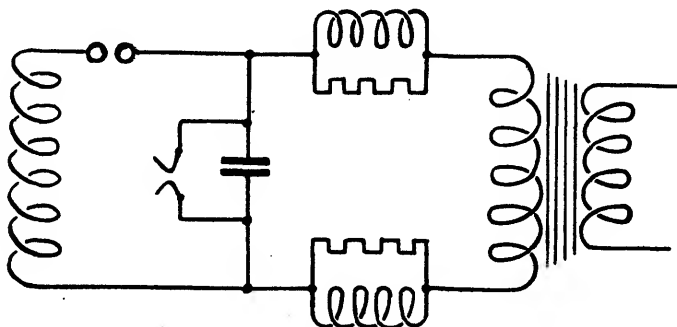
This is a form of trouble known as "Back Oscillations."

If this high-frequency oscillation is applied directly to the secondary turns of the transformer, it will set up a big inductive voltage to earth across the end turns, and the insulation of the transformer might easily be broken down.

To prevent this, coils of wire, termed "Protecting Coils," of about 200 to 300 mics inductance, are placed in series with each main (Fig. 226).

The back E.M.F. of these coils to the **low-frequency charging current** is very small indeed, nor does the addition of a few mics disturb the resonance of the charging circuit.

On the other hand, when the **high-frequency discharge** tries to send some of its current back along the high-tension mains, the reactance of the protecting coils becomes so large that they practically form insulators to high-frequency currents. (The insulation of these coils may be punctured, but they can easily be re-wound on board.)



Protecting Coils and Resistances.

FIG. 226.

Now, protecting coils alone, under certain circumstances, have proved worse than useless.

Being mounted on a baseboard, the two coils have a certain small capacity effect between them; also each coil has a certain self-capacity of its own.

This capacity, combined with the inductance of the two coils, forms an oscillatory circuit which may happen to have the same LC value, and therefore the same natural frequency as that of the primary oscillator.

Should this happen, then resonant currents will be set up in the protecting coils, and high-frequency currents will be impressed on the end turns of the transformer windings, with the result that those turns will have their insulation punctured.

To avoid this, the protecting coils are shunted with non-inductive resistances in the shape of **Carbon Rods**, upon which the energy of the high-frequency currents is expended.

The high-frequency current sets up a big P.D. across the inductance, which consequently tries to force a big current through the resistance, and the back oscillation is damped out.

463. Design of the Oscillatory Circuit.—The oscillatory circuit must be arranged so as to provide a certain range of LC values, according to the purpose for which the set is designed.

For example, a set might be required to give a range of all LC values between 50 and 1,200, which would give a range of frequencies between 676 kc./s. and 140 kc./s.

This might be arranged by having a condenser of 50 jars and an inductance which could be varied between 1 mic and 24 mics, but this would be bad electrical practice for the following reason :—

The energy stored in a condenser of C farads when charged up to a discharging voltage of V volts is $\frac{1}{2}CV^2$ joules.

From this it follows that, the larger the condenser, the smaller the voltage required to store a given amount of energy. (See para. 449.)

A high condenser voltage would mean a long spark gap, which would have a heavy damping effect. Also, the higher the voltage, the greater do insulation difficulties become.

Consequently we want to use the largest possible condenser and the smallest possible inductance.

A much better arrangement would be to provide a condenser composed of two elements of 100 jars each, which could be joined in series, giving a capacity of 50 jars, or in parallel, giving 200 jars. This arrangement is generally used in practice.

A primary inductance which could be varied between one and six mics would give a range of LC values from 50 to 300 in the series position, and from 200 to 1,200 in the parallel position.

464. The Condenser.—The condenser must be made with adequate dielectric strength to stand the greatest sparking voltage that is likely to be used. For instance, if the condenser referred to above is required to stand an eight-millimetre spark, and if one sheet of the dielectric used in it will only stand two millimetres safely, it would be necessary to make up the two elements of 50 jars each by joining four sections of 200 jars each permanently in series. It would therefore be composed as follows :—

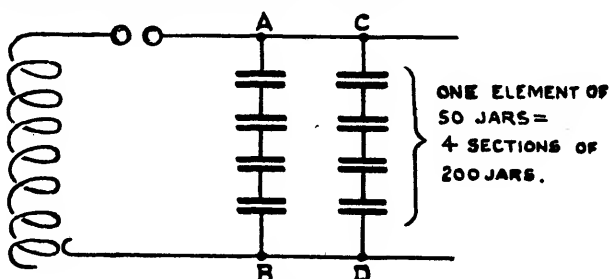


FIG. 227.

To join the two elements in series, connect terminals B and C together.

To join the elements in parallel, connect A to C and B to D.

The dielectrics generally used in the Service for spark transmitting condensers are ebonite, glass or mica.

The elements and sections composing the condenser are contained in an iron tank, which is kept brimfull of oil, to prevent brushing over the plate edges, and to keep the condenser cool.

However efficient a condenser is made, certain losses in it are unavoidable. These losses were referred to in Chapter III, under the general heading of "Hysteresis Losses."

When a condenser is being charged up and discharged, it gradually gets hotter and hotter as a result of these losses.

Arrangements have to be made for this heat to be radiated away, and also for the oil to expand without forcing its way under the edge of the lid.

465. The Primary Inductance.—This is a coil of copper tubing or flat copper strip, of large surface area, in order to obtain low resistance to high-frequency currents with a given required inductance and mechanical rigidity. It must have a surface of adequate area for the maximum oscillatory currents it will be required to carry.

The turns must be spaced sufficiently far apart to prevent sparking over between adjacent turns.

An adjustable connection, of very low resistance, must be provided so that the inductance in circuit can be varied gradually from the minimum to the maximum value, in order to give any required LC value between the limits for which the circuit is designed.

466. The Aerial Circuit.—This circuit, illustrated in Fig. 228, comprises the following :—

Aerial, Feeders and Deck Insulator, which are fully dealt with in Chapter XVIII.

(a) **The Aerial Coil.**—The aerial coil is provided in order to increase the LC value of the aerial circuit when transmitting waves longer than the fundamental wave of the aerial.

In spark transmitting circuits it is made of stout copper wire, the turns of which are wound on a former of insulating material, and spaced sufficiently wide apart to prevent sparking over between adjacent turns.

An important point is whether the idle turns of the aerial coil, *i.e.*, those not required for the particular frequency in use, shall be short-circuited, as in Fig. 229 (a), or left on open circuit, as in Fig. 229 (b). This question has already been discussed in para. 382.

The general practice is to short-circuit the idle turns of coils used in spark transmitting sets.

When making a tuning connection on the aerial coil, it is important to remember that all the current of a received signal has to pass through it, and therefore a thoroughly clean and tight connection should be made.

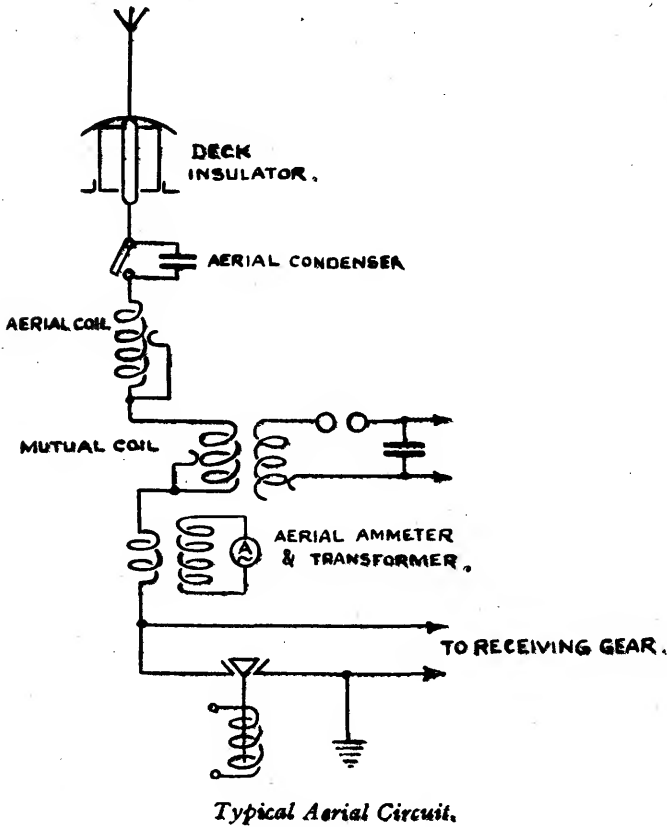


FIG. 228.

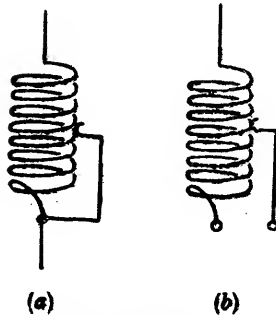


FIG. 229.

(b) **The Mutual Coil.**—The mutual coil is provided in order to transfer into the aerial circuit the high-frequency oscillations generated in the primary.

Its distance from the primary is made adjustable in order to allow the coupling to be varied. In some circuits its inductance is also made adjustable.

The loosest possible coupling consistent with giving readable signals to the receiving station should always be used.

It is better to use a long spark and a loose coupling than a short spark and a tight coupling.

(c) **The Aerial Condenser.**—For transmitting waves shorter than the fundamental wave of the aerial, a **series condenser** is generally used, being short-circuited with a link when not required.

467. (d) The Aerial Ammeter.—In order to enable an operator to tell that his circuits are correct, a hot-wire ammeter, termed the "Aerial Ammeter," is provided.

If joined directly in series with the aerial, it would have to be unduly large to carry the full aerial current in most aerial circuits. It is therefore usually joined across the secondary of a toroidal transformer (para. 377).

468. (e) Send-Receive Contact.—This consists of a make-and-break in the earth lead, across which the receiving gear is joined.

When the transmitting key is pressed this break is short-circuited, thus short-circuiting the receiving gear, and connecting the mutual coil to earth for transmitting purposes.

When the transmitting key is released the break is opened, thus putting the receiving circuit in series with the aerial.

This allows for what is termed "listening through," *i.e.*, listening between the Morse signals of one's own message to see whether anyone else is transmitting at the same time.

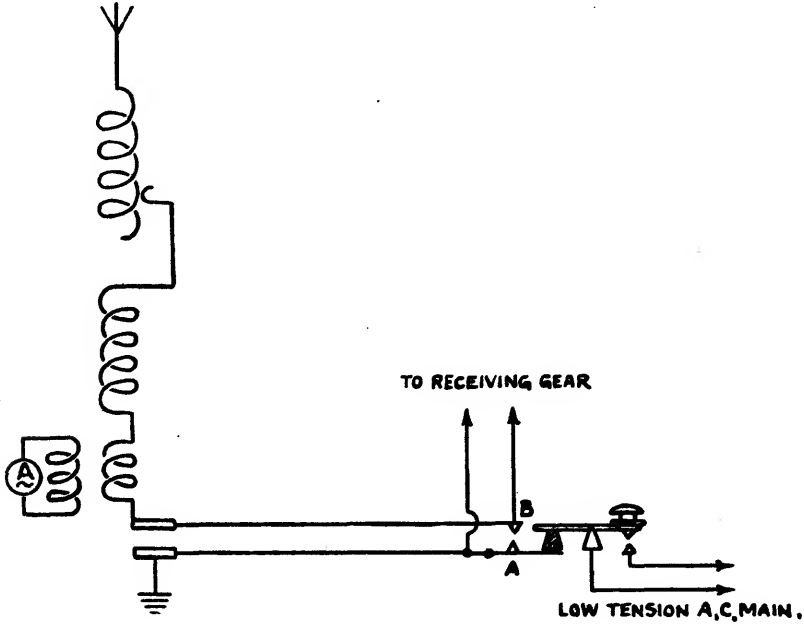
The requirements of the break are :—

- (a) It must make before, and break after, the low-tension charging circuit, to obviate the risk of sparking into the receiving gear.
- (b) It must either be very close to the earth connection or be joined to it by a non-inductive lead. If there were a long inductive lead between it and earth the voltage across it would be considerable, and the break would have to be a long one to prevent sparking taking place across it as it was made and broken.

A suitable arrangement is illustrated in Fig. 230. Here we have a hand key, fitted with two "back contacts," A and B. Contact A is carried on a flexible springy piece of copper.

A and B are connected to aerial and earth by a non-inductive lead of concentric cable, and across them is joined the receiving circuit. When the key is at rest an ebonite thimble on the toe of the key is bearing on the end of the copper strip that carries contact A. The aerial is then connected to the receiving gear.

When the key is pressed, however, its first motion allows A and B to make contact, and thus to complete the aerial circuit to earth, and to short-circuit the receiving gear before the "heel" contacts make.



"Listening Through" Device.

FIG. 230.

Its further motion closes its "heel" contacts, and completes the low-tension A.C. circuit.

CHAPTER IX.

THE POULSEN ARC.

469. Continuous Waves.—In the spark system of energising an aerial for transmitting purposes, discussed in the previous chapter, a condenser is charged to a high potential at regular intervals and allowed to discharge freely across a spark gap, setting up a high-frequency oscillation in a closed oscillatory circuit, which transfers its energy to the aerial circuit.

The resulting wave trains set up in the aerial are as illustrated in Fig. 231.

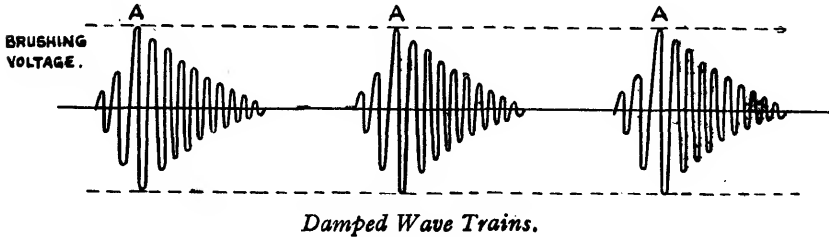


FIG. 231.

These are spoken of as damped wave trains, and they follow each other at the spark train frequency, which depends on the frequency with which the primary condenser is charged up and discharged.

The limit of power that can be put into an aerial is determined by the voltage at which the aerial begins to brush, and it is obvious from Fig. 231 that, if brushing losses are to be avoided, the aerial can be charged up to its maximum safe voltage once only for each wave train—near the beginning of the train of waves. For the

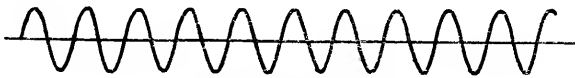
*A Continuous Wave.*

FIG. 232.

rest of the wave train the oscillatory voltage of the aerial is much less than this maximum safe value, and hence the aerial is only radiating energy at the maximum rate for which it is designed over a very small proportion of the time during which the key is pressed.

Quite apart from the variation in the energy radiated during the oscillatory action, the aerial is not radiating at all for the comparatively long intervals between wave trains. If, on the other hand, some means are adopted for maintaining what is termed a "continuous oscillation," or, rather, a uniform radio-frequency alternating current (as illustrated in Fig. 232), in the aerial circuit, the aerial may be charged up to a point near its brushing voltage and will be radiating energy at its maximum rate for the whole of the time during which the transmitting key is pressed.

These and other advantages of continuous waves over damped waves are summarised in the following sections :—

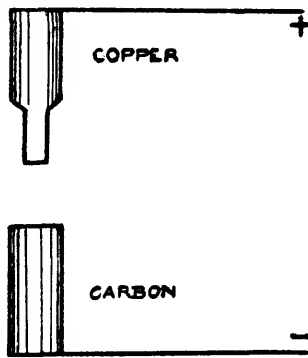
- (1) With the same permissible maximum voltage, more power can be radiated from an aerial system with C.W. owing to continuous transmission ; conversely, to radiate the same power, lower voltages are required in the aerial system, and hence there is less danger of brushing losses.
- (2) Experiments have shown that C.W. does not suffer so much from absorption in its passage through the atmosphere from transmitter to receiver.
- (3) Selectivity and efficiency in reception are much easier to obtain with the C.W. system, because—
 - (a) The pitch of the note heard can be controlled by the receiving operator to suit his ear and to avoid interference, as will be seen in Chapter X.
 - (b) Only one wave frequency is radiated, instead of energy being radiated over a band of wave frequencies, as shown by the energy distribution curves given in Chapter VII. This concentration of energy into one frequency gives increased range of transmission, and prevents interference with other stations.
 - (c) With damped wave trains the receiving aerial, in addition to the forced oscillations imposed on it, is "shocked" into oscillation at its own natural frequency, even although it is not tuned to the incoming wave. With C.W. this transient effect only lasts for a few cycles at the beginning or end of a train of waves, and if the receiving gear is mistuned to the incoming frequency the effect of the forced oscillation alone is very small.
- (4) High-speed automatic transmission and reception are possible with C.W., but not with spark.
- (5) Radio-telephony (R/T) has been made possible by the use of C.W.

Spark telegraphy only maintains its position in certain cases, such as small ship commercial work, because it is simpler and less

expensive, and the receiving apparatus need not be elaborate. In addition, the band of wave frequencies on which energy is radiated, and the general non-selectivity of the system, make it more likely that S.O.S. calls will be picked up. In the Service, spark attachments are fitted to C.W. transmitting sets for emergency purposes.

470. Methods of Generating Continuous Waves.—These are:—

- (a) The Poulsen arc, to be described in this Chapter.
- (b) The thermionic valve, to be described in Chapters XII and XIV.
- (c) The high-frequency alternator, not used in the Service, and not described in this Handbook.



Poulsen Arc Electrodes.

FIG. 233.

471. The Arc.—Suppose that two electrodes, one made of copper and one of carbon, are joined to a high-voltage D.C. supply, as illustrated in Fig. 233, and that the carbon is pushed in until it just touches the copper; then a large current will flow across their point of contact, and each electrode will tend to get intensely hot. In practice, the copper electrode is artificially cooled by means of water-cooling—otherwise it would melt—but the carbon gets white hot at its tip, and as a consequence liberates great numbers of electrons from its surface.

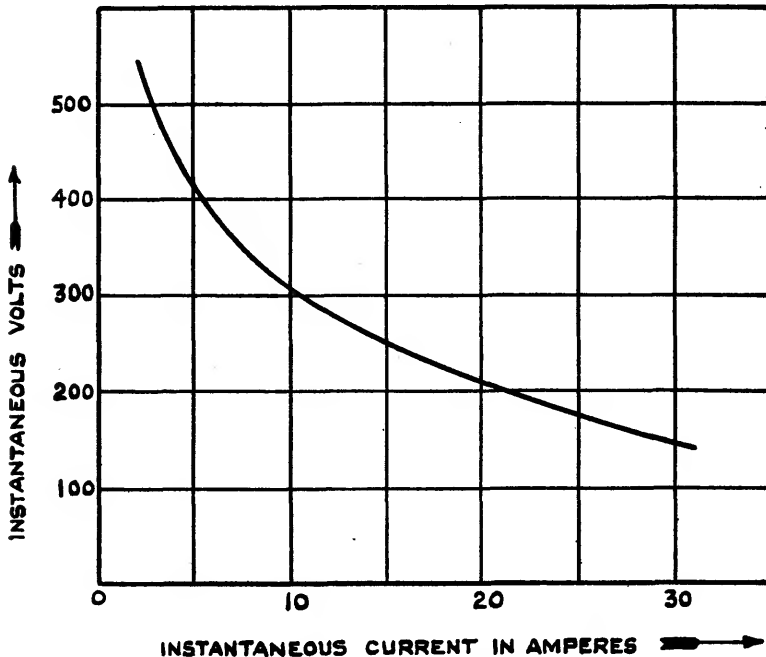
If now the carbon negative electrode—or cathode, as it is termed—is slowly withdrawn, these electrons will fly across to the positive (copper) electrode—or anode—through the intervening air. As a result, numerous collisions between electrons and molecules of air occur, and positive and negative ions are formed.

A gaseous arc is thus created which carries a convection current consisting of electrons and negative ions moving to the anode, and of positive ions moving to the cathode.

This arc can be drawn out in length until the energy supplied is insufficient to make good the energy radiated from it as heat and light. It will then collapse and have to be re-struck.

472. Characteristic Curve of Arc.—Such an arc as this has a property which is common in gaseous conductors of electricity—its resistance decreases as the current through it increases.

The explanation is as follows: The conductivity of the arc depends on its state of ionisation, which, in its turn, depends on the temperature of the electrodes. This temperature is determined by the current passing across the arc, so that, the greater the current, the greater the heating effect on the electrodes, and the



Static Volt-Ampere Characteristic Curve.

FIG. 234.

greater the ionisation. The more ionised vapour there is available, the "fatter" the arc becomes, and the greater is its conductivity and the less is its resistance.

Actually, **if the resistance decreases fast enough** with increase of current, the IR drop across the arc will diminish as I increases, so that the voltage across the arc is less for an increased current and greater for a decreased current. This is always the case with the Poulsen arc.

A physical explanation may be given for this peculiarity of the arc. If the current is increased, there is an increased accumulation of positive and negative ions near the negative and positive electrodes respectively; these are not immediately absorbed by the

electrodes and exert a back E.M.F., which is greater the greater the current. Hence the voltage drop across the arc decreases with increased current.

A graph of voltage drop across an arc plotted against current flowing is given in Fig. 234.

Such a curve is known as the "static volt-ampere characteristic curve" of the arc. The name arises from the fact that it is obtained from a series of fixed current values, with their corresponding voltages. We shall see later that if the current is changing rapidly, the voltage corresponding to each value of current is somewhat different from that given by the curve above.

It can be seen from this static curve that when the current across the arc is 5 amperes, the voltage drop across it is about 400 volts; for 10 amperes the voltage drop is 300 volts; for 15 amperes the voltage drop is 240 volts.

Within this range, therefore, its resistance varies from $\frac{400}{5} = 80$ ohms to $\frac{240}{15} = 16$ ohms.

The utility of the arc as a generator of continuous oscillations depends on the property discussed in this paragraph—that not only does the resistance decrease with increased current, but, in addition, the voltage across the arc decreases as the current increases.

473. Application of the Arc to an Oscillatory Circuit.—Fig. 235 shows an arc connected to a D.C. supply through two choke coils L_1 and L_2 . A steady current is flowing through the chokes and the arc, the actual value of the current depending on the arc length, the applied voltage and the resistance of the circuit.

Now suppose an oscillatory circuit LC to be connected across the arc when burning, by means of the switch X.

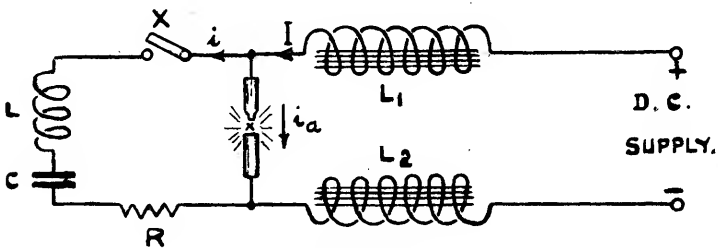


FIG. 235.

Two methods of treatment will be given—one using mathematical results given in Chapter VII, and the other without mathematical formulæ and similar to the explanation of oscillatory action given at the beginning of Chapter VII. The P.D. across the arc will start a current flowing into the condenser.

(a) Let this P.D. be v_a .

We have here conditions which correspond to some extent to the case in Chapter VII, in which an oscillatory circuit is attached to a D.C. supply. The differential equation (para. 427) representing voltage relationships in the circuit is in this case

$$v_a = L \frac{di}{dt} + Ri + \frac{1}{C} \int i \cdot dt,$$

where i is the instantaneous current in the LC circuit.

Let i_a be the instantaneous current across the arc, and I the current from the supply.

Because of the large inductance of the choke coils, I remains sensibly constant, whatever the value of the oscillatory current i .

In other words, we can take the equation, $i + i_a = I$, a constant, as determining the relationship between current in the LC circuit and current across the arc.

Differentiating the equation above :—

$$\frac{dv_a}{dt} = L \frac{d^2 i}{dt^2} + R \frac{di}{dt} + \frac{i}{C}.$$

Also

$$\frac{dv_a}{dt} = \frac{dv_a}{di_a} \times \frac{di_a}{dt}.$$

$\frac{dv_a}{di_a}$ is the slope of the characteristic curve of the arc, and from para. 472 is a **negative** quantity ; it will be written r_a .

$\frac{di_a}{dt} = - \frac{di}{dt}$ since $i_a = I - i$ and I is constant. Hence the differential equation becomes

$$L \frac{d^2 i}{dt^2} + R \frac{di}{dt} + \frac{i}{C} = - r_a \frac{di}{dt}$$

or

$$L \frac{d^2 i}{dt^2} + (R + r_a) \frac{di}{dt} + \frac{i}{C} = 0.$$

By the theory of Chapter VII, this represents an oscillatory action in which the damping factor " α " is $\frac{R + r_a}{2L}$.

But r_a is itself a negative quantity, and if it is **numerically** equal to R , then $R + r_a = 0$. The interpretation of this is that the oscillatory action is **undamped** or continuous, and, once started, will continue with the same amplitude. Hence, if the negative slope of the characteristic is at least equal to the positive resistance included in the LC circuit, it is evident that the arc can maintain continuous oscillations.

It is important to realise that, though the actual D.C. resistance of the arc is positive, the negative slope of the characteristic has the physical significance that, as regards **changes** in current and voltage, r_a is negative ; this may be expressed by saying that the

A.C. resistance is negative. In an oscillatory circuit we are dealing with R.F. A.C. conditions, and if the sum of the A.C. resistances (algebraically) is zero, oscillations will continue undamped.

(b) The non-mathematical treatment is as follows:—The current i flowing into the condenser means a decrease in current i_a across the arc, since $i + i_a =$ current I from the supply, and this is kept constant by the inductance of the choke coils.

The decrease in arc current means an increase in voltage across the arc, as will be seen from the characteristic curve. This increased voltage across the arc results in a further charge into the condenser.

The voltage across the condenser eventually rises to that across the arc, but the inductance in the oscillating circuit makes the charging current tend to persist, and raises the condenser voltage above the arc P.D.

When the magnetic field round the inductance collapses, the condenser voltage reaches its maximum, and, being now greater than that across the arc, the condenser starts to discharge and current flows in the opposite direction.

This current from the condenser cannot flow through the D.C. supply owing to the chokes. It must therefore flow through the arc. The arc current therefore rises above the steady current from the mains during this period; the P.D. across the arc decreases in consequence and facilitates the flow of current from the condenser.

When the condenser voltage has reached the same value as the P.D. across the arc, the inductance comes into play again, and keeps the discharge current flowing.

This finally leaves the condenser with its voltage less than that across the arc, so it commences to charge up again as soon as the discharge current has ceased, and the process is continued.

The current flow in the LC circuit is thus oscillatory, and provided the resistance in this circuit is small enough it is possible to get a constant amplitude of current or continuous oscillation.

474. Graphical Illustration.—The explanation of para. 473 (b) is illustrated in Fig. 236 by graphs which represent the various voltage and current changes during oscillatory action in the LC circuit.

Four curves are given, showing (a) the arc current, i_a , (b) the oscillatory current i , (c) the arc voltage v_a , and (d) the power delivered to, or returned by, the oscillatory circuit.

The arc current i_a in (a) varies about the steady D.C. value I , which it has when the LC circuit is not shunted across the arc.

The oscillatory current i in (b) varies about zero, representing current alternately flowing into, and out of, the condenser.

Since, algebraically, $i_a + i = I$, the constant value of supply current, the variations in curves (a) and (b) are exactly equal and opposite, and the algebraic sum of the two ordinates at the same point in the cycle is constant and equal to I .

The curve of arc voltage, v_a , given in (c) is simply derived by plotting from the "static characteristic curve" of the arc the values of voltage corresponding to the varying values of arc current in (a). It must be particularly understood that the arc voltage does not reverse in direction; it only varies above and below a certain mean value. Its variation is not sinusoidal, although the curve (a) is a sine curve, because the characteristic curve from which it is plotted is not a straight line.

When the arc voltage is above normal, current is flowing into the circuit and energy is imparted to it.

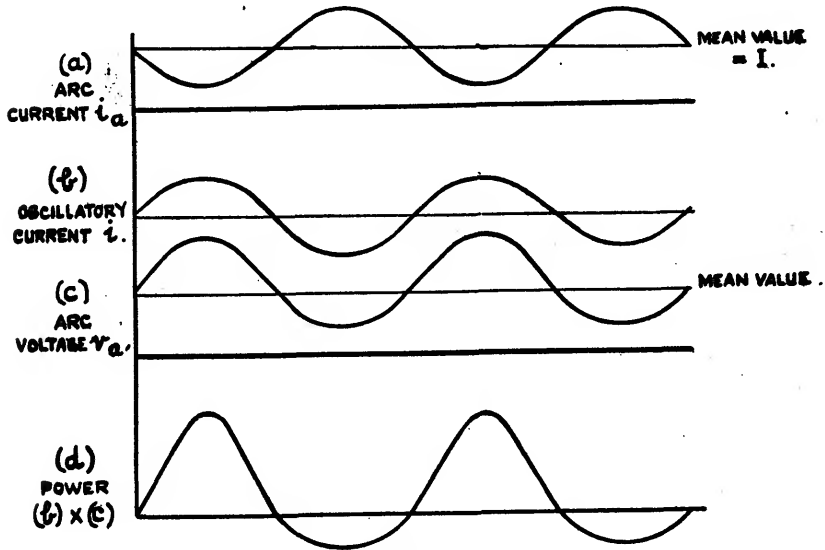


FIG. 236.

When it is below normal, the condenser is discharging and energy is returned by the circuit.

The power curve is given in Fig. 236 (d). If corresponding ordinates of curves (b) and (c), oscillatory current and arc voltage, are multiplied together, their product represents the rate at which energy is being given to, or taken from the circuit. If the product is positive, *i.e.*, during the half-cycles when curve (b) is positive, the power curve is positive; during the other half-cycles the power curve is negative.

Since the base is an axis of time, and the ordinates in curve (d) represent power, being the product of current and voltage, the area between the curve and the axis represents total energy imparted to the LC circuit (if above the axis) and total energy taken from the circuit (if below the axis).

It will be seen that the energy imparted exceeds that returned, because (b) is a sine curve and the ordinates of (c) for points in each

half-cycle at which the ordinates in (b) are equal and opposite are much greater in the positive than in the negative half-cycle.

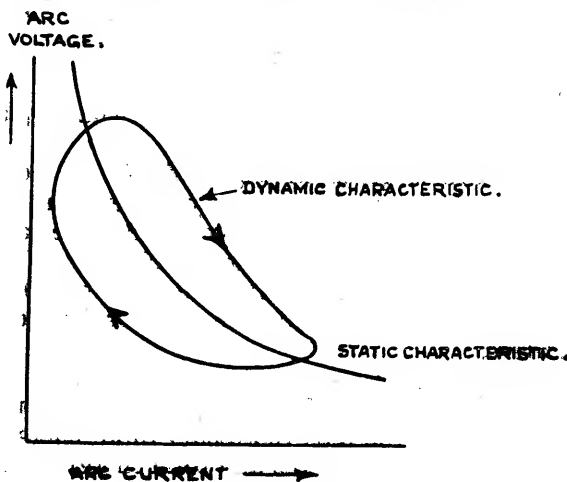
The excess energy is available for overcoming damping losses in the oscillatory circuit, and if it is sufficient to neutralise these, oscillations are continuous.

The steeper the characteristic curve of the arc, the greater is the difference between the heights of the crests and troughs in curve (c), and hence the greater is the excess energy imparted to the circuit. This treatment is thus brought into line with the mathematical treatment of para. (a), where it was shown that the greater the negative A.C. resistance of the arc, which is also the slope of the characteristic curve, the more likely are oscillations to be maintained.

In the above discussion the amplitude of the oscillatory current has been taken as less than the D.C. supply current, so that the arc current does not fall to zero, and the arc is never extinguished. This was the first type of oscillation produced with the arc as a generator, but it was not very efficient for wireless frequencies. In the Poulsen arc, which we shall consider in para. 477, the arc is actually extinguished once per cycle. The type of oscillation in which the arc does not go out is called an " α " oscillation and the other a " β " oscillation.

Before considering the improvements which Poulsen introduced, we shall consider the characteristic curve under "dynamic" conditions and the effect this change has upon the above theory.

475. The Dynamic Characteristic.—The static characteristic curve of relationship between arc current and arc voltage does not hold when the current is changing rapidly. If the current is



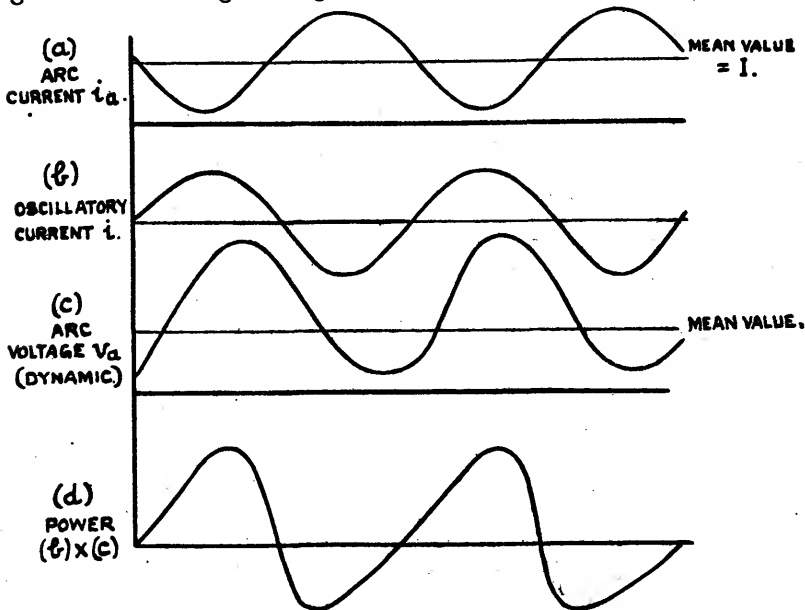
Dynamic Characteristic of Arc.

FIG. 237.

decreasing, say, at a rapid rate, the temperature of the electrodes and the consequent state of ionisation at any instant is really determined by the greater value of current which existed some little time before. In other words, changes in the state of ionisation do not instantaneously follow changes in the value of current flowing, because it takes an appreciable time for ions to form and to disappear. Hence, when the current is decreasing, the P.D. across the arc for any value of current is lower than the P.D. which would be determined from the static curve, and, conversely, when the current is increasing, the P.D. is greater than in the static case. A characteristic curve can be drawn to represent the relationship between the arc current and arc voltage under these "dynamic" conditions, and it takes the form of a loop, as shown in Fig. 237.

The conditions may be summarised by saying that changes in arc voltage lag behind changes in arc current.

476. Application to Oscillatory Action.—If we consider the graphical theory of para. 474 from the correct point of view of the dynamic characteristic, the curve (c), representing arc voltage, will be displaced to the right of the position it occupies in Fig. 236. This can be seen by plotting curve (c) from curve (a) and the dynamic characteristic, or simply from the fact that the dynamic characteristic expresses the lag of arc voltage behind arc current, and this lag means a displacement of the voltage curve, (c) to the right. The new figure is given below (Fig. 238).



Oscillatory Currents and Voltages under Dynamic Conditions.

FIG. 238.

When the power curve (*d*) is now drawn as the product of (*b*) and (*c*), the area above the axis, representing energy imparted to the circuit, is decreased, and the area below the axis, representing energy returned from the circuit, is increased. Hence, owing to the lag of voltage, the excess energy delivered to the LC circuit to overcome damping losses is diminished.

The lag of voltage on current can be regarded as a definite time interval, and depends on the construction of the arc. The primary object of all Poulsen's improvements to the arc, which we consider in the next paragraph, is to make this lag as small as possible, and consequently to ensure greater efficiency as regards maintenance of oscillations.

The higher the frequency of the oscillatory circuit, the less is the period of oscillation, and hence a given time lag of voltage on current means a lag of a larger proportion of one period. That is, in curve (*c*) above, the higher the frequency, the further is the voltage curve displaced to the right out of phase with curve (*b*), and the less is the excess of energy given to the circuit, *i.e.*, the difference between the positive and negative areas of curve (*d*).

Even with all the improvements made by Poulsen to the simple arc, the lag is such that there is a limiting frequency above which the excess energy becomes too small to maintain oscillations.

This frequency is about 250 kilocycles per second.

477. Poulsen's Improvements to the Arc.—The arc, as developed by Poulsen for wireless purposes, comprises a metal box with the two electrodes projecting through its ends, and two pole tips through its sides.

The copper anode and carbon cathode are mounted in heavy porcelain insulators. To ensure stability—that is, a constant arc length and adequate cooling, so that heat is radiated quickly—the following precautions are taken :—

- (a) The cathode, which consists of a short piece of carbon carried in a cathode sheath, is slowly rotated by means of a small auxiliary motor. This ensures that the carbon burns evenly all round, instead of burning only on one point.
- (b) The copper anode is cooled by internal water circulation. It has channels drilled in it, through which water is pumped. The water must be pure; otherwise it will be a conductor, and will put a dead earth on the anode. The sides and, in the case of large arcs, the ends and lid, of the arc chamber are watercooled. In addition to the watercooling, two other special precautions are taken to keep the temperature of the arc as low as possible and so to ensure that the state of ionisation alters rapidly in response to high-frequency changes.

These are :—

- (c) Burning the arc in a powerful transverse magnetic field.
- (d) Burning the arc in a hydrogenous vapour.

478. The Magnetic Field.—In order to produce a strong magnetic field across the arc, two field magnets are placed one on either side of it, as illustrated in Fig. 239 (a)

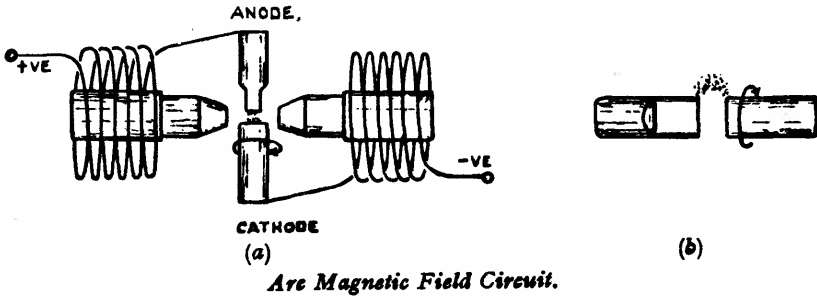


FIG. 239.

They are wound over soft iron cores, whose tips project close to the electrodes.

The windings are joined in series with the D.C. supply mains and the arc electrodes, and thus they act as the choke coils necessary to keep the D.C. supply steady (*see* para. 473). Consequently the arc burns in a powerful magnetic field which acts in such a direction as to bow the arc upwards (Fig. 239 (b)).

Four useful purposes are served by this :—

- (1) The arc is much longer for a given distance between the electrodes and thus has more chance of radiating its heat away.
- (2) It is made to burn on the top edges of the electrodes instead of being free to flicker about from point to point. It is thus stabilised.
- (3) The arc length is self-regulating. When the supply current is high the field strength is great, and the arc is well bowed up. With a lower supply current the field strength decreases and so the arc length is less. The adjustment of arc length to suit various values of supply current is thus simplified.
- (4) The magnetic field helps to extinguish the arc when the current across it falls. It was mentioned in para. 474 that with the " β " type of oscillation developed by the Poulsen arc, the amplitude of the oscillatory current exceeds that of the supply current, so that for a short period during each cycle the current across the arc is zero, and the arc goes out. This type of oscillation will be considered further in para. 480.

479. The Hydrogenous Vapour.—The arc is always burnt in a "hydrogenous" vapour instead of in air. This can be produced by allowing methylated spirit to drip on the copper anode and become "vaporised"—turned into gas—or by passing pure hydrogen or coal gas through the arc chamber. The first method is generally used, being simple and convenient.

The advantages are :—

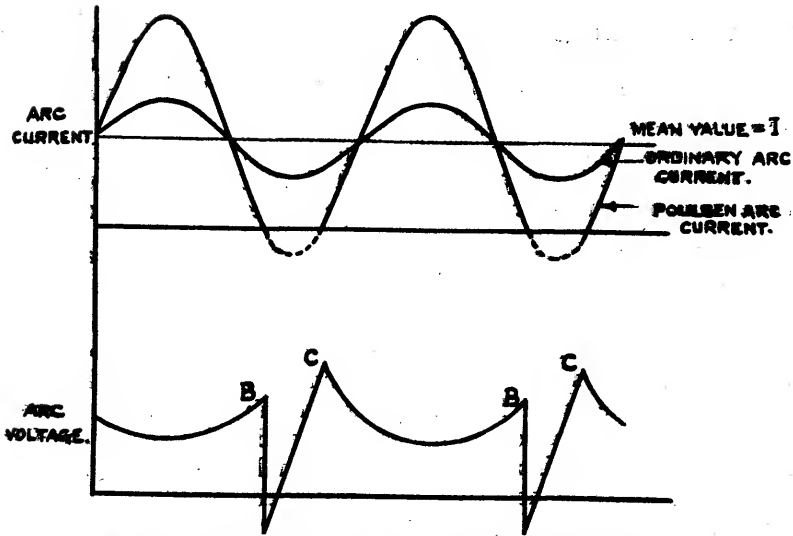
- (1) The heat conductivity of hydrogen is greater than that of air and so the temperature of the arc is lowered.
- (2) The hydrogen ions move very readily between the electrodes, and the state of ionisation therefore follows the changes in current more quickly than in the case of air. In addition, the negative slope of the characteristic curve is much steeper than when the arc is burnt in air, so that for a given current variation the variation in arc voltage is increased and more energy is imparted to the oscillatory circuit.
- (3) The gas helps to prevent the electrodes from becoming oxidised.

480. Oscillatory Action with Arc Extinction.—The type of oscillation so far described (in para. 474) is common to the original type of arc circuit and to the Poulsen arc when the latter is first started up. The oscillation builds up, however, in the Poulsen arc until it becomes of the " β " type, in which the arc goes out for a small portion of each cycle.

Fig. 240 illustrates the changes in arc current and arc voltage with this type of oscillation. As the arc current approaches zero, the potential across the arc increases until it reaches a certain value (at B), known as the "extinction voltage," at which the arc goes out through lack of current, being assisted to do so by the magnetic field. The arc voltage then falls immediately to the voltage across the condenser, which rises almost uniformly during the extinction period because the whole of the supply current is diverted into the condenser, charging it up. Because of the cooling and de-ionisation which goes on during the extinction period, the voltage at which the arc is re-ignited, (C), is higher than its extinction value. The current across the arc is now increasing, and the voltage falls from C onwards until the arc current reaches its maximum. The cycle of events is then repeated.

The behaviour of the Poulsen arc has not yet been thoroughly investigated.

The simplest method of looking at the behaviour of the normal Poulsen arc is to consider it re-struck immediately it goes out. It is found that the R.M.S. value of the oscillatory current in the LC circuit is, in practice, equal to the D.C. supply current divided by $\sqrt{2}$, or $\cdot 707$ times the D.C. supply current, when the arc is burning most steadily. This means that the peak value of oscillatory



Arc Current and Voltage under Extinction Conditions.

FIG. 240.

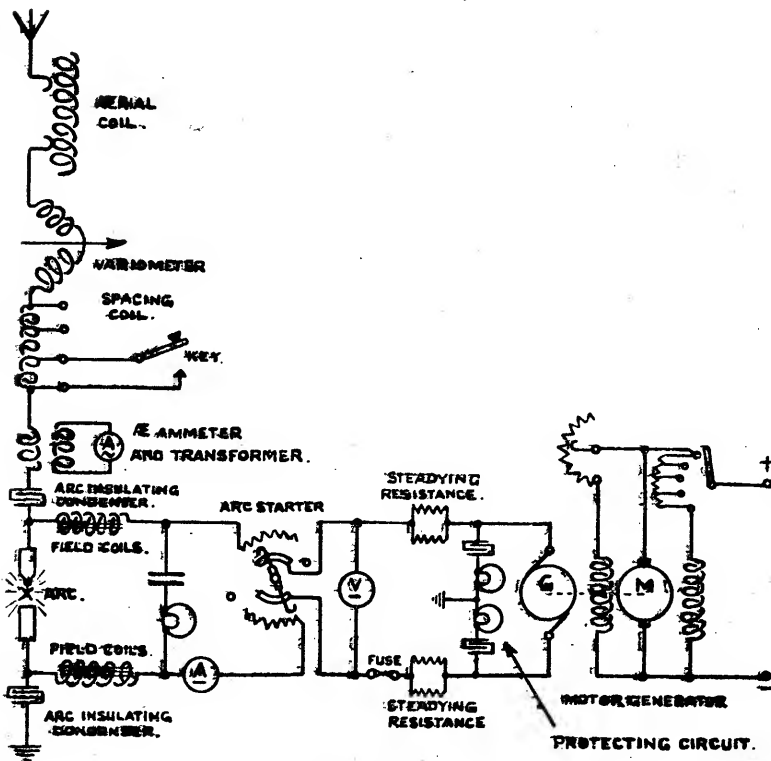


FIG. 241.

current, taken as sinusoidal, is just equal to the D.C. supply, and so the current across the arc just falls to zero once per cycle. This is the best working condition, and is arrived at by a careful design of the magnetic field strength.

If the magnetic field is too strong the arc will be extinguished too long, the oscillatory current will depart greatly from a sine curve, and the arc circuit will radiate undesirable harmonics.

481. Typical Poulsen Circuit.—Fig. 241 illustrates a typical Poulsen circuit. The various items comprised in it will be discussed in detail.

The Aerial Circuit.—It will be noticed that there is no primary circuit, as in the spark system, but that the aerial circuit itself is joined across the arc. The oscillatory circuit is therefore made up of the total inductance in the aerial circuit, the aerial capacity, the equivalent resistance of the aerial (including radiation resistance), and the negative A.C. resistance of the arc, in series. The frequency of the wave transmitted depends on the aerial capacity and the sum of the various inductances—aerial, spacing coil, variometer, etc.—which comprise the total inductance of the aerial circuit.

A coupled aerial is sometimes found in modern arc transmitters.

482. Methods of Signalling.—It is not possible to make and break the charging circuit in order to signal, as is done in spark telegraphy, because the arc will go out each time the circuit is broken.

Two methods of signalling are in use :—

- (1) Marking and Spacing Wave.
- (2) Back Shunt Circuit.

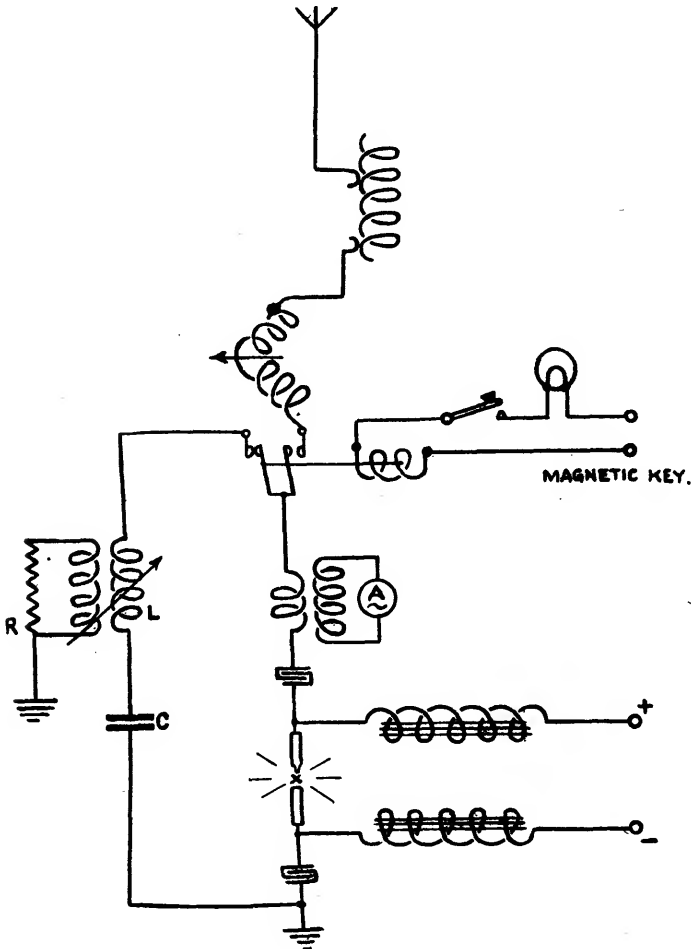
Marking and Spacing Wave.—In this system the aerial is kept oscillating continuously, but when the key (Fig. 241) is pressed, it short-circuits the "Spacing Coil" and so alters the wave-frequency radiated; therefore, one wave—termed the spacing wave—is being sent out when the key is open, and another wave (of higher frequency)—termed the marking wave—when the key is pressed.

If the receiving instruments of the ship receiving the message are tuned correctly to the marking wave, and are sufficiently selective, the spacing wave will not be heard. A difference of 2 kc./s. in frequency between marking and spacing waves is quite sufficient in practice. No difficulty is experienced in transferring from one frequency to another—actually the oscillating system transfers in this way without any transient effects.

The disadvantage of this method is that two different wave frequencies are used, but as they differ by probably only about 1 per cent., the interference with other stations is not excessive. From the Naval point of view a further disadvantage is that the aerial is always alive and an enemy can use direction-finding apparatus continuously.

Example 57.

How much inductance is required in a spacing coil to give a frequency difference of 2 kc./s. when transmitting on 150 kc./s. on a 1.5 jar aerial?



The Back Shunt Circuit.

FIG. 242.

Frequency of marking wave = 150 kc./s.

Frequency of spacing wave = $150 - 2 = 148$ kc./s.

$$\begin{aligned} \text{LC value of marking wave} &= \left(\frac{3 \times 10^4}{2\pi \times 150} \right)^2 \\ &= \left(\frac{100}{\pi} \right)^2 = 1013.2 \text{ mic-jars.} \end{aligned}$$

$$\text{LC value of spacing wave} = \left(\frac{3 \times 10^4}{2\pi \times 148} \right)^2$$

$$= 1040.4 \text{ mic-jars.}$$

$$\text{Total inductance required in aerial for marking wave}$$

$$= \frac{1013.2}{1.5} = 675.5 \text{ mics.}$$

$$\text{Total inductance required in aerial for spacing wave}$$

$$= \frac{1040.4}{1.5} = 693.6 \text{ mics.}$$

Hence the amount of extra inductance necessary for the spacing wave (and short-circuited when marking) is
 $693.6 - 675.5 = 18.1 \text{ mics.}$

The respective wavelengths radiated are

$$\frac{3 \times 10^5}{150} = 2000 \text{ metres (marking)}$$

$$\text{and } \frac{3 \times 10^5}{148} = 2027 \text{ metres (spacing).}$$

483. The Back Shunt Circuit.—A method of signalling which gets rid of the spacing wave is illustrated in Fig. 242. The dummy non-radiating circuit (LC) is called a "Back Shunt" circuit. The magnetic key connects the arc alternately to the aerial circuit and to the back shunt circuit. When the hand key is not pressed, the left-hand magnetic key contact is made, and the arc oscillates on the back shunt circuit.

When the hand key is pressed, the right-hand spring finger of the magnetic key is pulled on to the arc contact piece and the left-hand spring finger is pulled off; the arc then sets the aerial circuit in oscillation. Thus either the aerial circuit or the back shunt circuit is energised according to whether the hand key is pressed or released.

The advantages of this method of signalling, which is normally used in the Navy, are that no wave is radiated except when the key is pressed, and that only one wave frequency is used. The arc can be started up without any indication that a signal is to be made before the arc is in a fit condition to transmit.

In order to prevent sudden changes in the arc current it is necessary to ensure that the effective resistance of the back shunt circuit is equal to the effective resistance of the aerial. For this purpose the resistance R is provided, and is coupled inductively to L through a variable coupling coil. The energy absorbed by this resistance is equivalent to the energy which would be absorbed by a certain resistance in series with L and C.

The necessary adjustment is readily effected by varying the coupling until the current, as read on the ammeter A, is the same whether the hand key is pressed or released.

In the sets used in the Naval service, the back shunt circuit has only two possible LC values, so that, in general, the frequency at which it oscillates is different from the aerial circuit frequency. There is no difficulty in getting the arc to oscillate on two different frequencies, as was mentioned in para. 482, but the adjustment of arc length and equivalent resistance becomes more difficult. (See para. 487.)

484. The other instruments essential to the aerial circuit are shown in Fig. 241.

In order to insulate the direct-current mains from the transmitting earth and from the aerial, condensers of large capacity are placed above and below the arc, as shown.

As they are of large capacity they afford no obstacle to the oscillating current, but prevent :—

- (a) A shock being received from the D.C. supply if the aerial is touched ; and
- (b) A short-circuit if another earth occurs on the charging mains.

An accurately-reading Aerial Ammeter is necessary, and is fitted, as shown diagrammatically, across the secondary of a small "Toroidal Transformer," which is provided in order to step the aerial current down to a value suitable for passing through the ammeter.

The Variometer is provided as a means of fine tuning.

It was also formerly used to attract the attention of the operator of the receiving ship. His heterodyne circuit (see Chapter X) might be set exactly in tune with the transmitted wave ; he would then be on the "dead space" of the transmitting ship, and would hear nothing. The operator of the transmitting ship, therefore, before commencing his message, rotated his variometer, and thus altered his transmitted frequency. This caused the transmitted wave to move out of the receiving ship's dead space, and the attention of the receiving operator was attracted. This practice has now been prohibited owing to the interference it caused.

The Aerial Coil must have a large inductance, since the waves used with the Poulsen System are long ones. It must have a good current-carrying capacity, and good insulation between turn and turn.

485. **The Direct-current Circuit.**—The direct current is supplied by a motor generator or motor booster. A generator field regulator is fitted for varying the voltage applied to the arc.

A 220-volt supply may be sufficient if the insulation resistance of the aerial is high, if there are not many turns of the aerial coil in use, or if a large aerial current is not required.

If, however, the aerial is heavily damped—as it generally is in a submarine, for instance—or if a long wave is being used, necessitating the addition of a good deal of inductance (and its accompanying resistance), and if the maximum possible range—and therefore aerial current—is required, the applied voltage will have to be raised considerably.

A double-pole resistance—termed the Arc Starter—is fitted in the D.C. mains, to prevent a sudden rush of current when the arc is first struck. This resistance is gradually cut out as the arc is opened up.

A D.C. voltmeter is provided on the live side of the arc starter and a D.C. ammeter is joined in the negative main.

It should be noted that when the arc length is correctly adjusted the reading of the aerial ammeter should be the R.M.S. value of, *i.e.*, 0.7 times, the reading of the D.C. ammeter; *e.g.*, if the D.C. ammeter is reading 10 amps., the aerial ammeter should read 7 amps.

486. Arc Main Protecting Circuit.—It is essential to guard against the possibility of radio frequency oscillations finding their way back through the charging circuit and causing damage, in the same way as in spark circuits (Chapter VIII, para. 462). This is effected by providing an easy oscillating path for the radio-frequency currents both close to the magnet coils and also close to the generator; for this purpose one or more large-capacity condensers and lamp resistances are provided, and afford an easier path for these currents than the high inductance of the generator armature.

(Remember that a condenser is an obstacle to direct current or audio-frequency alternating current, but an easy path for radio frequency current, while an inductance is an obstacle to radio frequency current, but an easy path to direct current or audio-frequency alternating current.)

487. Adjustment of Arc Length, etc.—The adjustment of the arc length is dependent on the supply voltage and the LC value and damping of the oscillatory circuit.

For a given D.C. voltage, the arc must be opened out until the aerial current has risen to a maximum value and is perfectly steady. If more aerial current is required, the supply voltage should be raised and the arc length increased.

If the aerial is very efficient, the D.C. voltage required will be low and the arc length will be short. Under these conditions the arc is liable to become unsteady, as it becomes excessively hot. It may even pay to put some artificial resistance in series with the aerial, increase the D.C. voltage and increase the arc length.

When adjusting a fixed LC value back shunt circuit to give equivalent damping, the procedure is as follows:—

The arc is started up on the back shunt circuit. The arc length is adjusted until the aerial ammeter reads its maximum value.

If more oscillatory current is required, the D.C. voltage should be increased and the arc lengthened.

The correct value of oscillatory current having been obtained, the voltage, arc length and damping are reduced until the current is about two-thirds of its maximum value.

The key is pressed and the aerial circuit thus switched in. The arc length and voltage are again adjusted to give the desired value of aerial current.

Lastly, the back shunt circuit is switched in again and the damping adjustment altered until stable oscillations are obtained **with the arc length which is correct for the aerial**, the current value being as nearly as possible the same as that with the aerial in circuit. It is not always possible to get an exact balance of arc length as well as of current, and it is more important that the arc length should be correct for both circuits. With a correct arc length, the arc burns silently; if too short, it is noisy, with occasional hissing; and if too long, it is noisy and liable to collapse, particularly during signalling.

488. Special Conditions met with in Submarines.—It will be of interest to describe two circuit arrangements used under special conditions which arise with submarines.

The first condition occurs when a submarine first rises to the surface: her deck insulator is wet with salt water, which offers an easy conductive path straight to earth, and so throws a very heavy load on the arc, which may refuse to start oscillating at all.

The second occurs when the submarine is running awash and spray is intermittently splashing the deck insulator. This will momentarily lower the aerial insulation resistance, and will cause violent fluctuations in the arc current, sufficient perhaps to put the arc out.

489. The Drying-out Condenser.—To deal with the first condition, the drying-out condenser is used. With the ordinary arc circuit, the voltage across the deck insulator would be very great: this voltage, being applied to the low-resistance path across the damp surface of the insulator, would try to force a big current through this path: the arc would thus have a heavy load imposed on it, and would refuse to start up.

To get over this difficulty the condenser marked C (Fig. 243) is connected up for a minute or so. It is then in parallel with the aerial capacity, and so increases the LC value and decreases the natural frequency of the aerial circuit. Thus the voltage across the deck insulator per aerial ampere is decreased (being equal to ωL_1 volts, and ω being decreased); the arc has imposed on it a load with which it can easily deal, and a small oscillatory current flows over the surface of the deck insulator and quickly dries off the moisture. The condenser is then switched out and signalling is commenced.

The following practical figures may be of interest. If σ is 0.5 jar and $L_{\infty} = 2,400$ mics, the L.C. value of the aerial is 1,200 mic-jars, and the frequency of the circuit is 138 kc./s. The voltage at the deck insulator per ampere of current in the aerial is about 2,000 volts.

With a drying-out condenser, value 6.3 jars (as in a Service set), switched in, the total capacity becomes 6.8 jars, the L.C. value 13,600 mic-jars, and the frequency 37.4 kc./s. The voltage at the deck insulator per aerial ampere is 530 volts, so that there is much less current flowing in the shunt resistance and less damping in the circuit.

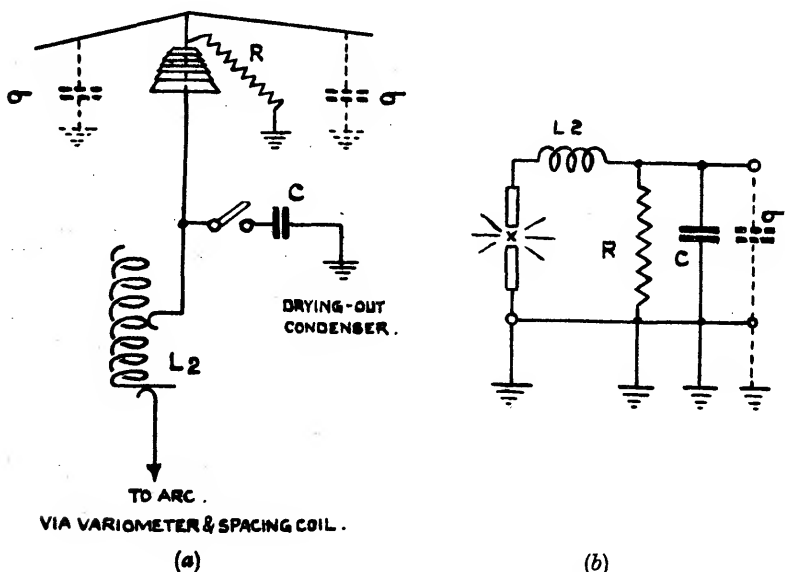


FIG. 243.

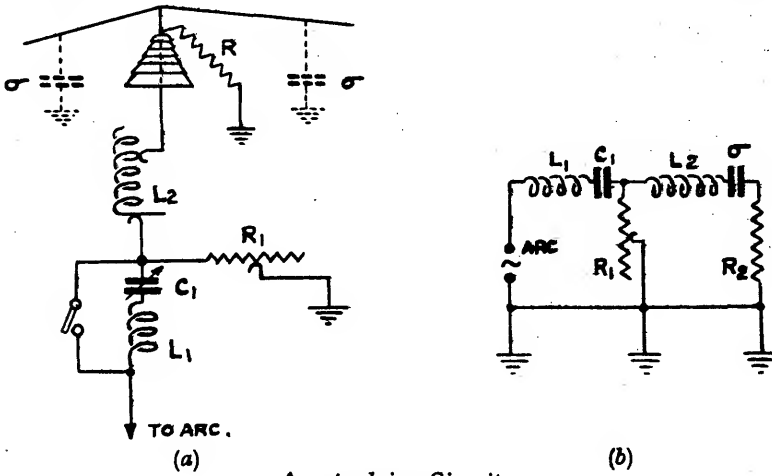
It was seen in para. 310 that a resistance R in parallel with a condenser can be replaced by a series resistance of value

$$\frac{1}{\omega^2 C^2 R} = \frac{LC}{C^2 R} = \frac{L}{CR}$$

In this case, L remains the same, and C is increased from 0.5 to 6.8 jars, so that **the equivalent series resistance of the leakage path is diminished** in the ratio $\frac{0.5}{6.8} = \frac{1}{13.6}$, or **inversely in the ratio of the capacity values**. The arc can start up much more easily with a circuit in which the resistance is decreased so much. The frequency which is transmitted during the drying-out period is much lower than any wave-frequency in use in the Service, and so no additional interference results.

490. The Arc-steadying Circuit.—The arc-steadying circuit is supplied to deal with the less serious case of aerial damping when the deck insulator is being splashed intermittently.

The additional apparatus, as shown in Fig. 244 (a), consists of a condenser C_1 , an inductance L_1 , and a resistance R_1 . $L_1 C_1$ and $L_2 \sigma$ are made equal and correspond to the frequency it is desired to transmit.



Arc-steadying Circuit.

FIG. 244.

The adjustable resistance R_1 is joined to earth from the mid-point of the two circuits. Fig. 244 (b) shows the circuit diagrammatically. The insulation resistance R , which is in parallel with the aerial capacity in Fig. 244 (a), is replaced, in accordance with the theory also used in the last paragraph, by its equivalent series resistance R_2 , which therefore represents the damping effect of the faulty insulation. R_2 varies inversely as R .

The arc will start up on the auxiliary circuit $L_1 C_1 R_1$ and the voltage drop across R_1 will set up oscillations in the coupled circuit $L_2 \sigma R_2$. Since the circuit $L_2 \sigma R_2$ is tuned to the same frequency as $L_1 C_1$, which determines the frequency of the oscillations, the theory of resistance-coupled circuits with a resonant applied frequency can be used. The equivalent resistance of the whole circuit is that of R_1 and R_2 in parallel, *i.e.*,

$$\frac{R_1 R_2}{R_1 + R_2}.$$

However great R_2 becomes, the combined damping effect of these resistances will always be less than that of the smaller of the two.

R_1 is thus the controlling factor so long as the aerial insulation resistance is low (R_2 high), and, however suddenly the aerial insulation varies, the arc load will remain fairly constant.

The aerial current depends on the value of R_1 . If R_1 is small, the arc will burn easily, but the coupling to the aerial will be loose, and not much current will flow in the aerial circuit. If R_1 is too large, the total equivalent resistance will be high, and the damping considerable whenever R_2 has a high value, reducing the oscillatory current in both circuits. For any given condition of aerial intermittent damping, the best value of R_1 must be found by trial.

The current in the aerial will fall off considerably whenever the insulation resistance gets very small, *i.e.*, R_2 large, so it will be necessary to repeat two or three times the message that is being made.

491. Advantages and Disadvantages of the Arc System.—As compared with the Spark System, the arc possesses the merits of giving greater ranges, power for power, and sharper tuning.

It is very straightforward in design, and arcs up to 1,500 kW. are in operation to-day.

The great drawback from the Naval point of view is that the arc is not suitable for quick signalling.

The arc has to be struck before signalling is commenced, and switched off again before reception—it is not possible to “listen-through” while it is burning—whereas in the spark and valve systems, signals can be received whenever the transmitting key is not pressed.

This point is not of such importance in point-to-point working between shore stations, as during transmission reception may take place simultaneously on another wave and on another aerial.

CHAPTER X.

RECEPTION AND DETECTION OF ELECTRO-MAGNETIC WAVES.

492. General Principles.—We have seen in the preceding chapters how radio-frequency currents are generated in the aerial circuit of the transmitting station by means of the spark and arc methods. In Chapter XIV the valve as a transmitter will be investigated.

These radio-frequency currents set up electromagnetic waves in the æther, which spread out radially in all directions from the transmitting station. The passage from transmitter to receiver, and the constitution of an electromagnetic wave, will be treated in more detail in Chapter XVII. It is sufficient here to state that an electromagnetic wave, which encounters a conductor, and more especially an aerial, induces in it **alternating voltages and currents of very small magnitude and of the same frequency as that of the wave itself**. The science of reception consists in devising apparatus which will make these minute currents perceptible to the senses, the eye or ear. If the amplitude or frequency of the currents in the transmitting aerial can be varied by means of a signalling key (W/T) or by means of the voice (R/T), and these effects can be reproduced at the receiving end, communication will be rendered possible. It must be clearly understood that exact measurement of the currents in the receiving apparatus is not necessary; all that is wanted is that starts and stoppages and variations of the transmitter currents should give corresponding results in the device which affects the senses (*e.g.*, the telephone for aural reception, or a morse inker for visual reception). We are concerned practically with aural reception, and the theory in this chapter will be based on that assumption.

Any station which wishes to receive a particular signal has two main objects in view :—

- (a) To receive a signal intended for it as loudly as is comfortable to read, *i.e.*, **audibility**.
- (b) To prevent interference with this signal by other signals not intended for it, *i.e.*, **selectivity**.

The ordinary receiving arrangements may be summarised broadly as consisting of :—

- (a) **A circuit** which will respond most readily to waves to which it is tuned, and will not respond to waves out of tune with it, actuating
- (b) **A detector**, which turns the high-frequency received wave trains into the most suitable form for energising

- (c) Some device (generally a pair of telephones) which will render the wave trains perceptible to the senses.

These three divisions of receiving gear will be dealt with in detail in this chapter.

At this point, only a brief description will be given of the circuit, this being dealt with more fully later on.

493. The Circuit.—Fig. 245 illustrates a typical form of receiving circuit, which may be used in a preliminary discussion of broad principles.

Since an electromagnetic wave induces in a receiving aerial E.M.F.s. of the same frequency as itself, the resultant oscillatory current in the aerial will have a maximum value if the aerial is tuned to the same LC value as that which corresponds to the

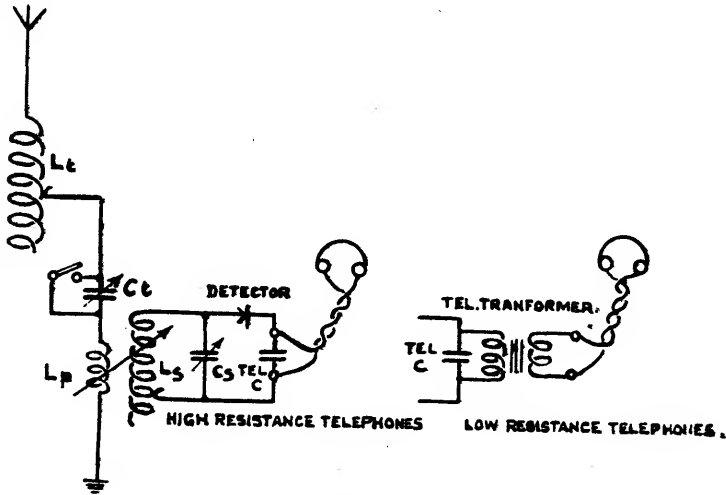


FIG. 245.

frequency of the wave—i.e., the same LC value as that of the transmitting aerial. The greater the oscillatory current in the receiving aerial, the greater will be the oscillatory voltage which can be applied to the detector, and, as we shall see, the greater will be the audibility of the signal. The first essential is, therefore, to tune the aerial circuit so that it is **resonant** to the voltage induced in it. We may regard the effect of the wave as being represented by an alternator inserted in the aerial circuit, giving a radio-frequency voltage of very small amplitude, which produces forced oscillations in the aerial.

Aerial Circuit.—The various components in Fig. 245 are as follows :—

“ L_t ” denotes the “**Aerial Tuning Inductance**” or **A.T.I.** for adding inductance in series with the aerial.

" C_t " denotes the "**Aerial Tuning Condenser**" for adding capacity in series with the aerial.

" L_s " denotes the **primary** of an inductive coupling to the "Secondary Circuit."

Unless the LC value of the incoming wave is the same as the LC value of the receiving aerial itself (as given by the product of its natural inductance and natural capacity), we shall have to add inductance in series, using the A.T.I. to increase the LC value, or else capacity in series (using the aerial tuning condenser) to decrease the LC value, and so make the aerial a resonant circuit.

Example 58.

It is required to receive (a) a wave whose LC value is 400; (b) a wave whose LC value is 20, on an aerial whose natural inductance is 30 mics., and natural capacity σ is 1.5 jars; the primary (L_s) of the inductive coupling to the secondary circuit has a value of 40 mics.

$$(a) \text{ Total inductance required } = \frac{LC}{\sigma} = \frac{400}{1.5} = 266.6 \text{ mics.}$$

$$\text{Inductance already in circuit} = 30 + 40 = 70 \text{ mics.}$$

$$\text{Additional inductance required on tuner} = 196.6 \text{ mics.}$$

$$(b) \text{ Total capacity required } = \frac{LC}{L} = \frac{20}{70} = \frac{2}{7} \text{ jar.}$$

$$\text{Capacity already in circuit} = 1.5 \text{ jars.}$$

Additional capacity (C_t) must be added such that

$$\frac{1}{C_t} = \frac{1}{C} - \frac{1}{\sigma} = \frac{7}{2} - \frac{1}{1.5} = \frac{7}{2} - \frac{2}{3} = \frac{17}{6}.$$

$$C_t = \frac{6}{17} = 0.35 \text{ jar.}$$

494. Secondary Circuit.—The detector, which is a voltage-operated device, must be inserted in the receiving circuit in such a way that across it are impressed as large variations of potential difference as possible.

It might be connected up across an inductance in the aerial circuit itself, but such an arrangement is very unselective.

In general, the detector is fed from a separate circuit of its own, inductively coupled to the aerial circuit.

This circuit is known as the "**Secondary Circuit**."

The inductance (L_s) consists of a large adjustable inductance, whose coupling to the primary can also be varied. It is arranged to have many more turns than the primary, and therefore by means of mutual induction a greater E.M.F. may be induced in the secondary circuit than the voltage across L_s itself.

Across L_s is joined an adjustable condenser (C_s), known as the "**Secondary Condenser**."

The secondary circuit is tuned to have the same LC value as the aerial circuit and the incoming wave.

The secondary circuit being thus resonant to the E.M.F. induced in it, the current in the secondary circuit has a maximum value. **The oscillatory voltage set up across the secondary condenser by this current is applied to the detector.**

If we wish to apply the greatest possible voltage to the detector, L_s is made as large as possible and C_s as small as possible, since the value of the voltage across the condenser is given by $I_s \sqrt{\frac{L_s}{C_s}}$ volts, where I_s is the secondary current in amperes.

This brief description of a typical form of circuit shows the mechanism by which high-frequency oscillatory voltages of as large amplitude as possible are available to affect a detecting device. Further reference to receiving circuits, and their different types, will be made later in the chapter.

495. The Telephones.—The theory of the **Telephones** will be taken next. A pair of telephones is the device most widely used for making W/T signals perceptible to the senses. Other devices are used for automatic reception, such as a dictaphone, an inkler making a mark on a piece of tape, or a moving light ray making a mark on a sensitised film; but these systems have the drawback that they cannot distinguish between the notes of various incoming signals.

The function of the telephones is to convert variations of electric current into audible sounds.

A telephone receiver consists essentially of a permanent magnet with two pole pieces N and S, on which are wound a very large number of turns of fine wire, and a diaphragm. The diaphragm is a circular piece of very thin sheet iron, supported all round the edge by the outer case or shell of the "ear-piece" as close to the faces of the pole pieces as possible without actually touching. Fig. 246 shows diagrammatically the arrangement of the component parts of a telephone.

Normally, with no current passing through the coils, the permanent magnet exerts a pull on the diaphragm, which is therefore slightly bulged inwards, but not sufficiently to touch the pole pieces.

Action.—The action, with current passing through the coils, is as follows:—

If a current is sent through the coils in such a direction that the flux due to the permanent magnet is increased (*i.e.*, temporary magnetisation assisting the permanent magnetisation), the pull on the diaphragm will be increased, and it will be attracted still closer to the pole pieces, taking up the position shown by the dotted line " D_1 ". If, on the other hand, a current is sent through the coil in the opposite direction, so that its effect is in opposition to that of

the permanent magnet, the pull on the diaphragm will be decreased, and it will recede further from the pole pieces than normally, and take up the position " D_2 ."

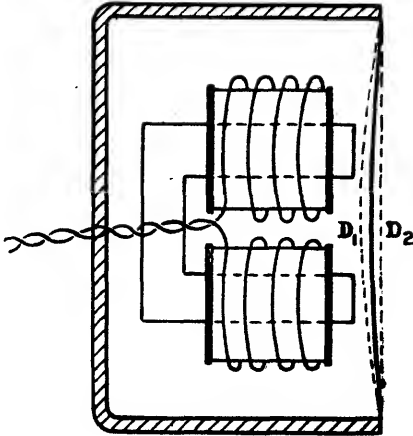


FIG. 246.

If these movements of the diaphragm are repeated at a frequency which corresponds to a note audible to the human ear, this note will be heard as a result of the wave motion set up in the air by the diaphragm vibration. Such a repetition of the diaphragm movement will occur if alternating current of a suitable frequency is passed through the coils, or if a steady current through the coils is either increased or decreased at an audible frequency.

496. Permanent Magnetisation.—The stronger the permanent magnet, the greater is the sensitivity of the telephones, up to a limit determined by the saturation point of the magnet.

This follows from the fact that the pull on the diaphragm is proportional, not to the flux density itself, but to its square. The amplitude of movement of the diaphragm from its unflexed position may be taken to be proportional to the magnetic pull.

We may therefore represent the distance through which the diaphragm centre is pulled by the permanent magnet as being given by KB^2 , where B is the flux density due to the permanent magnet.

If a current i flows through the coils so as to increase the flux density by an amount b per unit of current the new flux density will be $(B + ib)$, and the distance the diaphragm is strained will be $K(B + ib)^2$.

The distance through which the diaphragm moves consequent upon the current i flowing is therefore given by

$$K(B + ib)^2 - KB^2 = 2KBib + Ki^2b^2.$$

If the permanent magnet were not present the displacement of the diaphragm would only be Ki^2b^2 . The displacement is therefore

increased by an amount $2KBib$, and if B is large compared with b , as is frequently the case, the contribution of the term Ki^2b^2 by itself to the total displacement is comparatively negligible.

Taking the term $2KBib$ only, the amplitude of movement is seen to be directly proportional to the exciting current i ; to the flux due to the permanent magnet represented by B ; and to b , which is the additional flux density per unit current, and hence is proportional to the number of turns of wire. Actually " ib " may be grouped together and regarded as the **ampere-turns** of the coil.

It is not possible to increase the amplitude of vibration indefinitely, because as B increases, saturation effects set in; and if the iron becomes saturated as a result of the permanent magnetisation, the currents flowing through the coil produce only a very minute variation of flux density (b is nearly zero), and a correspondingly minute variation of the position of the diaphragm.

497. High and Low-resistance Telephones.—As was seen in the last paragraph, the diaphragm movement is proportional, amongst other things, to the number of ampere-turns in the coils.

The telephones have to be joined in series with the detecting device, which has usually a very high resistance, with the result that only minute currents flow through the telephone receiver coils. Thus to get any appreciable effect with these small currents it is necessary to make receiver coils with a great many turns, winding them with a great length of wire, which must be very fine in diameter to go into the small space of the receiver. In other words, the telephone coils must have a **high inductance**. Unfortunately, a great length of fine wire means that the coils will have a high resistance, and we find that each ear-piece may have a resistance of 4,000 ohms, or 8,000 ohms for the two joined in series. A custom has arisen of speaking of such telephones as "high-resistance telephones," as if there were some merit in their being of high resistance, whereas what we really want (and use) are "high-inductance" telephones.

Generally a pair of low-inductance—or "low-resistance"—telephones are used in combination with a **telephone transformer**, which steps voltage down and current up, to compensate for the decrease in the number of turns. This is indicated on the right of the typical diagram, Fig. 245.

The following are the **advantages** consequent upon the use of a telephone transformer and low-resistance telephones:—

- (1) There are fewer turns in the windings, which can therefore be more robust and better insulated, given the same space.
- (2) There are lower voltages across the windings, and hence less liability to break down the insulation, which may get damp.

- (3) There is no constant current in the telephones, which is generally present when the telephones are connected up in the detector circuit. The disadvantage of a constant current is that, if the telephones are joined up the wrong way round, it will produce flux in opposition to that of the permanent magnet, and demagnetise the latter.

The net result of the above is that there is economy due to durability and the lower initial cost of low-resistance telephones. The fact that low-resistance telephones are used **does not increase the signal strength**, because the increase in current is counterbalanced by the decrease in turns of wire.

The only **disadvantages** are that extra space is needed for the transformer, and a little power may be lost in the transformer.

DETECTORS.

498. The Necessity for a Detector.—It has already been explained that wireless waves are generated in two forms—Damped Waves and Undamped Waves.

Damped Waves are generated by the Spark and Interrupted Continuous Wave (valve) systems; Undamped Waves by the Poulsen Arc, Valve and High-frequency Alternator systems.

Wireless waves in general consist of currents at **oscillatory (or radio) frequency**, which may be anything from 30,000 to 30,000,000 cycles per second.

It is obvious that the high inductance of the telephones will not allow the passage of current at radio-frequency, nor, would the diaphragm vibrate at such a high frequency, nor, again, could the human ear respond to such radio-frequency vibrations, as they would be above the limit of audibility.

Audio-frequency.—The human ear can distinguish sounds caused by air vibrations between limits of 16 and 20,000 cycles per second, but much prefers frequencies between 800 and 3,000.

Most telephone receivers will also respond to frequencies up to 5,000 cycles, but give the best response at frequencies in the neighbourhood of 800—1,000 cycles.

In order to produce an audible sound it is necessary :—

- (a) To **modulate** the wireless wave, *i.e.*, to break it up into a series of groups or pulses which follow one another at audio-frequency.
- (b) To **rectify** the modulated pulses, the effect of which is to turn each group of radio-frequency waves into **one** variation of current through the telephone windings, and hence produce **one movement of the diaphragm** as the **cumulative effect** of the many constituent radio-frequency waves in the pulse.

In the **spark system** the wave is modulated at the transmitter, because oscillations only occur each time the spark gap breaks down ; the sparking rate, or spark train frequency, depends on the alternator frequency and on the rate of revolution of the spark gap (in the case of rotating gaps). The sparking rate generally lies between 300 and 1,000 wave trains per second.

In the **I.C.W.** (interrupted continuous wave) **valve system** the wave is again modulated at the transmitter, as explained in Chapter XIV.

In the case of **continuous** waves, whether produced by the arc, valve, or high-frequency alternator system, the modulation is effected in the receiving circuit, as explained in para. 519 of this chapter.

499. Rectification.—The **rectification** mentioned as necessary in the last paragraph is achieved by means of a **detector**.

The essential characteristic of a **detector** is that it should have **unilateral, or one-sided, conductivity**, so that equal amplitudes of voltage applied to it in opposite directions should result in unequal currents passing through the detector. In other words, the resistance of a detector does not conform to Ohm's Law ; or the curve of voltage against current is not a straight line.

A modulated pulse, or wave-train, of radio-frequency voltage variations, applied to such a device, will give, for each single complete cycle of voltage variation, an excess of current in one definite direction ; and for the whole pulse these excess currents will be additive ; and so, by a suitable arrangement of circuits, **this cumulative uni-directional current** may be made to pull the telephone diaphragm out of position **once only** as a result of the whole wave-train.

500. Types of Detector.—In order to detect a signal, many different devices have been used. The most important, in historical order, are the Coherer, the Electrolytic Detector, the Magnetic Detector, the Crystal Detector, and the Valve Detector.

These differ widely in their appearance and nature, but all achieve the same purpose, viz., to turn a group of high-frequency oscillations into a single current variation, and hence a number of groups themselves occurring at audio-frequency into an audio-frequency current variation, through the device which renders signals perceptible to the ear.

The only two which are in use to-day are the Crystal Detector and the Valve Detector, and the latter is rapidly replacing the former.

As, however, the crystal detector illustrates very conveniently the principle of rectification without the complication produced in a receiving circuit by the introduction of a valve as detector, a full description of its action will now be given. The Valve Detector will be discussed in Chapter XII. The essential feature for detection

is the same in both devices, viz., a conductance that varies with the applied voltage.

501. The Crystal Detector.—A peculiar property of certain combinations of two crystalline substances in contact with each other, or of a crystal in contact with a metal, is that they form a conducting path whose resistance varies according to the direction and also the amplitude of the voltage applied across them. In other words, (a) for equal applied voltages, they allow more current to pass in one direction than the other, and (b) the ratio of voltage to current, for varying voltages in the same direction, is not a constant.

The actual values of resistance, for crystals commonly employed, are high, of the order of 10,000 to 100,000 ohms. (For small applied voltages the current may be zero and the resistance infinite.) Such combinations are bornite and zincite, zincite and tellurium, or carborundum and steel, to name the ones most generally used. Many other combinations are used, but the above will serve our purpose for discussion.

Such combinations as the above are known as "Crystal Couples."

A crystal and metal combination is usually of the following form:—A fragment of the crystal is mounted in a small brass cup, and the metal contact is held against it by a spring.

Usually the sensitivity of the combination depends very much on the contact pressure, and it is also liable to be affected by oxidation. A certain amount of searching for a sensitive spot may thus be necessary.

The reason for the variation of resistance in crystal couples is not yet thoroughly understood.

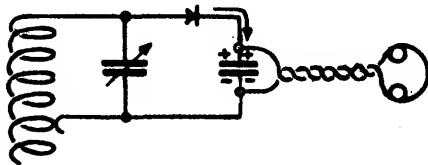


FIG. 247.

Fig. 247 shows the way in which a crystal couple is joined up in series with the telephones, so that the oscillatory voltages across the condenser of the secondary circuit are applied to it.

502. The Potentiometer.—It will here be of advantage to describe an instrument which has not been mentioned so far, but which is extensively used in receiving circuits, namely, the "Potentiometer."

A potentiometer, as shown in Fig. 248, consists of a high resistance R connected across a cell Q , and provided with two tapping points, viz., the tapping C at the middle point of the

resistance, connected to terminal D, and the sliding tapping E, making contact on any point in the resistance, connected to terminal F.

The resistance R should be large enough to prevent the current passing through it from discharging the cell rapidly. In practice a resistance of 200–300 ohms is suitable, and will allow a 2-volt cell to maintain its voltage for many weeks with continuous working.

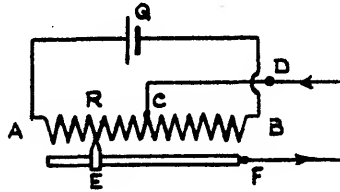


FIG. 248.

If the voltage of the cell Q is 2 volts, then the P.D. between A and B will be 2 volts, and there will be a steady fall of potential along the whole resistance.

The P.D. between A and C , and between C and B , will be 1 volt in each case.

The P.D. between terminals F and D will depend on the relative position of the slider E to the point C .

- (a) If E is opposite C , there will be no difference of potential between F and D .
- (b) If E is opposite A , there will be a P.D. of 1 volt between F and D , and F will be positive to D .
- (c) If E is opposite B , there will again be a P.D. of 1 volt between F and D , but D will be positive to F .

Hence, as E is slowly moved from A to B , the P.D. across any circuit joined between F and D will vary from a maximum in the direction F to D , down to zero, and up to a maximum in the direction D to F .

In short, a potentiometer is an arrangement for varying the strength and direction of voltage applied to any circuit joined across it.

It should be noted that the resistance of the external circuit is in parallel with the resistance of the potentiometer between C and E . This will modify the actual values of the P.Ds. quoted above, but the principle remains the same. The higher the resistance of the external circuit, the more nearly do the above values represent the true state of affairs.

An alternative arrangement for wiring a potentiometer, if the resistance has no centre tapping point, is shown in Fig. 249.

The terminal D is now joined to the centre point of the battery Q . The left-hand cell is trying to force a current through the resistance between A and E , and through the outside circuit in the direction of F to D .

The right-hand cell is trying to force a current through the outside circuit in the direction D to F, through the resistance E to B.

The resultant current will be the difference between the two, i.e., if the resistance A to E is less than the resistance E to B, current in the outside circuit will be from F to D and *vice versa*.

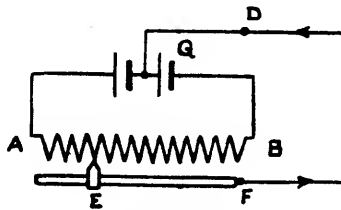
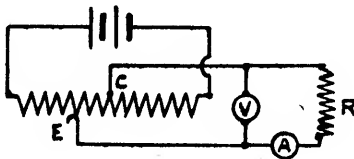


FIG. 249.

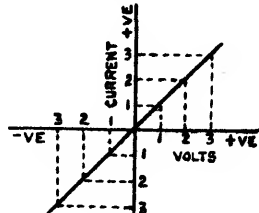
As there is a resultant current in the outside circuit, a steady voltage will be applied across this circuit, equal to its resistance multiplied by the resultant current flowing.

503. Characteristic Curves.—The action of a crystal couple can only be thoroughly understood by considering its "characteristic curve," which is a graph of current against voltage, for varying values of voltage in both directions.

First let us join up any ordinary resistance in series with an ammeter and across a potentiometer, as in Fig. 250 (a).



(a)



(b)

FIG. 250.

Let us term the direction from C to E "positive" and the direction from E to C "negative."

Fig. 250 (b) shows current plotted against voltage; this curve is called a "characteristic curve" of this resistance.

From it we see that when C is 1 volt positive to E, a current of 1 ampere flows from C to E through the resistance, for 2 volts 2 amperes flow, and so on.

Similarly, when C is 1 volt negative to E, a current of 1 ampere flows in the reverse direction, and so on.

From this we gather that the resistance R is given by

$$R = \frac{V}{I} = \frac{1}{1} = \frac{2}{2} = 1 \text{ ohm.}$$

If we replace the resistance R by a crystal couple, we get totally different curves, depending on the couple used.

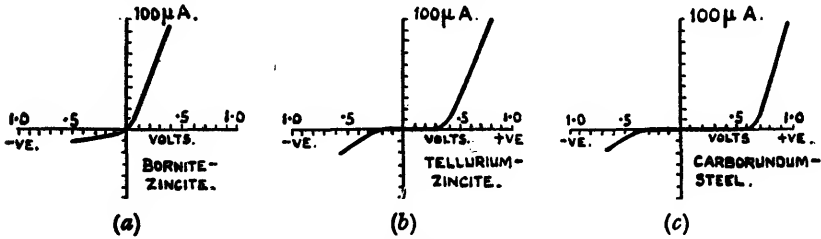


FIG. 251.

Fig. 251 (a), (b) and (c) show characteristic curves given by bornite-zincite, tellurium-zincite and carborundum-steel couples respectively.

Let us refer to the direction from bornite to zincite as "positive," and that from zincite to bornite as "negative."

Fig. 251 (a) tells us that when a voltage is applied to a **bornite-zincite couple** in a positive direction, the resistance of the crystal is low, *i.e.*, 0.1 volt gives a current of 20 micro-amperes, 0.3 volts gives 72 micro-amperes, &c.

When voltage is applied in a negative direction, the resistance of the couple increases enormously; 0.1 volt gives about 3 micro-amperes, and 0.5 volt only gives about 12 micro-amperes.

Hence the resistance of the couple to voltages applied in a positive direction is much less than its resistance to voltages applied in a negative direction.

Fig. 251 (b) gives a curve for **tellurium-zincite**. On the positive side no current can flow at all until 0.3 volt is applied, while after that point the current increases very rapidly for an increase in voltage.

On the negative side the resistance of the crystal is much greater, and as the voltage applied is increased, the increase of current is much slower.

The characteristic curve for **carborundum-steel**, Fig. 251 (c), is very similar, except that the critical point at which the curve bends sharply upwards occurs when the applied voltage is about 0.7 volt, instead of 0.3 volt as in the case of the tellurium-zincite detector.

504. The Process of Rectification in detail.—Let us now consider what happens when we join up a bornite-zincite couple in series with the telephones and across the condenser of the secondary circuit as shown in Fig. 247.

An incoming wave-train will set up an oscillation in the secondary circuit, which will apply an oscillatory E.M.F. directly across the crystal, as shown in Fig. 252 at (a).

It is important to notice the manner in which Fig. 252 is drawn.

Fig. 252 (a) shows a characteristic volt-ampere curve of a crystal, with the voltage of a damped wave-train being applied.

By running perpendiculars from the oscillatory voltage curve to cut the volt-ampere curve, the corresponding current curve can be determined.

The axes of coordinates are made to serve both as axes of current and voltage, and as axes of time.

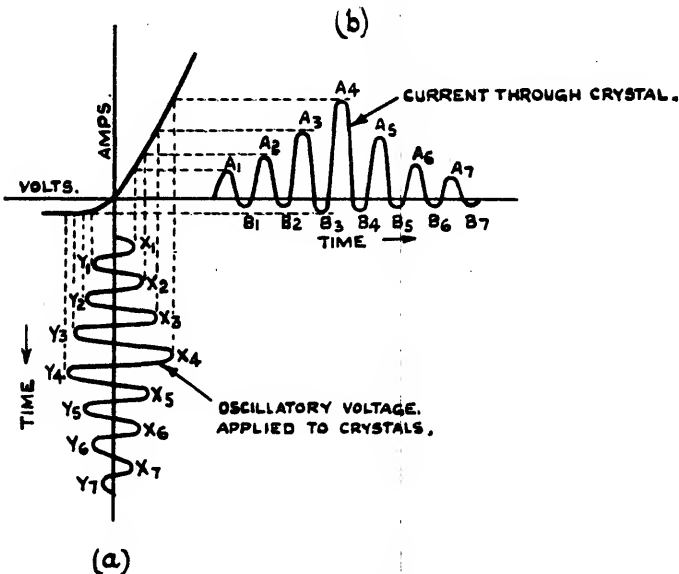


FIG. 252.

From the figure it is obvious that the first result of the introduction of the detector is **to produce an asymmetrical variation of current from a symmetrical variation of applied voltage**, the frequency with which the current varies being the same as the frequency of the incoming waves. The signals are not yet made audible, since the current variations are still in the form of radio frequency pulses.

A condenser is inserted in the circuit in parallel with the telephones, as shown in Fig. 247. The effect of this is that the condenser receives unequal positive and negative charges during each two successive half-cycles, and therefore accumulates a resultant charge, which sets up a P.D. across the condenser, and forces a unidirectional current through the telephones, making the diaphragm move out of position once only as a result of the whole wave-train.

An alternative explanation is as follows :—

The unsymmetrical current wave-form resulting from Fig. 252 may be split up into two components as below.

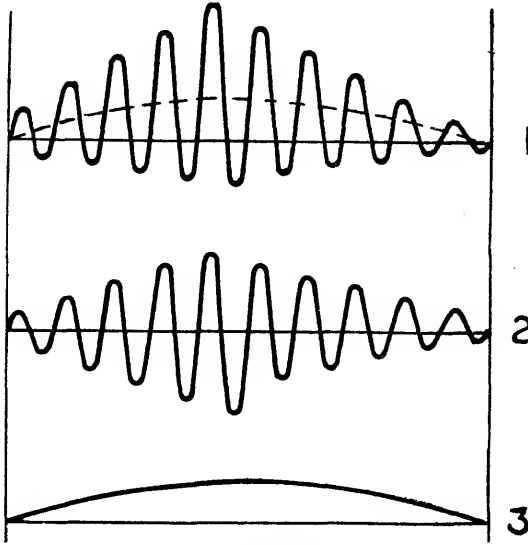


FIG. 253.

It is equivalent to :—

- (a) A symmetrical variation at radio-frequency as in Fig. 253 (2), superimposed on
- (b) A variation of current at audio-frequency (the frequency of the wave-trains), one single variation corresponding to one wave-train being represented as in Fig. 253 (3).

These two components may be separated by providing two circuits in parallel, one suitable for radio-frequency and unsuitable for audio-frequency, and the other *vice versa*. This is effected by the introduction of a condenser of suitable capacity across the telephones. The reactance of the condenser to radio-frequency is small compared with that of the highly-inductive telephones, whereas to the audio-frequency component the inductance of the telephones offers much less reactance than the capacity of the condenser.

The reactances are respectively ωL and $\frac{1}{\omega C}$.

If ω is large, ωL is large compared with $\frac{1}{\omega C}$.

If ω is small, ωL is small compared with $\frac{1}{\omega C}$.

The radio-frequency component of the complex wave-form is therefore passed by the comparatively small reactance of the

condenser, while the variation at audio-frequency alone passes through the telephones and deflects the diaphragm at its own frequency, that of the wave-trains.

The shunting condenser is called the "by-pass" or telephone condenser.

Where a condenser is not fitted, the self-capacity of the telephone windings may be considered as fulfilling the duties of the condenser. A capacity in parallel is absolutely necessary.

The conception of the resolution of an unsymmetrical wave-form into a symmetrical wave-form superimposed on a varying mean value is of great importance in wireless work.

Rectification may therefore be considered as made up of two parts :—

- (1) The production of an asymmetrical current variation from a symmetrical voltage wave-form.
- (2) The analysis, by a suitable circuit, of this current into its components, one of which only, the audio-frequency component, is necessary for the production of sound in the telephones.

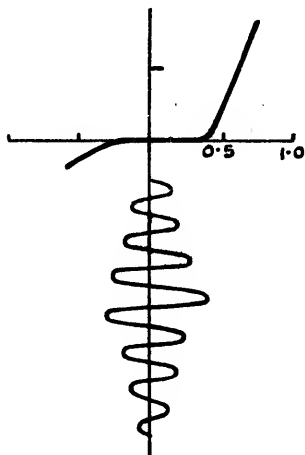


FIG. 254.

505. Fig. 254 shows the oscillatory voltage of a wave-train as applied to the characteristic curve of a tellurium-zincite couple. It can be seen that unless the voltage applied is greater than 0.3 volt, no current flows at all. The same negative result would occur with a carborundum-steel couple.

In each case, unless the oscillatory voltage applied is of large amplitude, it is insufficient to force any current through the high-resistance contact.

This renders necessary the use of the potentiometer.

It is joined up as shown in Fig. 255 (a) so as to pass a steady current round the circuit from C, through the crystal, secondary inductance and telephones, back to E.

Another way of joining it up, which will give the same result, is shown in Fig. 255 (b).

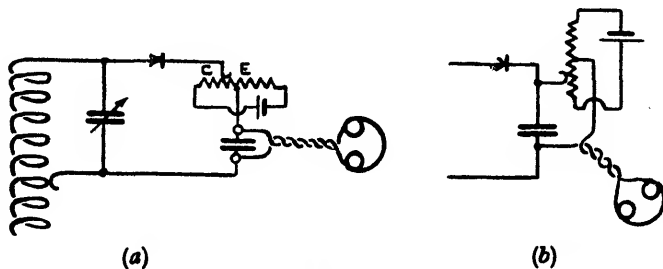


FIG. 255.

This latter arrangement has the advantage that the **oscillatory** currents need not pass through the resistance of the potentiometer, but as this resistance is comparatively small compared with that of the crystal, the advantage gained is unimportant.

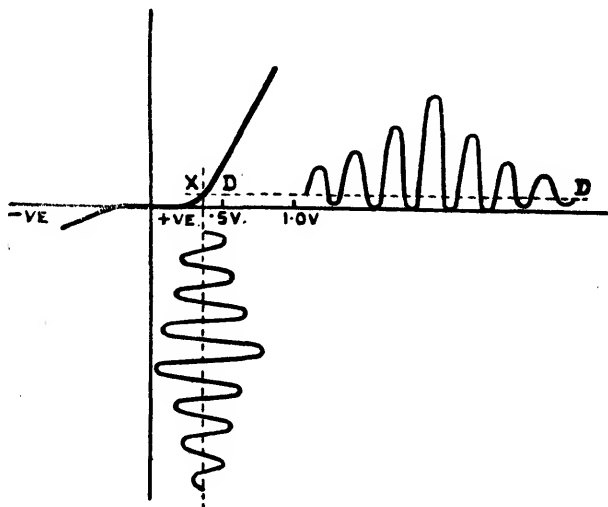


FIG. 256.

There are now two voltages acting on the crystal :—

- (a) The steady voltage of the potentiometer.
- (b) The oscillating voltage of the incoming wave.

The current flowing through the junction will be that due to the resultant of these two.

Let us apply a steady positive voltage from the potentiometer equal to that at which the curve turns up sharply (at the point X), *i.e.*, 0.35 volt (Fig. 256).

Then a steady current will flow through the telephones as shown by the line DD.

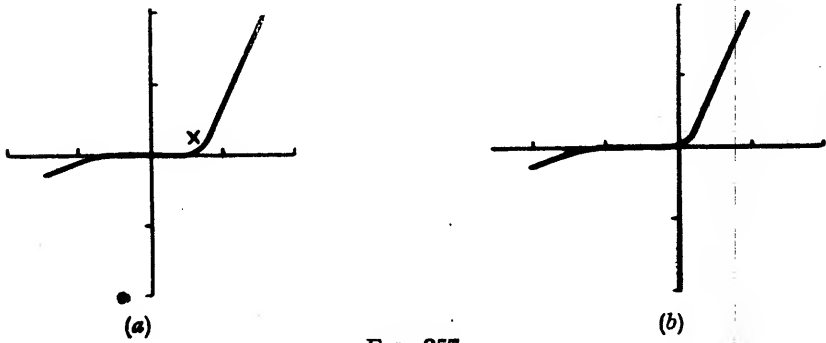


FIG. 257.

When an oscillatory E.M.F. is applied, as in Fig. 256, the current wave-form will be as shown.

It now consists of an asymmetrical variation superimposed on the steady value D. As before, this asymmetrical variation may be resolved into a radio-frequency component by-passed by the condenser, and an audio-frequency component which passes through

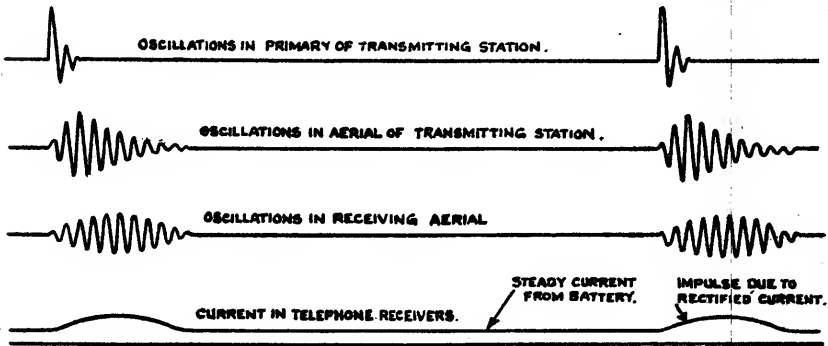


FIG. 258.

the telephone windings in addition to the steady current already there. The diaphragm will therefore move at audio-frequency and produce audible sound waves. The same argument applied to the carborundum-steel curve will give a similar result.

We see, therefore, that by means of a potentiometer we get the crystal into the most sensitive condition for reception by making use of the sharp bend of its characteristic curve.

In short, the duty of the potentiometer is to heat up the junction of the crystal couple to the point where the latter suddenly becomes **unidirectionally** conductive.

An ideal characteristic curve is one that turns up very steeply on one side of the line of origin, and is as flat as possible on the other side, for then the resulting rectification will be as great as possible.

A very convenient way of regarding the application of the potentiometer is to think of it as a means of sliding the axis of the ordinates to the right or left at will, so that it cuts the characteristic curve at the point where it bends most sharply.

For instance, the curve of Fig. 257 (a) with a potentiometer adjusted to point X is the same as the curve of Fig. 257 (b).

The process of rectification described in the preceding paragraphs may be illustrated as in Fig. 258.

RECEIVING CIRCUITS—TUNERS.

506. Receiving Circuit Arrangements.—We can now return to the question of receiving circuits, and consider them more carefully.

The main objects in view are (1) audibility and (2) selectivity (*see* para. 492).

With the help of amplifiers (described in Chapter XIII) it is possible to bring up any signal, however weak, to audible strength, so that it is not so vitally important to have an arrangement of receiving circuits which will apply the maximum possible voltage to the detector, as to have an arrangement which will give greatest selectivity combined with the maximum oscillatory voltage obtainable in the circumstances.

Selectivity means the avoidance of interference from atmospherics or from signals on a frequency different from that which it is desired to receive. Although a receiver may be efficient as regards amplification it is useless if it is unselective, because the interfering signals are amplified along with the wanted signal.

507. Tuning the Aerial Circuit.—The requirements of the last paragraph are met with to some extent by tuning the aerial circuit, as already described in para. 493.

If the frequency to be received is less than the resonant frequency of the aerial, inductance may be added in series by means of the A.T.I.; if greater, capacity may be added in series by means of the A.T.C. Combinations of these methods may be employed.

With very low frequencies it is sometimes inconvenient to use more than a certain amount of inductance in series; in this case the condenser may be joined up in parallel with the inductance, so that it is in effect in parallel with the aerial capacity. The total capacity is then the sum of the separate capacities.

This arrangement may also be used as a means of fine tuning, where the extra inductance necessary for tuning is adjustable only in steps, while the condenser is continuously variable.

Again, for fine tuning, the tuning condenser may be placed in series with the aerial, reducing the total capacity, with the A.T.I. also in series with the aerial. With such arrangements a very large range of wave-frequencies can be covered.

A further method is given in Fig. 259 (c) below, using both a series and a parallel condenser, as well as an inductance.

These circuit arrangements are shown in Fig. 259.

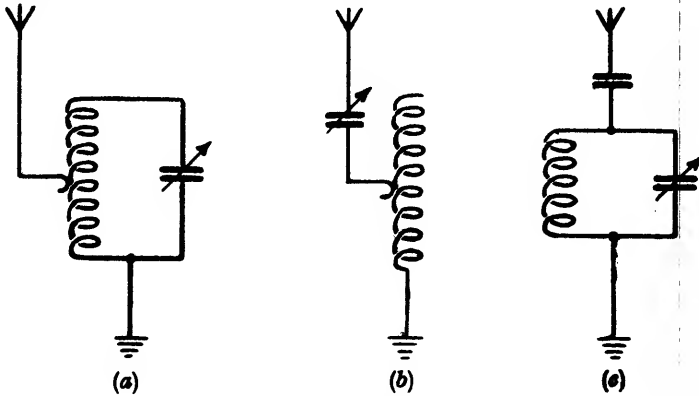


FIG. 259.

By means of a double-pole two-way switch arrangements (a) and (b) above can be produced with the same inductance and condenser, as illustrated in Fig. 260.

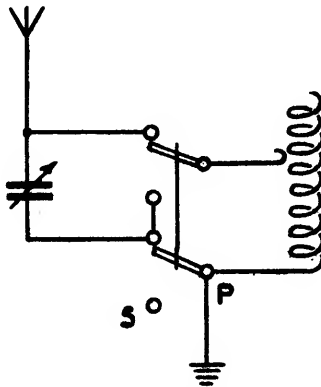


FIG. 260.

508. Detector Connections—Direct Coupling.—So far we have considered the tuning of the aerial circuit only. The connection of the detector must now be taken into account.

The detector may be connected across a portion of the aerial tuning inductance. This is the simplest method of bringing it into the circuit. Such a connection, applied to two different aerial arrangements, is shown in Fig. 261

This circuit is extremely easy to handle, but it is very unselective and susceptible to all forms of interference, because with only one tuned circuit voltages induced at non-resonant frequencies may set up currents of considerable amounts, especially if the "resonance curve" of this single circuit is flat-topped (*see para. 302*).

The detector circuit, being joined directly across the inductance, increases the resistance of the aerial circuit, in accordance with the usual theory that a resistance in parallel can be replaced by an equivalent series resistance. The greater the amount of inductance across which the detector is joined, the greater

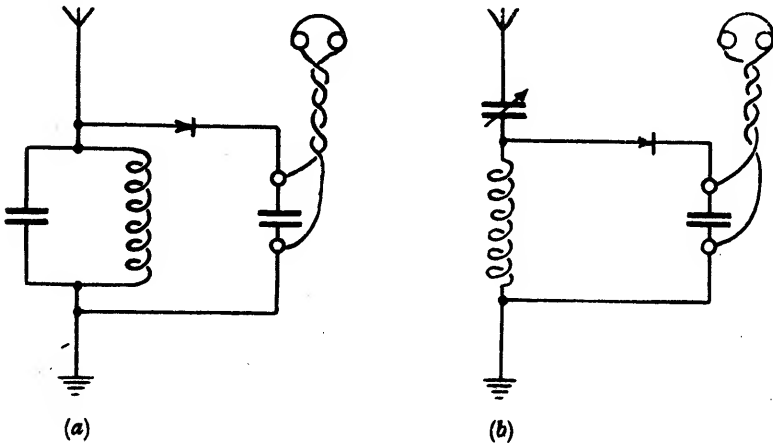


FIG. 261.

is the equivalent series resistance. It may be shown that the power delivered to the detector is a maximum when the total aerial resistance with the detector joined up is twice the resistance without the detector, and there is therefore a best value for the amount of inductance to be tapped off by the detector connection.

The increase in effective aerial resistance due to the detector connection means that the aerial circuit is rendered less selective, for the following reason (*see also para. 302*).

An E.M.F. at resonant frequency gives a current which is reduced in inverse ratio to the increased resistance; while an E.M.F. at non-resonant frequency gives a current which is not proportionately reduced, because in the formula for impedance the reactance is the same and only the resistance increases. Therefore the increased resistance means a flatter resonance curve and less selectivity.

If it is desired to make the direct-coupled arrangement as selective as possible, the coupling (*i.e.*, the amount of inductance across which the detector is joined) should be reduced below its best value as defined above from considerations of audibility.

In general, the direct-coupled attachment is only used when searching for signals in what is called the "stand-by" position. Once a signal has been picked up a more selective device is called for, and this is provided by a tuned secondary circuit coupled mutually to the aerial circuit; this was mentioned briefly at the beginning of the chapter and will now be considered in more detail.

509. Detector Connections—Mutually Coupled Secondary Circuit.—Instead of the detector being joined straight across a portion of the aerial circuit, it is joined across a separate circuit, which may consist of an inductance only, or, more generally, an inductance and capacity, whose LC value is made to correspond with that of the incoming wave.

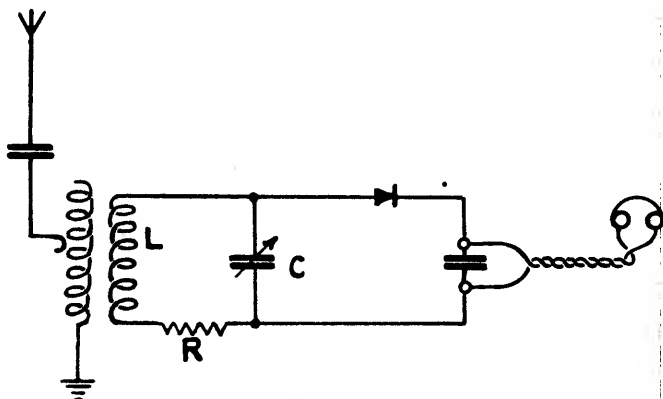


FIG. 262.

Oscillatory currents in the aerial set up high-frequency alternating voltages in the secondary circuit through the mutual coupling. It must be clearly understood that the secondary circuit is a **series** circuit as far as such voltages are concerned. The voltage introduced into the circuit can be regarded as that of a small alternator joined in the secondary circuit in series, and if the secondary is tuned to resonance there will be an alternating current in the secondary, given by the voltage divided by the ohmic resistance. The actual voltage applied to the detector is the voltage across the secondary condenser consequent upon this current flowing.

If the secondary induced voltage is E , the current is $\frac{E}{R}$ and the voltage applied to the detector is $\frac{E}{R} \times \frac{1}{\omega C}$, which may therefore be much greater than E .

$\frac{1}{\omega C}$ may be written $\sqrt{\frac{L}{C}}$, so that the voltage applied to the detector is $\frac{E}{R} \sqrt{\frac{L}{C}}$.

For this to be large, L must be large and C small, so that by increasing L and diminishing C , while keeping their product constant, **audibility** is increased. In connection with this, it may be noted that an increase of L increases the ohmic resistance R , which is mainly the resistance of the inductance, so that the voltage across the condenser does not increase exactly in proportion to $\sqrt{\frac{L}{C}}$.

The second point to be considered with regard to this arrangement is that of **selectivity**. Because of the fact that there are two tuned circuits instead of one, the tuned secondary circuit arrangement is more selective. Forced oscillations, set up in the first

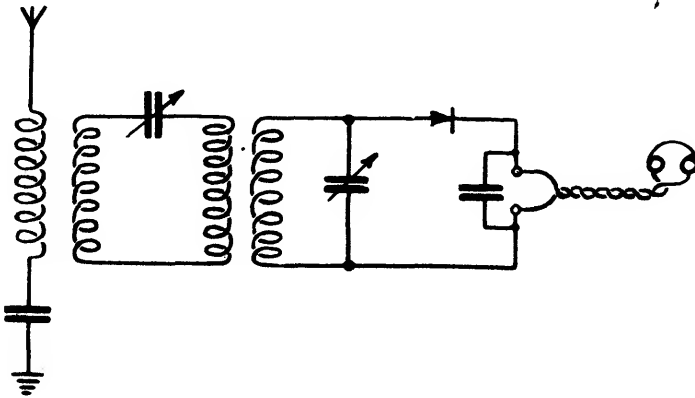


FIG. 263.

circuit, the aerial, by incoming waves of non-resonant frequency, produce smaller currents than a resonant wave of the same amplitude, and since the secondary circuit is also tuned to the resonant frequency, the voltage they induce in it gives a current whose value is still further cut down. If increased selectivity is required, the number of circuits may be increased, as in Fig. 263, but usually one tuned circuit in addition to the aerial circuit is found to be sufficient, and every additional circuit means more complicated tuning and greater losses.

Such additional circuits which are introduced between the aerial circuit and the detecting device, or, if an amplifier is used, between the aerial circuit and the amplifier, are usually regarded in Service sets as a separate unit of the complete receiver. The various inductances and condensers are assembled in a separate box, and

are referred to as the "tuner." A tuner is designed to work on a certain band of wave-frequencies, and an amplifier used with it is usually designed for the same frequencies and introduces additional selectivity.

As regards the effect on selectivity of changes in the ratio $\frac{L}{C}$, it follows from the ordinary theory of series (or acceptor) circuits, that the greater this ratio the greater is the impedance at non-resonant frequencies and the less the current. **Therefore, for both audibility and selectivity, L in the secondary circuit should be large and C should be small.**

A circuit with a large $\frac{L}{C}$ ratio is called a "stiffly-tuned" or "stiff" circuit. If $\frac{L}{C}$ is small it is called a "flatly-tuned" or "flat" circuit.

510. Selectivity.—So far, selectivity has only been considered as regards its dependence on the number of tuned series circuits through which the energy received from the ether has to pass before arriving at the detector (or amplifier), and on the ratio of inductance to capacity in the additional circuits.

Modern tuners generally consist of **one** secondary circuit in addition to the aerial, with a switching device for cutting out this circuit and using direct coupling when searching for signals, and with efficient coils and condensers all the selectivity necessary can be obtained by this simple device.

The principles explained in para. 509 with respect to the secondary circuit can obviously also be applied to the aerial circuit, and older receiving outfits used in the Service have elaborate devices for increasing the selectivity of the aerial circuit itself. These may be classified as follows :—

- (a) Use of acceptor circuits in series.
- (b) Use of rejector circuits in series and parallel.

A very important factor as regards selectivity is the degree of coupling which exists between the aerial and the secondary circuit in the normal arrangement, and greater selectivity is obtained by the use of loose coupling.

These devices will now be investigated.

511. Acceptor and Rejector Circuits.—The principles of acceptor and rejector circuits have already been discussed in Chapter V.

Some different circuit arrangements are shown in Fig. 264.

(a) Fig. 264 (a) shows two acceptor circuits in series. L_1 is the aerial tuning inductance, which, in combination with the aerial capacity σ (shown dotted), is in resonance with the incoming wave.

The circuit $L_2 C_2$ is separately tuned to the same LC value as the circuit $L_1 \sigma$.

The whole circuit, consisting of two acceptor circuits in series, is then correctly tuned as a whole.

The merit of this arrangement is that L_2 can be increased and C_2 diminished, at will, to make the lower circuit, and hence the complete circuit, as stiff as may be required.

The inductance L_2 may conveniently be used as the primary of the inductive coupling to the secondary circuit.

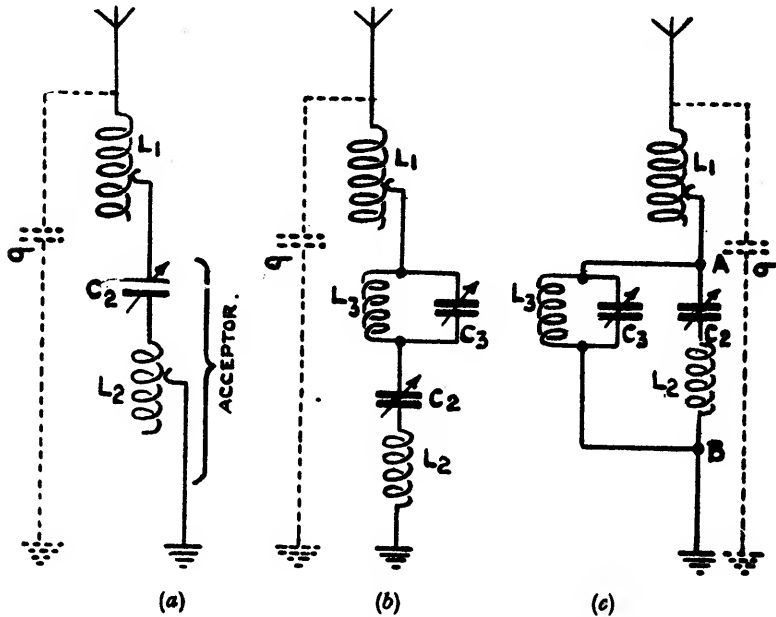


FIG. 264.

(b) Fig. 264 (b) shows a rejector circuit $L_3 C_3$ joined in series with the aerial to bar some particular interfering wave.

The circuits $L_1 C_2$ and $L_2 C_2$ are tuned to the wave required, whereas $L_3 C_3$ is tuned to the interfering wave and bars its passage, while permitting the required wave, to which it is not tuned, to pass easily.

The series rejector circuit only deals, of course, with one wave, and it does not actually increase the selectivity of the receiver in general. It is known as a wave-trap.

(c) **Acceptor-Rejector Circuit.**—Fig. 264 (c) shows a rejector circuit shunted across an acceptor circuit, to act as a by-pass for all interfering waves.

All three circuits are tuned to the same LC value.

Let us consider how this circuit will behave with regard to the resonant wave, and to interfering waves.

The Resonant Wave.—The only oscillatory voltage required between A and B to maintain the oscillatory current is that necessary to overcome the effective resistance of the acceptor and secondary circuits combined. That is to say, A is a point in the aerial circuit which is very nearly at earth potential (para. 305).

Thus there is only a very small voltage acting across the rejector, which will only cause a negligible current to flow through it, provided that its resistance is low.

Action of Circuit on Interfering Waves.—Now consider the action of the circuit when waves of other frequencies strike the aerial.

If these waves are sufficiently powerful, as is the case with a station transmitting on full power a few miles away, then some current of the interfering frequency is set up in the aerial circuit.

This current has two paths to earth :—

- (a) Through the rejector circuit.
- (b) Through the acceptor circuit.

Consider (a) first. Since the frequency of the interference is different from that to which the rejector is tuned, it follows that a large current will flow through the rejector for a small applied voltage. This current has no direct influence on the secondary circuits and will not lead to signals in the telephones.

Now consider (b). The acceptor circuit is not in tune to this interference, and consequently relatively large voltages will be required to cause any appreciable current to flow.

But no large voltages are available, because even the large aerial currents set up by powerful waves can escape to earth through the rejector with quite small voltages across it.

Moreover, the interfering current has to pass first through the aerial circuit, which is a stiffly-tuned acceptor, and will be considerably cut down in its passage.

The action of the acceptor and rejector circuits with interference is thus exactly the reverse of their action with the waves to which the circuits are tuned.

In the first case the rejector acts as a by-pass to earth for practically all the aerial current at the interfering frequency, and the acceptor, being out of tune, carries practically none of the current.

But, in the second case, with the in-tune oscillations, the rejector becomes a non-conducting path and the acceptor a conducting path, with the result that practically all the current passes through the acceptor and none is wasted through the rejector.

(d) **Drain Circuit.**—Another method, besides (b) of dealing with interference from a particular wave, is to fit a "drain" circuit for it.

A "drain" circuit is a series, or acceptor circuit, tuned to the interfering frequency, and tapped off from a low potential point.

Fig. 265 illustrates a receiving set with this attachment.

The aerial circuit down to A is tuned to the wave required, as is the circuit from A to earth, and also the secondary circuit. A is therefore a low potential point to earth, and for the received wave frequency (500 kc/s.) very little energy is wasted in the 1,000 kc/s. path.

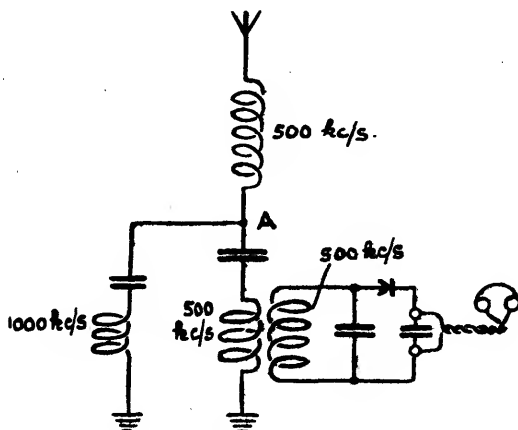


FIG. 265.

The interfering 1,000 kc/s. wave will expend whatever energy it has left at the point A in setting up an oscillation in the drain circuit which is tuned to it, and should cause no interference in the telephones.

512. Coupling.—The strength of signals and the degree of selectivity both depend on the coupling between the aerial and the secondary circuit.

(a) **Strength of Signals.**—If the coupling is very loose, the aerial is unable to hand over to the secondary circuit much of the energy it absorbs from the aether, and most of it is dissipated in IR losses in the aerial circuit, a certain amount being re-radiated into the aether.

As the coupling is increased, the secondary circuit takes a greater percentage of energy from the primary, or aerial, circuit; while its effect is equivalent to the insertion in the aerial circuit of an increasing resistance.

After the coupling is increased beyond a certain point, the increased resistance in the aerial circuit due to the presence of the secondary circuit cuts down the aerial current to such an extent that the increased percentage of energy taken by the secondary circuit may give less actual energy in the secondary than its value for a looser coupling.

In addition, a double-frequency effect is set up with tight coupling, and by the theory of forced oscillations in coupled circuits,

the circuits will be resonant to two frequencies, one higher and one lower, than the actual frequencies to which they are separately tuned.

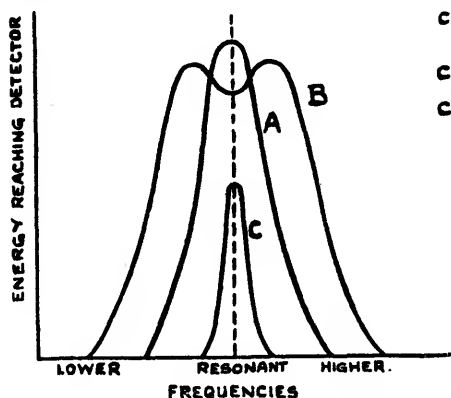
These statements may be summed up as follows :—

The aerial and the secondary circuit form two tuned coupled circuits, on the primary of which is impressed an E.M.F. of the same frequency as that to which the circuits are tuned ; under these conditions the current in the secondary has a maximum value for a certain critical, or optimum, coupling, and, for tighter or looser coupling than this the current is reduced.

(b) **Selectivity.**—It may further be proved that, if the coupling is diminished below the optimum value, the selectivity is increased : while, if the coupling is increased, the selectivity is decreased.

It is generally advisable, therefore, to use a coupling which is less than the critical value, and so to gain selectivity, for strength of signals can always be increased by amplification.

These effects are illustrated in Fig. 266.



CURVE A: BEST COUPLING FOR STRONGEST SIGNALS.
CURVE B: TOO TIGHT COUPLING.
CURVE C: LOOSE COUPLING.

FIG. 266.

Curve A illustrates what happens with critical coupling. The energy reaching the detector has an optimum value at resonance for this type of coupling, as compared with curves B and C. The selectivity obtainable is illustrated by the slope of the curve.

Curve B shows what happens with coupling so tight that a double-frequency effect is set up. The coupling being tighter than the critical value, the energy at resonance is less than for curve A, while the peak values are obtained for frequencies a little higher and a little lower than the resonant one. The curve is much flatter than curve A and therefore the selectivity is less.

Curve C shows the effect of a coupling looser than critical. The energy at resonance is less than for curve A, but the curve is much steeper and hence very little energy indeed reaches the detector at non-resonant frequencies—i.e., the selectivity is better than for critical coupling.

513. Capacitive Coupling.—A tuned secondary circuit coupled to the aerial by direct capacitive coupling is illustrated in Fig. 267.

The secondary circuit $L_2 C_2$ is coupled to the aerial circuit by the condenser C' .

If the capacity in the aerial circuit is C_1 , the coupling factor (para. 333) is $K = \frac{C'}{\sqrt{(C' + C_1)(C' + C_2)}}$, and hence the greater the value of the coupling condenser, the greater is the coupling factor.

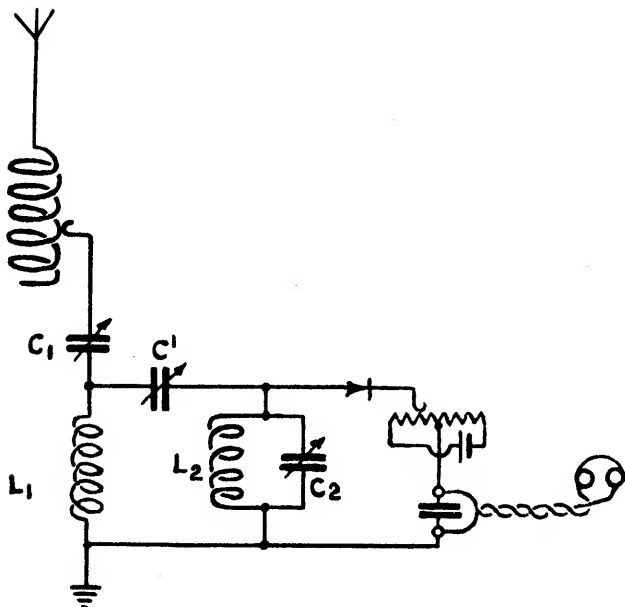


FIG. 267.

The variation of C' affords a very delicate adjustment of coupling, provided that L_2 is so screened from L_1 that there is no mutual inductance between them. This may be partly ensured by placing their axes at right angles to one another, but metallic screening may also be necessary.

514. Tuning Receiving Gear.—As a general principle, a list of receiving adjustments for all waves should be available in the silent cabinet.

When first picking up a wave whose correct adjustments are not known, the whole circuit should be made as unselective as possible; that is, all series acceptor circuits should have small inductance and large capacity, the rejector circuit (if included in the set) should not be switched in, the coupling to the secondary circuit should be tight, and the secondary inductance should be small and the capacity large (the secondary being effectively a series circuit).

When the wave has been picked up, to gain selectivity, the aerial circuit is stiffened, the rejector switched in, the coupling loosened, and the secondary circuit adjusted to have large inductance and small capacity.

515. Shock Interference.—All the preceding arguments have been based on the ordinary theory of alternating currents, as if we merely had to consider the effects of regular alternating voltages of different frequencies applied between aerial and earth.

With continuous-wave valve transmitters we do get these conditions very nearly, and to a certain extent with arc transmitters, but with spark systems we have to deal with damped trains of waves, often starting with a very big initial amplitude if the transmitting station is close to the receiving station and is using a tight coupling.

Although such waves, striking an aerial which is not tuned to their frequency, are quickly damped out, they are able to expend a certain amount of their energy in setting it oscillating at its own natural frequency in the same way as a bell is set ringing when struck with a hammer.

Such interference cannot be cut out by any amount of selective tuning, and the only way for the operator to deal with it is to "over-read" the note of the signal he wishes to receive.

516. Receiving through Atmospherics.—"Atmospherics" are a source of interference universally experienced. They are the result of electric waves set up in the aether by an electric disturbance of some sort, such as flashes of lightning in a thunderstorm, and the source of origin may be at a great distance from the receiving station.

Various elaborate circuits have been tried for reducing them, but generally without much success, as they are a form of shock interference of no particular frequency and merely set the aerial in oscillation at **its own** natural frequency.

Receiving aerials which have directional properties (to be described in Chapter XIX) are not so much affected by atmospherics provided they arrive in a different direction from that of the signal the aerial is adjusted to receive.

Statics.—Another form of atmospheric trouble that sometimes arises is that known as "Statics." These are stray electrical charges that are picked up by the aerial, and if the latter happens to be left insulated, for instance, by an aerial condenser, these charges will accumulate and will eventually break down the insulation of the condenser and spark through it.

A coil known as an "Aerial Discharge Coil" having very high resistance and inductance is frequently connected across the receiving gear to earth.

On account of its high impedance, only a minute current due to the incoming wave flows through it, but it allows static charges to leak slowly to earth.

Inductances and condensers are often protected from lightning by joining "safety points" or "lightning arresters" across them.

517. Wiring of Receiving Circuit.—In wiring receiving gear we have two important losses to guard against :—

- (a) I^2R or ohmic losses in conductors.
- (b) Leakage losses through bad insulation.

Now, naturally, wherever we have a **large current** we shall have to be specially careful to provide a good conductive path.

This will be the case in all circuits which have large capacity and small inductance.

In all such circuits, therefore, it is important to provide low-resistance conductors of good surface area.

Whenever we have a **big voltage** we shall have to be specially careful to provide a very good insulation.

There will be very big voltages to earth at the high potential points of all circuits having a large ratio of inductance to capacity.

A very good test to find the high potential points (paras. 305 and 306) of a receiving circuit is to see whether signals disappear when any particular point is touched with the finger.

All leads should be as short, direct, and as non-inductive as possible ; it is not a good thing, however, to run two leads side by side with the idea of making them non-inductive, as there will be a considerable capacity between them, which will make tuning to short waves difficult.

It is particularly important to see that the receiving earth is good. This matter is discussed in Chapter XVIII.

518. Reception of Continuous Waves.—When considering the rectification of a damped wave-train in para. 504, it was shown that the current waveform passed by the detector could be analysed into a symmetrical high-frequency oscillation superimposed on a low-frequency variation of the mean value, this latter occurring at the frequency of the wave-trains. By passing this current through a circuit consisting of an inductance and capacity in parallel, only the low or audible frequency passes through the inductance (of the telephones) and produces an audible note.

From the theory there described and the figure appended (Fig. 268), it should be evident without further explanation that if continuous waves of steady amplitude are being detected, the resultant asymmetrical current can be likewise resolved into a high-frequency variation about a new **steady** mean value. With the same circuit the high-frequency oscillation would be passed by the condenser, and the only effect in the inductance would be an increase or decrease of current at the beginning and end of the signal, *e.g.*, a morse dash, the altered value of current lasting the whole time the transmitting key is held down. The only effect in the telephones would therefore be a click at the beginning and end

of the signal as the diaphragm moved from one fixed position to another.

There would be no audio-frequency component in the rectified current wave form, which is an essential for producing an audio-frequency movement of the diaphragm and hence an audible note, because the received signal would not possess in itself any audio-frequency characteristic.

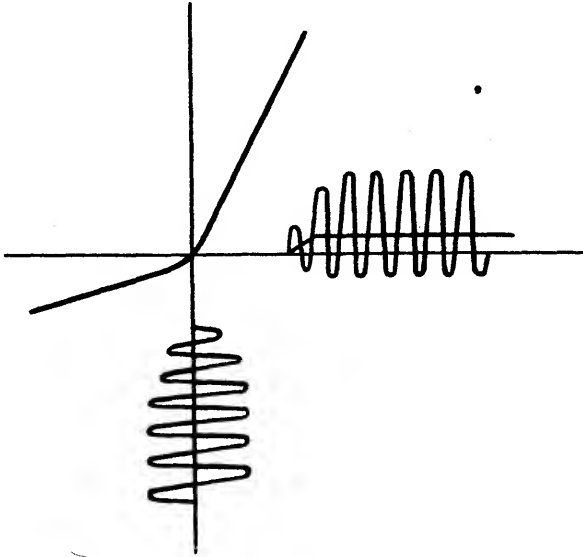


FIG. 268.

In order, therefore, to make the telephone diaphragm vibrate at audio-frequency, **it is necessary to break up a long train of continuous waves into groups succeeding each other at audio-frequency**, each group giving one pull on the diaphragm as the resultant effect of the asymmetry introduced by the process of detection and the fact that the telephones are shunted by a condenser. This result may be achieved as follows :—

- (a) At the receiving station the current in the receiving circuit may be interrupted or varied at audible frequency by a mechanical method. An arrangement for doing this is called a "Tikker," and is generally either an arrangement for breaking and making the circuit, or for periodically mistuning the circuit by breaking or making an additional circuit or rotating a closed coil coupled to the inductance in the receiving circuit.

This method does not give such satisfactory results as :—

- (b) Heterodyne reception.

519. Heterodyne Reception.—In the heterodyne system of reception a separate local circuit—known as the “heterodyne circuit”—is coupled loosely to the detector circuit, as illustrated in Fig. 269. The heterodyne circuit consists of an inductance and capacity, in which a continuous radio frequency oscillation of small amplitude is maintained by some suitable device—a valve acting as an oscillator—as explained in subsequent chapters.

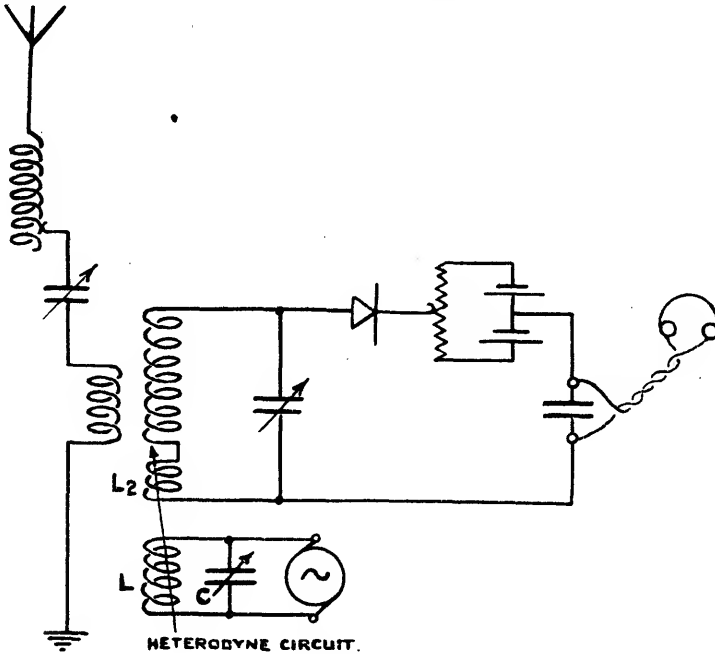


FIG. 269.

For the present we shall merely assume that this oscillation is set up, and that its frequency, which depends on the LC value of its circuit, is variable between wide limits by adjustment of the variable condenser C.

When there is no incoming signal, a supersonic frequency voltage variation of constant amplitude is being applied to the detector by the local oscillator. By the theory of the last paragraph, no sound is heard in the telephones, a steady rectified current passing through them.

While a C.W. incoming signal is being received, two radio-frequency oscillatory voltages are applied to the detector; one at the frequency of the incoming wave and one at the frequency of the local oscillator, or heterodyne circuit.

Now, unless these oscillatory voltages are of exactly the same frequency, they will not rise and fall in time with each other, but will get into step and out of step alternately.

This action is illustrated in Fig. 270. Curve A shows the voltage due to the incoming wave at the frequency of the transmitting station which generated it.

Curve B shows the voltage due to, and at the frequency of, the heterodyne circuit.

Curve C shows the combined effect of these two voltages as applied to the detector.

Curve D shows the rectified current, and the dotted line through the high-frequency variations of current represents the audio-frequency variation of mean value of current on which we may consider the high-frequency symmetrical oscillations superimposed; this audio-frequency component actuates the telephones.

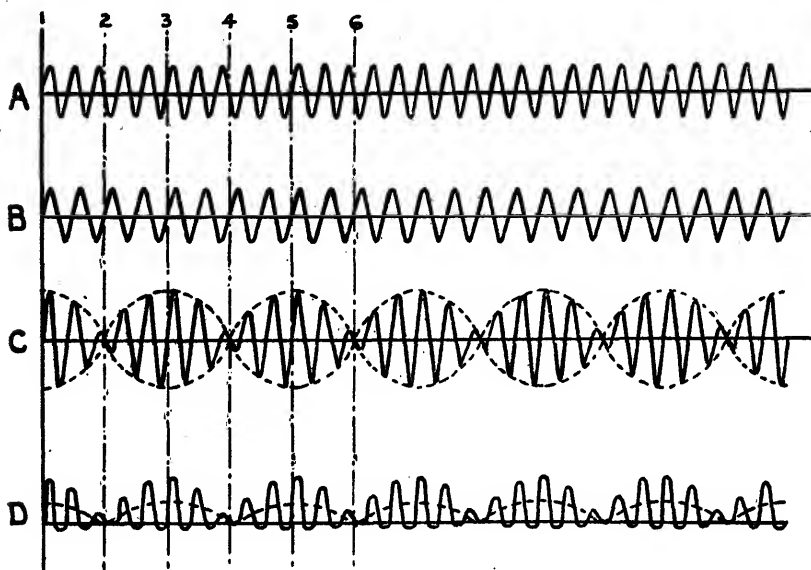


FIG. 270.

Each of the sections or groups into which the wave form of curve C is divided gives one **variation** in the pull on the diaphragm, instead of the constant additional pull which would be the result of applying either wave form A or B separately.

The wave form illustrated in C is exactly similar to that familiar in acoustics, where a throbbing effect is heard when two organ pipes of nearly the same frequency are energised simultaneously. The coming in and out of step in such cases is known as the "**beat**" effect.

520. Beat Frequency.—The frequency of the "**beats**" of voltage illustrated in curve C, which is also the frequency at which the telephone current varies, and the frequency of the note heard, is

given by the difference of the frequencies of the two component radio-frequency waves A and B.

This may be seen from the curves given. Between moments 2 and 4 there are 5 complete cycles in curve A and 4 complete cycles in curve B, while curve C, the resultant, passes through one complete set or group of variations.

An accurate proof of this statement, not depending on results obtained by careful drawing, is appended.

*521. Suppose that the E.M.F. introduced into the detector circuit by the incoming signal is $a \sin \omega_1 t$, and that due to the local oscillation is $b \sin \omega_2 t$.

The combined E.M.F. is $a \sin \omega_1 t + b \sin \omega_2 t$.

Let $(\omega_1 - \omega_2)$ be written ω .

Then the combined E.M.F. is $a \sin (\omega_2 + \omega) t + b \sin \omega_2 t$
 $= (a \cos \omega t + b) \sin \omega_2 t + (a \sin \omega t) \cos \omega_2 t$
 $= A \sin (\omega_2 t + \theta),$

where

$$A^2 = (a \cos \omega t + b)^2 + (a \sin \omega t)^2 = a^2 + b^2 + 2ab \cos \omega t,$$

and

$$\tan \theta = \frac{a \sin \omega t}{a \cos \omega t + b}.$$

The resultant can therefore be considered as a sine curve of **varying amplitude A**.

Since $A = \sqrt{a^2 + b^2 + 2ab \cos \omega t}$, it reaches a maximum value every time $\cos \omega t = +1$, i.e., at a frequency given by $\frac{\omega}{2\pi}$.

The separate frequencies of the two waves are given by $\frac{\omega_1}{2\pi}$ and

$\frac{\omega_2}{2\pi}$, and their difference is $\frac{\omega_1 - \omega_2}{2\pi} = \frac{\omega}{2\pi}$.

Therefore the frequency with which the amplitude of the wave form in curve C reaches its maximum, i.e., the **beat frequency**, is the **difference of the two separate radio-frequencies**.

The range of variation in the amplitude of the combined wave-form is from $(a + b)$, when $\cos \omega t = +1$, to $(a - b)$, when $\cos \omega t = -1$.

When $a = b$, as in the figure given, the range is from $2a$ to zero.

It may further be observed that, in this case, the value of θ becomes

$$\tan^{-1} \frac{\sin \omega t}{\cos \omega t + 1} = \tan^{-1} \left(\tan \frac{\omega t}{2} \right) = \frac{\omega t}{2},$$

so that the combined E.M.F. may be written

$$\begin{aligned} A \sin \left(\omega_2 + \frac{\omega}{2} \right) t &= A \sin \left(\omega_2 + \frac{\omega_1 - \omega_2}{2} \right) t \\ &= A \sin \frac{\omega_1 + \omega_2}{2} t \end{aligned}$$

where $A = \sqrt{2a^2 + 2a^2 \cos \omega t} = 2a \cos \frac{\omega}{2} t$,

and therefore

- (1) The frequency of the **radio**-frequency oscillations contained in curve C is the mean of the separate radio-frequencies ;
- (2) The amplitude variation is simple harmonic and so gives a pure note without harmonics.

If a is not equal to b , the radio-frequency in the resultant wave form is itself a varying quantity and harmonics are introduced into the beat frequency note.

522. Numerical Illustration.—To take figures, the following table gives the beat frequency corresponding to various adjustments of the heterodyne circuit, if a wave at a frequency of 48,000 cycles per second is being received :—

Incoming Wave.	Heterodyne Circuit.	Note heard or Beat Frequency.
Frequency in Cycles/sec.	Frequency in Cycles/sec.	
48,000	49,000	1,000
	48,750	750
	48,500	500
	48,250	250
	48,000	0
	47,750	250
	47,500	500
	47,250	750
	47,000	1,000

From an inspection of this table we may deduce that when the frequency of the heterodyne circuit is the same as that of the incoming signal, no sound is heard in the telephones. There will be a space on either side of this point where the beat note is so low as to be inaudible. This occurs if the beat frequency is less than about 20 cycles per second. This space is called the “**Dead Space.**”

As the LC value of the heterodyne circuit is increased or decreased relatively to the LC value of the signal, a note of increasing pitch is heard.

Eventually the note becomes so high as to be beyond the limit of audibility.

The same note can be obtained at two points, *i.e.*, points at equal differences of frequency above and below the dead space, *e.g.*, in the above example a 500-cycle note is heard when the heterodyne circuit is set to 48·5 or 47·5 kc./s.

The great advantage of the heterodyne system of reception can now be seen—that the note is absolutely under the control of the

receiving operator. By a slight variation of the heterodyne condenser he can adjust the note in his telephone to any pitch that suits his hearing.

In addition, it is very easy to get over interference by over-reading.

For example, if an operator wished to receive a signal of 100 kc./s. frequency, and interference was experienced at 99 kc./s., he could either :—

- (1) Set his heterodyne to 100·7 kc./s. and read the signal as a 700-cycle note, while the interference would be heard as a 1,700-cycle note (which would be easy to over-read).
- (2) Set his heterodyne to the frequency of the interference, or close enough to it to be inside the dead space ; in this case to 99 kc./s., giving the wanted signal as a 1,000-cycle note, while the interference would be inaudible.

523. Adjustment of Heterodyne Circuit.—It is important to note that, when working with high-frequency signals, whose LC value is small, a very small difference in LC value in the heterodyne circuit as compared with that of the signal will give a big frequency difference and a high-pitched note ; while, conversely, when receiving low-frequency signals, *i.e.*, signals corresponding to a large LC value, the heterodyne circuit has to be thrown considerably out of tune with the incoming signal to produce a readable note.

For example, to receive a signal from a transmitting station set to 30 LC at a 500-cycle note, the heterodyne circuit should be set to 29·965 or 30·035 LC, while for receiving a 10,000 LC wave at a 500-cycle note, the heterodyne circuit has to be set at 9,795 or 10,210 LC.

It may be proved that the change in LC value to give a note of a certain pitch is proportional to the pitch and to the $3/2$ th power of the LC value, and as the LC value is inversely proportional to the square of the frequency, the change is proportional to the pitch and inversely as the cube of the frequency.

In the above example, this is equivalent to saying that

$$\frac{30\cdot035 - 30}{10,210 - 10,000} = \left(\frac{30}{10,000} \right)^{3/2}$$

which may easily be verified.

Hence great accuracy is required when working with high frequencies, as a small variation in the LC value of the transmitting circuit, with a fixed adjustment of heterodyne at the receiving end, will cause a big variation in the note heard in the telephones.

Such changes in the transmitted frequency may arise from very small alterations in the aerial capacity such as occur, for instance, with an aerial swaying in the wind.

524. Comparison between C.W. and Spark.—From the point of view of reception the following contrasts may be drawn (*also see* para. 469):—

- (a) C.W. reception requires the production of a local oscillation, which is unnecessary with spark or I.C.W. as they are modulated at the transmitter.
- (b) Spark has the advantage that the transmitting station is defined by its note, or spark train frequency; but the fact that the operator can adjust the note to suit his ear and to cut out interference, when receiving C.W., more than compensates for the loss of a characteristic note inherent in the signal itself.
- (c) Circuits can be made much more selective for the reception of C.W., as any wave form which is modulated at the transmitting end may be regarded as being in itself the resultant of several C.W. oscillations at different frequencies. This point will be referred to again under radio-telephony.
- (d) Receiving apparatus, equipped with a heterodyne circuit, can be made more sensitive as regards reception of C.W. than when receiving spark. The reason for this will be explained in the chapter on the valve as detector, Chapter XII.

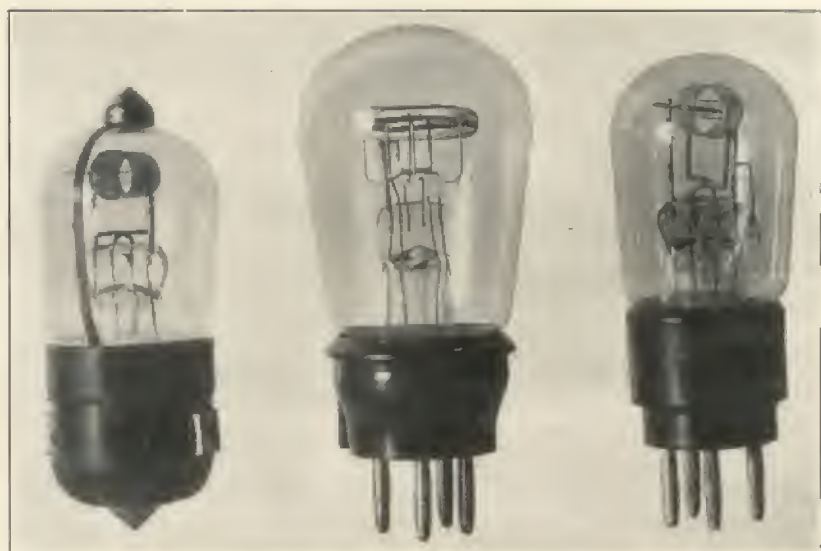
CHAPTER XI.

THERMIONIC VALVES, THEIR CONSTRUCTION AND CHARACTERISTICS.

HISTORICAL INTRODUCTION.

525. The Fleming Valve.—In the eighties of the 19th century, Thomas Edison, who was a pioneer in the development of the electric incandescent lamp, found that a current passed across the space between a hot filament and a metal plate which he had sealed into an evacuated lamp bulb. The current, which was of the order of one or two milliamperes, was detected by connecting a galvanometer in the external circuit between the hot filament and the metal plate. This effect, known as the "Edison Effect," was studied by many experimenters, and J. A. Fleming in particular carried out researches on the subject over a period of many years. In 1899, J. J. Thomson announced his discovery of electrons, and this gave a clue to the nature of the Edison Effect. Fleming had shown that the current was due to something travelling from the hot wire to the collecting plate, and realised that the unilateral nature of the current could be applied for the detection of electrical oscillations. In 1904 he took out a British Patent covering the use of this device for the rectification of both radio and audio frequencies. This was an entirely original application of the Edison Effect, and it definitely marks the introduction of the vacuum **valve** to wireless engineering. In its original form the Fleming valve had a cylindrical anode made of aluminium, and a hair-pin carbon filament. It was a soft valve (*see* para. 531) and functioned solely as a rectifier or detector of oscillations. It was used as a substitute for crystals, etc., for detection, but since the development of greatly improved processes of evacuation, the modern high vacuum type of two-electrode valve has chiefly been used as a means of rectifying high alternating potentials, and, through suitable circuits, supplying smoothed high-tension direct current.

526. The Three-Electrode Valve.—In 1907 Lee de Forest patented in America a thermionic valve with a third electrode—the grid—in addition to the original filament and plate of the Fleming valve. De Forest had done a considerable amount of work on various modifications of Fleming's original oscillation detector, and his discovery of the influence of the grid produced an enormous increase in the potentialities of the apparatus, although it was not until several years later that improved methods of evacuating valves enabled these to be fully developed. Since the later years of the



Types of Small Receiving Valve.

FIG. 271.

war the three-electrode valve has completely revolutionised the technical development of wireless apparatus; in addition, it has become an important instrument in the measurement and detection of small effects in electrical work generally, and its use is by no means exclusively confined to wireless applications.

In wireless telegraphy the three-electrode valve is used:—

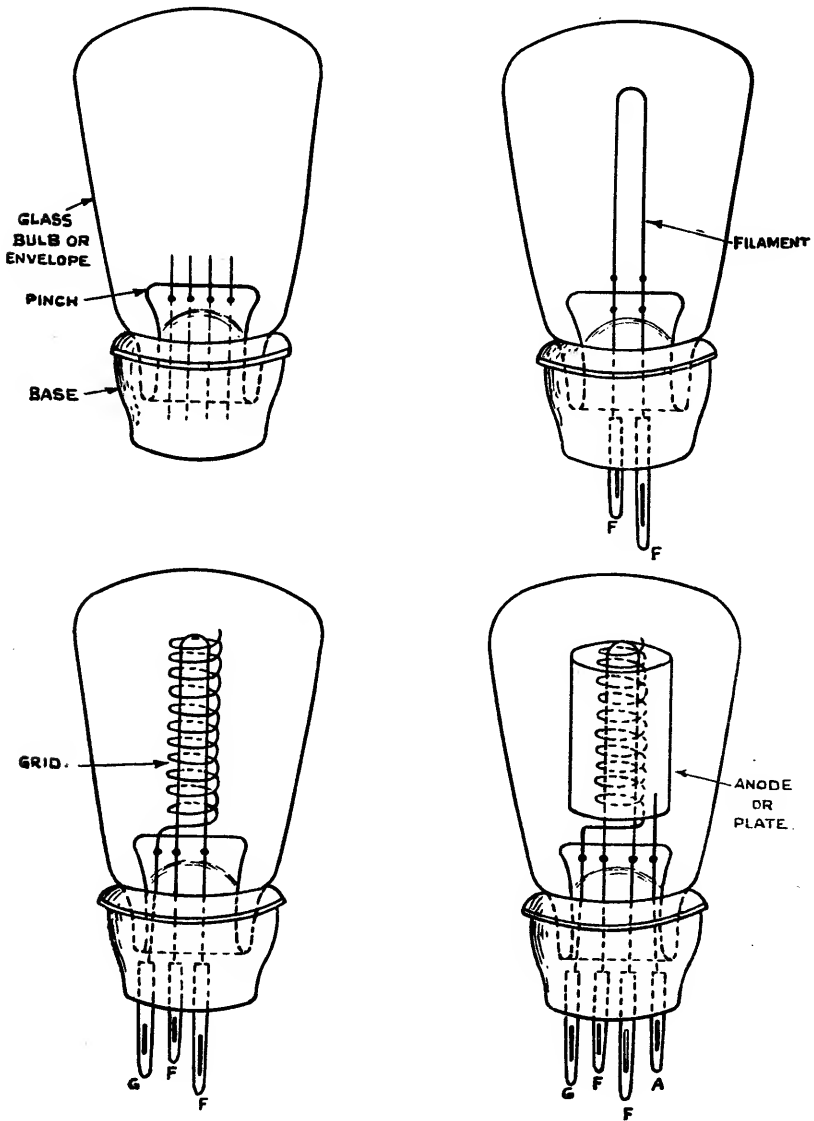
- (1) As a **Detector**.
- (2) As a **Local Oscillator** for heterodyne purposes when receiving C.W.
- (3) As an **Amplifier**. The use of the valve as an amplifier is probably the most marked advantage which has followed its invention. It has increased the efficiency of receivers to an enormous extent and made possible the use of loud-speakers. The improvement in sensitivity, amongst other things, has enabled the use of loop aerials to be developed.
- (4) As a **Transmitter**. For this purpose the valve is rapidly replacing all other methods of generating oscillations and energising an aerial for both telegraphy and telephony. Its operation is fundamentally the same as for (2) above, as in both cases the valve is maintaining oscillations in a circuit, but in (2) the amplitude required is only a minute fraction of the amplitude of the oscillations produced by a transmitter for communication over wide areas of the earth's surface.

Valves have also been developed with four, five or more electrodes. The additional electrodes gives rise to various properties which are applied for particular purposes; some of these will be discussed later. The words diode, triode, tetrode and pentode are used to indicate two-electrode, three-electrode, four-electrode, and five-electrode valves respectively. Valves with more than five electrodes have not come into general use, and no generally adopted names have therefore been applied to such types. The symbols used throughout this book to represent the different valves are given in Fig. 306 at the end of this chapter.

GENERAL THEORY OF THE THREE-ELECTRODE VALVE.

527. The Elements of the Valve.—Three types of small receiving valve are illustrated in Fig. 271. These are triodes of the types most commonly used in the Naval service. The valves in actual use at the present day usually have their glass bulbs covered with a metallic deposit (which will be described later), making it impossible to see the electrode system clearly. The examples selected for Fig. 271 were chosen so that the internal parts could be compared with the following description and with the constructional diagrams of Fig. 272.

The envelope, if of glass, has a re-entrant tube in the base for supporting the electrodes. The wire supports and leads are welded



The Elements of a Triode.

FIG. 272.

into the inner end of the re-entrant tube, which is softened and pinched on the wires for this purpose. This flattened portion of the re-entrant tube or stem is termed the "pinch."

Essentially a valve consists of the following components (*see* Fig. 272) :—

- (a) **A suitable "envelope,"** usually a glass bulb, which is exhausted to a very high degree of vacuum.
- (b) **The Filament.**—This consists of a straight wire or a loop of tungsten, thoriated tungsten (*i.e.*, tungsten impregnated with thoria) or a wire or ribbon coated with certain materials which will be described later in this chapter. The two ends of the filament are brought to two contacts or pins outside the envelope.
- (c) **The Grid.**—This consists of a spiral of metal wire, or a fine wire mesh cylinder, surrounding the filament. A single lead from the grid is brought to a contact or pin outside the envelope.
- (d) **The Anode (or Plate).**—This consists of a metal cylinder placed outside and surrounding the grid. As in the case of the grid, a single lead is brought from the anode to the outside of the envelope.

In modern receiving valves the grid and anode are usually box-shaped, or rectangular in section instead of circular, to enable improved characteristic properties to be produced. An example of this is seen in the middle valve of Fig. 271.

The typical valves shown in Fig. 271 illustrate the usual methods of securing electrical contact between the electrodes and external connections. One valve has the filament terminal contacts at the extremities of the envelope and the grid and anode terminals on the sides, connection being made to the external circuits by inserting the valve in a suitable socket with sprung metal fittings. The other valves show the more common type with four terminals in the form of sprung pins on the base cemented to the envelope ; the electrical connections are made by inserting these pins in a holder which has four corresponding tubular sockets. The pins are not disposed symmetrically, and so it is impossible to put the valve in the holder with the contact pins in the wrong sockets.

In general, valves used for transmitting purposes are larger and are subjected to much higher voltages than valves used for receiving (*i.e.*, detecting, amplifying, heterodyning). The essential features are the same, except that, in transmitting, the power dissipated in the valve is so great that precautions to prevent melting of the electrodes or softening of the envelope constitute an important feature in the design.

The requirements involved in the design of receiving valves are as follows :—

Long life, durability and robustness.

Small filament current consumption.

Constancy in performance throughout the life of the valve.

Suitability for use in heterodyne circuits.

Effective operation as detectors.

Freedom from spurious noises for use in amplifiers.

Low capacity between the electrodes.

Similarity of performance between all valves of the same type.

Certain other requirements depending on the geometry of the electrodes have also to be considered, the reasons for which will become evident after this chapter has been read. Some of these cannot be achieved in one and the same valve, so that the tendency has been for many years to manufacture valves for special positions in sets, *e.g.*, R.F. amplification, detection, etc.

528. The Filament.—The various components of the valve will now be considered in detail, starting with the filament.

It was explained in Chapter II that the mobile electrons present in a substance which is an electrical conductor are continually moving about from one atom to another, and that, if the substance is heated, the velocity of the electrons is increased. If the temperature is raised sufficiently, it is believed that in certain substances

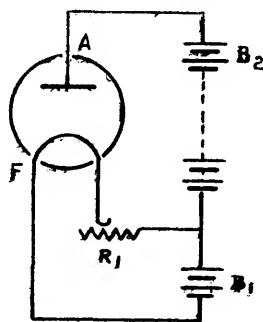


FIG. 278.

the electrons are agitated to such an extent that some of them leave the surface of the substance. This evaporation of electrons from the surface of a body is very similar to the evaporation of molecules of liquid at a surface exposed to air, and a similar quantitative law has been found to apply. The effect is known by the term "thermionic emission."

If there is no external electric field acting on the escaped electrons, they return to the parent substance, which their absence has left positively charged.

Thus, any conductor, when heated above a certain temperature, is surrounded by a cloud of electrons, which are continually being shot out of it, and attracted back into it. This emitting property of a hot metal is the basis of the thermionic current in the valve. The hot metal is the filament and it is made of a material which, raised to a suitable temperature less than its melting point, gives adequate electron emission for the purpose to which the valve is to be put.

The method used to heat the filament is to pass a current through it. As we shall be concerned in valve theory with batteries connected between the various electrodes, and the resulting electrical conditions of voltage and current, it is important to grasp clearly the idea that the battery which is connected across the ends of the filament is there simply because it is the most convenient way of raising the temperature, and the heating current applied to the filament is entirely distinct from the emission current, or the currents that flow from one electrode to another.

A low-voltage battery of 2, 4 or 6 volts is used for heating the filament of a small receiving valve. (See B₁, Fig. 273.) In certain cases an adjustable resistance R₁ is connected in series with the filament to regulate the heating current flowing. Large transmitting valves are generally supplied with filament power from machines and transformers, and in order to preserve the correct voltage on the filaments the filament rheostat is essential and must be carefully used in conjunction with a reliable voltmeter.

529. The Emission Current.—At ordinary temperatures there is no measurable emission from the surface of materials. The effect can only be observed at comparatively high temperatures and, once started, it increases very rapidly as the temperature is raised. This is illustrated in Fig. 274, which gives the emission current for tungsten over the range of temperatures at which this metal emits a useful quantity of electrons. Thermionic emission is a surface effect and it depends on the physical nature of the surface as well as on the temperature. Later on in this chapter filaments will be described which are made with surfaces able to produce useful quantities of emission at lower temperatures than tungsten. For the moment it suffices to say that there are three classes of emitting materials:—

1. Pure tungsten, operating at 2,400° A. to 2,500° A.
2. Thoriated tungsten, operating at 1,800° A. to 1,900° A.
3. Certain "oxides" operating at 1,100° A. to 1,300° A.

NOTE.—The absolute scale of temperatures which is generally adopted for filament temperatures has its zero at -273°C . The degree intervals are the same on both scales, so that a reading on the absolute scale (indicated by °A.) is obtained by adding 273 to the corresponding reading on the Centigrade scale.

The general form of the curve shown in Fig. 274 is the same for all three types. It agrees closely with the following formula

$$i = AT^2e^{-\frac{b}{T}}$$

where i = electron emission in amperes per sq. cm.

T = temperature in degrees absolute.

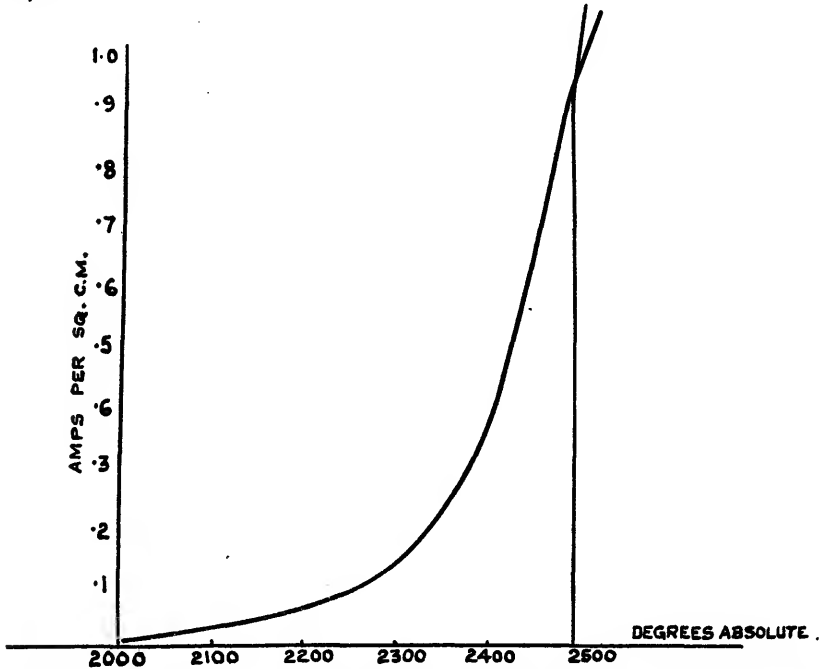
e = the base of Napierian logarithms = 2.71828.

A and b are constants which vary with different emitting materials.

For pure tungsten, $A = 60$, and $b = 52,400$.

For thoriated tungsten, $A = 3$, and $b = 30,500$.

The values of A and b for the various "oxides" have been investigated by many experimenters, but their results do not agree very closely; A appears to be about 0.001 and b from 12,000 to 20,000.



Filament Emission per sq. cm. of Tungsten Surface Plotted against Temperature.

FIG. 274.

The operating temperature for pure tungsten is approximately the same as the temperature of the filament in an electric lamp. Thoriated tungsten operating at 1,800° to 1,900° A is distinctly less bright, though definitely glowing, and hence arose the name "dull emitter" when this type of filament was introduced. The same term is used for the third class, the oxide-coated filament, and is more justified in this case, for the operating temperatures are so low that in certain types of valves the glow from the filament is practically invisible. Further information on these filaments will be found in paras. 533 and 534.

530. The Anode and the Anode Current.—If another electrode, A (Fig. 273), is placed close to the filament, and raised to a positive potential with respect to the filament, for example, by a battery B_2 , it will attract to itself some of the electrons which have been emitted by the filament. Such a collecting electrode is called an "anode."

As the electrons have a certain velocity on breaking through the surface tension of the filament, it is possible for an anode to collect those which are driven on to it by their velocity, without a difference in potential being necessary; but this effect is negligible and will not be further considered. In general the anode is made considerably positive to the filament. The anode voltage of a receiving valve is from 50 to 200 volts, and is generally provided by joining a high-tension battery between the outside terminal of the anode and one of the terminals of the filament (for standard measuring purposes, the negative terminal). The anode voltage of transmitting valves may be from a few hundred to ten or twenty thousand volts, and may be provided from high-tension D.C. machines, or from alternators through rectifiers.

Referring again to Fig. 273, when the negative electrons arrive at the anode they pass along the conductor, through the battery and back to the filament. Thus there is a current round the circuit comprising battery, valve and leads.

Attention may be directed again to the difference between the direction of the actual flow of electrons round the circuit and the flow of electrical current as conventionally assumed. The electrons travel through the valve from negative to positive, whereas the flow of an electrical current has always been assumed to be from the positive terminal of a source of current to the negative. Thus in ordinary electrical parlance the anode current is spoken of as flowing from the positive terminal of the battery, through the valve, and back to the battery at the negative terminal. In the case of receiving valves the anode current is generally of the order of a few milli-amperes; in a large transmitting valve it may be many amperes.

It will now be evident that one essential property of the valve is that current can only flow through it **in one direction**. Hence, obviously, arose the use of the term "valve" for this piece of apparatus.

The electron current across the valve to the anode, from a filament with a copious supply of emitted electrons, depends in value on the shape, size and relative position of these two electrodes and also on the voltage between them. As this relative voltage, commonly referred to as the **anode voltage** (with respect to the filament as zero) is steadily increased, the **anode current** up to a point steadily increases also. At that point the anode voltage is such that every electron emitted by the filament is attracted to the anode, and, no matter how much further the anode voltage is increased, no increase in current is possible because the supply is limited by the rate of emission (Fig. 275).

This limiting value of anode current is called the "**saturation current**," and equals the total available filament emission. The anode current can only be increased for higher anode voltages by

increasing the filament temperature and thus increasing the available emission current.

The rise in anode current with fixed filament temperature and increasing anode voltage, which follows the curve illustrated in Fig. 275, is found to obey the law

$$I_a = \frac{K}{r} \cdot V_a^{\frac{3}{2}}$$

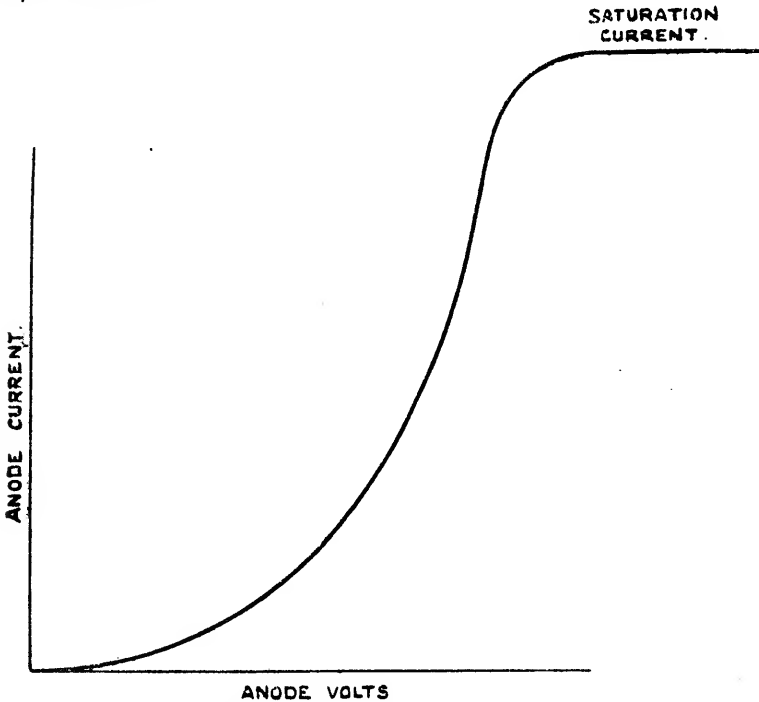
where K = a numerical factor.

r = radius of cylindrical anode surrounding the filament.

I_a = anode current.

V_a = anode voltage.

This is an important relationship, and is known as the "three-halves power law."



Anode Current Plotted against Anode Voltage ($\frac{3}{2}$ Power Law).

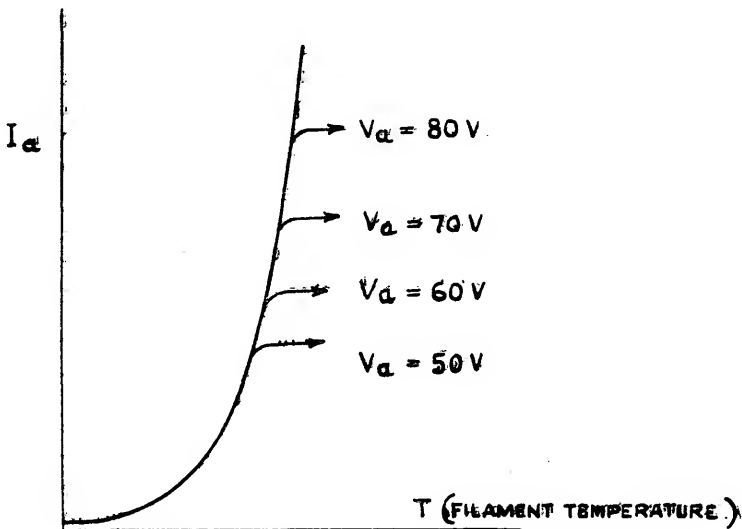
FIG. 275.

The curve rises until the anode current attains the saturation value, when it bends over and becomes horizontal. This condition, however, is never reached in a serviceable thermionic valve, as the filament is designed to give ample emission, easily covering the highest anode voltage which may be applied. Thus in order to observe the saturation effect in a standard type of valve it is

necessary to reduce the filament current and so limit the available emission.

In a triode the emission curve is obtained by connecting the grid and anode terminals together and thus treating them as one collecting electrode. Under normal working conditions the grid is not connected to the anode, and the anode current flowing through the mesh of the grid does not exactly follow the formula given above. This matter will be dealt with later in the chapter; at present only the collection of filament emission by a surrounding anode is being considered.

An important characteristic of the graph in Fig. 275 is that it is **curved**. It has already been shown in Chapter X that a non-linear relationship between voltage and current is a necessary and sufficient



Anode Current Plotted against Filament Temperature.

FIG. 276.

condition for detection to take place, and so it is evident that the simple two-electrode valve may be used as a detector of wireless signals; but the three-electrode valve is a much more effective detector and has almost entirely displaced the diode for this purpose.

The behaviour of the anode current may be investigated from another point of view. Let us consider the voltage between anode and filament as constant, and raise the filament heating current, and hence the filament temperature. For low values of filament temperature the available emission is so small that the applied anode voltage will attract all the electrons, and these conditions will hold with increasing filament temperature until it rises to such a point that the emission current just equals the current the anode is capable

of attracting. As the temperature is increased still further, only a portion of the emitted electrons are collected by the anode and the remainder fall back towards the filament. The emitted electrons form a negatively charged cloud round the filament which is known as the "**space charge**." When the filament is heated further, the space charge becomes more dense, but the anode current remains at the steady value it has reached.

This effect is shown in Fig. 276, which illustrates the increase in the value of maximum anode current for different anode voltages. This curve is obviously similar to that of Fig. 275, with the addition of the horizontal portions representing the maximum anode currents which the respective anode potentials are able to collect under the conditions of the particular valve employed.

531. The Vacuum. Hard and Soft Valves.—Modern valves are exhausted to a very high degree of vacuum, possibly to a pressure of the order of $\cdot 0000001$ mm. of mercury. Such valves, known as "**hard**" valves, were not in existence prior to the introduction of new types of high vacuum pumps during the period of the war.

In the early days of the use of valves in wireless telegraphy, "**soft**," or only partially-exhausted valves were used. It was generally considered that traces of gas assisted the operation of the valve. Special "**inert**" gases—argon, helium and nitrogen—were introduced to endeavour to obtain uniform conditions and to obviate the erratic phenomena which the presence of oxygen and its compounds had been found to produce. The conditions which determine the electron flow between electrodes in a soft valve are very different from those in a hard valve. The electrons do not have an entirely unobstructed passage to the anode. Collisions occur between the electrons and the molecules of the residual gas and the impact of the electrons may be sufficient to ionise the molecules. The positive ions formed move off towards the filament and may unite with electrons from the space charge and so become neutral once again. The presence of positive ions has the effect of neutralising the space charge and of enabling more electrons in the vicinity of the filament to come under the influence of the attractive force of the anode. This increases the anode current, which is further increased by the electrons liberated from the gas. Many of the positive ions reach the filament, and the heat generated by their impact has the effect of raising the temperature of the filament and increasing the emission, but unfortunately the impact of positive ions causes disintegration of the filament and it burns out more quickly. The blackening of the bulb of a valve (*e.g.*, a glass transmitting valve) is caused by the deposit on the envelope of particles of filament material driven off by disintegration.

If a high anode voltage is used or if the quantity of residual gas is sufficient, the "**ionisation by collision**" may be so intense that it is visible as a **blue glow**. If this should be heavy and be allowed

to persist for any length of time the valve will be ruined by the very rapid disintegration of the filament or melting of the anode.

Soft valves gave excellent results as detectors, but they required careful handling. The anode current was very much influenced by the degree of softness, or the pressure of the residual gas, and devices were adopted in certain types to endeavour to control this pressure by applying heat externally.

The great advantages of valves exhausted to an extreme degree of hardness are :—

- (a) They have a much longer life.
- (b) Individualities are eliminated and valves can be manufactured in large quantities to a very close specification so as to give reasonably similar results when in use. This enables replacement of valves to be carried out in complicated circuits without difficult adjustments, and, further, valves may also be used in parallel without individual adjustments.

532. The Grid. Secondary Emission.—The third electrode of the valve to be considered is the **grid**, which, as its name indicates, consists of an open wirework—either a spiral or mesh—and which is placed between the filament and the anode. The potential of the grid relative to the filament exercises a controlling effect on the electron current from filament to anode, while, due to its open construction, it does not act to an appreciable extent as a collecting electrode. In practice the potential of the grid is varied, *e.g.*, incoming signals produce voltage variations in the tuner which are applied between grid and filament, with consequent variations of the anode current at the same frequency.

The very important position which the three-electrode valve occupies in wireless engineering and in allied sciences is due to the remarkable control which the grid exerts on the anode current. This is illustrated by the graph, Fig. 277, which shows how the anode current of a standard type of receiving valve varies with changes in grid voltage.

Starting with the grid potential at a considerable negative value with respect to the negative terminal of the filament, and gradually reducing this negative value, it is observed that no anode current will flow, although the anode potential is 50 volts, until the grid potential reaches -5 volts. As the grid potential changes from -5 volts to zero the anode current rises to about 0.7 mA., and it continues to rise as the grid voltage increases in the positive direction. The anode current in the example shown attains the saturation value of approximately 1.5 mA. when the grid voltage reaches 6 volts positive. This curve describes the relationship, in this particular valve, between the anode current and the controlling grid voltage. It is usually termed a **Mutual Characteristic** of the valve, and a family of such curves taken at various anode voltages can be

used for ascertaining the behaviour of the valve under various circuit conditions (para. 543).

The grid itself begins to take current at about zero grid voltage, but this current is **very** small at first. In the case shown in Fig. 277, for example, the grid current is only about 100 microamperes when the anode current has attained its maximum value of 1.5 m.A. at about 5 to 6 volts. Thus it would appear that for negative and small positive values of grid voltage the grid exercises considerable control over the anode current with practically no expenditure of energy itself, *i.e.*, with a minimum of damping in the input circuit to the grid. But this question is complicated by the inter-electrode capacities (*see* para. 553), owing to which the output circuit may

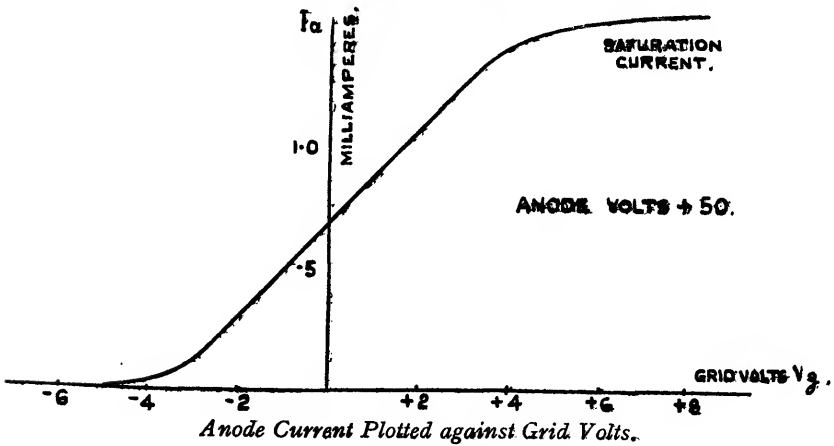
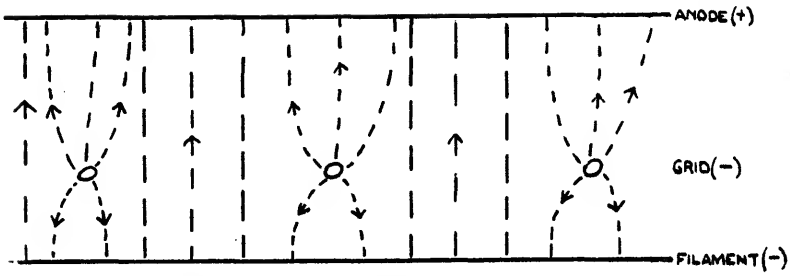


FIG. 277.

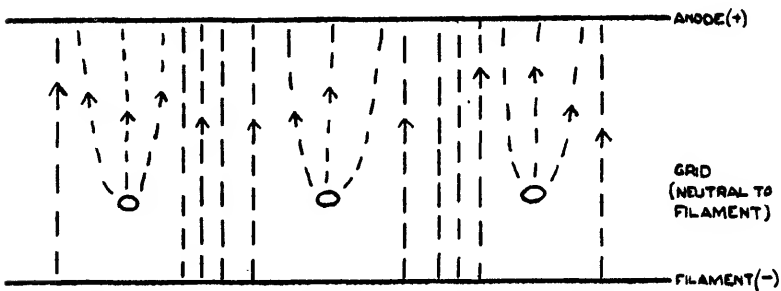
be responsible for power being consumed by the input circuit when no grid current is flowing. However, with properly-designed apparatus this effect can be made a minimum and the fullest advantage be taken of the amplifying property of the triode. This is the property of the thermionic valve with a control grid which gives it its unique position as an amplifier in wireless reception.

A mental picture of the action of the control grid may be obtained with the conception of lines of force as illustrated in Fig. 278, which shows enlarged diagrams of portions of the three electrodes. These lines represent the idea of strain (para. 99), or attraction between positive and negative charges at the ends of the lines. Thus lines of force connect positive charges on the anode with negative ones on the filament or in its vicinity. In Fig. 278 the electrostatic field is shown between the electrodes without the presence of electrons, in order to make the position more easily understood. The arrowheads indicate the direction in which electrons tend to move, and not the direction of conventional current.



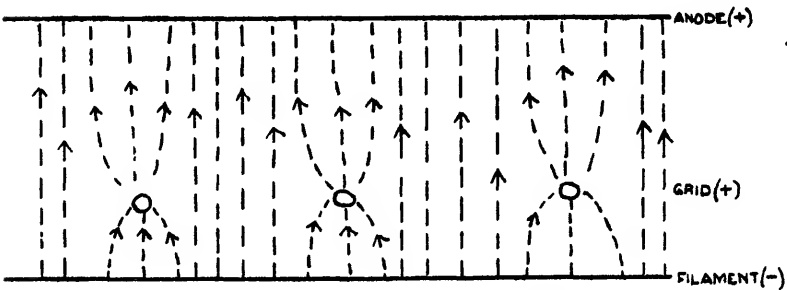
GRID NEGATIVE WITH RESPECT TO FILAMENT.

(a)



GRID AT THE SAME POTENTIAL AS THE FILAMENT.

(b)



GRID POSITIVE WITH RESPECT TO THE FILAMENT

(c)

Electrostatic Field in a Triode.

FIG. 278.

When the grid is negative with respect to the filament (Fig. 278 (a)), the lines of force between the grid and filament represent the strain in the reverse direction, *i.e.*, electrons are forced back to

the filament by the negative grid potential. If the grid potential is sufficiently negative the influence of each wire will extend along the greater proportion of the filament and entirely suppress the emission current.

When the grid potential is neutral with respect to the filament there is no electrostatic strain between the two electrodes and electrons are therefore removed solely by the strain represented in Fig. 278 (b) by the lines of force joining anode and filament between the grid wires.

When the grid potential becomes positive, Fig. 278 (c), the direction of the field between grid and filament is everywhere such as to draw electrons away from the filament, but because of the large spaces between the grid wires, the majority of the lines of strain connecting filament and grid are curved. All the electrons which are initially given an outward impetus by the grid potential travel faster and faster under this attraction, and because of their acquired momentum the majority of them do not keep to the original **curved** lines of strain, but are shot through the open spaces between the grid wires. Practically only the electrons which are drawn straight to the grid wires actually reach them, and hence the grid current is small. All the others pass through the interstices of the grid and travel on to the anode. The greater the grid potential, the greater becomes the field intensity near the filament, and the quantity of electrons removed to the space outside the grid is correspondingly increased.

At high positive values of grid potential—approaching the anode potential—the concentration of the field considerably increases the grid current, and if the grid potential exceeds the anode potential there will be a reverse field outside the grid tending to send back to the grid the electrons which have passed through. The grid current will then be very high. Such an effect does not occur in valves operating in receiving apparatus, but conditions approaching it sometimes occur in transmitting valves.

It has been shown by numerous experimenters that when electrons hit an electrode, such as the grid or anode of a valve, they may cause the ejection of other electrons by their impact. These "secondary electrons" may actually exceed in quantity the primary electrons which produce them. Obviously they can only be detected by being collected on an appropriate electrode at a higher potential than the target electrode. Secondary electrons emitted by the anode of a triode at high potential are attracted back to the anode by this potential, but secondary electrons emitted by a grid, which happens to be at a potential sufficiently high to cause the production of secondary emission, may pass to the anode which is normally at a still higher potential, and thus the secondary electrons are included in the anode current. The effect, in general, is very undesirable, and numerous efforts have been made to prevent

it by coating the grid with various materials, but no satisfactory solution is known at present. A particular effect which sometimes arises with a transmitting valve, and is known as "blocking," is possibly the outstanding example of limitation of operation by secondary emission. The conditions of a transmitting circuit cause the grid potential to swing positive and negative. If these conditions produce such a high positive swing that the grid current becomes very large, the resulting secondary emission from the grid may make the grid potential so much more positive that it cannot return. The result is a cessation of oscillations of the circuit and a great increase in both anode and grid currents at high static potentials with consequent great risk of overheating one or both electrodes.

533. The Tungsten Filament.—Reference has already been made to the three main classes of emitting materials, viz. :—

- (1) Pure Tungsten, operating at 2,400°–2,500° A.
- (2) Thoriated Tungsten, operating at 1,800°–1,900° A.
- (3) "Oxides," operating at 1,100°–1,300° A.

Tungsten wire came into very general use as the filament material of incandescent electric lamps during the first decade of this century. It was introduced because its physical properties enabled it to withstand higher temperatures than the carbon filament which had preceded it, and consequently made it a better source of light. Tungsten has a very high melting point compared with other metals, as is shown by the following table :—

Melting Points.

Tungsten	3,400° C.
Molybdenum	2,450° C.
Platinum	1,756° C.
Iron	1,530° C.
Nickel	1,452° C.
Copper	1,084° C.
Gold	1,060° C.
Silver	960° C.

As the emission of light increases with temperature, it is obvious that the material which can be produced in a suitable form and can also stand the highest temperature is the most suitable one to use as a lamp filament. However, the very high melting point of tungsten indicates also that it is a difficult metal to work. It is, in fact, one of the most refractory metals which metallurgists have to deal with, and a considerable amount of research was carried out before good filament wires of tungsten were successfully produced. This experimental work was done for electric lighting filaments, and it was natural that tungsten should very soon be used in the subsequent development of the thermionic valve when it was realised that the

properties which rendered it an effective source of light made it an equally effective source of electrons.

In order that the filament of an incandescent lamp should have a long life, it was found to be essential to exhaust the bulb to the highest possible vacuum. The same condition was soon found to be desirable for preserving good and uniform conditions of electron emission. Thus in two important features the valve industry profited by the experience of the lamp industry, and as similar methods of assembly, using identical machinery, could be employed to a very large extent for both, the two industries have been closely associated for research and production purposes. In recent years, however, it has been found possible to operate tungsten filaments at still higher temperatures, and so produce more light, by filling the bulbs with an inert gas, such as argon, after all traces of oxygen and water vapour have been removed. The majority of thermionic valves, on the other hand, still require the highest possible vacuum for efficiency, so that there is now not so much in common in the modern products of the two industries.

Tungsten filaments have been used in a wide range of diameters from about 0.05 mm. in receiving valves up to 1.3 mm. in the largest transmitting valves; the corresponding range of heating currents is from about half an ampere to 75 amperes.

534. The Thoriated Tungsten Filament.—Thoriated tungsten wire is a type of lamp filament introduced during the lamp development work referred to in the previous paragraph. Pure tungsten filaments were liable to fracture suddenly, generally on switching the current off, and this was traced to a change in the crystalline nature of the material. It was found that the addition of a trace of thorium (thorium oxide) to the tungsten during manufacture prevented this defect to a considerable extent, and it was during observations on valves with filaments constructed with thoriated lamp wire that the enhanced electron-emitting properties of this material were discovered. According to modern theory, based on extensive researches, the thorium oxide is decomposed in the filament, and the pure metal thorium forms a layer on the surface of the tungsten and is responsible for the emission of electrons. Thorium has the property of emitting copiously at a temperature five or six hundred degrees lower than tungsten, consequently a thorium-coated filament requires less heating current than a pure tungsten one. The filament is distinctly bright, but by no means so brilliant as the normal "bright filament."

The layer of thorium is not extremely robust, and is easily removed by excessive temperature. Further, if traces of gas remain in the valve, the filament becomes subject to a bombardment by the positive ions formed by ionisation of the gas, and this also wears away the surface of the filament. The surface of a pure tungsten filament is similarly worn away by excessive temperature and ionic

bombardment, but in this case the remaining surface is normally hot enough to continue to emit; on the other hand, if a thoriated filament loses its surface of thorium the emission current fails, because the new surface consists of tungsten, and at the operating temperature of thorium which the specified working voltage would maintain, this tungsten surface has a negligible emission. The ionic bombardment of the filaments is greatly intensified by high anode voltages and on this account the thoriated filament is not suitable for use in transmitting valves. By using special manufacturing processes, and taking extreme measures to ensure the highest possible vacua, valves with thoriated filaments have been made to withstand anode potentials of 1,500 volts, but, in general, thoriated filaments in receiving valves should not be subjected to more than about 150 anode volts. Frequently manufacturers specify the filament voltage so that the wire is hot enough to cause a continual diffusion of thorium from the inside of the wire to the surface, and so to replenish the emitting coating. This precaution is not absolutely necessary, as the coating will last for many thousands of hours if the filament current is kept low and the valve has been properly evacuated. If the emission begins to fail it usually dies away gradually over a period of many hours and causes a gradual weakening of signals. **Sudden failure under normal conditions of use is never observed.** The emitting coating can be easily renewed by the following procedure:—Apply about two and a-half times the normal filament voltage for 20 seconds with no anode potential; this “flashing” of the filament is supposed to decompose the thoria and so provide a large store of thorium; then apply a voltage of about 20 per cent. above normal for some time with no voltage on the anode or with only about 20 anode volts; this is the “reactivation” process which produces the new coating of thorium on the surface of the filament. This treatment can be applied many times to a thoriated filament (although the quantity of thoria originally introduced in the filament wire is usually less than 1 per cent.), because a coating only one molecule thick is sufficient for the emission of electrons.

The thinnest thoriated filament made operates at 0.06 ampere, but this is not very much in use. The normal values are from 0.1 to 0.3 amps.

535. Oxide-coated Filaments.—The oxides of barium, strontium and calcium are copious emitters of electrons at a moderate red heat, and consequently filaments coated with these materials are very economical in battery power. The core is usually a metal ribbon, which shape obviously exposes most surface for a given quantity of core metal to be heated. The usual core metals are platinum, nickel-platinum, nickel, tungsten and molybdenum; nickel and molybdenum are in most common use. The heating currents are from 0.075 amperes upwards.

There are two main processes for coating filaments with oxides, and a number of variations in the details of the methods employed. In one type the oxides in the form of powder are mixed with some binding agent and painted on the core ribbon before the elements of the valve are assembled. In the second and more modern process the active material is caused to settle on the filament wire after the latter has been assembled in the valve. A barium compound—barium azide—is incorporated in a “dope” painted on the inside of the anode. In the manufacturing process the anode is heated, the dope volatilises, and some settles on the filament, which has been specially prepared and is heated slightly by a suitable current to enable the most effective adhesion of barium compound to take place. Generally speaking, this depositing process produces a more robust and more efficient emitting surface than the painting process, but the latter has been brought to a very high degree of success by one or two firms.

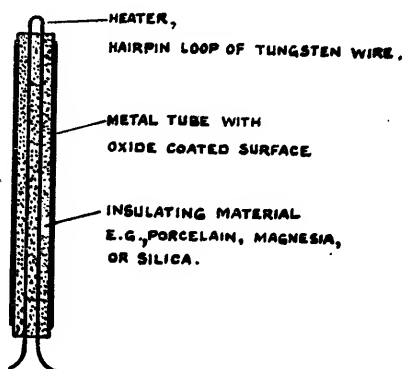
The modern view is that the compounds are decomposed by the heating and electrical treatment to which they are subjected, and that the actual emission of electrons takes place from pure barium metal set free by these changes. There is much experimental evidence to support this view, but it has not yet been proved conclusively.

Coated filaments are liable to lose their emitting materials by excessive heating current, ionisation of residual gas, and high anode voltages in the same way as thoriated filaments, but, unlike the latter, they cannot be reactivated.

As coated filaments are not heated so strongly as the other two types they do not expand so much, consequently it is possible to bring the grid wires much closer to them and so to increase the control of this electrode. Thus, generally speaking, valves with coated filaments have much better operating characteristics than those with the brighter filaments.

536. Indirectly-heated Cathodes.—Indirectly-heated cathodes are the most recent type, introduced to enable alternating current to be used direct on receiving valves. If an ordinary filament is supplied with alternating current at commercial frequencies, the fluctuations of voltage along its length produce corresponding fluctuations of the valve characteristics. The detector valve is particularly susceptible to this trouble, which produces a hum associated with the frequency of the power supply; this is, of course, amplified by the following stages of the receiving set. The indirectly heated cathode overcomes this trouble. The filament itself is not the emitter, but provides the heat necessary to produce emission from a tube which surrounds it. The heater is a stout hairpin tungsten filament with the standardised rating of 4 volts, 1 ampere. It is coated with an insulating material such as magnesia, porcelain, or silica, and over this is placed a closely-fitting tube whose external

surface is oxide coated (Fig. 279). This metal tube is the cathode ; it is usually made of nickel, and the coating is applied by one of the various painting processes before assembly in the valve. There is, of course, an extra lead from this electrode which is connected to an additional pin situated in the centre of the base of the valve. The heater is never regarded as one of the electrodes of the valve, being merely an auxiliary device which enables the cathode to emit



Indirectly Heated Cathode System.

FIG. 279.

electrons. Thus a valve with anode, grid, and indirectly-heated cathode is a "triode" and not a "tetrode." The cathode is a rigid tube, and consequently the grid may be brought extremely close to it. This allows very good operating characteristics to be obtained. The heater takes about half a minute to warm up the cathode to the emitting temperature, a point which has to be remembered when switching on amplifiers using these valves. On account of the high wattage of the heater, it is obvious that valves with indirectly-heated cathodes cannot be used in large numbers on batteries.

537. Materials for Grids and Anodes.—For grids and anodes metals are used which are clean, can withstand the heat generated both in the process of manufacture and in use, and can be relied upon not to exude gases which will spoil the vacuum of the valve. The first metal to come into universal use was nickel, which satisfies the above requirements very well. The limiting factor for nickel is its melting point, and consequently it has been superseded by molybdenum in very many types of valves.

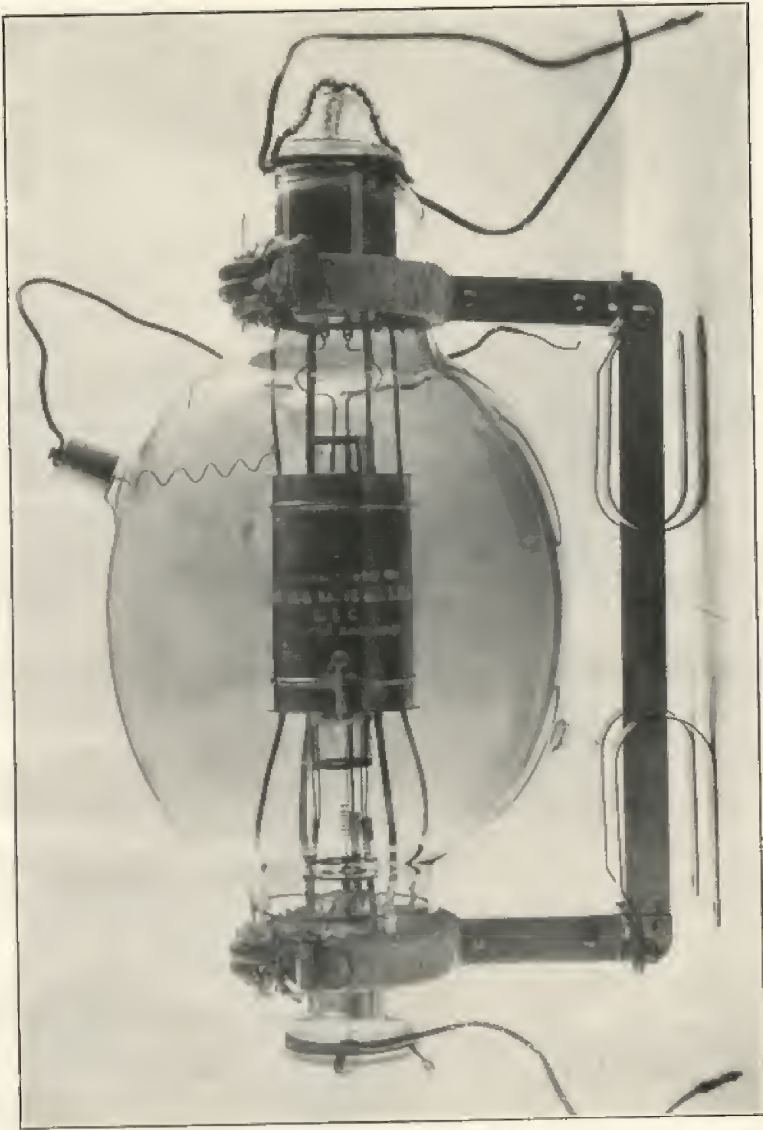
Molybdenum, being a highly refractory metal, is expensive to roll into sheets, and it is therefore the practice to weave cylinders, basket fashion, of molybdenum strip to form anodes for high power silica valves (see example in Fig. 281). The strip is about 1.5 mm. wide, and is produced by rolling thin wire, a comparatively cheap

operation. This basket construction has the advantage of presenting more surface, for the volume of metal employed, than a corresponding sheet anode, and it is therefore more easily out-gassed in manufacture. Moreover, it is not perfectly rigid, and can accommodate itself to changes of dimensions through heating without fracturing rigid insulating supports.

When electrons are driven across the space from the filament to the anode they bombard the anode and heat it in exactly the same manner as bullets heat a target, *e.g.*, if one ampere is flowing and the potential difference is 1,000 volts, then the rate at which energy is converted into heat in the anode is 1 ampere \times 1,000 volts = 1 kW. Valves have to be designed with anodes capable of dealing with the heat generated in this manner. In small receiving valves a few milliamperes flow under a P.D. of 100 volts or so, and the anodes are only required to deal with a fraction of a watt or perhaps a few watts. But in large transmitting valves the power thus wasted as heat per second may be many kilowatts, and it is necessary that the shape, size, and nature of the anode should enable this heat to be dissipated safely without injury to any parts of the valve.

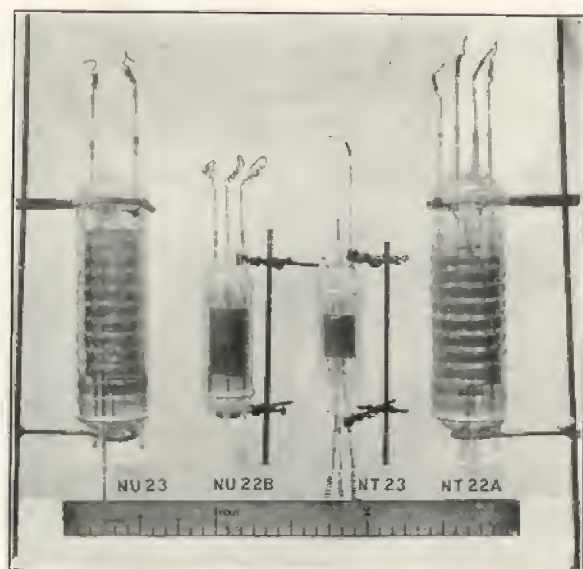
When any metal or other substance is heated it is liable to evolve gas which has become occluded in it during its previous history. All metals in valves are heated as much as possible during manufacture, and the evolved gases pumped away; but if in use an anode should become heated more than during manufacture, it is probable that more traces of gas will be evolved and the vacuum become "soft." Another danger is the risk of actually melting the anode. This frequently happens in small transmitting valves with nickel anodes. As a very general rule it may be stated that from one-third to one-half of the energy supplied to an oscillating transmitting valve is unavoidably wasted in heat in the anode. Thus a transmitting unit is limited in power by the heat which the valve can dissipate, and the trend of valve development for many years has been to produce larger and larger valves with improved devices for successfully conveying away this wasted energy. The power which can be safely dissipated by a valve when in use in an oscillatory circuit is generally regarded as the "rating" of the valve.

538. Envelopes.—The first bulbs containing the electrodes of thermionic valves were naturally made of glass. These became of larger and larger dimensions as the design of transmitters progressed. The largest standard type with a nickel anode is about 500 watts, *i.e.*, the anode is able to dissipate by radiation through the glass that amount of power as heat per second. Fig. 280 illustrates a transmitting valve with a nickel anode rated at 450 watts. By the use of molybdenum anodes and special heat-resisting glass, this type has been increased to 1,500 watts rating, but bulbs bigger than this are impracticable.



450-watt Glass Transmitter Valve (Type NT4A) in Spring Type Holder.

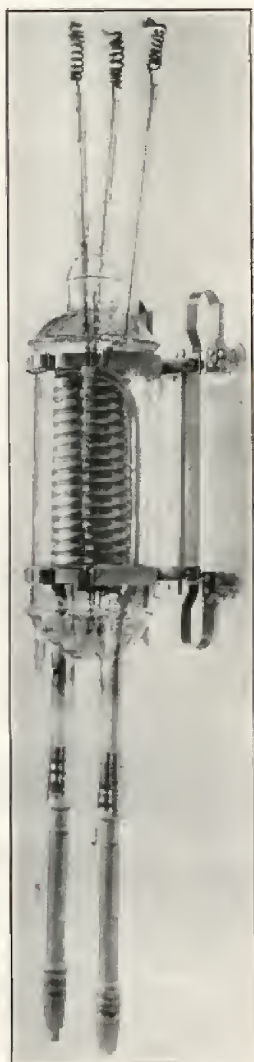
FIG. 280.



Silica Valves.

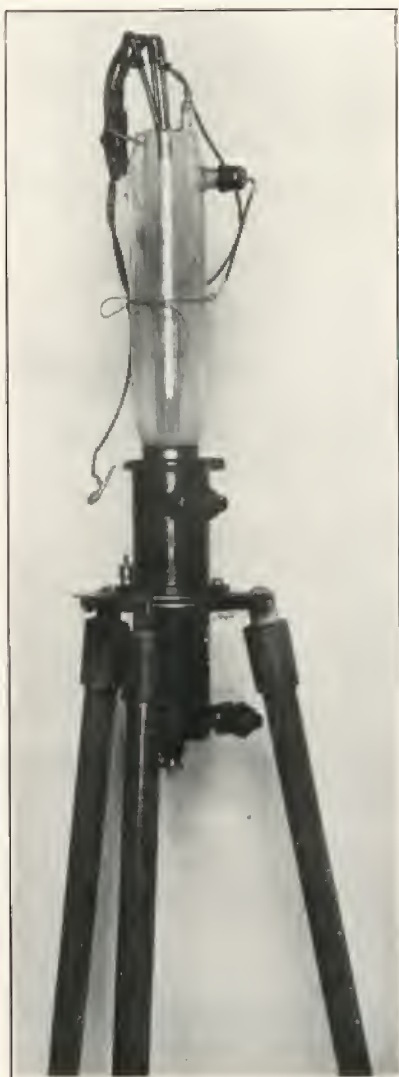
<i>Rectifying Valve</i>	<i>NU 23</i>	15 kW anode.
"	"	<i>NU 22B</i>	..	4 "
<i>Transmitting Valve</i>	<i>NT 23</i>	2.5 "
"	"	<i>NT 22A</i>	..	15 "

FIG. 281.



Silica valve with tubular metal spiral anode for water cooling.

FIG. 282.



Metal-glass valve with water-cooled anode.

FIG. 283.

Fused silica was suggested for the envelopes of valves about 1918. This material, which is obtained by fusing crushed quartz in very high temperature electric furnaces, does not expand appreciably under change of temperature, and is therefore never cracked by heat; moreover, it does not soften until its temperature reaches $1,400^{\circ}\text{C}$. Glass softens at about 500°C ., and, as is well known, readily cracks on sudden heating or cooling. Thus it has been possible to make envelopes of silica for valves smaller in size than the 1,500-watt glass valves, but capable of radiating from their enclosed molybdenum anodes as much as 6 kW. Larger valves have been standardised up to 15 kW. rating in larger silica envelopes. Fig. 281 shows some typical silica valves of power rating from 2.5 kW. to 15 kW.

In order to produce valves of still higher powers, water cooling of the anode has been introduced. Silica valves have been made whose anodes are tubular metal spirals (Fig. 282). Another type of valve which has been most successful employs as anode a copper tube which itself forms part of the envelope. This has a glass tube welded to it, and carrying the fittings and leads for the grid and filament (see Fig. 283). The copper anode is surrounded by a water jacket, which provides the necessary cooling. This type of valve has been made up to 100 kW. rating.

The silica envelope has an advantage over the other types in that it can be opened easily for repair. The silica is cut with a carborundum wheel, and, after the necessary repair (such as replacement of a burnt-out filament) has been effected, it is re-fused with the oxy-hydrogen flame and completed exactly like a new valve. Defective silica valves are thus nearly equal in value to new ones, and should be handled with just the same care.

539. Seals.—In all types of valve, electrical conductors must pass through the insulating material—glass or silica—constituting the envelope or portion of the envelope. These positions are termed the “seals.” In the copper-glass water-cooled valve there is an additional large seal where the copper anode is welded to the glass portion which forms the upper half of the envelope.

It will readily be seen that the seals present special problems; one obvious difficulty is the difference between the coefficients of expansion of the metal and the vitreous insulator. In small receiving valves, constructed like electric lamps, all seal conductors were formerly platinum wires pinched in the glass stem which supports the internal parts. Platinum has practically the same coefficient of expansion as glass, and, moreover, the molten glass adheres to it, so that a good vacuum-tight seal can easily be made. In most modern valves of this type a substitute for platinum is employed. A nickel-iron alloy has been made with approximately the same coefficient of expansion as glass. The wire and glass, however, do not adhere, and so the wire is coated with copper,

which has been found to weld satisfactorily to glass. This "platinum substitute," or "copper-clad" wire, is considerably cheaper than platinum. It cannot be used, however, in the larger types of valve, as it appears that with the higher currents necessary the deposited coating of copper becomes porous, and the valves tend to soften. The seals in the copper-glass water-cooled valves are made with copper tubing sharpened to an acute edge and welded to the glass tubing. Here it appears that the materials are in forms which enable strain through differences of expansion to be taken up mechanically, while the two materials satisfactorily adhere and form a vacuum tight joint.

In silica valves the seal is made by means of lead, which has the unique property of adhering to silica when melted. In order to obtain perfect joints the metal is melted and the seal made in a vacuum which prevents the formation of oxide (scum) on the molten metal. The expansion difficulty is overcome by placing the lead plug in a thick-walled silica tube. The silica appears to be strong enough, and the lead perhaps sufficiently ductile, to stand the strain due to difference of expansion. A solid conductor, usually a molybdenum rod, constitutes the internal conductor from the seal, and a short length of copper flex is inserted in the outer end of the plug for the purpose of making external connections. As lead melts at 327°C . the seals cannot be baked with the valve during manufacture (*see* para. 540), consequently they are placed at the ends of tubes protruding at one or both ends of the valve (*see* Fig. 281). They are thus beyond the limits of the ovens, and are kept cool by strong air blasts. Most silica valves also require air-cooling on the seals when in use, but stronger seals are being introduced and in certain types of valve render this precaution unnecessary. The increasing use of very high frequencies has given rise to a serious problem in some types of glass seals in recent years. The electrostatic strain in the insulator surrounding the conductor causes it to crack. Silica has proved to be immune for all frequencies and input powers so far employed, but special protecting devices are necessary with high-power valves which have glass-metal seals.

540. Evacuation Processes.—Reference has already been made to the very great importance of the high vacuum which is required for uniform characteristics and long life, and also to the fact that all materials evolve gases when heated. Hence the evacuation processes aim at the elimination of all occluded gas, and not only that filling the bulb itself. All the internal parts of the valve are scrupulously cleaned before use, and the valve assembled with as little contamination as possible. It is then mounted on a pumping bench, and connected by tubing to a series of pumps. The pump nearest the valve is a mercury vapour pump. Briefly, this functions somewhat on the injector principle, *i.e.*, a stream of vapour from

boiling mercury carries the gases along with it to a position where the mercury vapour condenses while the accumulated gases are pumped away by another type of pump. At a convenient position between the mercury vapour pump and the valve a flask of liquid air surrounds the connecting tube, and prevents mercury vapour from passing into the valve.

The second pump, backing the mercury vapour pump, is usually a mechanical one of rotary type, oil sealed, and is itself backed by a main pump, termed the "rough pump," which works on the usual piston and valve principles. The rotary pump only works efficiently if backed by a good rough pump, and the mercury vapour pump only functions properly if it in turn is backed by a very good backing pump. The rotary pump is capable of producing a vacuum of about 0.0001 mm., which was the order of vacuum used in electric lamps before the mercury vapour pump was developed. The latter increases the vacuum to 0.0000001 mm. or less.

While the pumps are working, the valve is baked in an electric or gas oven to as high a temperature as possible, the limit being the softening temperature of the envelope. If this is approached the atmospheric pressure outside the valve will cause the envelope to cave in. This temperature varies from 400° C. to 650° C. for the various types of glass and copper-glass valves. Silica valves are baked to 1,100° C., and evolve correspondingly larger quantities of gas in this process. When the gauges indicate that the evolution of gas at the maximum baking temperature has ceased, the baking is stopped and the electric oven is removed. The metal parts inside the valve are then heated by electrical methods. These are of two main types, "Bombardment Methods" and "Eddy Current Methods."

When bombardment is used, the filament is first run at its full rated current for a short time to drive off its occluded gas. Gradually increasing positive potentials are then applied to the grid and anode (independently), so that these elements are bombarded by the electron stream emitted by the filament. This process is carried on with the pumps working until the metals are much hotter than they are ever likely to be when the valve is in use. As each puff of gas is evolved by the hot metals there is the risk of ionisation producing bombardment of the filament by positive ions, with consequent filament disintegration, and so this process has to be very carefully carried out. On account of this risk it is not satisfactory for dull emitting filaments, and this led to the introduction of the second method, *i.e.*, "Eddy Current Heating" (E.C.H.). This method makes use of the heat developed by the eddy currents induced in metals when they are placed in an alternating magnetic field (para. 204). A small coil carrying a radio-frequency current is placed round the valve, and in a few seconds all the metal parts in the valve become red hot. The intensity of the process is, of

course, adjusted to the dimensions of the metal parts being treated, and to avoid damaging the filament its circuit is left open, so that very little current can be generated in it.

After the conclusion of the heating processes, a flame (for glass valves), or an electric arc (for silica valves), is applied to the exhaust tube quite close to the bulb. This softens the tube, the external atmosphere compresses, and so closes, it, and the valve is pulled off the pumping set at this point. This is known as "sealing off" the valve. A "pip" remains on the valve. This used to be visible on the top of receiving valves and at the end of electric lamp bulbs, but in recent years it has been concealed in the base or cap of the article.

An additional device is now universally used in receiving valves to produce the high vacuum more economically and more effectively. A small piece of magnesium is welded on the outside of the anode during assembly; when the anode is heated by the E.C.H. process, the magnesium volatilises and settles on the glass bulb as a silvery deposit. In settling down, the deposit absorbs all remaining traces of gas, and continues to function in this manner in the event of any subsequent slight evolution of gas in the valve. This process is so effective that some manufacturers seal off the valves before applying the E.C.H. process, and the magnesium deposit (or "getter," as it is called) is relied upon to absorb all the gas evolved by the hot metals when subsequently treated by the E.C.H. process. The exhausting processes are applied to small receiving valves by automatic machinery, and the time taken is about 10 to 30 minutes. Transmitting valves, however, and large amplifying valves, are treated individually and carefully watched. The processes for their evacuation usually last at least five or six hours, while for the most powerful valves two or three days are required.

541. Demountable Valves.—The normal end to the life of a transmitting valve is brought about by the "burning out" of the tungsten filament, *i.e.*, the wire becomes thinner by volatilisation and eventually fractures at the hottest portion. Hence for many years a design of valve has been aimed at which would lend itself to opening and refitting of a new filament at the transmitting station. This necessarily involves the use of high vacuum pumps on which the valves can be run continuously, and obviously the power of the unit has to be high enough to justify the expense of the pumping system. For several years, while only mercury vapour pumps were available, the difficulty of having liquid air at hand (para. 540) made this an impracticable proposition, but two types of pumps have been introduced which do not require the use of liquid air:—

(a) The **Holweck** system depends on a special rotary type of pump on which the valve stands with a wide glass connecting tube for insulation. The parts are separated by releasing conical ground

joints between the metal parts and glass insulators, these joints being greased to facilitate this action. The valve is stated to operate successfully up to input powers of 150 kW. at low radio-frequencies.

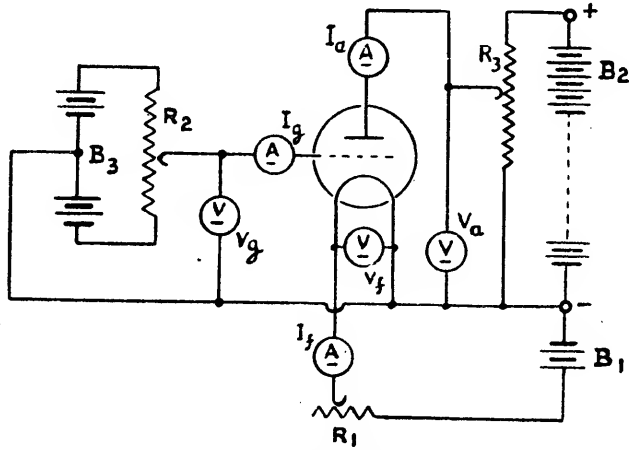
(b) The **Metro-Vick** valve is similar in general construction, but employs a special porcelain instead of glass for the insulating portions. The pump is similar in principle to the mercury vapour pump, but employs a specially-prepared oil instead of mercury for providing the vapour stream by boiling. This is backed by an ordinary rotary pump. The input power claimed for this valve is 500 kW. Its success is entirely due to the discovery of the oil, which has such a low vapour pressure that it can be used in the vapour pump without the liquid air trap between pump and valve. This discovery may prove to be of far-reaching importance in transmitting valve work.

Demountable valves appear to have a great future in industrial sets generating oscillatory currents for eddy current heating of metals. They offer speed and accuracy of temperature control, which cannot be obtained with any of the usual lagged oven devices. On the other hand the delay in starting up the pumps, and the much longer delay immediately following the replacement of a filament, may be disadvantageous for radio purposes, except in stations where there are ample facilities and space for reserve sets.

CHARACTERISTICS AND CONSTANTS OF TRIODES.

542. Graphical Representation of Valve Characteristics. Valve Testing Circuit.—An introductory description of the current variations in a triode, when varying voltages are applied between the electrodes, was given in paras. 529 to 532. The two currents, one in the anode circuit and the other in the grid circuit, depend upon the filament emission, the grid voltage and the anode voltage. It is usual to represent the relationships between these five variable but mutually dependent quantities by graphs, some of which are of considerable value in estimating the behaviour of the valve under working conditions. These graphs are termed the "characteristics" of the valve. Each graph is a curve of current variation plotted against voltage variation. Obviously several types of such curves may be produced, and it is necessary to select those of most value for the particular purpose for which a given valve is to be used. Fig. 284 shows the standard circuit employed for testing valves. The figure is self-explanatory, and indicates that variable voltages may be applied to the filament, grid, and anode, and the corresponding currents observed in suitable ammeters. There are two important points to note. Firstly, there is a standard convention to state all voltages with respect to the negative terminal of the filament as zero. Secondly, it is important to place the instruments

measuring the anode and grid currents in such a position that they do not carry, and give inaccurate results by indicating, the currents flowing through the corresponding voltmeters. This is particularly important in the grid circuit, where the current between filament and grid through the valve may be only a few micro-amperes.



Valve Testing Circuit.

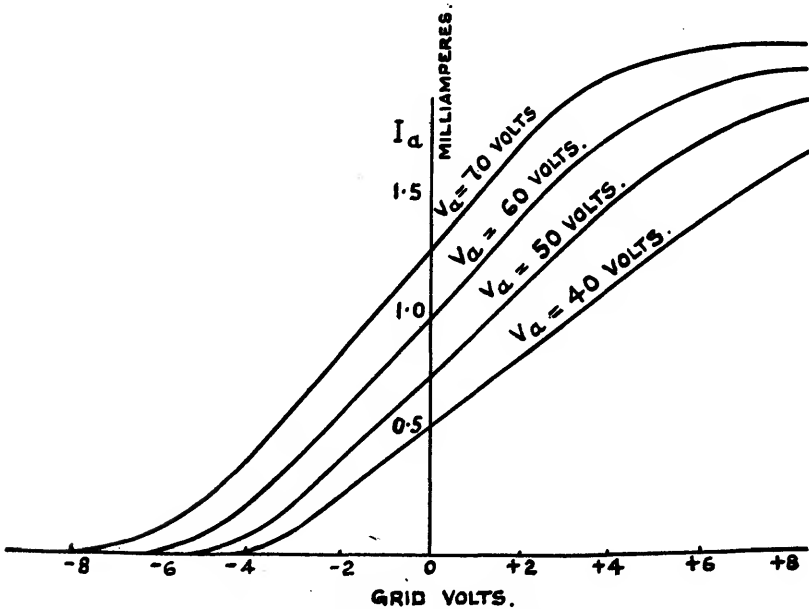
FIG. 284.

The graphs indicating the effects of changing the filament voltage are not of practical use. This subject has already been discussed in para 529, and will only be referred to occasionally in the following paragraphs. In practice the maker of a valve specifies the voltage which should be applied to the filament; as this voltage is always the one which will enable the user to obtain best performance from the valve, including reasonably long life, it is obviously of little interest to depart from the rated filament supply.

543. Mutual Characteristics.—The curves of most general use for triodes are those (already referred to in para. 532) showing the effect on the anode current of changes in the grid voltage. These curves are termed the mutual characteristics of the valve, and a typical set for a standard valve is given in Fig. 285. In the example shown, the anode voltage is kept fixed while the grid voltage is changed in steps from some four to eight volts negative to about the same value of positive potential with respect to the negative terminal of the filament. The anode current is recorded at each step and the mutual characteristic corresponding to the fixed anode voltage is obtained by plotting the observations.

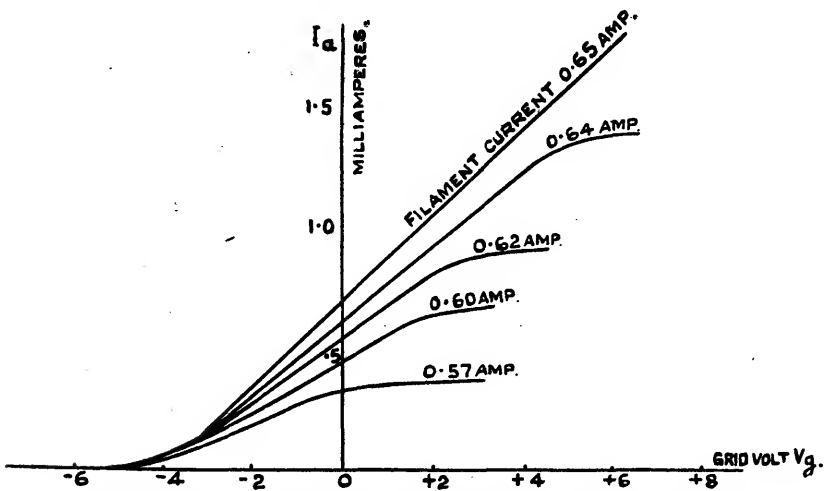
The lower portion of the characteristic follows approximately the "Three Halves Power Law" (para. 530), but the graph becomes a straight line and continues as such until the anode current

approaches the value of the maximum available emission, or saturation current, when it curves over and becomes horizontal. As a general rule it also falls as the positive grid potential is increased, because the grid itself collects electrons and so reduces the current



Mutual Characteristics of Triode.

FIG. 285.



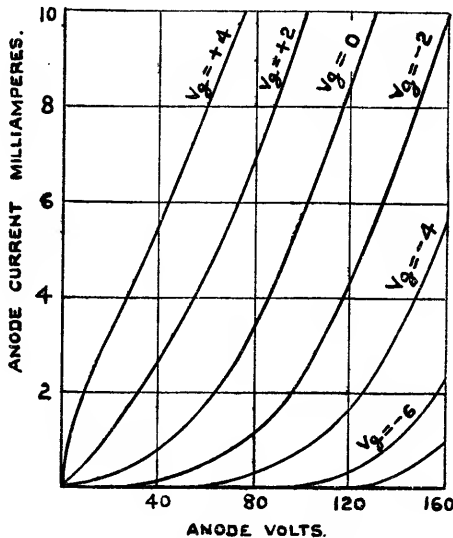
Anode Current—Grid Voltage Characteristic Curve (Varying I_f).

FIG. 286.

flowing to the anode. This will be referred to later. If another graph is produced with the anode at a higher potential, it is of a similar form to the first, but displaced towards the left on the chart, as shown in Fig. 285. The straight portions are nearly always of increasing steepness, and attain higher saturation values as the anode voltage is increased.

If the filament current is increased, the maximum value of the anode current is raised; but, in addition, the slope of the straight portion of the characteristic is slightly increased (see Fig. 286). These two effects, viz., increase of maximum current and increase in "slope," either together or separately, have the effect of improving the operation of a receiving or transmitting set, as will be understood from subsequent discussion. There is therefore a great temptation to overrun filaments to obtain better results. Occasions may arise when this technical offence may be justified by the necessity of achieving certain results, but the risks which have previously been mentioned in this chapter will have to be remembered.

544. Anode Characteristics. The Load Line.—The anode characteristics form a useful family of curves, and are obtained by plotting anode current against anode voltage with the grid voltage kept constant for each curve.



Anode Characteristics of Triode.

FIG. 287.

Fig. 287 shows these characteristics for a standard type of Naval receiving valve, over the normal working range of voltages up to the maximum which the valve is specified to withstand. The

filament of the valve in the example shown supplies an ample quantity of emission, and therefore no saturation is reached on these graphs, which indicate at least 10 milliamperes of available emission. If the filament had, say, only 8 milliamperes of available emission, the curves would bend over and become horizontal at that value; they would, in fact, be special cases of the general form shown in Fig. 275.

The anode characteristics are in general probably more useful than the mutual characteristics for determining the behaviour of a given triode under various circuit conditions, but the mutual curves have become the general type for publication by valve manufacturers. The anode curves, however, can easily be drawn from the mutual curves. Thus, in Fig. 285, a series of readings is taken where the characteristics cut the vertical ordinates at (say) $V_g = +2$, $V_g = 0$, $V_g = -2$, etc., and each group gives the data for one of the curves in a family such as Fig. 288. When taking characteristic curves of a valve it is more convenient to take the series with fixed grid volts and vary the anode volts than *vice versa*, because, as the anode current changes, the drop of potential in the anode potentiometer changes the anode voltage. If the mutual curves are being produced by the method described in the last paragraph, it is necessary to re-adjust the anode voltage with every change of grid voltage. It is therefore simpler to take the series of readings for anode characteristics with fixed grid volts which are not changed by change of anode current over the useful range of the characteristics.

As an example of the use of anode characteristics, consider the simple circuit of Fig. 288 (a), and the corresponding graphs, Fig. 288 (b). A triode has a load equivalent to a resistance R in the anode circuit, with power supplied from a high tension battery of voltage V . Let V_g represent the voltage applied to the grid.

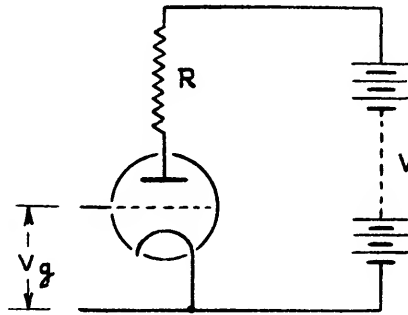
If V_g is adjusted to a negative value so that I_a falls to zero, the voltage on the anode will be V . Let A (Fig. 288 (b)) represent these conditions.

If a current I_a is allowed to flow in the anode circuit by adjusting the grid bias, the voltage drop between the high tension supply and the anode will be $I_a \times R$, and the anode voltage becomes $V - RI_a$. Suppose the point C on the graphs represents the new conditions. Draw the straight line A B C D E F, cutting the characteristics at the points indicated by these letters. Draw CH perpendicular to OA. OH obviously represents the voltage on the anode, which is $V - RI_a$, and CH represents the corresponding current I_a , while AH represents the voltage drop in the external load, *i.e.*, $AH = RI_a$.

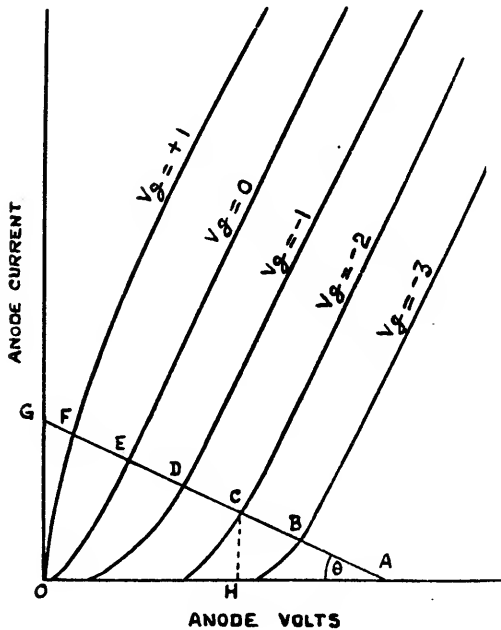
$$\therefore R = \frac{RI_a}{I_a} = \frac{AH}{CH} = \cot \theta$$

R is constant, *i.e.*, $\cot \theta$ is constant, therefore the series of conditions brought about by a swing of grid voltage of amplitude AF, will be

represented by the **straight** line A B C D E F. This is termed the **load line**. It is a characteristic of the circuit, not of the valve alone,



(a)



(b)

FIG. 288.

and is employed in this manner, *i.e.*, by being drawn across the anode characteristics of the valve at an angle representing external

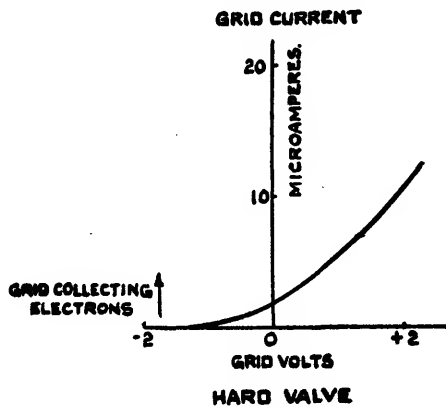
conditions, in order that information may be obtained concerning amplitude of grid swing and the corresponding conditions of anode current and voltage.

Thus, suppose a mean grid bias of -1 volt is applied with the conditions described above, the point D represents the mean conditions, and the load line is drawn through D at the angle θ determined by $\cot \theta = R$.

The range within which the load line is cut at equal sections by the characteristics on both sides of C will be the range of undistorted amplitude of oscillation. A swing to the left past the characteristic corresponding to $V_g = 0$ will bring about a flow of positive grid current (para. 532), while a swing towards A will be over curved portions of the characteristics; thus inequalities of anode current variation will result, *i.e.*, there will be distortion and rectification.

The condition is more complicated when the external circuit includes reactance. The straight load line is then replaced by an ellipse, but as in practice it is usual to aim at conditions where the external load is equivalent to a resistance, no useful purpose would be served by a full investigation in this chapter.

545. Grid Characteristics.—While the grid functions primarily as a controlling and not as a collecting electrode (especially in receiving sets and all types of amplifiers), yet in oscillatory circuits and in certain detectors the flow of grid current is essential. When

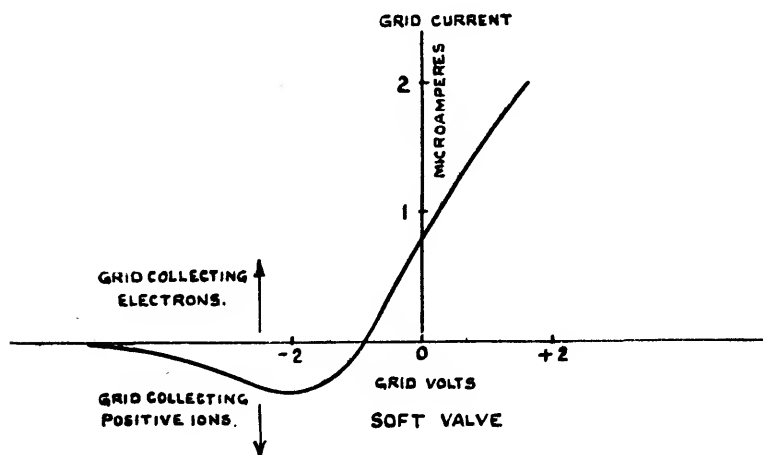


Grid Current Curve of Triode.

FIG. 289.

its potential is more than a fraction of a volt negative, the grid does not collect any electrons. The electron current begins in the vicinity of zero grid voltage, the actual point of commencement being dependent upon the nature of the materials constituting grid and filament. By virtue of the phenomenon known as "contact

difference of potential," there is a potential difference between the surfaces of the grid and the filament whenever these two surfaces are of different materials; for they are in metallic connection through the grid circuit. Thus when the filament is tungsten and the grid nickel or molybdenum, there is a trace of grid current even when the applied potential on the grid is slightly negative (see Fig. 289). With a coated filament, however, the grid current does not start until the grid potential is half a volt or so positive. The value of the current is very small—a few microamperes only—for low grid potentials, because of the comparatively small dimensions of the grid wires and the distribution of the field as discussed in para. 532. It is common practice to show the grid characteristics on the chart of mutual characteristics, and so a magnified scale is necessarily used for the grid current. As the grid voltage is made more and more positive the current increases, and when the voltage exceeds that on the anode the grid current may be very much in excess of the anode current.



Grid Current Curve of a Soft Valve.

FIG. 290.

It is not usually possible to make observations on high grid currents with the apparatus shown in Fig. 284, on account of the risks of the production of secondary emission or overheating of the grid by electron bombardment. It is therefore not usual to draw grid characteristics beyond a few positive volts. The small trace of grid current shown on Fig. 289 is responsible in one method of rectification for the behaviour of the three-electrode valve as a detector, and is therefore of great importance.

The grid characteristic is a very delicate indicator of the state of the vacuum inside the valve. If any ionisation occurs, the positive ions travel away from the anode towards the grid and filament.

If the grid is at a negative potential with respect to the filament, it attracts these ions and collects their positive charges. The resulting current is in a direction opposite to that due to collection of electrons, and is therefore termed "the reverse grid current." The term "backlash" has also been used for this reverse current, but it is gradually being replaced by the former expression.

Fig. 290 shows the grid characteristic of a slightly soft valve. It is standard practice to specify the maximum value of the reverse grid current at a fixed value of anode current and/or corresponding anode voltage. Thus receiving valves in general are tested at -2 grid volts with 100 volts on the anode, when the reverse grid current must not, for example, exceed two microamperes. In the case of a certain high-power transmitting valve, the corresponding limit is 50 microamperes at -20 grid volts, with 1 ampere of anode current at about 7,000 volts.

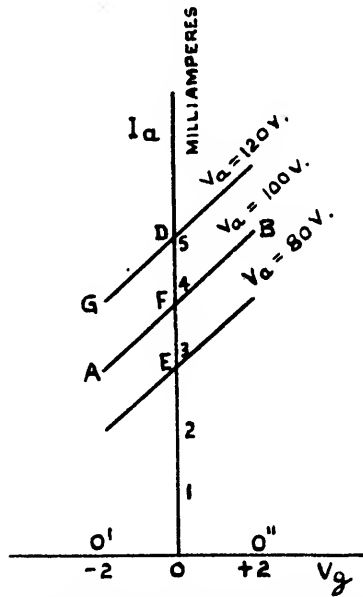
The reverse grid current falls to zero as the anode current is reduced by the increased negative potential of the grid. This is shown in Fig. 290 for a small receiving valve, where the current is reduced to zero with only a few negative volts. In the case of a large transmitting valve, a negative grid bias of several hundred volts may be required to suppress the current.

546. The Valve Constants.—As has been explained in the earlier paragraphs of this chapter, the outstanding property of the triode is the control of the anode current by small changes of grid voltage. The form of the mutual characteristics (Fig. 285) allows certain relationships to be derived between the grid and anode voltages and the anode current, and these are of prime importance. These relationships are called the constants of the valve, and are deduced from the straight portions of the mutual characteristics. The straight portions are also very nearly parallel, and from their geometry the required valve constants may be easily obtained, though it is necessary to specify the conditions of anode and grid voltage at which the observations are made. For receiving valves these are usually $V_a = 100$ volts and $V_g = 0$.

Mutual Conductance (g_m).—The most important constant describes the rate of change of anode current with change of grid voltage, the anode voltage remaining constant. It is termed the **Mutual Conductance** (symbol g_m). It is the gradient or slope of the mutual characteristic and is usually expressed in milliamperes per volt. Thus, in Fig. 291, which represents portions of typical mutual characteristics, the graph AB rises from 3 mA to 5 mA as the grid voltage changes from -2 volts to $+2$ volts. The "slope" or "mutual" conductance is therefore 0.5 mA per volt. It is to be noted that **changes** in current and voltage are used for obtaining the constant, not the **actual values** of current and voltage.

Although the mutual conductance represents the control of the grid voltage over the anode current, for consistency it is measured

under the static condition of constant anode voltage; but with an external impedance in the anode circuit, the voltage drop across the impedance alters with changes in anode current, and so, therefore, does the anode voltage. Consequently a given change of grid voltage cannot produce in such a circuit the full change in anode current, which is indicated by $g_m \times \text{change in } V_g$.



Middle Portions of Mutual Characteristics of a Triode.

FIG. 291.

A.C. Resistance or Impedance (r_a).—The anode current may be changed by changing the anode voltage, while keeping the grid voltage constant. In the early days of the use of triodes the “anode conductance” was therefore deduced by observations on change in anode current at zero grid volts. But it is more convenient to use the reciprocal of this when considering the behaviour of the valve and its associated circuits, for it is usually a change of anode current which causes a change of anode voltage, and not the reverse. The term “Impedance” has been widely used for many years for this constant. However, many authorities have pointed out that the term Impedance includes the effects of inductance and capacity, and is dependent on frequency, while the constant of the valve to which the term has been applied excludes inductance and capacity, and is always determined from steady current readings, so that it does not depend on frequency. Several terms including the word “resistance” have been proposed, and it appears that “A.C. Resistance” is likely to meet with the most general agreement. The symbol is r_a .

Referring again to Fig. 291, the anode current at zero grid volts rises from 3 mA at 80 anode volts to 5 mA at 120 anode volts. The A.C. resistance or impedance at 100 volts is therefore $\frac{40 \text{ volts}}{2 \text{ mA}} = 20,000 \text{ ohms}$.

The A.C. resistance may be defined as the reciprocal of the rate of change of anode current with anode voltage when the grid voltage is kept constant. It is to be noted that **changes** of anode voltage and current are referred to, **not** the actual values of voltage and current.

Amplification Factor (m).—A change in anode current brought about by a change in grid voltage is much greater than the change in anode current brought about by the same change in anode voltage. The ratio of the two is called the **Amplification Factor** (symbol " m "). An example of the practical derivation of this constant may be simply illustrated on the graphs of Fig. 291. Suppose the conditions are as defined by the point F on the 100-volt characteristic. Let the grid voltage be changed to -2 volts while the anode voltage remains steady. The anode current falls to the point A. Now raise the anode voltage (keeping the grid potential at -2 volts) until the current attains the value it had under the first conditions. Let G be the point representing this final condition. It is seen from the characteristics that a change of 20 anode volts is required to neutralise the effect of a change of 2 volts on the grid. In other words, the grid control is ten times as effective as the anode control, or the amplification factor is 10. The amplification factor is a ratio of two similar units, and is therefore a pure number, for which no units can be specified.

547. Relation between g_m , r_a and m .—From the above discussion the simple relationship between the three constants is quite clear. By definition, the amplification factor is the ratio between the mutual conductance g_m , and the anode conductance. Now the anode conductance is the inverse of the A.C. resistance r_a , hence $m = g_m \times r_a$. The reader may deduce this relationship geometrically from Fig. 291 as a simple exercise, and check the result with the data used in the two previous paragraphs.

The mutual characteristics become steeper with increasing anode voltage; in other words, the mutual conductance increases; also it clearly follows that the graphs taken at equal increments of anode voltage cut the zero grid volts ordinate in increasing lengths of sections, *i.e.*, the A.C. resistance decreases with increasing anode voltage. The product of g_m and r_a , however (*i.e.*, m), is constant over a wide range of anode voltage, but falls slightly at very low values.

548. The valve constants are determined by the geometry of the electrodes, and fairly accurate design formulæ have been developed by various authorities. The calculations are beyond

the scope of this book, but the general principles are as follows : The amplification factor is high if the mesh of the grid is "close" (*i.e.*, the wires close together), and if the distance from the filament or cathode to the anode is great compared with the distance to the grid. The mutual conductance increases with the value of m , but is nearly inversely proportional to the distance between the emitting surface and the grid. The A.C. resistance increases with increasing values of m , and also increases with increasing distance between emitting surface and grid. The greater the proportion of anode surface actually collecting electrons, the greater is the mutual conductance and the smaller the A.C. resistance.

In general, for operation in most circuits it is desirable to have g_m as high as possible and r_a as low as possible, but the final design of an electrode system is invariably a compromise between the desirable characteristics and a practicable form of construction. Very much ingenuity has been shown in the invention of electrode designs, and innumerable patents have been taken out on the subject.

549. For many years it has been the practice to design valves for particular functions in wireless apparatus, and the constants of valves may be used to determine their most suitable functions. Thus a valve with a comparatively high impedance can be accepted for use in a stage whose output is resistance-capacity coupled, provided its amplification factor is correspondingly high ; a low impedance valve with a good value of mutual conductance is desirable in the last or output stage ; and so on. In the Service a multiplicity of types of valve has hitherto been avoided so as to simplify storage conditions, and it has been the custom to use a general purpose valve ; but the remarkable properties of modern valves of special designs render this general rule on technical grounds no longer possible, and valves for special purposes are now being introduced to a limited extent. The following table shows the range of values of the constants of the most widely used commercial types of triode.

Constants of Triodes.

(i) Filament Valves.

Purpose.	A.C. Resistance. (Ohms.)	Amplification Factor.	Mutual Conductance (mA per volt).
R.F. and Detector..	20,000 to 50,000	15 to 50	0.7 to 1.5
A.F. stage	4,000 to 12,000	10 to 30	0.9 to 1.8
Output valve ..	1,000 to 4,000	2 to 10	2.0 to 4.0

(ii) Indirectly Heated (A.C.) Valves.

Purpose.	A.C. Resistance. (Ohms.)	Amplification Factor.	Mutual Conductance (mA per volt).
R.F. and Detector..	8,000 to 20,000	15 to 85	3.0 to 6.5
Output valve ..	2,000 to 5,000	5 to 13	2.5 to 4.0

550. Total Change in I_a due to Simultaneous Changes in V_g and V_a .—The steady value of the anode current I_a is determined by the steady values of grid voltage V_g and anode voltage V_a . Suppose now that the grid voltage is changed by an amount v_g . If the anode voltage were kept constant, the corresponding change in anode current, i_a , would be given by

$$i_a = g_m v_g,$$

provided the change were confined to the straight part of the appropriate mutual characteristic.

It has been seen, however, that unless special arrangements are made to keep it constant, the anode voltage will be affected by the change in grid voltage, owing to the different current flowing in the external circuit. This change in anode voltage in turn affects the nett change in anode current. Suppose that the anode voltage, either from the above reason or by a designed alteration in the H.T. supply, changes by an amount v_a . The change in I_a from this cause is connected with the change in V_a by the relation

$$i_a = \frac{v_a}{r_a}.$$

The total change in I_a due to simultaneous changes in V_g and V_a is therefore given by the sum of these two effects,

$$i_a = g_m v_g + \frac{v_a}{r_a}.$$

To prevent misunderstanding, it may be added that v_a would be negative if due to an impedance in the external circuit. In the relation as given, the changes i_a , v_g and v_a are all supposed to be positive, or increases in the steady values.

***551. The Characteristic Surface.**—The relationship between the static values of grid voltage, anode current and anode voltage may be conveniently illustrated by the wire model shown in Fig. 292.

The model is in three dimensions, the (rectangular) axes being V_a , V_g and I_a . The origin of these is **not** at the point A where three straight wires meet but at the point O in one of the opposite sides. If we place the model as shown in Fig. 292 (a) these axes

are viewed as shown in Fig. 292 (b), in which the positive directions only are indicated.

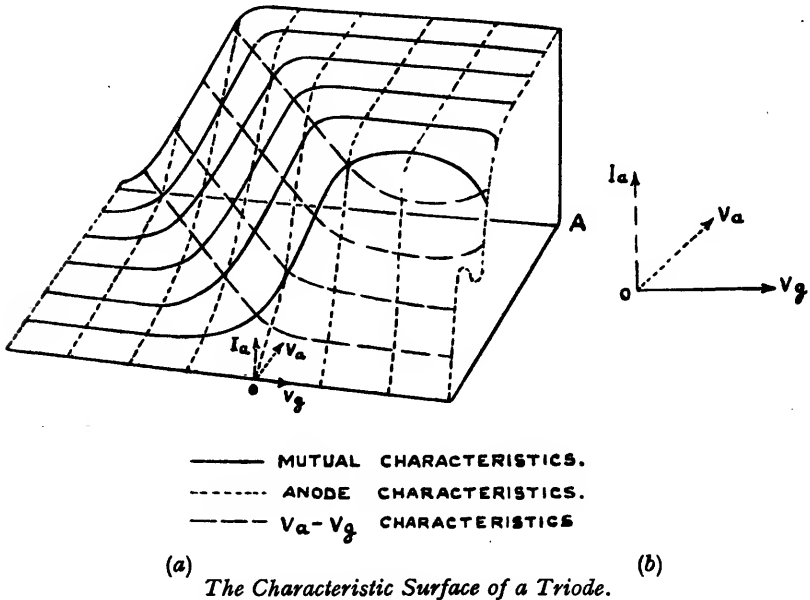


FIG. 292.

(1) Looking along direction OV_a (Fig. 292), we see in outline the $V_g - I_a$ family of curves, i.e., the mutual characteristics, for different constant values of V_a (Fig. 293). For negative values of

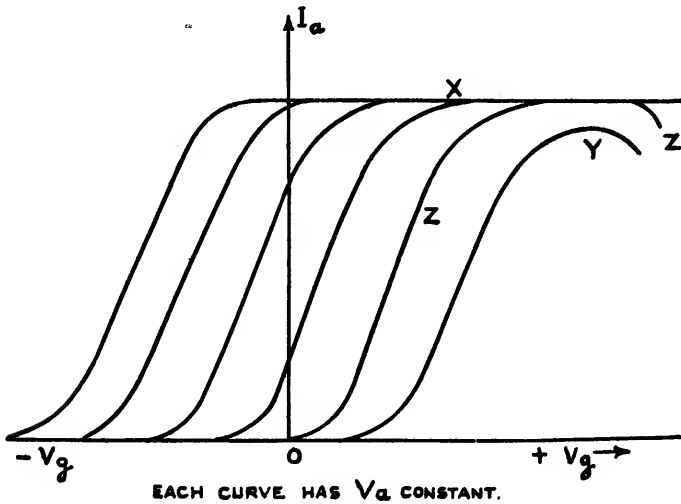


FIG. 293.

grid voltage these rise in nearly parallel lines to a nearly common level X, the saturation value; but, when V_g becomes comparable with V_a , the grid takes more and more current and the curve begins to fall (at Y) without attaining saturation.

The other curves, such as Z, which have reached saturation will also commence to fall when V_g attains a value near the (constant) V_a of the curve in question, for the sum of the anode and grid currents remains approximately constant as long as V_a is constant, i.e., along any one mutual characteristic.

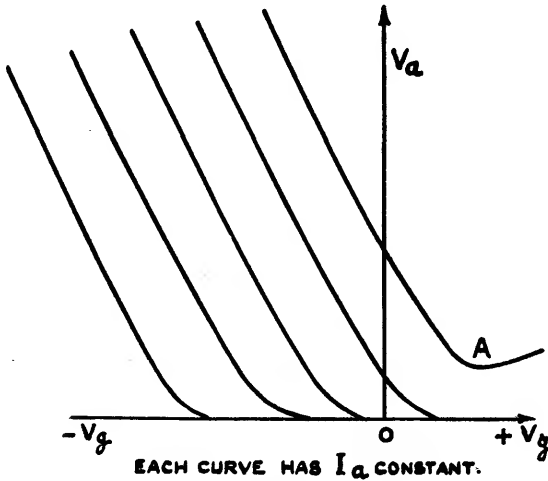


FIG. 294.

(2) Looking vertically downwards on the model (Fig. 292) we see the $V_a - V_g$ system of curves for various constant values of I_a (Fig. 294). To maintain I_a constant it is necessary, to vary both V_a and V_g . As the negative grid bias is reduced it is necessary to reduce the value of V_a to retain constant I_a . When V_g becomes positive, however, and grid current commences to flow, V_a must increase again (as at A) to make up for the current taken by the grid. These curves are straight almost along their whole length and so justify the generalised valve equation,

$$I_a = F(V_a + mV_g),$$

for along any one curve, I_a is constant, and so $V_a + mV_g$ is constant. This is the equation of a straight line connecting the variables V_a and V_g , the slope of the line being equal to m . These characteristics have some useful applications in the case of transmitting valves.

(3) Looking *towards* the origin along V_aO we see the $I_a - V_g$ characteristics, i.e., the anode characteristics for various constant values of V_a (Fig. 295).

Here again saturation is attained along XX for all values of V_a and V_g . (V_g is decreasing as we move to the right, so that OZX represents a positive, and OYX a negative, value of V_g .) There is a curious reversal effect at Z which is the commencement of

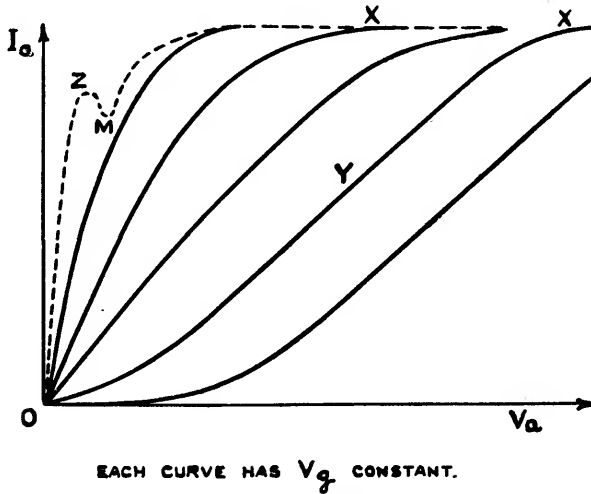


FIG. 295.

a crevasse that becomes deeper and wider as we move upwards out of the sketch. It is explained as follows. With large positive values of grid voltage, and zero anode voltage, there is a grid current flowing. The bombardment of the grid releases many secondary

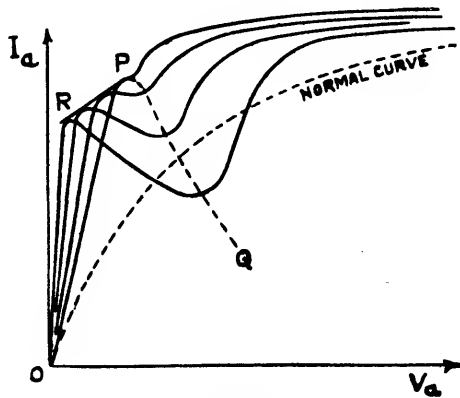


FIG. 296.

electrons, which form a dense space charge round the grid. The smallest V_a will draw away a large number of these from the outer part of the cloud owing to repulsion from those inside; and so I_a rises rapidly just at the start. The effect of raising V_a is to draw

more primary electrons to the anode, and this reduces the grid bombardment. On this account the copious supply of secondary electrons falls off and, as these are much in excess of the primaries, I_a will tend to fall. The two effects—increasing primary and decreasing secondary electrons to the anode—are in opposition and, owing to their greater number at first the latter effect prevails, causing a dip in I_a . Ultimately, of course, as the anode voltage increases, the increasing number of primary electrons reaching the anode more than compensates for the decreased secondary emission from the grid, and I_a goes to saturation along XX, having previously fallen to a minimum at M. With greater values of V_g , the effect is more marked and the crevasse grows larger. For clearness this is illustrated in a separate diagram (Fig. 296) with only one of the "normal" anode characteristics. The anode characteristics rise higher and higher as V_g increases until the dip appears at P as described above. As V_g increases the dip becomes broader and deeper, forming a crevasse of which the base or col is PQ and the ridge PR.

552. Dynamic Characteristics.—It has been emphasised in the preceding paragraphs that the valve constants refer to static conditions and are indications of the suitability of valves for particular purposes, but that the results are modified when the valves are used under various circuit conditions. The ordinary characteristics described above are therefore usually referred to as "static characteristics" in contra-distinction to those which may be drawn to represent the behaviour of a valve under particular circuit conditions and which have been termed "Dynamic Characteristics." A useful example of dynamic characteristics has been given in para. 544, where the use of a "load line," (representing external circuit conditions), on the anode characteristics was described. Another form, and the one generally intended to be understood by the term "dynamic characteristic," is produced by drawing the load line on the mutual characteristics. Fig. 297 shows the circuit used, and a typical family of dynamic characteristics.

The discussion is similar to that employed in para. 544 for the anode characteristics. Suppose that the load in the anode circuit is R ohms. When the grid bias is sufficiently negative to prevent the flow of anode current, the potential of the anode is the same as that of the H.T. battery, which in Fig. 297 is 100 volts, and the conditions are represented on the family of mutual characteristics by the point A. Now if a current I_a is allowed to flow in the anode circuit by reducing the grid bias, there will be a voltage drop in R of $I_a R$ volts, so that the anode potential will be less than 100 volts by that amount. Suppose R is 40,000 ohms, and $I_a = 0.25$ mA. The drop will be 10 volts and the new conditions will be represented by the point C on the 90 volt characteristic where $I_a = 0.25$ mA and $V_g = -8.5$ volts.

Now suppose that the grid bias is further reduced so that the anode current is still further increased to 0.5 mA . The voltage drop is now 20 volts ($40,000\text{ ohms} \times 0.5\text{ mA}$) and the new conditions are represented by a point D on the 80 volt characteristic with grid bias approximately -6 volts. The line joining the succession of points A C D E is the dynamic characteristic corresponding to the

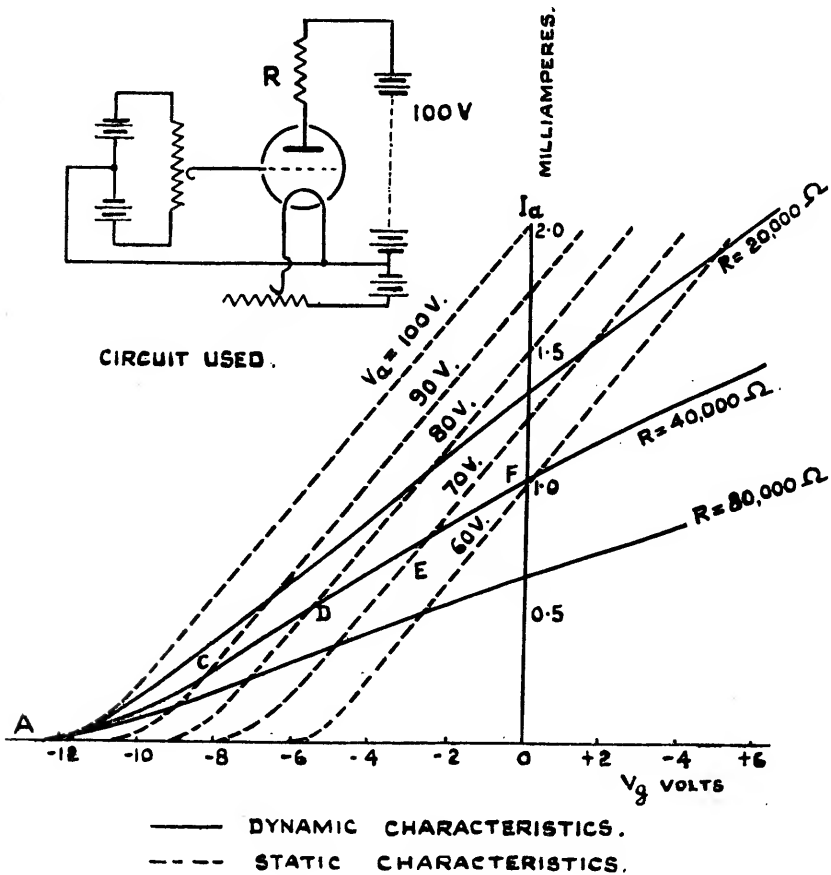


FIG. 297.

external load of $40,000\text{ ohms}$. It is nearly straight except for a small portion at the extreme end near A and from near this region to the point F where it cuts the zero grid voltage ordinate it represents the range of alternating grid voltage and corresponding anode current variation which can occur together without distortion. The figure also shows the dynamic characteristics for external resistances of $20,000$ and $80,000\text{ ohms}$.

If the external impedance contains a reactive component the dynamic characteristic becomes a loop. It must be clearly understood that the dynamic characteristics are primarily characteristics of the circuit and not of the valve alone. This subject will be further discussed when dealing with the valve as amplifier and transmitter (Chapters XIII and XIV).

553. Inter-Electrode Capacity.—The electrodes of valves are conductors in more or less close proximity to one another, consequently each pair acts as a small capacity. The actual values of the inter-electrode capacities are small, but they are of considerable importance in wireless circuits. The effects of inter-electrode capacity will be described in the appropriate sections of this book ; at the moment only the nature, extent, and variation of the property are being considered.

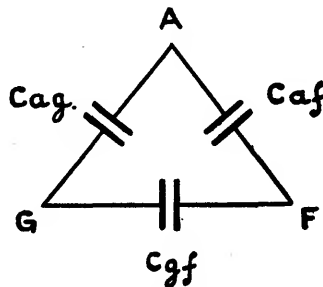


FIG. 298.

Not only are the electrodes themselves responsible for the apparent inter-electrode capacities but their supporting wires and the leads to the external contacts enter very largely into the problem. The method of assembling the electrode system on wires secured in a pinch, invariably used for receiving valves and described earlier in this chapter, is responsible for a considerable addition to the true inter-electrode capacity and it is therefore avoided as far as possible in valves made expressly for operating on very high frequencies.

The capacities between the three electrodes of a triode may be represented by the diagram of Fig. 298. A, G and F represent the anode, grid and filament respectively, and C_{ag} , C_{af} , C_{gf} represent the capacities between the three electrodes. It is obvious that when considering any one of these, the effect of the other two in series cannot be neglected. The capacities are usually measured by a bridge method using A.C. of say 500 or 1,000 cycles sec., with telephones as a balance indicator. Three observations are taken with the electrode terminals connected in pairs successively to eliminate one of the capacities. Thus with G and F connected together the first observation (i) = $C_{ag} + C_{af}$. With A and F

connected together the next observation (ii) = $C_{ga} + C_{gf}$. And with A and G connected together the last observation (iii) = $C_{af} + C_{gf}$.

$$\begin{aligned}\text{Hence } (i) + (ii) - (iii) &= 2 C_{ga} \\ (i) + (iii) - (ii) &= 2 C_{af} \\ (ii) + (iii) - (i) &= 2 C_{gf}\end{aligned}$$

The values obtained for the capacities of a small receiving valve when measured in this manner obviously include the capacities between the metal contacts of the socket in which the valve is placed for the test and due allowance must be made for these. Similarly the capacities of the flexible leads of transmitting valves must also be taken into consideration.

The approximate values of the inter-electrode capacities of the types of valve most generally used are given in the following table :

—	C_{ga}	C_{gf}	C_{af}
	$\mu\mu\text{F.}$	$\mu\mu\text{F.}$	$\mu\mu\text{F.}$
Small receiving valves	2.0 to 8.0	2.5 to 4.5	2.5 to 3.5
Glass transmitting valves—			
150 watts	5	8	3
450 watts	13	5	9
Silica valves—			
2.5 kW.	11	9	7
4.0 kW.	14	11	9
15.0 kW.	26	20	10

When the filament is alight the presence of electrons in the space surrounding it slightly increases the inter-electrode capacities, and when potentials are applied to the other electrodes the thermionic currents, which represent more electrons in the intervening space, add still further to the capacities. It therefore follows that any adjustments which are made to circuits to reduce the detrimental effects of inter-electrode capacity must be carried out with the valve filaments alight and the usual potentials applied.

It is clear that the grid-anode capacity will cause changes of potential on the anode to react on the grid. In other words, the grid-anode capacity acts as a direct capacitive coupling device between the output and the input circuits. As the potential changes on the anode are primarily caused by potential changes on the grid, it follows that the back coupling can cause instability if the circuit conditions allow (*see* below, para. 600). For example, if the potential changes fed back to the grid are in phase with the input variations there will be regeneration, and a valve which is only required to **amplify** oscillations will **produce** oscillations instead. This is most marked in valves of high amplification, as is shown in the following equation which has been deduced to show how the

effective input capacity C' depends on the constants of the valve and the equivalent resistance load r_o in the anode circuit.

$$C' = C_{af} + C_{aa} \left(1 + \frac{m r_o}{r_a + r_o} \right).$$

It can also be shown that the back coupling is sometimes responsible for power losses in the input circuit even though no electron current is flowing to the grid. This is referred to again in Chapter XIII (para. 601).

The instability of a radio frequency amplifying valve has been a well-known phenomenon even from the early days of valve amplifiers when triodes were very much inferior to the present day types, and several devices have been introduced to prevent it. All these aim at neutralising the grid-anode capacity or reducing it to the smallest possible amount. The various methods of neutralising will be discussed in Chapter XIII, and the construction and operation of a tetrode known as the "screen grid valve" in which C_{aa} is reduced to a minimum will be described later in this chapter.

CHARACTERISTICS AND CONSTANTS OF DIODES.

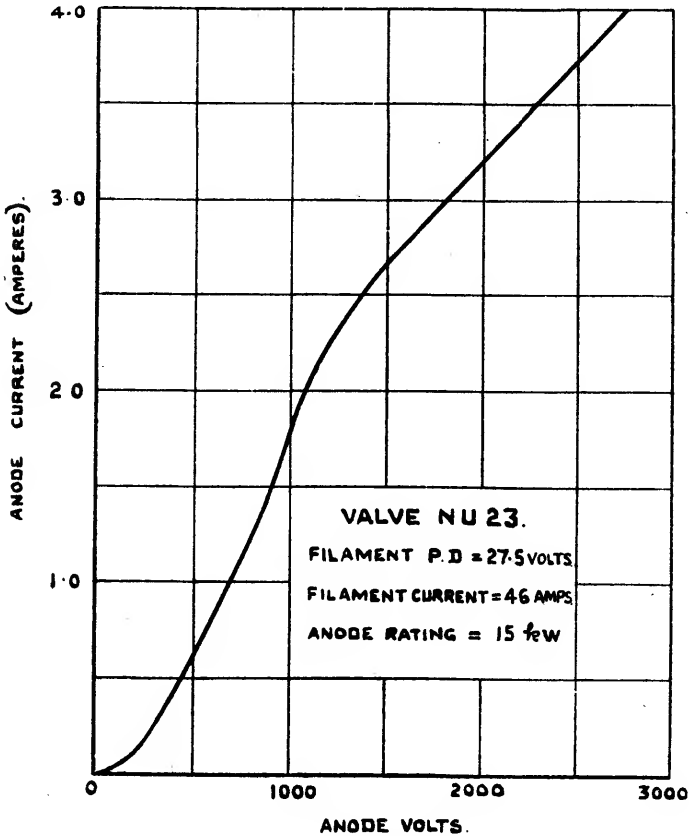
554. Characteristics of Power Rectifiers.—The form of the characteristic curve of a diode showing the relationship between the electron current flowing from filament to anode and the anode voltage is identical with the simple curve shown in Fig. 275. It is described by the three halves power law given in para. 530.

Diodes are now almost solely used for rectifying alternating current. Small diodes in glass bulbs are used for supplying currents of the order of 0.1 amp. for high tension purposes in receiving apparatus, while larger ones with silica or water-cooled metal envelopes are used for delivering several amperes at many thousands of volts to transmitting sets.

An interesting static characteristic of a high power silica rectifier is shown in Fig. 299. The curve shows signs of reaching saturation at about 2.5 amperes emission with about 1,500 volts on the anode, but instead of becoming horizontal it continues to rise. This is because the anode is becoming hot by the bombardment of the electron current, and consequently the loss of heat from the filament is restricted. The filament becomes hotter and it emits more electrons. This type of valve is used in high power rectifying panels and would normally be supplying power with a loss of about a thousand volts in the diode. The mean current would then be of the order of 2 amperes and the A.C. resistance of the valve approximately 500 ohms. In order to meet the requirements of the transmitter it would be necessary to pass probably 8 or 10 amperes at the peak values. The filament has to be designed to carry these momentary

currents safely, but static characteristics cannot be taken up to such values on account of extreme overheating. If necessary the saturation currents can be obtained by the use of oscillographs.

In recent years a very successful type of "soft" diode has come into use for power rectification. The "gas" remaining in the valve is mercury vapour. Its use is due to the discovery that



Diode Characteristic.

FIG. 299.

positive ions produced by the ionisation of one of the common inert gases, and impinging on a cathode at a fall of less than about 20 volts, do not disintegrate the cathode surface. Thus if a diode contains a gas ionising at less than 20 volts, and has a cathode (or filament) capable of the maximum emission required for whatever purpose the valve is being used, the voltage across the valve will not rise above the safe value for avoiding cathode disintegration. It is then possible to use oxide coated emitting

surfaces, and such have been constructed for a wide variety of applications from small filament emitters passing one or two amperes in battery charging apparatus up to power rectifiers passing hundreds of amperes.

The obvious advantage of the mercury vapour thermionic rectifier over the high vacuum type is the higher efficiency obtained because of the very low potential drop across the tube. This is only about 15–20 volts in the mercury vapour diode whereas it is from several hundreds to three or four thousand volts in the high vacuum type. But the soft valves suffer from the drawback—for naval purposes—of delayed action on starting. The cathodes require 20 seconds or so to heat up and therefore it is dangerous to apply H.T. for at least half a minute since there may not be sufficient electron emission to prevent the voltage across the tube rising above the value which will disintegrate the filament or cathode coating. Premature application of H.T. will therefore ruin the valve instantly. This delay is a disadvantage in ships, where it is necessary to switch off machines, etc., between transmitting in order to receive satisfactorily.

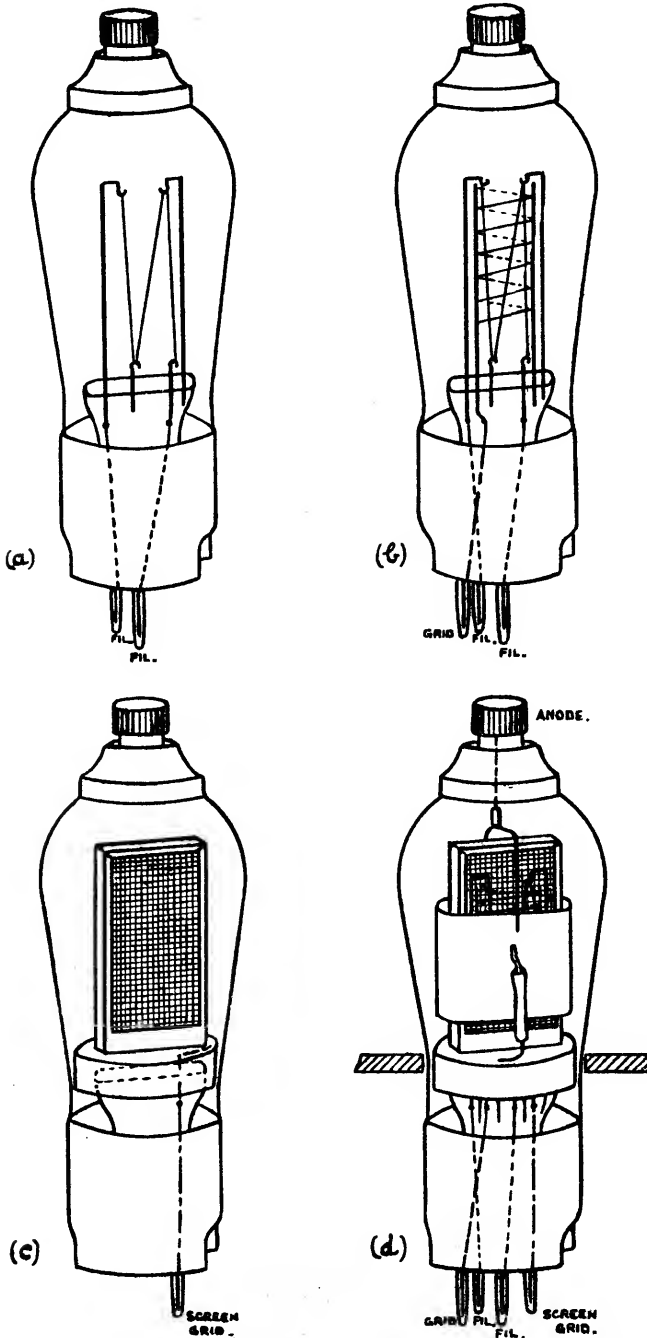
THE SCREEN GRID VALVE.

555. Characteristics and Properties of the Screen Grid Valve.—

A four-electrode valve which has come into very general use for R.F. amplification is one with two grids between the filament (or cathode) and the anode. It is called the screen grid valve. The grid near the filament performs the same function as the grid of a triode and is therefore frequently referred to as the "Control Grid" while the second grid acts as an electrostatic screen between the control grid and the anode and is therefore termed the "Screen Grid," or simply the "Screen."

The construction of a screen grid valve is illustrated in Fig. 300.

- (a) represents the valve with all electrodes removed except the filament, which consists of three strands forming the letter N and is supported by nickel wires. (Only the filament contact pins on the base are shown.)
- (b) shows the addition of a grid consisting of a flat spiral of wire wound on two upright supports, with its lead to the grid pin on the base.
- (c) shows the screen only. This consists of a rectangular mesh with a sheet metal skirt or cylindrical flange at its lower end. The screen lead is brought to the fourth pin on the base—the one which on a triode is the anode pin.
- (d) shows the complete assembly with a flattened cylindrical anode placed round the screen. The anode is supported by glass rods from the skirt of the screen and is connected by wires to a special cap on the top of the bulb.



Construction of a Screen Grid Valve.

FIG. 300.

The magnesium getter invariably used in the manufacture of small valves is placed on the anode and consequently deposits only on the glass above the flange of the screen. A light wire welded on the screen presses on the interior of the bulb so as to put the magnesium film in contact with the screen. For clearness the wire is not shown in the figure.

The reason for the interposition of the screen will be fully understood after the chapter dealing with the valve as an amplifier has been read, but it may be stated here that it is a device to reduce the grid-anode capacity to the lowest possible value.

The screen is kept at a high potential approaching that of the anode. Its function may be best understood by considering the lines of force between the electrodes. If the screen were a sheet of metal instead of a mesh or grid it is clear that all lines of force from the control grid and filament would end on the screen and there would be no capacity between the control grid and the anode. It may be considered that with the screen an open mesh and nearly at the same potential as the anode, practically all the lines of force from the grid and filament end on the screen. When the potential of the screen is below that of the anode there is a field between them, and a few lines also join the grid and anode, *i.e.*, there is a slight residual grid-anode capacity. Although the majority of the lines of force from the grid end on the screen, the large majority of the electrons travelling towards the screen from the vicinity of the grid are projected by their momentum through the spaces of the mesh and so come under the screen-anode field and are collected by the anode. Thus the screen grid valve can function in the same manner as a triode though its characteristics are peculiar to itself. In commercial types of screen grid valve, the residual grid-anode capacity is from $0.001 \mu\mu\text{F.}$ to $0.02 \mu\mu\text{F.}$, while the corresponding value for triodes is about 2 to $8 \mu\mu\text{F.}$; the screening effect of the fourth electrode is therefore very pronounced.

In order to have the screening as complete as possible it is also necessary to separate the grid and anode circuits external to the valve by a similar electrostatic screening device. For this purpose the valve is inserted in a hole in a metal screen, indicated in Fig. 300 (*d*). The cylindrical flange on the screen grid is brought very close to the bulb to enable the continuation of screening to be as complete as possible.

It will be clear from the above discussion that slight changes in anode voltage have practically no effect on the field between the grid and the screen which is responsible for driving the anode current across this region. In other words, the impedance or A.C. resistance of the valve is very high. The grid control, however, is obviously practically the same as if the screen and anode together formed a collecting electrode as in a triode; *i.e.*, the mutual conductance is of the same order as for a triode. Hence from the relationship between the valve constants (para. 550) the amplification

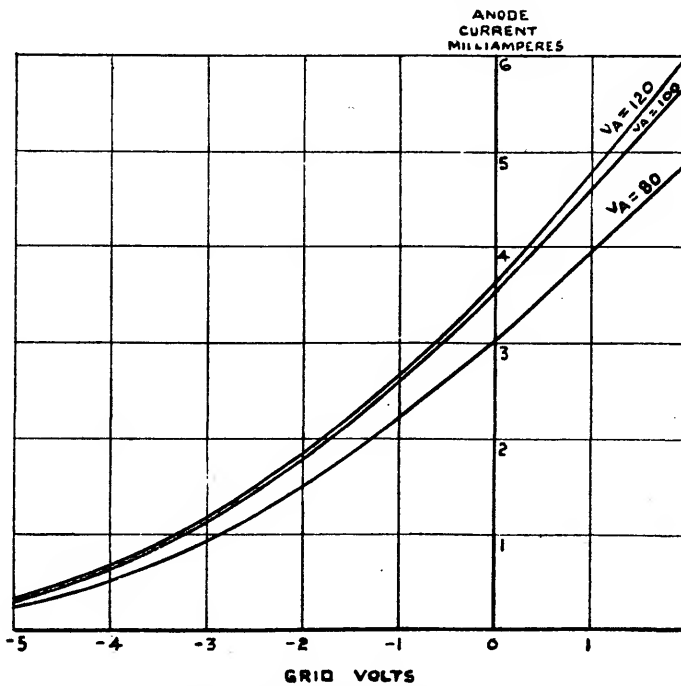
factor is high. The following table of practical values may be compared with that given for triodes in para. 549; as in the case of triodes the valves with indirectly heated cathodes have the highest amplification factors.

Constants of Screen Grid Valves.

A.C. Resistance. (Ohms.).	Amplification Factor.	Mutual Conductance. (mA per volt).	Grid-anode Capacity. $\mu\mu\text{F.}$
200,000 to 1,000,000	100 to 1,500	0.5 to 3.5	0.001 to 0.02

(H.T. voltage, 120 to 150 volts; Screen voltage, 70 to 90 volts.)

In order to take the fullest advantage of the high amplification factor of the screen grid valve it is essential that the anode circuit should also have a very high impedance. This will be clear from a consideration of the discussion on the load line in para. 544. Unless the anode circuit impedance is very high the changes in anode current brought about by changes of grid voltage will not produce the desired changes in anode potential.



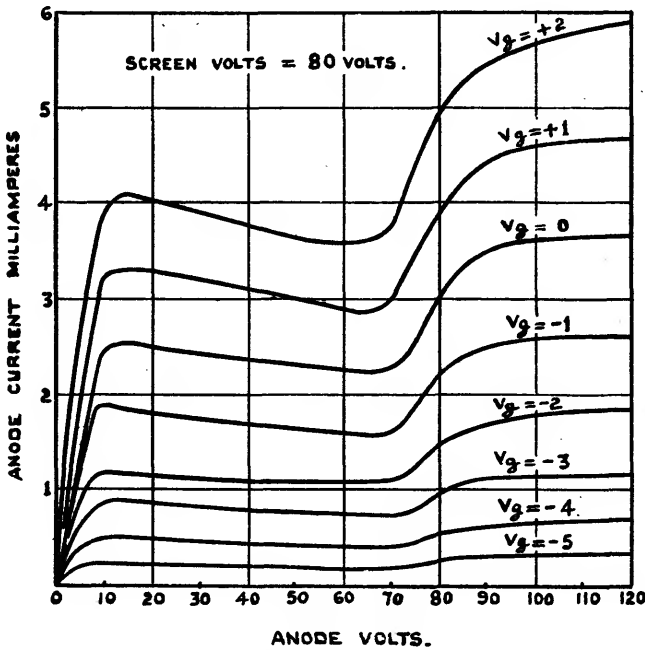
Mutual Characteristics of a Screen Grid Valve.

Screen Volts = 80.

FIG. 301.

The mutual characteristics of the screen grid valve (Fig. 301) are similar in shape to those of the triode, but the curves for different values of anode voltage are closer together, indicating the high A.C. resistance. Obviously the only valuable curves are those for anode voltages higher than the screen voltage.

The anode characteristics (Fig. 302) are very different from those of the triode; as the anode voltage is raised from zero, keeping the screen at the rated value for the valve, the anode current rises until in the vicinity of 12–20 volts it attains a maximum value and then falls. This fall is explained by the emission of secondary electrons



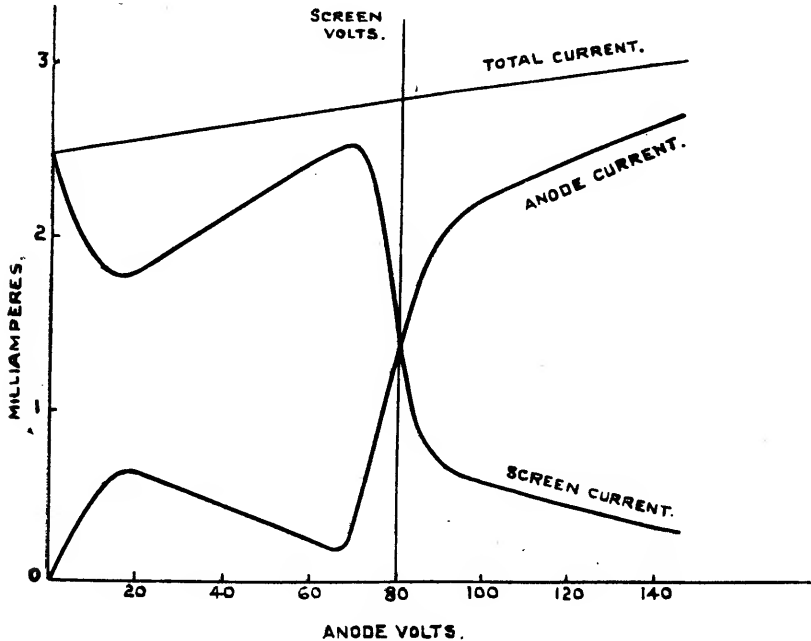
Anode Characteristics of a Screen Grid Valve.

FIG. 302.

from the anode, due to the impact of the primary electrons. The secondary electrons travel to the screen which is at a higher potential than the anode. The number of secondary electrons emitted by the anode depends on the number of primary electrons and on the voltage through which they have fallen on reaching the anode (*i.e.*, on their velocity). If the screen voltage is made high enough, it is actually possible to produce a current of secondary electrons from the anode larger than the primary current, *i.e.*, the curve falls below the anode voltage base. Under ordinary adjustments, the anode current falls as the anode voltage is raised until the latter approaches the screen voltage. Then the reverse condition occurs. The primary electrons

impinging on the screen produce secondary electrons which travel to the anode, while those emitted from the anode return to it and are not detected.

Fig. 303 shows the variations of screen and anode current with anode voltage, the screen voltage being fixed at 80 volts, and it also shows the rising total current which is the actual value of the current passing through the control grid plus secondary emission currents. When the valve is in use as a radio frequency



Screen Volts = 80.

Grid Volts = 0.

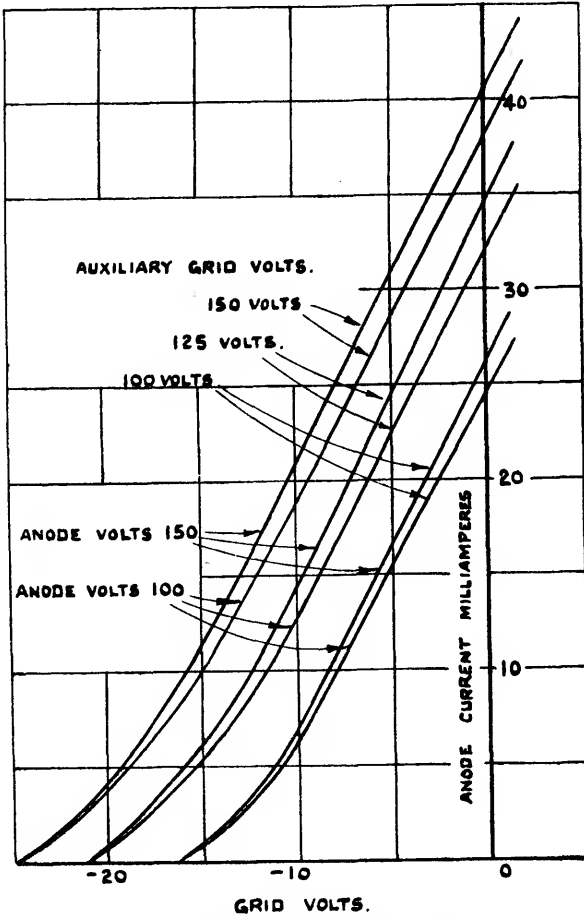
Anode and Screen Currents plotted against Anode Voltage.

FIG. 303.

amplifier it is obvious that to avoid effects due to secondary emission from the anode, the adjustment of potentials must be made so as to produce conditions corresponding to the nearly horizontal portions of the anode characteristics, *i.e.*, when the anode voltage is higher than the screen voltage. The best adjustment is when the screen voltage is about two-thirds of the anode voltage. It may be obtained either by a tapping on the H.T. battery or by inserting an appropriate resistance of some 50,000 ohms or so between the screen and H.T. positive. The value of the resistance depends on the screen current, which is about 0.5 mA in the majority of screen grid valves operating with 120–150 volts on the anode.

THE PENTODE.

556. Characteristics and Properties of the Pentode.—A five-electrode valve, the Pentode, has recently come into very general use for the last, or power output, stage of receiving circuits. This valve has three grids between the cathode or filament and the anode. Numbering from the filament, the first grid is the usual control grid,

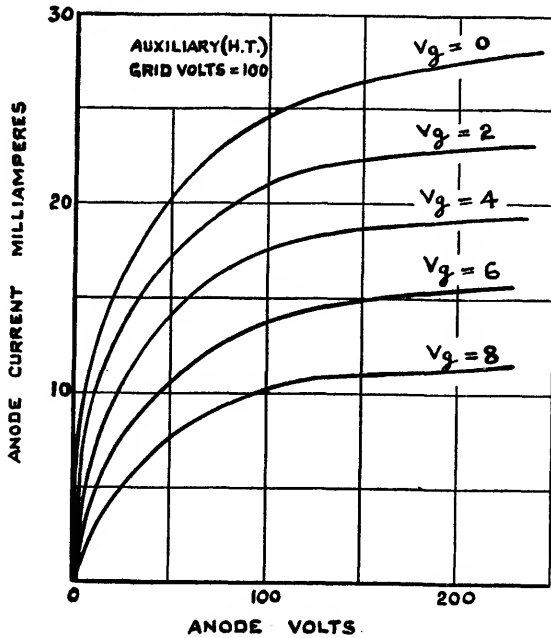


Mutual Characteristics of Pentode.

FIG. 304.

the second is termed the auxiliary or priming grid and is connected externally to a source of high potential, the third is connected inside the valve to the filament, or to the cathode in an indirectly heated type of valve, and is usually termed the "earthed grid." The lead from the auxiliary (H.T.) grid is connected either to a screw terminal on the side of the base or to a fifth pin situated exactly in the centre of the base.

The function of the earthed grid is to suppress the emission of secondary electrons either by the anode or the auxiliary grid. This function is clearly understood when it is realised that secondary emission is only important when it can be collected by an adjacent electrode at a higher potential than the emitter. The pentode thus has certain characteristics somewhat similar to the screen grid valve, but it has no negative resistance or unstable properties due to secondary emission, and it is not constructed with the H.T. grid as an electrostatic screen.



Anode Characteristics of Pentode.

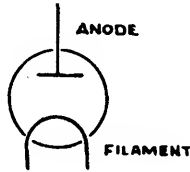
FIG. 305.

Figs. 304 and 305 show the characteristics of a typical commercial pentode; the anode curves (Fig. 305) are seen to be similar to those of the tetrode (Fig. 302), without the secondary emission kink. The constants of commercial types of pentode are of the following order :—

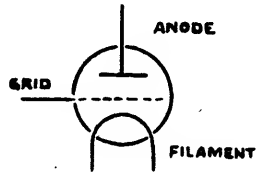
Constants of Pentodes.

A.C. Resistance. (Ohms.)	Amplification Factor.	Mutual Conductance (mA per volt).
30,000 to 100,000	50 to 100	1.3 to 4.0

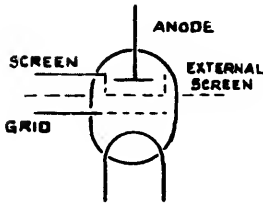
557. Symbols.—The symbols used throughout this book to represent the various types of thermionic valve are shown in Fig. 306.



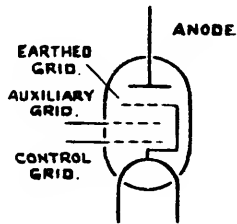
2 - ELECTRODE VALVE.
(DIODE)



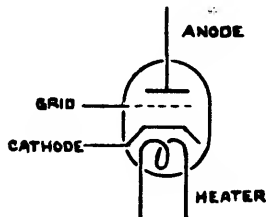
3 - ELECTRODE VALVE
(TRIODE)



SCREEN GRID
VALVE.



PENTODE



TRIODE WITH INDIRECTLY
HEATED CATHODE.

FIG. 306.

CHAPTER XII.

RECEIVING CIRCUITS—THE VALVE AS DETECTOR.

558. Methods of Rectifying Symmetrical Oscillations.—The three-electrode valve may be used instead of a crystal for rectifying and thus detecting, *i.e.*, making audible, either the damped wave-trains produced by a spark or I.C.W. transmitter (Chapter XIV), or R/T transmitter (Chapter XV), or the "beats" consequent upon superimposing a local heterodyne oscillation on an incoming C.W. signal.

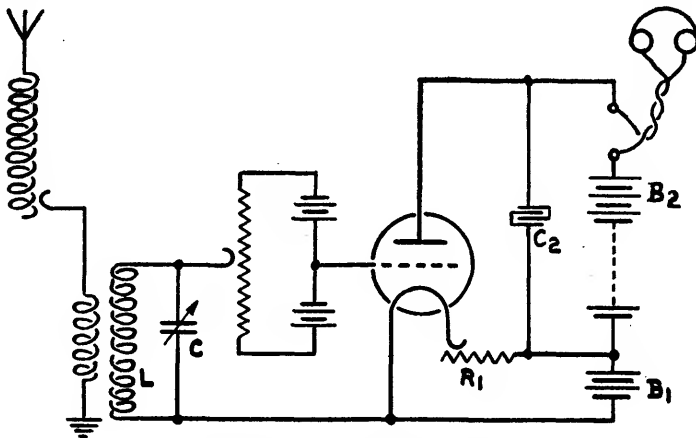
The principle of **rectification**, which consists in the production of asymmetrical current variations from symmetrical voltage variations (Chapter X), is applied by taking advantage of:—

(a) The non-linear characteristic curve of anode current plotted against grid voltage, utilised at either its upper or lower bend, where the curvature is variable. This is termed **Anode Rectification**.

(b) The non-linear characteristic curve of grid current plotted against grid voltage, the circuit employed in the previous case being altered by the insertion of a condenser in the grid circuit as a grid current integrating device, with a resistance of the order of megohms inserted between grid and filament to allow the device to reset itself after each train of waves. This is termed **Cumulative Grid Rectification**.

(c) Possible combinations of the effects (a) and (b) above. These will now be examined in detail.

559. Anode Rectification.—The circuit used is that of Fig. 307.

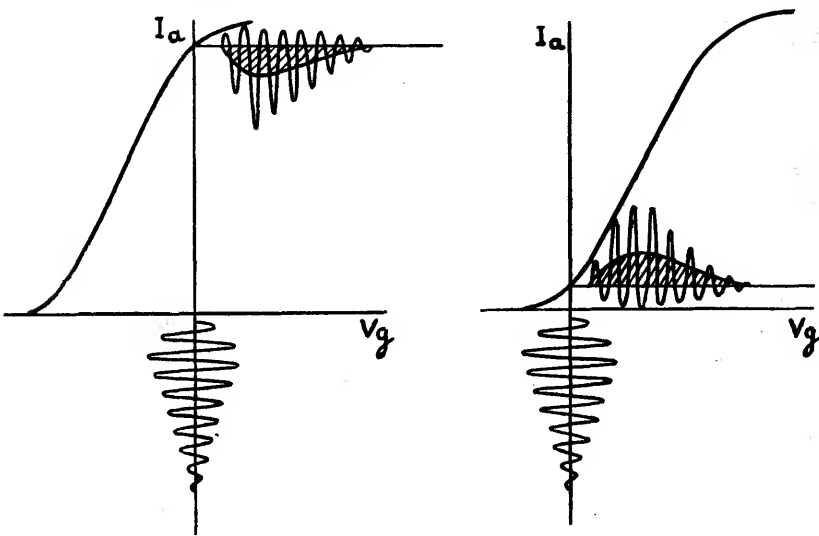


Circuit for Anode Rectification.

FIG. 307.

The secondary circuit LC is joined directly between the grid and filament of a valve. The steady potential of the grid relative to the filament can be adjusted by means of the potentiometer. The battery B_1 provides the filament heating current, which is controlled by the rheostat R_1 . Between the anode and filament of the valve are joined a battery B_2 , and a pair of telephones, shunted by a condenser C_2 .

Note.—In all cases where telephones are joined in series with a high tension battery, an unfair strain is thrown on their insulation, which is liable to suffer in consequence. It is better to use telephone transformers in combination with low resistance telephones. For the sake of clearness, however, the former arrangement has been adhered to in diagrams.



Illustrating Anode Rectification.

FIG. 308.

Action.—The essential feature of any rectifying device is that it should produce an asymmetrical current variation from an applied sinusoidal voltage variation. In the circuit shown, symmetrical voltage variations are applied between grid and filament, being superimposed on any steady potential already in that circuit, and the object is to make these voltage variations produce changes in the anode current flowing which will be asymmetrical. It is obvious that the parts of the (I_a, V_g) characteristic where the curvature is changing rapidly are the best suited for this purpose. Suppose that the necessary adjustments of anode and grid voltage (to be examined later) have been made so that the ordinate corresponding to the steady value of grid voltage cuts the (I_a, V_g) curve at its upper or lower bend.

Before a signal arrives, a steady anode current will be flowing through the anode circuit, of amount given by the length of the ordinate.

A sinusoidal, and therefore symmetrical, variation of grid voltage due to an incoming signal will produce changes in anode current as illustrated in Fig. 308. If we are working on the upper bend of the curve, the current decreases more during the negative half-cycles of oscillatory grid voltage than it increases during the positive half-cycles; on the lower bend the reverse is the case. As in crystal detection, the inductance of the telephone windings is sufficient to prevent each individual variation of current from having any effect on the diaphragm, and, in any case, its mechanical inertia would prevent it vibrating at such a high frequency. A condenser is therefore shunted across the telephones, and the unsymmetrical current wave form can be considered to be split up into two components:—

- (i) A change in the mean value of the anode current.
- (ii) A symmetrical radio frequency oscillation about this new mean value.

The symmetrical changes in current at radio frequency are passed by the condenser, and the telephones are affected by the changes in the mean value of the anode current.

Waves Modulated at the Transmitter, Spark Signals, etc.—In these cases the mean value is itself a varying quantity, varying at the audible frequency impressed on the wave form at the transmitter. For example, with spark signals, there is one increase or one decrease of mean anode current for every train of waves, and the diaphragm therefore vibrates at the frequency of the spark trains, which is made to correspond to audibility.

Continuous Waves Unheterodyned.—Incoming unheterodyned C.W. oscillations of voltage applied to the grid give, as before, asymmetrical variations of anode current. Considering them as symmetrical variations about a new mean value, this mean value is simply a constant increase or decrease above or below the original current flowing. The only response given by the telephones during a group of C.W. oscillations representing a morse dot or dash is therefore a click at the beginning, when the diaphragm moves from one steady position to another, and similarly at the end of the signal.

Heterodyned C.W. consists of high-frequency oscillations whose amplitude varies at audible frequency, and the resultant effect corresponds to that of spark signals; the mean value of the anode current itself varies at audible frequency, and the diaphragm vibrates at the same frequency, that of the beat.

560. Adjustments for Anode Rectification.—The necessary adjustment for good rectification is that the normal grid voltage should correspond to a point on the characteristic at either the upper

or lower bend. This condition may be obtained, in the case of **Upper Bend Rectification**, by :—

- (i) Filament control. Alteration of heating current to the filament alters the point on the (I_a , V_g) curve where it approaches saturation. In practice, the filament is dulled by increasing the resistance in its heating circuit.
- (ii) Alteration of anode potential. This shifts the characteristic to the left when increased, and to the right when diminished.
- (iii) Alteration of grid potential by potentiometer control.

For **Lower Bend Rectification** the following methods only are applicable :—

- (i) Alteration of anode potential. This shifts the characteristic to the left when increased, and to the right when diminished.
- (ii) Alteration of grid potential by potentiometer control.

An advantage of using lower bend rectification is that the normal adjustment makes the grid negative to the filament, and no grid current flows. The effect of grid current flowing is that energy is absorbed from the input circuit, and its damping is increased. (Note that the flow of grid current is the essential condition on which cumulative grid rectification depends, but it is not necessary to use it with anode rectification.) Another advantage of lower bend rectification is that less current is taken from the H.T. battery.

Characteristic used.—The effective characteristic used is the dynamic characteristic, because of the impedance in the anode circuit. Hence the characteristic is flatter than in the static case, and, when using lower bend rectification, the difference in slope of the curve on the two sides of the bend will not be so great as if it were the static characteristic. The efficiency of the circuit as a rectifier for weak signals is greater the greater the curvature or the rate of change of slope at the rectifying point, as will be shown mathematically in para. 562. Hence a valve should be used for anode rectification whose A.C. resistance is high compared with the external impedance, so that the slope of the dynamic characteristic does not differ greatly from that of the static. In addition, the steeper the slope of the actual static characteristic, the sharper will be the bend in the curve. Hence the valve should have a high value of r_a and a high value of g_m , and in consequence a high value of m . This gives also the advantage that the characteristic falls to zero at a voltage which is not too negative, and so reduces the amount of grid bias necessary.

561. Value of Rectified Current.—In general, the value of the rectified current depends in a complicated manner on the amplitude of the applied signal voltage, and no simple relation between the

two can be stated. For incoming spark, I.C.W. or R/T signals of small amplitude, the rectified current is very approximately proportional to the square of the applied voltage. For incoming C.W. signals of small amplitude, which must, of course, be heterodyned before detection, the value of the rectified current is proportional to the product of the amplitude of the incoming signal voltage and that of the local oscillation. It is therefore proportional to the first power of the incoming signal voltage.

Thus, for small applied signal voltages, the sensitivity of the detector is much greater for heterodyned C.W. than for signals modulated at the transmitter, and this is the chief reason why greater ranges were obtained by the introduction of C.W. transmission.

The value of the rectified current is also proportional, of course, to the curvature of the characteristic as discussed in the previous paragraph.

It should be emphasised that these remarks only hold for small voltages applied to the detector. In modern receivers with a number of radio frequency amplifying stages the results may be very different.

The proof of the above statements will be given in the next paragraph.

***562.** For a small part of its length, round a point where its slope is changing, a curved characteristic can be taken as approximating to a parabola, and so can be represented by an equation of the form $I_a = a + bV_g + cV_g^2$, where a , b and c have certain definite values depending on the curvature of the dynamic characteristic in this region.

Let the polarising, or steady, voltage applied to the grid before a signal arrives be V_0 . Then a steady anode current I_0 flows, given by $I_0 = a + bV_0 + cV_0^2$.

Let an oscillatory signal voltage, $V_s \sin \omega t$, be applied to the grid.

This, being superimposed on the steady voltage V_0 , gives a grid potential $V_0 + V_s \sin \omega t$. The corresponding current is then

$$\begin{aligned} I_a &= a + b(V_0 + V_s \sin \omega t) + c(V_0 + V_s \sin \omega t)^2 \\ &= a + bV_0 + cV_0^2 + bV_s \sin \omega t + 2cV_0 V_s \sin \omega t \\ &\quad + cV_s^2 \sin^2 \omega t \\ &= I_0 + bV_s \sin \omega t + 2cV_0 V_s \sin \omega t + \frac{cV_s^2}{2}(1 - \cos 2\omega t) \\ &= I_0 + \frac{cV_s^2}{2} + (bV_s + 2cV_0 V_s) \sin \omega t - \frac{cV_s^2}{2} \cos 2\omega t. \end{aligned}$$

There are two radio frequency terms in the above expression, whose mean value over a complete cycle or number of cycles is zero. In other words, they are by-passed by the telephone condenser.

The term $\frac{cV_s^2}{2}$ represents the change in the **mean** value of current

passing through the telephones. Now $I_a = a + bV_g + cV_g^2$,

$$\therefore \frac{dI_a}{dV_g} = b + 2cV_g,$$

$$\text{and } \frac{d^2I_a}{dV_g^2} = 2c.$$

Hence the increase in the mean anode current, $\frac{cV_g^2}{2}$, may be written

$$\text{as } \frac{V_g^2}{4} \times \frac{d^2I_a}{dV_g^2}.$$

This shows that the rectified current is proportional to the square of the amplitude of the applied voltage, and also to the second differential coefficient of I_a with respect to V_g , *i.e.*, to the rate of change of slope of the (I_a, V_g) curve. The most sensitive position for rectification is thus where the slope is changing most rapidly, *i.e.*, at the upper and lower bends of the curve.

It also follows from this result that, with weak signals, it is advisable to amplify before detection, because of this "square law" which connects the final current change through the telephones, and the strength of audible signal, with the value of applied voltage.

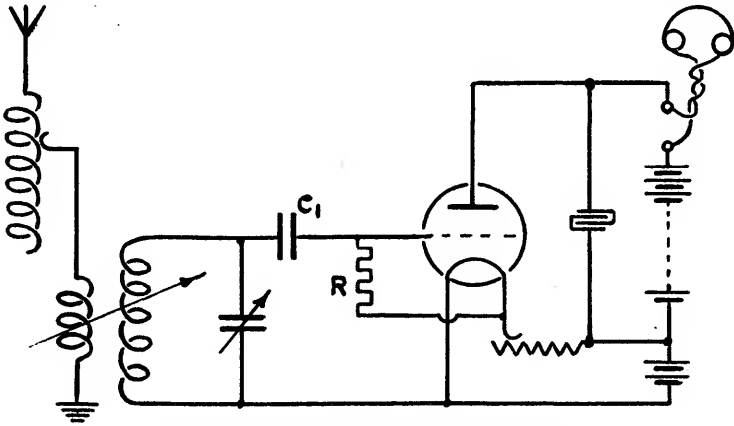
The same method of investigation can be applied to the case of heterodyned C.W. oscillations, in which case the total voltage applied to the grid at any instant can be represented by the expression $V_0 + V_s \sin \omega_s t + V_h \sin \omega_h t$ (V_h and ω_h corresponding to the amplitude and frequency of the heterodyne oscillation). On substitution in the formula for I_a , it is found that the mean value of current is increased by a steady amount $\frac{V_s^2}{2} + \frac{V_h^2}{2}$ which does not, of course, produce a note in the telephones. In addition there are radio-frequency terms, which are by-passed by the condenser, and a slow variation of the mean value, represented by a term of the form $c V_s V_h \cos (\omega_s \sim \omega_h)t$. The frequency of this variation of current is therefore the beat frequency, and its amplitude is proportional to the rate of change of slope of the curve, and to the product $V_s V_h$. Being at the beat, or audio-frequency, this variation affects the telephones. The audible response is therefore proportional to the first power of the amplitude of the incoming oscillations and to the first power of the heterodyne amplitude. Its strength cannot, however, be indefinitely increased by increasing the heterodyne voltage, because the approximation as regards parabolic shape of the curve does not hold good for more than a slight distance along it. There is an optimum value of heterodyne amplitude, above which rectified signal strength again decreases.

It is obvious that sensitivity is greatly increased by using heterodyned C.W. instead of modulation at the transmitter, since a first power law is substituted for the "square law."

563. Cumulative Grid Rectification.—A circuit suitable for cumulative grid rectification is as shown below. The essential difference from the circuit for anode rectification is that an insulating condenser, C_1 , is inserted between grid and filament, with a resistance R , of the order of megohms, joined in parallel with it. This resistance is termed a "grid leak."

The complete theory of the operation of the circuit is best approached by **assuming the grid leak to be omitted.**

If this is the case, the valve will adjust itself before signals commence so that the voltage on the grid is that at which the (I_g, V_g) curve just reaches zero value. If this curve starts at a slightly negative value of V_g , and the initial potential between grid



Circuit for Cumulative Grid Rectification.

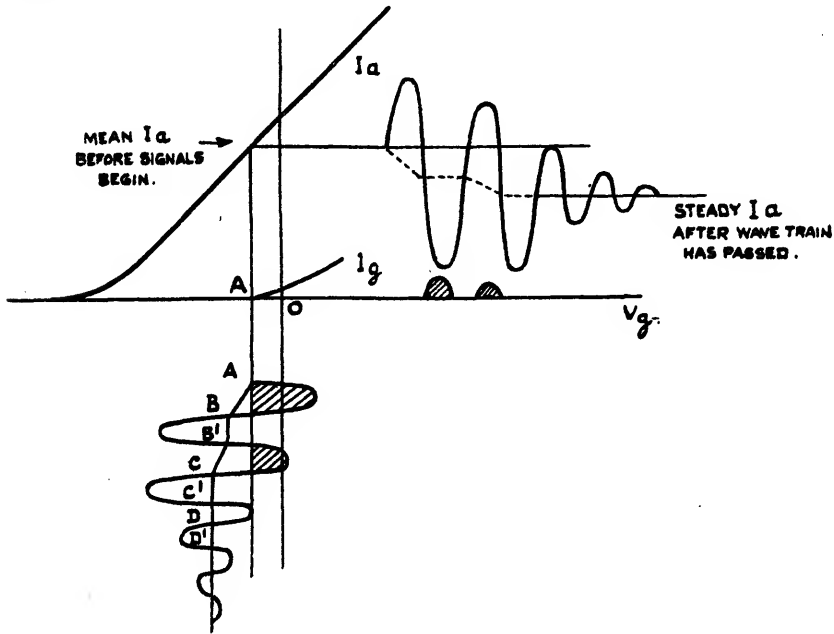
FIG. 309.

and filament is zero, electrons are picked up by the grid and constitute a negative charge on the right-hand plate of condenser C_1 . This continues until the resulting negative potential of grid to filament is such that no more electrons are absorbed by the grid; in other words, the potential of the grid, apart from incoming variations, is that corresponding to zero grid current.

Incoming oscillations make the grid alternately positive and negative to this initial position. When it is positive an electron current will flow from filament to grid. Due to the insulating condenser, this current remains as a negative charge on the right-hand plate, so that at the end of a positive half-cycle of voltage impressed from the incoming signal the grid is left at a more negative potential than at the start (corresponding to point B in Fig. 310, instead of A).

The negative half-cycle of oscillatory voltage immediately following does not cause any grid current to flow, and so the grid, at the end of this half-cycle, returns to B' , which has the same negative value as B.

The next positive half-cycle starts from B', and at the end of it the potential of the grid corresponds to C. Each successive oscillation makes the grid increasingly negative, until it reaches a point where the incoming positive voltage variation superimposed on the mean grid potential is never great enough to cause any grid current to flow. Assuming perfect insulation, the grid is then left at a steady negative potential, and, while it is reaching this point, there is a



No Grid Leak.

FIG. 310.

diminution in anode current. Succeeding signals, however, of the same (or less) amplitude have no effect as regards variation of the mean value of anode current, which remains at its steady lowered value.

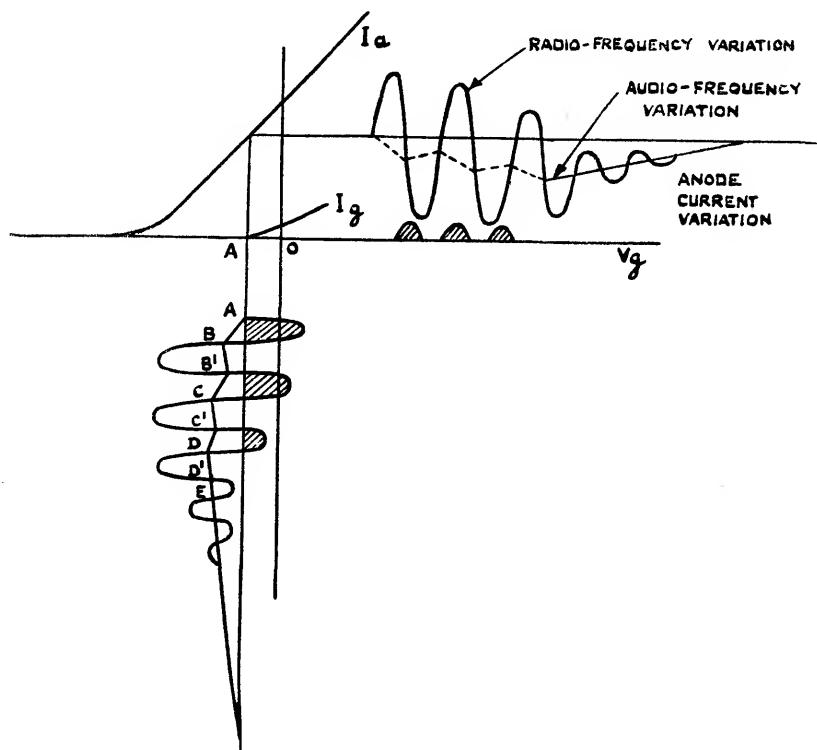
It is therefore necessary to reset the device after each train of waves.

Introduction of Grid Leak.—To drain away the accumulated charge on the grid, a grid leak of the order of megohms is used.

Charging.—As soon as the condenser starts to build up on itself a negative charge, making it more negative than its initial value, the leak comes into action, allowing the electrons to return gradually to the filament. The initial rate of charging of the condenser is not so rapid, but the result is to allow a bigger flow of electrons into the grid during successive half-cycles of positive oscillatory voltage than in the previous case (without leak).

In consequence, the total time of charge differs very little in the two cases, whatever the value of leak, provided it lies within certain rather wide limits, so that the function of the latter may be regarded as essentially that of arranging for the return of the accumulated charge before the next wave-train.

Discharging.—At the end of a wave-train, or as soon as the condenser has gone so negative that succeeding oscillations have no effect because they do not make the grid less negative than at the



Rectification with Grid Leak.

FIG. 311.

start of the operation, the charge on the condenser leaks away gradually through the resistance and enables the grid to return to its initial voltage.

The voltage applied to the grid during to an incoming spark wave-train may, therefore, be represented as in Fig. 311, its mean value going negative, and finally returning to its original position. The symmetrical radio-frequency voltage variations may be regarded as having been made asymmetrical as a result of the introduction of the leak and condenser.

Effect on Anode Current.—A succession of incoming wave-trains will give, therefore, a succession of asymmetrical voltage variations applied to the grid, which can be regarded as R.F. symmetrical oscillations about a mean value which varies at audio frequency, the frequency of the wave-trains.

The resulting effect on the anode current is as shown in Fig. 311. It is assumed that the potential of the grid, during the whole amount of its fluctuation, corresponds to a straight portion of the (I_a, V_g) characteristic. I_a varies at both radio and audio frequency. The telephones are unaffected by the R.F. variations of current superimposed on the A.F. variations of average value, the former being "passed" by the telephone condenser. The audio frequency changes, however, affect the telephones.

It should be clearly understood that, owing to asymmetry having been introduced into the variations of grid-filament voltage, there is no necessity to work opposite a bend of the (I_a, V_g) characteristic curve, as in Anode Rectification. Possible combinations of both types of rectification are examined in para. 565. It may be shown that, as in Anode Rectification, the strength of signals in the telephones—i.e., the strength of the rectified component of current—is proportional to the square of the amplitude of the incoming voltage. The formula is involved, and depends, as might be expected, on the slope and rate of change of slope of the grid current characteristic, as well as on that of the anode current curve.

Incoming C.W. oscillations, applied to the complete circuit with condenser and leak, cause the grid to take up a state of equilibrium at some potential negative to its original value, at which the charge put into the insulating condenser during the part of each cycle when grid current flows is just equal to the total discharge of electrons through the leak during the whole of each cycle. The result, as regards anode current through the telephones, would be a drop from one value to another.

With heterodyned C.W. oscillations, where the amplitude varies, the mean potential of the grid varies at audio frequency, and the resulting anode current is similar to that produced by damped or I.C.W. wave trains.

In transmitter theory, it will be found that the grid condenser and leak are utilised to maintain the grid at a steady mean negative potential, just as in the case of C.W. oscillations above, the essential difference being that the radio frequency variations of grid-filament voltage in a valve transmitter are supplied by the circuit itself.

564. Values of C and R.—The determination of the most suitable values for C and R is a matter of some difficulty. It is essential for audibility that sufficient asymmetry should be introduced into the applied voltage variations—in other words, the mean value of the grid potential should go sufficiently negative. It is also essential

that the charge should leak away during the later portion of the audio frequency cycle. Such considerations give the following results :—

- (i) C should be small, so that small accumulations of electrons may charge it up to a considerable potential.
- (ii) The product CR should be adjusted to give the requisite leak away during the portion of the cycle available. CR is, of course, the "time constant" of the condenser and the leak resistance (*see* para. 174). The time taken for the grid potential to change from one value to another is proportional to CR.
- (iii) The resistance R should be such that its value is much higher than the reactance of the condenser to the signal voltage oscillations, so that the voltage actually applied between grid and filament is as large as possible. This indicates a high value of R and a high value of C.

Since considerations (i) and (iii) give opposite results as regards C, its value is a compromise, and the value of R is chosen to satisfy (ii) as well as possible.

Obviously the leak-away period is dependent on the spark train frequency, on the damping of the oscillations, and on the frequency of the oscillations, as all these affect the time available for the grid to recover its normal potential. Any "CR" value adopted for use in a receiver must therefore be chosen so as to give an average result, as the values of C and R are not made variable. Again, as regards the resistance, R should not be too low or the rectified charge will drain away practically as fast as it collects. Nor must R be too high, or an atmospheric striking the aerial will induce a big negative charge on the grid which will take a long time to drain away through this high resistance, and the valve will be "thrown off," or rendered unable to rectify at all for several seconds, due to the fact that no anode current is flowing; thus part of a signal may be missed.

565. Combination of above effects.—With cumulative grid detection, the audio frequency decrease in anode current through the telephones is greater, the greater the slope of the anode characteristic. So far as we have only considered the action as taking place opposite the straight part of the curve.

Working point too low down.—Let us suppose that the anode current curve is such that, instead of the straight steep portion of the curve, the lower curved portion is opposite the asymmetrical grid voltage variations. In this case there is less response in the telephones, for two reasons.

Firstly, the slope of the characteristic is less, and, secondly, there is an anode rectification effect due to the concavity upwards of the characteristic. Now cumulative grid rectification always

gives an audio frequency **decrease** in anode current, while lower bend anode rectification gives an **increase**, and hence the two effects are in opposition.

Adjustment.—To move the working point further up the characteristic, the anode voltage may be increased, or the grid leak may be connected to the positive instead of the negative side of the filament. This last method may be explained by the fact that, in cumulative grid rectification with a leak fitted, the initial voltage on the grid is not exactly that at which no grid current flows (as we saw it was with the condenser only), but some other voltage determined by the value of the leak itself.

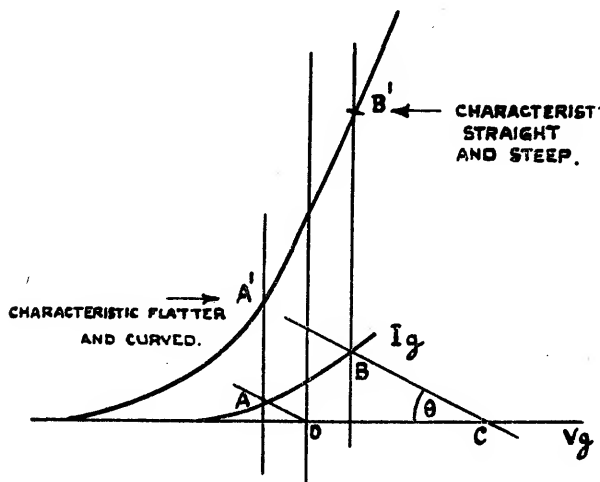


FIG. 312.

The steady voltage on the grid with no incoming signal is actually determined by the intersection of the (I_g, V_g) curve and the line OA drawn from zero grid volts (when the grid is connected to the negative side of the filament), in such a direction that its slope is given by $\cot \theta = R$, where R is the value of the leak resistance. In other words, the potential of the grid with respect to the filament is given by $V_g = -I_g R$. In addition, V_g and I_g are connected by the relationship expressed by their characteristic curve, and with both conditions to be satisfied, the construction of Fig. 312 determines the initial potential of the grid.

In the figure shown, the point of intersection A is opposite to A' on the (I_g, V_g) curve, which is rather low down. If, however, the grid were connected to the positive end of the filament, OC being the voltage drop along the filament, then by the same argument, when angle BCO = θ , the steady voltage on the grid is B, opposite a point B' on the (I_g, V_g) curve, where the latter is steep and straight.

Working point opposite the top bend of the (I_a , V_a) curve.—Since top bend anode rectification, which is bound to occur in this case as well as cumulative grid rectification, gives a **decrease** in mean anode current through the telephones, the effects are here **additive**, instead of being in opposition, as in the last case.

It is not usually possible to achieve this combination of results because :—

(i) Cumulative grid rectification depends on grid current, and it is usual to employ low values of anode voltage to give bigger values of grid current. This means that the (I_a , V_a) characteristic is moved to the right and so the upper bend tends to be away from the region of operations.

(ii) If it is proposed to overcome the difficulty above by reducing the filament current so as to bring the upper bend of the (I_a , V_a) curve to the left, similar difficulties arise. To get good values of grid current **flowing**, a large filament heating current is necessary, which increases the saturation current and moves the upper bend to the right.

Both from the point of view of filament current and anode voltage, therefore, the conditions which are suitable for upper bend anode rectification are antagonistic to the production of the values of grid current necessary for cumulative grid rectification.

566. Regenerative Amplification, Heterodyne Circuits, etc.—As mentioned above, the anode current undergoes not only the audio-frequency variation which operates the telephones, but also a radio-frequency variation, which is by-passed by the telephone condenser. This radio-frequency component is mainly of the same frequency as that of the incoming wave, but contains also higher harmonics.

This component may be usefully employed by causing it to pass through an inductance in the anode circuit, which is coupled to the inductance L of the input LC circuit. A radio frequency oscillatory voltage will then be induced across L . It will be shown below that this induced voltage can be arranged to be in phase with the current flowing in the LC circuit, and hence it represents an introduction of power into this circuit; if the power introduced is sufficient to balance the damping losses present in the circuit, an oscillatory current in the circuit will not die away, but will be maintained continuously at the same amplitude; if the power introduced is less than the rate at which energy is being wasted in damping losses, the oscillation will not be continuous, but will be of greater amplitude and duration than if no anode coupling coil were used.

The first of these conditions is that used in the employment of a circuit of this nature as a **heterodyne unit**, and the second is that used to increase and prolong the effect of an incoming signal, and is known as **regenerative amplification**.

The inductance in the anode circuit is known as a "reaction coil."

We shall first of all investigate the phase relationships of radio-frequency currents and voltages in this type of circuit, and then examine the various uses to which it can be put according as the coupling between the reaction coil and the oscillatory circuit is varied.

567. Reaction necessary for Self-Oscillation.—For the investigation of phasing and coupling the simple circuit, as illustrated below, without telephones, etc., is all that is necessary.

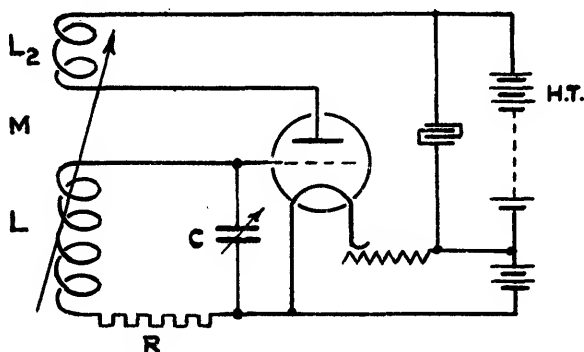


FIG. 313.

Suppose that an oscillation is set up in the LC circuit at its own natural frequency, so that the R.M.S. value of oscillatory current in the circuit is I amperes. Then energy is being dissipated in this circuit at a rate given by I^2R watts, and, with no reaction effect, the oscillation would die away at a rate depending on the damping factor $R/2L$ (see Chapter VII).

With the given circuit, however, an oscillatory current I sets up an oscillatory voltage across the condenser C of value $V_g = \frac{I}{\omega C}$ volts, and this voltage lags 90° on the current. This oscillatory voltage across the condenser is applied between grid and filament of the valve, and it therefore varies the anode current flowing through the valve at the same frequency and in phase with itself. If we assume that the reaction coil is small enough for us to neglect its effect on the anode current changes, and therefore to derive the value of the current variations from the static (I_a, V_g) characteristic curve, their R.M.S. value is given by $g_m V_g$ amperes, or $\left(g_m \times \frac{I}{\omega C}\right)$ amperes, where g_m is quoted in amperes per volt.

Let us call this anode current variation I_a ; it is in phase with the grid voltage. If M is the mutual inductance between the grid and reaction coils, this oscillatory component of anode current will give rise to an induced voltage in the inductance L , of value $\omega M I_a$.

volts. The induced voltage will either lag or lead by 90° on the oscillatory anode current, and therefore be in phase or 180° out of phase with the current I in the input circuit, according as M is positive or negative, *i.e.*, according to the direction of the windings of the separate coils L_2 and L_1 , their relative position, and the way in which the terminals of L_2 are joined up. A vector diagram representing these relationships is appended.

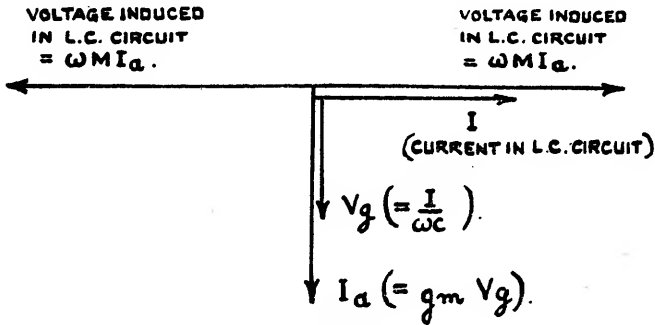


FIG. 314.

Assuming the reaction coil to be joined up so that the voltage induced in the LC circuit is **in phase** with the oscillatory current already present there, this represents an introduction of energy into the circuit at the rate of $(I \times \omega M I_a)$ watts. But, on account of damping losses, energy is being dissipated in this circuit at the rate of $I^2 R$ watts. Therefore the net loss of energy is at the rate of $(I^2 R - I \times \omega M I_a)$ watts.

Now $I_a = \frac{g_m}{\omega C} I$, and so $\omega M I_a \times I = \frac{g_m M I^2}{C}$. Hence the nett rate of loss of energy is $I^2 \left(R - \frac{M g_m}{C} \right)$ watts. If this is a positive quantity, *i.e.*, if $R > \frac{M g_m}{C}$, oscillations set up in the LC circuit will die away, but at a slower rate, because the effective resistance of the circuit may be considered to be reduced from the value R to the value $\left(R - \frac{M g_m}{C} \right)$. The energy derived from the reaction coil has partially neutralised the damping of the oscillatory circuit. If, however, $R = \frac{M g_m}{C}$, the effective resistance of the circuit and the damping factor both become zero, and oscillations set up in the circuit are maintained at a constant amplitude.

If R is less than $\frac{M g_m}{C}$, the effective resistance is negative, and oscillations will increase in amplitude up to a point where limiting

conditions, hitherto neglected, come into play, and prevent the increase going on indefinitely. These will be examined in the next paragraph.

In the latter two cases we have utilised the valve as a medium for maintaining self-oscillations in an oscillatory circuit; in other words, the valve has been acting as a generator of C.W. oscillations. The actual circuit shown is not suitable for high power transmission, but it is very useful for producing the small local oscillations necessary to beat with incoming signal C.W. oscillations and to act as a heterodyne unit.

The conditions for maintenance or otherwise of oscillations can be stated in a slightly different form. They depend on the relative values of R and $\frac{Mg_m}{C}$. Since it is the closeness of coupling and hence the mutual inductance, M , between L_2 and L_1 , which is the adjustable variable, it is usual to express the conditions in the form of a comparison between M and the other quantities involved.

This is easily seen to be as follows:—

If M is greater than $\frac{CR}{g_m}$, oscillations increase in amplitude.

If $M = \frac{CR}{g_m}$, oscillations are maintained at constant amplitude.

If M is less than $\frac{CR}{g_m}$, oscillations die away, but at a less rapid rate than if no reaction were introduced.

568.—Limiting Conditions.—When $M > \frac{CR}{g_m}$, oscillations increase in amplitude, because the effective resistance of the circuit is negative, *i.e.*, the damping factor, $\frac{1}{2L} \left(R - \frac{Mg_m}{C} \right)$, is negative. The following factors limit the growth in amplitude to a definite amount.

(i) So far we have assumed the slope " g_m " to be that of the static characteristic. If, however, the grid voltage variations increase to large values and give correspondingly large changes in anode current, the effect of the reaction coil must be taken into account. The result is that the curve which represents the actual anode current plotted against grid voltage is the dynamic characteristic, instead of the static characteristic. Now the average slope of the dynamic characteristic is always less than that of the static, and hence " g_m " has a lower value than that hitherto assumed, and the preponderance of M over $\frac{CR}{g_m}$ is not so large. This, however, does not account for the ultimate cessation of the increase in amplitude, but it describes more correctly the inequality hitherto found.

(ii) Again, we have not taken into account the fact that large grid voltage variations are bound to give a considerable flow of grid current during the time the grid is positive, and so to produce an increased damping effect on the oscillatory circuit. In other words, R increases above the value hitherto assumed, which was simply taken as the ohmic resistance in the LC circuit.

(iii) Finally, there is the most important condition from the point of view of limitation, that the anode current curve, whether static or dynamic, is not continuously straight, but reaches points, both at the top and bottom of its straight part, where it flattens out and becomes horizontal. Therefore, after the oscillatory grid

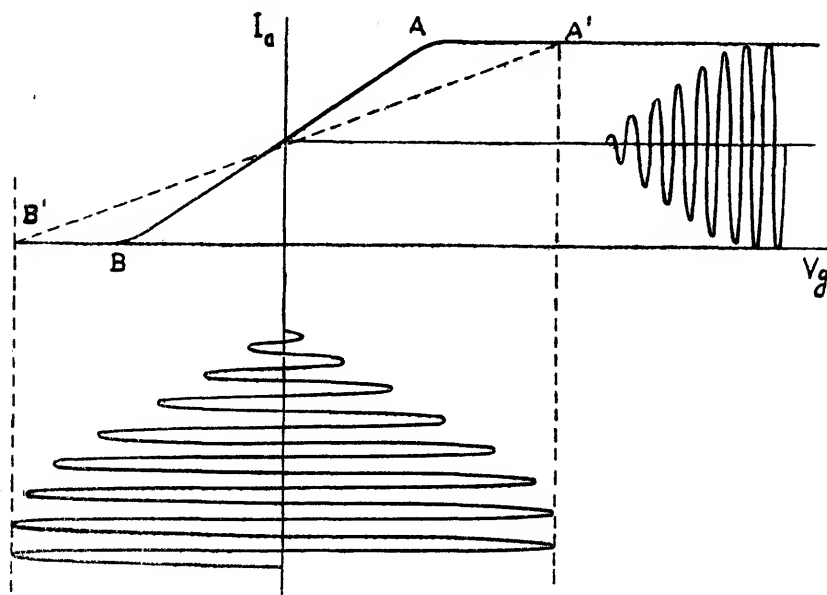


FIG. 315.

voltage reaches a certain amplitude, it is incorrect to say that the corresponding amplitude of oscillatory anode current is given by the product of the mutual conductance, g_m , and the amplitude of grid voltage swing. As soon as the increasing V_g variations reach either bend, further increase of I_a in that direction is prevented.

This is shown in Fig. 315, where increasing V_g variations are accompanied by I_a variations of proportionate amount until the latter are limited by saturation and zero values. We might express this fact by saying that, after the amplitude of the grid swing goes beyond the straight portion of the characteristic, the ratio of anode current variation to grid voltage variation is given by the slope of the line $A'B'$, which is less than g_m , the slope of AB . In other

words, the slope of A' B' should be substituted for g_m in the expression $\frac{CR}{g_m}$, and, as the amplitude of oscillation increases, the denominator of this expression will get steadily less and the value of the expression greater. Hence, although at the beginning of operations, the inequality $M > \frac{CR}{g_m}$ is true, an amplitude of oscillation is reached sooner or later at which it is no longer an inequality, but an equality, and then oscillations will be continuously maintained at this amplitude. The same result may be obtained by comparing the damping losses with the power reintroduced into the circuit. The damping losses are given by $I^2 R$, and the power introduced by reaction is the product of I and the voltage induced in the oscillatory circuit. This voltage is given by ωM multiplied by the variation in anode current, which cannot exceed a certain value, and hence, as I increases, the power introduced into the circuit is ultimately proportional to I . The damping losses are, however, proportional to I^2 , and therefore increase at a greater rate, until a point is reached at which there is a state of equality.

Hence, if the coupling is greater than the critical value necessary for self-oscillation, oscillations will increase in amplitude until a condition is reached when limitation of the value of oscillatory anode current flowing has the effect of making the coupling just sufficient to maintain oscillations at this definite amplitude.

569. Uses of Circuit with Reaction.

(a) **Regenerative Amplification.**—The circuit of para. 567, with the addition of telephones, coupling to a receiving aerial and arrangements for securing anode or cumulative grid rectification, can be used for the reception of spark waves or C.W. modulated at the transmitter. For this purpose the coupling of the reaction coil is adjusted to be **less than that necessary to set up self-oscillations** in the LC circuit, so that its effect is to diminish the effective resistance present in the circuit, and to give larger amplitudes of oscillatory current and of voltage applied to the grid for a given signal voltage induced into the circuit from the receiving aerial. In addition, the oscillations from an incoming signal are prolonged because of the diminished damping factor.

The effect of an incoming spark wave-train with and without reaction may be illustrated as in Fig. 316.

This process is known as **Regenerative Amplification**. The coupling can be increased up to the point when each wave-train dies away just before the next one starts. If the coupling is increased too much a continuous oscillation will be set up, quite irrespective of the incoming wave, and the spark note in W/T, and the speech in R/T, will be spoilt.

It should also be noted that the use of reaction, by decreasing the effective damping, increases the selectivity of the receiver.

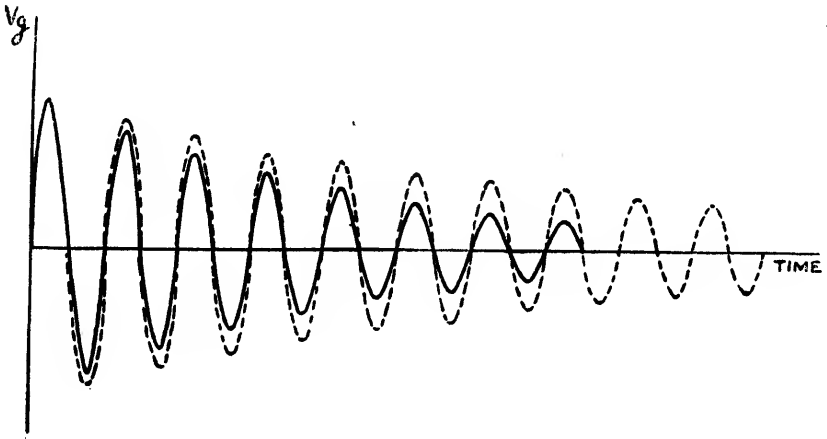


FIG. 316.

(b) **Heterodyne Oscillator.**—The simple circuit of para. 567 can be used as a means of generating the local oscillations necessary for heterodyning. In this case the coupling must be sufficient

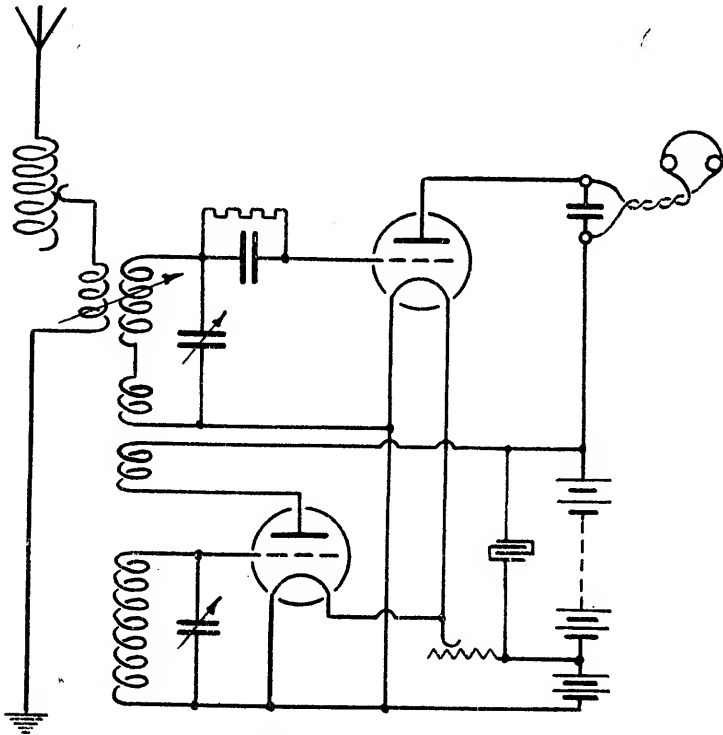
*Heterodyne and Detector.*

FIG. 317.

for self-oscillations to be set up. The circuit in which the heterodyne oscillation is generated must, of course, be mistuned from the frequency of the incoming signal to the detector valve, so as to give the necessary beat note. A complete circuit for the reception of C.W., using a detector valve and a separate heterodyne unit, might be arranged as in Fig. 317. The anode coil of the heterodyne oscillator is mutually coupled to the input inductance of the detector.

(c) **Autodyne or Auto-heterodyne Circuit.**—The same complete circuit as that used for regenerative amplification, and re-drawn below, may be used conveniently both to heterodyne an incoming continuous wave, and also to rectify the resultant "beats" due to the interaction between the incoming wave and the heterodyne oscillation.

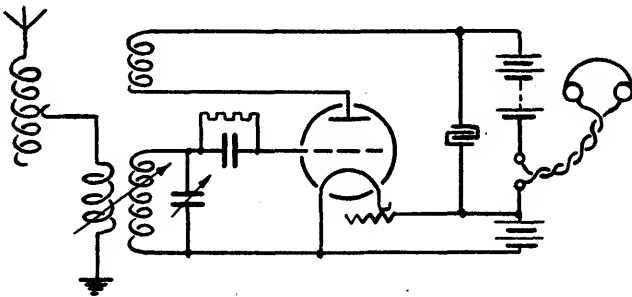


FIG. 318.

The necessary conditions in this case are that the **coupling must be sufficient to give self-oscillation, and the LC circuit between grid and filament must be mistuned** from the incoming oscillations. The circuit is known as an **autodyne** circuit, and performs the functions of local oscillator and detector.

Action.—Before a signal arrives, the valve is maintaining oscillations at a frequency f_1 determined by the LC value of the oscillatory circuit, and of amplitude limited by the conditions of para. 568. The presence of the grid leak and condenser, necessary for cumulative grid detection, simply means that the grid's steady potential, about which oscillations take place, is somewhat negative. (See para. 563.) A steady current is passing through the telephones, the radio frequency variations of anode current being by-passed by the condenser.

An incoming C.W. oscillation will set up induced voltages and currents in the LC circuit, at a frequency f_2 , which differs from f_1 by the beat note. It should be noted that the amplitude of oscillatory current and resulting voltage between grid and filament at frequency f_2 is less than if the LC circuit were tuned to resonance with f_2 . We have now two high-frequency oscillatory voltages

applied to the grid at frequencies f_1 and f_2 , and, being superimposed, their resultant effect is a high-frequency oscillation whose amplitude varies at the "beat" frequency, $f_1 - f_2$.

This will result in the mean potential of the grid varying at the beat frequency, as in the ordinary theory of cumulative grid detection, and hence an audio frequency rise and fall in the mean anode, battery and telephone current, giving an audible note in the telephones.

The best condition for combined heterodyning and detecting is when the self-oscillation is not unduly strong compared with the incoming signal, and this condition may be obtained by reducing the reaction, the anode voltage, or the filament current, until a small amplitude of self-oscillation is just maintained. Both filament current and anode voltage control operate by diminishing slightly the slope of the mutual characteristic (para. 543).

570. Comparison of Autodyne and Separate Heterodyne.—The **advantages** of the autodyne arrangement over the separate heterodyne can be summed up in the statement that it involves one circuit instead of two, occupying less space, and it needs less adjustment. The **disadvantages** are, however, considerable.

(a) It is difficult to control effectively the strength of the local oscillation. If local oscillations are to be maintained, these will increase until the variations of grid voltage cover a large portion, if not all, of the straight part of the (I_a, V_a) characteristic. With a separate heterodyne the amplitude of the local oscillations introduced into the detector circuit can be adjusted by varying the distance apart of the coupling coil of the heterodyne circuit and the inductance in the detector circuit.

(b) In order to obtain the "beat" note, the LC circuit in the autodyne receiver must be thrown out of resonance with the incoming wave, whose effect is consequently weakened. This loss is particularly serious in the case of lower frequencies, as a given frequency difference represents a greater percentage of mistuning. For these it is generally best to use a separate heterodyne.

(c) With an autodyne, it is difficult to prevent radiation of the locally produced oscillation from the aerial, which must, from the nature of the circuit, be coupled to the inductance of the oscillatory circuit; interference is consequently caused to other receiving sets in the neighbourhood working on the same or adjacent frequencies. This difficulty may be overcome to some extent in the case of a multi-stage circuit (i.e., amplifiers + detector) by setting into oscillation a tuned circuit in the receiver at a later stage, where it is not in close contact with the aerial.

Similar trouble arises when a receiving set, not intended for C.W. reception, generates self-oscillations through reaction being pushed too far, or from various factors which are liable to give the same effect, and which will be examined in the chapter on amplifiers.

CHAPTER XIII.

RECEIVING CIRCUITS—THE VALVE AS AMPLIFIER.

571. An amplifier in a receiver is a combination of valves and suitable circuits for increasing the strength of signals.

A signal is made audible (*see* last chapter) by applying between the grid and filament of a detector valve the variations of voltage set up in a circuit coupled to the receiving aerial. If, however, the resultant effect on the telephone current produces too small a movement of the diaphragm for audibility, steps must be taken to amplify or increase the effect.

The exceptional utility of the three-electrode valve in performing this function has already been pointed out in Chapter XI. It was there seen that changes in grid voltage had a much greater effect than the same changes in anode voltage in producing changes in anode current. In the case of a valve with negligible external impedance in its anode circuit, the amplification factor m is the measure of this effect. In the practical case, however, the external impedance in the anode circuit must be at least as large as the internal impedance, and generally must be considerably larger if full advantage is to be taken of the anode current changes. Thus the static characteristics and constants of the valve are no longer a direct indication of its behaviour. The modifications caused by this will be evident from the special cases discussed below. They amount essentially to making use of the dynamic characteristics of the valve and its associated circuit.

572. Amplification may be considered under two general heads according to its purpose :—

- (1) **Power Amplification.**—The ultimate function of a receiver is to reproduce the transmitted signal in a form directly perceptible to the senses, *e.g.*, as sound in a pair of telephones. The measure of the effect produced, *e.g.*, the loudness of the sound, is the power expended in the indicating device. Thus the ultimate aim in reception is to produce maximum power expenditure in the indicating device. For various reasons, which will be seen later (para. 591), this is limited in the valve stage directly supplying power to the indicating device, and so methods have to be devised of increasing the effect of the incoming signal before it is applied to this stage. These methods lead to

- (2) **Voltage Amplification** of the incoming signal voltage. The final power stage is operated by a voltage applied between grid and filament (input circuit) of the valve whose output circuit contains the indicating device. Thus the earlier stages consist of valve circuits designed to make the voltage change between grid and filament of the final valve as large as is feasible. Provision must also be made for detection of the radio frequency oscillation, as explained in Chapter XII, so that the power supplied to the indicating device is at a suitable frequency, *e.g.*, audio frequency in the case of a pair of telephones.

573. Types of Amplifier.—Voltage amplification may be arranged both before and after the detector stage. This gives rise to two general types of amplifier :—

- (a) **Radio-frequency Amplifiers.**—In these the radio-frequency voltage produced in the aerial circuit by an incoming signal is increased in amplitude, *i.e.*, radio-frequency amplifiers are voltage amplifiers.
- (b) **Audio-frequency Amplifiers or Note Magnifiers.**—In these the audio frequency voltage produced in the anode circuit of the detector is further amplified. In the last stage of note magnification, the aim in the anode circuit is to produce maximum power. The earlier stages are voltage amplifiers.

Both types of amplifier have certain advantages and certain drawbacks, which may be mentioned here briefly.

Radio-frequency amplifiers give better results as regards strength of weak damped wave, I.C.W., or R/T signals, because of the general characteristic of detectors that the value of rectified current is proportional to the square of the amplitude of applied voltage variation where the latter is small. If, therefore, this voltage variation is amplified ten times before detection, the resulting rectified current will be increased a hundred times.

With an audio-frequency amplifier designed to give equal amplification, the rectified current would only be increased ten times.

Radio-frequency amplifiers can be used to increase the selectivity of a receiver. They usually contain circuits which, for best results, have to be tuned to the radio-frequency being received, and therefore act as additional devices for diminishing the strength of interfering signals at non-resonant frequencies, in the same way as the aerial and secondary circuits.

Note magnifiers are apt to amplify, not only the audio-frequency variations of current and voltage from the detector, but also any audio-frequency changes in the voltage supply to the valves, etc.

It might seem, from the above, that radio-frequency amplification should always be used ; but serious difficulties arise in the

design of radio-frequency amplifiers, as compared with note magnifiers. The higher the frequency, the more difficult it is to prevent the generation of self-oscillations (when these are not wanted) by reason of energy being returned through unintentional coupling to the input circuit of the amplifier sufficient to reduce its effective resistance to zero. This effect increases also with the amount of amplification, so that the amount of radio-frequency amplification possible is limited. If further amplification is desired, it is found convenient to carry out part of the operation before rectification, and part after it.

574. Voltage Amplification Factor.—Before examining the different types of amplifier used in practice, we shall investigate more exactly the behaviour of a valve with an impedance in its anode external circuit.

The Voltage Amplification Factor (usually written **V.A.F.**) of a valve and an associated impedance in its anode circuit is defined as the **ratio of output voltage variation across this external impedance to the input voltage variation between grid and filament of the valve.**

The external impedance may be a non-inductive resistance, an inductance or choke, a combination of these, or a parallel circuit of inductance and capacity. In addition, the anode circuit may contain the primary winding of a transformer, the voltage across the secondary of which is taken as the output voltage of the system. The effect of these impedances will now be considered. No account will be taken of the effect of inter-electrode capacities in the argument. The results obtained are thus approximate only, but represent the truth fairly well at low frequencies where inter-electrode capacitive reactance is considerably higher than the other impedances in the circuit. It should be borne in mind, however, that the instability produced by regeneration through inter-electrode capacity, and other forms of coupling between stages, is generally the factor that limits the amplification in practice. This effect will be considered later.

(a) Resistance R in the Anode Circuit.—Let us apply an oscillatory voltage variation, of R.M.S. value V_g , between grid and filament of a valve. (In the following theory the notation V_g , I_a , etc., is taken as representing the R.M.S. values of oscillatory changes, not the steady values about which these take place.)

As a result oscillatory variations will take place in the anode current flowing, but we cannot state that these are given by $I_a = g_m \times V_g$, where g_m is the slope of the static mutual characteristic.

The variations in anode current themselves cause variations in the voltage between anode and filament.

Thus, let the steady anode current be I_0 and the voltage of the high-tension battery be V_0 . The actual potential difference between anode and filament is then $(V_0 - I_0 R)$.

If, now, I_0 has added to it an oscillatory component I_a , the new P.D. between anode and filament is $V_0 - (I_0 + I_a) R$.

That is, the change in anode current I_a results in a change in anode potential given by $V_a = - I_a R$.

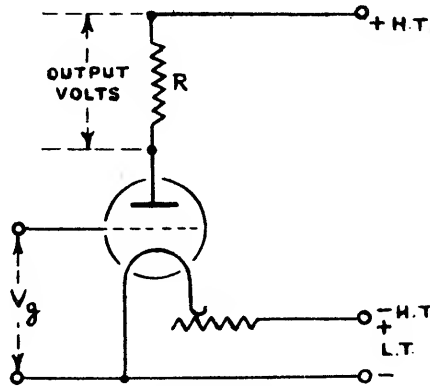


FIG. 319.

The general formula for the determination of I_a , as given in para. 550, must therefore be used, and combined with the last result to give the actual value of I_a . Thus, the two equations following are both applicable to the circuit :—

$$I_a = g_m V_g + \frac{1}{r_a} V_a$$

$$V_a = - I_a R.$$

Solving these, we obtain, by substitution for V_a in the first,

$$I_a = g_m V_g - I_a \frac{R}{r_a}$$

$$I_a \left(1 + \frac{R}{r_a} \right) = g_m V_g$$

$$I_a = \frac{g_m}{1 + \frac{R}{r_a}} V_g \dots \dots \dots (1)$$

Also output voltage across R

$$= I_a R = \frac{g_m R}{1 + \frac{R}{r_a}} V_g$$

Hence, the voltage amplification factor, which is $\frac{\text{output voltage}}{\text{input voltage}}$, is given by

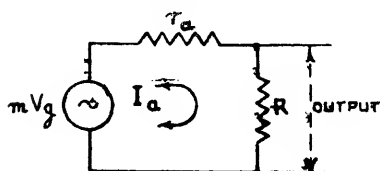
$$\frac{I_a R}{V_g} = \frac{g_m R}{1 + \frac{R}{r_a}} = \frac{g_m R r_a}{R + r_a} = m \frac{R}{R + r_a} \dots \dots \dots (2)$$

Equation (1) is an expression connecting the anode current variation with the grid voltage variation.

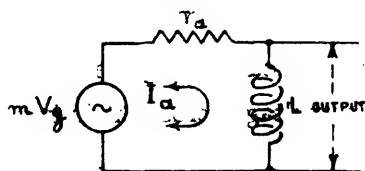
Instead of the ratio being g_m (its value in the static characteristic derived from a valve test board), it is now

$$\frac{g_m}{1 + \frac{R}{r_a}}$$

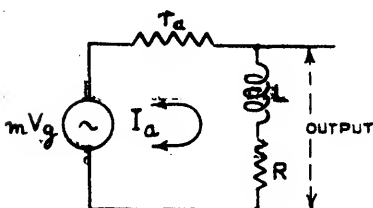
The latter quantity is, of course, the slope of the dynamic characteristic with a resistance R in the anode circuit, the circuit we have taken and the consequent dependence of anode voltage on anode current being similar to those of para. 552. As was stated there, the greater the external resistance, the less is the slope of the dynamic as compared with the static characteristic.



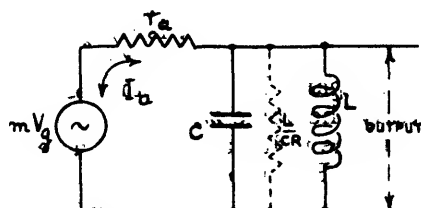
(a) RESISTANCE



(b) PURE INDUCTANCE



(c) INDUCTANCE AND RESISTANCE



(d) REJECTOR CIRCUIT.

Simplified Circuits for Amplification.

FIG. 320.

Equation (1) might also be written:—

$$I_a = \frac{g_m r_a}{R + r_a} V_g = \frac{m V_g}{R + r_a}$$

or, the anode current variation is that which would be given by a voltage equal to m times the voltage applied between grid and filament, if this voltage were applied to a circuit containing the A.C. resistance of the valve and the external resistance in series. This simplified method can be extended to the calculation of the anode current and the V.A.F. for the other types of external impedance used in amplifiers.

Equation (2) shows that the V.A.F. is always less than the amplification factor, m , of the valve, with this type of external impedance.

By increasing the value of the external resistance R relatively to the A.C. resistance r_a , of the valve the V.A.F. can be made to approximate closely to m , but this course is attended by difficulties in securing adequate voltage on the anode owing to the heavy IR drop in the resistance. This will be referred to later under Resistance Capacity-Coupled Amplifiers.

(b) **Inductance L in the Anode Circuit.**—Using the result of last section, that the anode current change is equal to that in a simple series circuit with m times the input voltage applied across r_a and the external impedance in series,

$$I_a = \frac{mV_g}{\sqrt{r_a^2 + (\omega L)^2}},$$

and the current lags behind the voltage by an angle $\tan^{-1} \frac{\omega L}{r_a}$.

The voltage across the inductance L is given by

$$\omega L I_a = \frac{m \cdot \omega L \cdot V_g}{\sqrt{r_a^2 + (\omega L)^2}}.$$

This is the "output voltage" and V_g is the "input voltage."

Therefore the V.A.F. is $\frac{m\omega L}{\sqrt{r_a^2 + \omega^2 L^2}}$.

Here, again, by increasing ωL , the V.A.F. can be increased up to nearly the value m .

If the inductance L has a resistance R , which is not negligible, the V.A.F. can be easily seen to be

$$\frac{m \sqrt{R^2 + \omega^2 L^2}}{\sqrt{(R + r_a)^2 + \omega^2 L^2}}.$$

(c) **Tuned Anode Circuit.**—Another form of output impedance is the "tuned anode," which consists of a tuned circuit of inductance and capacity in parallel in the anode lead.

If we assume the resistance R of this circuit to be entirely contained in the inductance side, the impedance of the circuit becomes a pure resistance to a resonant applied voltage, the effective value of the resistance being $\frac{L}{CR}$ (see Chapter V, para. 318).

We can, therefore, use the formula already obtained in section (a) of this paragraph to give the V.A.F. in this case, which is

$$\text{V.A.F.} = m \frac{\frac{L}{CR}}{r_a + \frac{L}{CR}}.$$

By making the tuned circuit "stiff," i.e., with a big ratio of L to C , its effective resistance can be made very large, and a voltage amplification factor obtained which is very nearly equal to m , the amplification factor of the valve.

For frequencies to which the parallel combination is not resonant, its impedance is much less than the value quoted above, so that these frequencies are not amplified to nearly the same extent, and the circuit introduces, as might be expected, an extra measure of selectivity.

(d) **Transformer in Anode Circuit.**—The mathematical theory of the transformer as an output circuit is more involved than in the preceding cases, and will be referred to briefly under Transformer Coupling. It may be mentioned now that a V.A.F. greater than m can be obtained with this type of coupling.

575. Intervalve Couplings.—Amplifiers are generally classified according to the type of output circuit employed, the amplified voltage variations across which are applied between the grid and filament of the next valve in the model. If several valves are used for amplifying purposes, before or after detection, they are said to be "in cascade." In such a case the amplified variations of voltage from the first valve are further amplified by the second, and so on. The **Voltage Amplification Factor** of several valves and their associated circuits is defined as the ratio of the output voltage across the impedance in the anode circuit of the last valve to the input voltage across grid and filament of the first valve, and is obviously given by the product of the V.A.F.s of each valve and its external circuit taken separately.

The types of output circuit or **Intervalve Coupling** used are the same as those whose V.A.F.s are investigated in para. 574, and are generally described as follows:—

- (a) Resistance-Capacity.
- (b) Choke-Capacity.
- (c) Tuned Choke-Capacity or Tuned Anode-Capacity.
- (d) Transformer.
- (e) Tuned Transformer.

The reason for the addition of the term "capacity" in the description will be made apparent later. With any of these, reaction may be applied, if considered necessary to give further amplification. They will now be considered in turn.

576. Resistance-Capacity Coupling.

Fig. 321 represents a resistance-capacity coupled amplifier. The input voltage causes variations in anode current, which set up variations in voltage across the output circuit R_1 of greater amplitude. This output voltage has to be applied between grid and filament of the second valve, and difficulty arises with regard to the method by which this transference of voltage is to be achieved. If

we join one end of the output resistance in the anode lead of valve V_1 to the grid of valve V_2 , the other end can be considered connected to the filament of valve V_2 through the H.T. battery, and the radio frequency variations of voltage will be impressed between grid and filament of V_2 .

This, however, would mean that the grid of the second valve would have a mean potential equivalent to that of the H.T. battery, whereas it should have a mean potential in the neighbourhood of that of the filament.

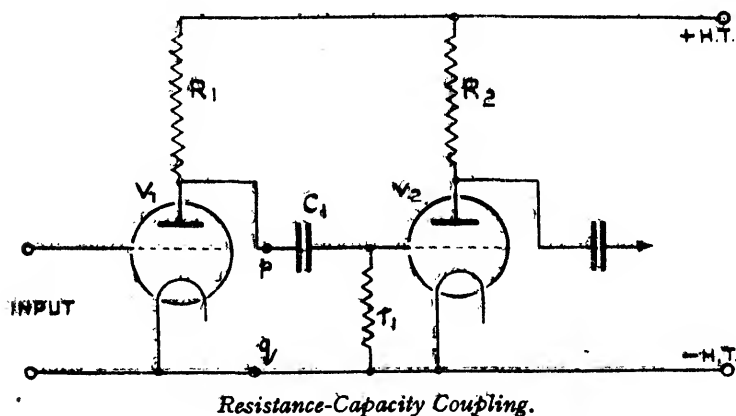


FIG. 321.

A condenser, C_1 , is therefore inserted in the lead joining the output resistance R_1 to the grid of V_2 , to insulate the grid from the steady potential of the high-tension battery. The condenser C_1 acts as a low impedance to the oscillatory voltage, but is an infinite resistance to the direct voltage of the battery.

With the condenser C_1 inserted, an additional resistance, r_1 , is necessary between grid and filament of V_2 to serve the usual purpose of preventing an accumulation of negative charge on the grid; if the latter were left completely insulated, it would in time accumulate such a negative charge on itself that the anode current of V_2 would fall to zero.

577. Values of Grid Condenser and Leak.—The insulating condenser and leak have values which are determined by somewhat similar considerations to those of para. 564, where their use in a detector was investigated.

In the case of an amplifier, the essential feature of the apparatus is to pass on to the second valve a waveform of voltage which is the same as that received by the first valve, so that it is necessary to minimise as much as possible the typical action of a condenser and leak, which is to introduce asymmetry into the waveform if grid current flows. There is therefore no definite reason for having a

small value of C_1 , the grid condenser, as was shown to be advisable for cumulative grid detection in para. 564, section (i), because here we want to prevent variation in the mean potential of the grid.

On the other hand, the statement of para. 564, section (iii), is very relevant to the case of the amplifier.

The amplified output voltage, which is available between points p and q , in Fig. 322, is impressed across a circuit which consists of the condenser C_1 , and a parallel combination between grid and filament of the next valve, made up of the leak resistance r_1 , the grid-filament capacity, C_{gf} , of the valve, and the A.C. resistance of the valve between grid and filament, R_{gf} .

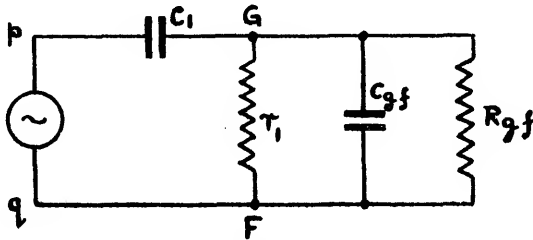


FIG. 322.

We want the voltage across GF (see Fig. 322), which is that actually applied between grid and filament, to be as large a fraction of that across pq as possible. Hence, the potential fall across C_1 should be negligible, or C_1 should have a small reactance compared with the parallel circuit GF. This means that the capacity of C_1 should be large.

Now consider the parallel circuit. The A.C. resistance, R_{gf} , is usually of the order of a megohm, and the reactance of C_{gf} is also high at low frequencies, because of the small value of C_{gf} . The leak, r_1 , should therefore be of the same order, so that the total effective impedance between G and F is not sensibly diminished by a low value of leak. At higher frequencies, the reactance of C_{gf} is the limiting factor.

The conditions are therefore similar to those in cumulative grid detection; except that there is no definite reason for having C_1 small. We obtain the general result, that in amplifiers the condenser has a larger value than in detectors, while the leak is generally somewhat smaller, so as to allow the accumulated electron charge on the condenser, which cannot entirely be avoided unless by grid bias (see para. 578), to leak away in the same time (depending on CR).

The only practical limit to the increase in value of C_1 is that, if it is made too large, its reactance becomes small for stray audio-frequency voltages in the amplifier, and these are passed on sensibly undiminished in the same way as the radio frequency variations.

The coupling and insulating condensers used in note magnifiers have, of course, to be much larger than those used in radio frequency amplifiers.

578. Use of Negative Grid Bias. Distortionless Amplification.—If the mean grid potential of a valve is such that the oscillatory voltage input gives instantaneous values of grid potential corresponding to points on the curved regions of the dynamic mutual characteristic, rectification takes place (Chapter XII). The anode current waveform, and therefore the output voltage waveform, is not a replica of the input voltage waveform, *i.e.*, distortion has been produced. The first requisite for distortionless amplification is therefore that the range within which the grid voltage varies owing to an oscillatory input voltage should be confined to the straight part of the characteristic.

Distortion is also caused if grid current flows during any part of the input voltage cycle, for the damping of the input circuit, and hence the oscillatory grid voltage, is thereby caused to vary. To prevent this, the grid may be kept at a steady negative potential (bias), large enough to prevent the peak grid voltage during the positive half cycle of the input from extending into the region where grid current flows.

If the grid is biased negatively to a point half-way between the voltage at which grid current starts to flow and the voltage at which the dynamic characteristic curve starts to bend, the best condition is achieved; provided the amplitude of the grid swing is less than the amount of bias, distortionless amplification will then be secured.

In a multi-valve amplifier, the grid swing increases from valve to valve, so that longer straight portions of the mutual characteristic curve on the negative side of zero grid volts are necessary in the later stages of amplification, especially in note magnifiers; this is secured by having valves of low amplification factor m , with the inherent disadvantage that the V.A.F. is thereby diminished. This is unimportant in the last stage, where power amplification is required, provided g_m is high.

579. H.T. Voltage with Resistance Coupling. Resistance-Capacity Coupling has one great disadvantage compared with other types of coupling.

We saw, in para. 574, that the V.A.F. is given by $m \frac{R}{R + r_a}$, and consequently, by increasing R , the V.A.F. can be increased.

With this type of coupling, however, it is essential to take into consideration the steady component of anode current which passes through the external resistance, as well as the oscillatory component.

The mean anode voltage is given by the voltage of the H.T. battery, minus the $I_a R$ drop in the external resistance, where I_a

represents the mean value of anode current. As R is increased, this $I_a R$ drop increases, and the voltage on the anode diminishes.

A fairly high anode voltage is necessary if the straight part of the mutual characteristic is to extend over the working region for distortionless amplification, and, even if distortion be accepted, it is still the case that the A.C. resistance of the valve increases as the working point approaches the lower bend, and so the V.A.F. of the stage is diminished. Hence, both for quality and sensitivity, the anode voltage should be high.

In practice, this is achieved by increasing the H.T. voltage, and as there are practical limits to the extent to which this can be done, it is seldom that the value of R is greater than three or four times the value of the A.C. resistance of the valve. Taking the typical figure of $R = 3 r_a$, the V.A.F. = $\frac{4}{3} m$.

Example 59.

In a resistance-coupled amplifier (one valve) there is a V.A.F. of 4 with a resistance of 20,000 ohms in the anode lead. With the resistance changed to 40,000 ohms, the V.A.F. is 5. Find the mutual conductance g_m of the valve.

$$\text{V.A.F.} = m \frac{R}{R + r_a}$$

so that the following equations hold :—

$$4 = \frac{20,000m}{20,000 + r_a}$$

$$5 = \frac{40,000m}{40,000 + r_a}$$

$$\therefore \quad 80,000 + 4 r_a = 20,000 m.$$

$$\text{and } 200,000 + 5 r_a = 40,000 m.$$

Multiplying the first by 2, we obtain

$$160,000 + 8 r_a = 40,000 m.$$

$$40,000 - 3 r_a = 0.$$

$$\therefore r_a = 13,333.3 \text{ ohms.}$$

$$\text{Also } 80,000 + 4 (13333.3) = 20,000 m.$$

$$13,333.3 = 20,000 m.$$

$$m = 6.7.$$

$$\therefore g_m = \frac{m}{r_a} = \frac{6.7}{13,333.3 \text{ ohms}} = \frac{1}{2,000 \text{ ohms}}$$

$$= \frac{1 \text{ amp.}}{2,000 \text{ volt}} = 0.5 \frac{\text{mA}}{\text{volt.}}$$

*Example 60.

The static mutual characteristic curve of a valve for $V_g = 50$ volts can be represented as a straight line from zero value of I_a at $V_g = -1$ volt to a value of $I_a = 1 \text{ mA}$ at $V_g = +3$ volts before becoming horizontal.

The amplification factor (m) of the valve is 8, and grid current starts flowing as soon as V_g is positive.

The valve is to be used in a resistance amplifier, giving a V.A.F. = 6. Find the voltage of the H.T. battery if it is desired to obtain distortionless amplification with an oscillatory grid voltage of amplitude 2 volts and a mean current of 0.5 mA flowing through the valve.

(a) The first step is to find the external resistance and hence the voltage drop across it.

$$\text{V.A.F.} = \frac{mR}{R + r_s}$$

$$\therefore 6 = \frac{8R}{R + r_s}$$

$$g_m = \frac{1 \text{ mA}}{4 \text{ volt}} = \frac{1}{4,000} \frac{\text{amp}}{\text{volt}}$$

$$\therefore r_s = \frac{m}{g_m} = 8 \times 4,000 \frac{\text{volt}}{\text{amp.}} = 32,000 \Omega$$

$$\therefore 6R + 6 \times 32,000 = 8R,$$

$$2R = 6 \times 32,000,$$

$$\therefore R = 96,000 \Omega.$$

$$\text{Voltage drop in } R = 96,000 \times 0.5 \text{ mA} = 48 \text{ volts.}$$

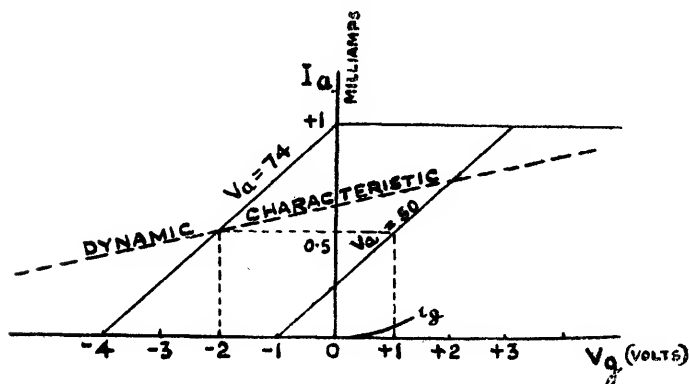


FIG. 323.

(b) The steady grid voltage V_g , corresponding to an anode current of 0.5 mA must be -2.0 volts to prevent grid current flowing. On the given characteristic, 0.5 mA corresponds to +1 grid volts, so the characteristic has to be shifted to the left through three grid volts. As we are dealing with static characteristics, this corresponds to an increase of $3m = 24$ volts on the anode. The anode voltage required is, therefore, $50 + 24 = 74$ volts, and the H.T. voltage required is $74 + 48 = 122$ volts.

A portion of the dynamic characteristic is indicated in the figure.

580. Resistance-Capacity Amplifiers—General Remarks.—Resistance-capacity coupling is used in both R.F. amplifiers and in note magnifiers.

It possesses the great advantage that it gives equal amplification over the full range for which the amplifier is designed, the V.A.F. being independent of frequency. We shall see that some other types of coupling give varying results in this respect.

No tuning is necessary to give best results.

The absence of tuned circuits in the amplifier means, however, that no further selectivity is introduced into the receiving model by its use.

The amplifier as used in audio-frequency work is sensibly the same as for radio-frequency, except that the coupling condenser must be larger to avoid losses (para. 577).

Resistance-capacity amplifiers are unsuitable for high frequencies. At such frequencies, the resistances used have self-capacities in parallel with themselves, and in addition the anode-filament capacity of the valve and the grid-filament capacity of the succeeding valve form a low impedance in parallel with the resistance. The effective value of the external impedance is thus reduced and a smaller V.A.F. is obtained.

The resistances used are also liable to give rise to internal noises in the amplifier, as their values are not exactly stable, but generally vary to a small extent. The higher the value of the resistance, the more likely is this to occur.

Lastly, the most serious disadvantage of the resistance amplifier is the fact that higher anode voltages than usual are necessary to compensate for the voltage drop in the external circuit. This is avoided in all other types of amplifier coupling.

581. Choke-Capacity Coupling.—Fig. 324 represents a choke-capacity coupled amplifier.

The general arrangements of the circuit are similar to those for resistance-capacity coupling. The amplified voltage variations across the output circuit (in this case the inductance L_1) are applied between grid and filament of the next valve, and so on.

For the same reason, the insulation of the grid of valve V_2 from the direct voltage of the H.T. battery, a coupling and insulating condenser, C_1 , is inserted; this makes it necessary to complete the circuit from grid to filament by a leak resistance R , to prevent an accumulation of negative charge on the grid.

The grid may, of course, be biased negatively so as to ensure that there is no flow of grid current even when the oscillatory potential between grid and filament has its maximum positive value.

The values of the condenser and leak are determined by considerations similar to those for resistance coupling.

It is unnecessary to have a much higher voltage in the H.T. battery than is actually required between anode and filament, because the resistance of the choke may be considered small, and hence the IR drop across it due to the steady component of anode current is small.

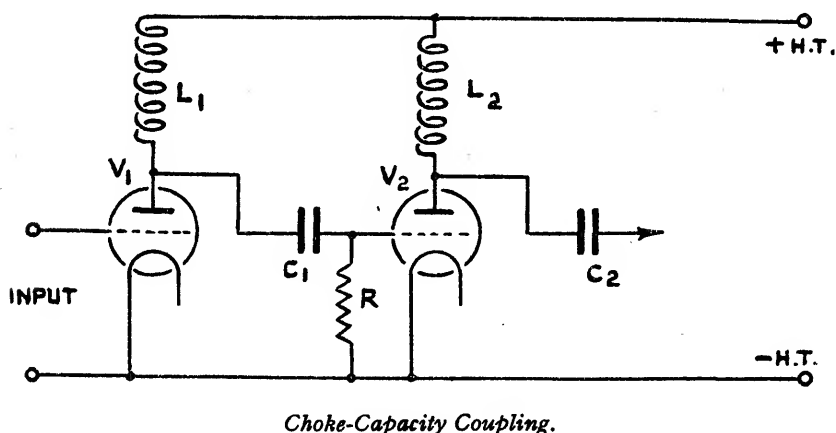


FIG. 324.

This is in no way contradictory to the statement that the reactance of the choke to the oscillatory component of anode current may be very large.

582. V.A.F. with Choke-Capacity Coupling.—We saw in para. 574 that the V.A.F. is given by :—

$$\text{V.A.F.} = \frac{m\omega L}{\sqrt{r_a^2 + \omega^2 L^2}}$$

By increasing the value of " ωL ," we can make the V.A.F. approximate closely to m , without the previous difficulty as regards H.T. supply.

The following conclusions can be drawn from the formula :—

(1) The chokes used in note magnifiers must be very much larger than those used in radio-frequency amplifiers, to give corresponding values of amplification. In practice, in note magnifiers, iron-cored chokes are used, whose inductance is of the order of henries.

For example, with an audio-frequency of 500, ω is about 3,000, and a choke of about 20 henries would be necessary to give a reactance of 60,000 ohms.

(2) For equal values of external reactance and resistance the choke amplifier gives better amplification than the resistance amplifier.

If we write $R = \omega L = x \times r_a$, the V.A.F. for the choke amplifier may be written in the form $\frac{mx}{\sqrt{1+x^2}}$ and for the resistance amplifier in the form $\frac{mx}{1+x}$.

If $x = 1$, the first result is $\frac{m}{\sqrt{2}} = 0.71 m$, and the second $\frac{m}{2} = 0.5 m$.

If $x = 3$, the first result is $\frac{3m}{\sqrt{10}} = 0.95 m$, and the second $\frac{3m}{4} = 0.75 m$.

(3) **The amplification secured from a given choke varies with the frequency amplified.**—This is a great disadvantage with this type of amplifier, and the unequal results achieved are made even more pronounced by the possibility that the inductance may form a resonant parallel circuit with the inherent self-capacity of the coil, and give exceptionally good amplification for one particular frequency as compared with all others. This point is discussed in the next paragraph.

583. Peak Amplification.—If the self-capacity, as above, combines with the inductance to form a resonant parallel circuit for one particular frequency included in the range of the amplifier, it is no longer correct to assume that the reactance of the choke is ωL . Actually the impedance of the circuit now becomes a pure resistance, whose effective value is given by $\frac{L}{CR}$, where R is the ohmic resistance of the coil.

If R is low, as is generally the case, this value is probably very much greater than the value of " ωL ," calculated for other frequencies within the range of the amplifier lower than and considerably removed from the resonant case. This results in specially good amplification at the particular resonant frequency. This effect is known as "**Peak Effect**" and is an undesirable feature in an amplifier.

Generally, therefore, steps are taken to destroy the peak effect, or to utilise it by arranging for it to occur at some desired frequency.

It may effectively be **avoided** by winding the choke with high-resistance wire, which reduces its impedance at resonance, and, in addition, tends to equalise the amplification at non-resonant frequencies, because of the introduction of a resistance term into the expression for V.A.F.

The full expression (para. 574) for the V.A.F. is

$$\text{V.A.F.} = m \frac{\sqrt{\omega^2 L^2 + R^2}}{\sqrt{\omega^2 L^2 + (R + r_a)^2}}.$$

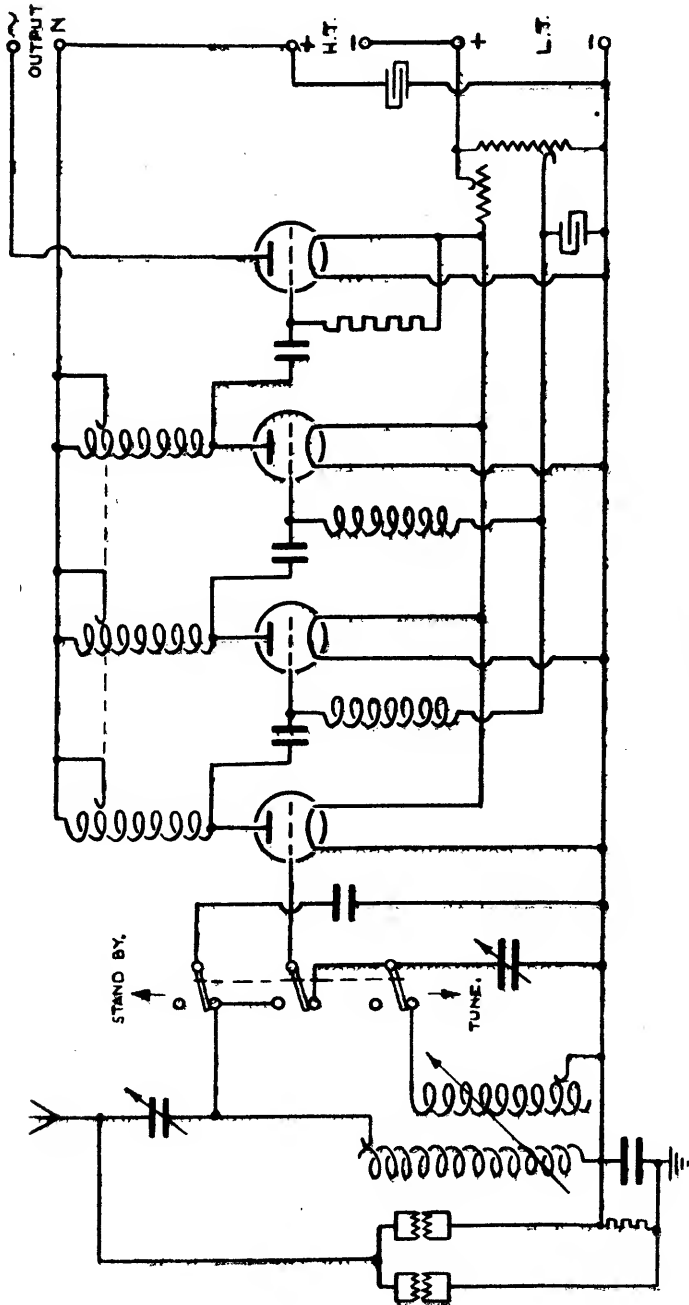


FIG. 925.

If R is comparable with ωL , the variation in V.A.F. for different values of ω is not nearly so great.

It is better, however, to try to take advantage of the peak effect for different frequencies, and this can be done in two ways. Obviously, it is necessary to secure the resonant parallel circuit condition.

(a) The inductance may be large enough, in combination with its total self-capacity, to be resonant to a wave-length longer than, or a frequency smaller than, the limiting value for which the amplifier is designed.

If the inductance is then provided with several tapping points, it is possible to get a rough approximation to resonance for any frequency desired with a portion of the inductance and the corresponding self-capacity.

Fig. 325 illustrates the tuner and amplifier stages of an early Service receiver designed on this principle for medium and intermediate frequencies. The three chokes are tuned by a single handle as indicated by the dotted line. This simplifies the tuning considerably, but it will be seen that the first two grid leaks are inductive and the third (for the detector valve) is a non-inductive resistance. As these leaks (in series with the coupling condensers) are shunted across their respective chokes, the resonant frequency of the first two amplifier stages is different from that of the third stage. The three stages can thus never be all in tune together and efficiency is sacrificed.

(b) The inductance may be so small that, in combination with its total self-capacity, the resultant LC value is less than that necessary for resonance at any of the frequencies passing through the amplifier. If, in this case, we join an artificial condenser across the inductance, whose value can be varied, it will be possible to make the parallel circuit exactly resonant to any desired frequency, the condenser capacity being in parallel and therefore additive to the self-capacity of the choke.

This method of coupling is known as Tuned-Choke or Tuned-Anode Coupling, and will be discussed in the following paragraph.

584. Tuned-Choke-Capacity Coupling.—Fig. 326 represents an intervalve coupling in which the addition of the condenser C_1 across the anode choke L_1 makes it possible for the circuit $L_1 C_1$ to be tuned to resonance at any frequency desired.

If the ohmic resistance of the circuit is R , its effective resistance is $\frac{L_1}{C_1 R}$ which may be very large indeed.

The V.A.F. is therefore (para. 574) equal to

$$m \cdot \frac{L_1}{C_1 R} \\ r_a + \frac{L_1}{C_1 R}$$

The **advantages** of this type of output circuit are :—

(1) By increasing the stiffness of the circuit, *i.e.*, by having a large value of L and a small value of C , and by using low resistance wire, the V.A.F. can be made very nearly equal to m . In practice a limit is set to the decrease of C , because it consists partly of the unavoidable self-capacity of the choke and the inter-electrode capacities of the valve in parallel with the artificial condenser provided for tuning purposes. Further, the design of a tuned anode stage is ruled more by considerations of stability and selectivity than of the maximum possible amplification.

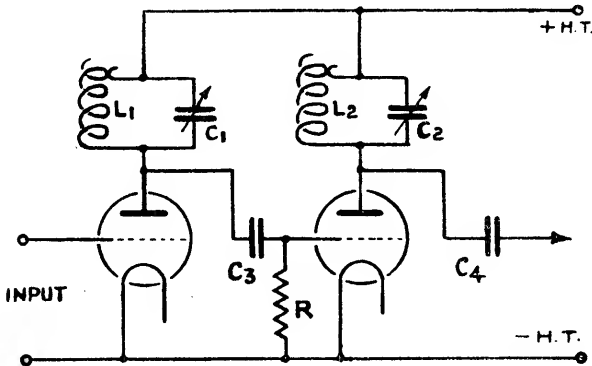


FIG. 326.

(2) At the same time, the circuit is free from the disadvantage attendant on pure resistance coupling as regards H.T. supply, because the steady component of anode current flows through the inductive arm of the parallel circuit, whose ohmic resistance is small, and hence there is no big IR drop to be compensated for by extra high-tension voltage.

(3) A **high measure of selectivity** is achieved.

For interfering frequencies, to which the circuit is definitely mistuned, the impedance of the circuit becomes, instead of a high resistance, a reactance (inductive or capacitive) of considerably less value. If the interfering frequency is less than the resonant frequency, the reactance is inductive; if greater, it is capacitive. The further the interfering frequency is from resonance, the less is the value of the reactance. A very high measure of amplification is thus obtained over a narrow band of frequencies in the vicinity of resonance.

With several stages of amplification of this type, the selectivity can be increased to a very great degree.

The **disadvantages** of this type of coupling are :—

(1) To ensure the marked selectivity mentioned above, various tuned circuits have to be adjusted accurately to the incoming

frequency, which means longer time spent in adjustment and difficulty in moving from one frequency to another.

(2) The tuned circuits involved in this type of amplifier make it very difficult to prevent the generation of self-oscillations.

This subject is treated further in para. 601.

Tuned-Choke Capacity Coupling used in note magnifiers is similar to that in radio frequency amplifiers, except that the values of inductance and capacity are much larger so that the circuit may be resonant to audio-frequencies. Coupling condensers are necessary, as before, their values being much higher for note magnifiers than for radio frequency amplifiers.

585. Examples on Choke and Tuned-Choke Coupling.

Example 61.

Find the V.A.F. of a choke amplifier wound with high resistance wire, $L = 7,500$ mics., $R = 20,000$ ohms.

The valve has $r_a = 16,000$ ohms and $m = 10$. The applied frequency is such that $\omega = 2 \times 10^6$.

With the above data, find also the R.M.S. value of I_a in micro-amperes, when a voltage of 0.1 volt (R.M.S.) is applied between grid and filament.

(a)

$$\text{V.A.F.} = m \frac{\sqrt{R^2 + (\omega L)^2}}{\sqrt{(R + r_a)^2 + (\omega L)^2}}$$

$$\omega L = \frac{7,500}{10^6} \times 2 \times 10^6 = 15,000 \text{ ohms.}$$

$$\begin{aligned} \therefore \text{V.A.F.} &= 10 \frac{\sqrt{(20,000)^2 + (15,000)^2}}{\sqrt{(36,000)^2 + (15,000)^2}} = 10 \frac{\sqrt{20^2 + 15^2}}{\sqrt{36^2 + 15^2}} \\ &= \frac{10 \times 25}{39} = \frac{250}{39} = 6.41 \end{aligned}$$

$$\begin{aligned} (b) I_a &= \frac{mV_g}{\sqrt{(R + r_a)^2 + (\omega L)^2}} = \frac{10 \times (0.1)}{\sqrt{(36,000)^2 + (15,000)^2}} \text{ amperes.} \\ &= \frac{10^6}{\sqrt{(36,000)^2 + (15,000)^2}} \mu\text{A} = \frac{10^6}{39,000} \mu\text{A} \\ &= 25.64 \mu\text{A} \end{aligned}$$

Example 62.

In a tuned-anode amplifier the coil has $L = 360$ mics., and its resistance is 6 ohms, C being 0.5 jar.

A signal voltage of 0.15 volt at resonant frequency is applied to the grid of the valve, which has $m = 8$ and $g_m = 0.4 \frac{\text{mA}}{\text{volt}}$.

Ten per cent. of the stepped-up voltage is lost in the coupling condenser to the next valve. What is the voltage available between grid and filament of the next valve?

$$\frac{L}{CR} = \frac{\frac{360}{10^6}}{0.5 \times \frac{9 \times 10^8}{9 \times 10^8}} \text{ ohms} = \frac{360 \times 9 \times 10^8}{3 \times 10^6} = 108,000 \text{ ohms.}$$

$$r_a = \frac{m}{g_m} = \frac{8}{0.4 \frac{mA}{\text{volt}}} = 20,000 \text{ ohms.}$$

$$V.A.F. = 8 \times \frac{108,000}{108,000 + 20,000} = \frac{8 \times 108}{128} = 6.75$$

\therefore voltage applied to the next valve is

$$\left(6.75 \times 0.15 \times \frac{9}{10}\right) \text{ volts} = 0.91 \text{ volt.}$$

The **selectivity** of the tuned anode amplifier can be illustrated numerically.

The resonant frequency of the circuit above is very nearly equal to $\frac{3 \times 10^4}{2\pi\sqrt{LC}}$ kc/s.

$$\begin{aligned} \text{Substituting for L and C, } f &= \frac{3 \times 10^4}{2\pi\sqrt{180}} \text{ kc/s.} \\ &= 356 \text{ kc/s.} \end{aligned}$$

Let us find the impedance of the circuit for incoming frequencies of 100 kc/s and 500 kc/s respectively.

In the former case, the frequency is so much below the resonant frequency that the impedance will be sensibly the reactance of the inductance alone.

$$\omega L = 2\pi \times 100 \times 1,000 \times \frac{360}{10^6} = 72\pi = 226 \text{ ohms.}$$

$$\frac{1}{\omega C} = \frac{9 \times 10^8}{2\pi \times 100 \times 1,000 + 0.5} = \frac{9,000}{\pi} = 2,845 \text{ ohms.}$$

The parallel combination (neglecting resistance) gives a make-up current $I = V \left(\frac{1}{\omega L} \sim \omega C \right)$ and therefore a reactance of $\frac{\omega L}{1 - \omega^2 LC}$.

$$\text{At 100 kc/s, this reactance} = \frac{226}{1 - \frac{226}{2,845}} = 245 \text{ ohms, practically}$$

that of the inductance alone.

This reactance is so low compared with the impedance of the valve that no amplification is secured.

In the same way, with a frequency considerably above that of resonance, the reactance of the parallel circuit is effectively that of the capacity.

For a frequency of 500 kc/s, $\frac{1}{\omega C} = 569$ ohms, again so low that no amplification is achieved.

586. Transformer Coupling and Tuned Transformer Coupling.

—Fig. 327 shows a type of intervalve coupling in which the primary of a transformer is included in the anode circuit of the first valve, and the voltage variations induced across the secondary are applied to the grid of the next valve.

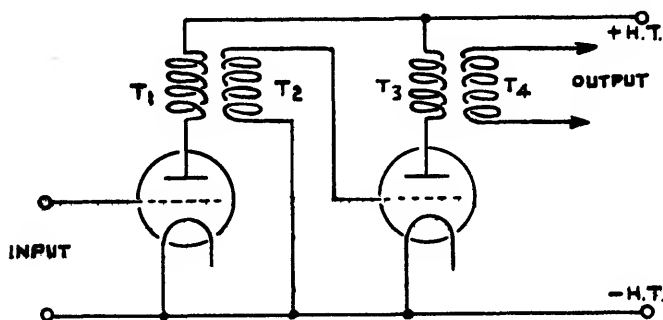


FIG. 327.

The essential difference between the transformer circuit and those previously dealt with is that no coupling and insulating condenser and grid leak are necessary, because there is no direct connection from the anode of one valve to the grid of the next valve.

If a condenser and leak are found in the secondary circuit of a transformer coupling, it is a definite indication that the next valve is a detector valve using cumulative grid detection.

Now, in the transformer, as in the case of choke coupling, there are self-capacities between the turns of the primary and also of the secondary, and similar results, as regards "peaky" amplification, may be expected.

The same steps can be taken either to eliminate the peak effect, or to utilise the exceptionally good amplification it gives by tuning either the primary or secondary inductances, or both, so that they form resonant circuits to the received frequency.

Thus, to avoid peak effect, resonance effects may be annulled by winding the transformers with high-resistance wire.

To utilise the peak effect, either

- (a) the primary or secondary windings, or both, may be provided with several tapping points, so that the inductance included in the circuit together with its self-capacity may form a roughly tuned circuit to the frequency being amplified; or

- (b) A variable condenser may be joined across either, or both, windings of the transformer.

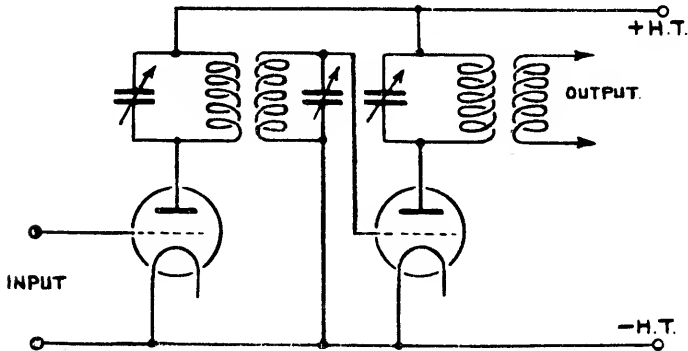


FIG. 328.

With several transformer couplings in the model, these modifications make tuning a slow process, unless it can be arranged for one handle to adjust the condensers, or the tapplings, simultaneously. Selectivity is, of course, greatly increased.

587. Transformer Step-up and Construction.—The transformers used in radio-frequency amplifiers and those used in note magnifiers differ in certain respects.

In **note magnifiers**, the transformers are wound on an iron core, which gives the result that the magnetic leakage is negligible, or the coupling is approximately perfect (100 per cent.).

In addition, the ratio of secondary to primary turns may be as much as 5 to 1, or 7 to 1, giving a bigger voltage variation across the secondary than the primary circuit, and hence a high voltage amplification factor.

In **radio-frequency amplifiers**, the transformers are wound on a former of insulating material with as little self-capacity between turns as possible. The usual iron core would be unsuitable, on account of the excessive eddy current and hysteresis losses which would occur at the high frequencies involved. This incurs the disadvantage that there is considerable leakage, and the coupling is very much looser than if iron were used.

In addition, it is found that very little advantage is gained by increasing the number of secondary turns above the number of primary turns, because of effects which arise through the capacities inherent in the coils and between the valve electrodes.

588. Transformer Coupling Theory.—The theory of the V.A.F. obtainable with transformer coupling is complicated, and only an introduction to it will be given here. It should further be noted that the type of amplification required is very different according

as it is intended for W/T or R/T signals. For W/T signals amplification at one particular frequency is desired but for R/T the essential necessity is uniform amplification over a band of frequencies and for this purpose advantage is largely taken, in the design, of the effects of magnetic leakage, and coil and interelectrode capacities. R/T reception is considered in Chapter XV. The remarks below should be taken as applying to W/T reception only.

In transformer theory, as we saw in Chapter VI, the performance depends very much on the nature of the load across the secondary circuit. In the case we are considering, the secondary is closed across the grid and filament of a valve. If the grid is separately biased so that no grid current flows, the secondary circuit is closed by the inter-electrode capacity; if grid current is flowing, the secondary circuit is closed by a resistance, which decreases in value as the grid of the valve is made more positive to the filament. It is generally assumed that, even with a large negative bias, the A.C. resistance between grid and filament, which we may call R_g , is never infinite, and has in fact a value of the order of one to ten megohms.

Let us take, first of all, the case of the **Audio-Frequency Transformer**, and assume an ideal case, in which the leakage resistance and reactance are zero, the coupling factor 100 per cent., the magnetising current zero, and the secondary load a pure non-inductive resistance, R_g , the grid to filament A.C. resistance of the next valve.

By ordinary transformer theory, such a resistance can be replaced by an equivalent resistance in the primary circuit, equal to the secondary load divided by the square of the transformation ratio.

The equivalent circuit is therefore as follows :—

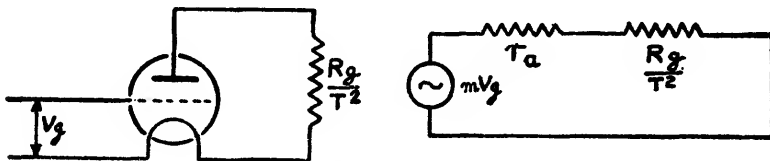


FIG. 329.

As in para. 574, the anode current variation is the same as if m times the input voltage were applied across a circuit consisting of r_a and $\frac{R_g}{T^2}$ in series.

$$\text{i.e., } I_a = \frac{mV_g}{r_a + \frac{R_g}{T^2}}$$

and the voltage variation across $\frac{R_p}{T^2}$, in the primary circuit, is

$$\frac{m \frac{R_p}{T^2} V_p}{r_s + \frac{R_p}{T^2}}$$

The voltage across the secondary is T times this, and is therefore

$$\frac{m \frac{R_p}{T} \cdot V_p}{r_s + \frac{R_p}{T^2}}$$

$$\text{The ratio } \frac{\text{output volts}}{\text{input volts}} = \frac{m \frac{R_p}{T}}{r_s + \frac{R_p}{T^2}}$$

If $\frac{R_p}{r_s}$ be written a , this is equivalent to $\frac{Tma}{T^2 + a}$.

The V.A.F. therefore varies directly as m , as might be expected, and also depends in a complex manner on T and a .

It increases with an increase in " a ," and hence it is desirable to make the grid-filament A.C. resistance of the next valve as high as possible by biasing the grid.

As regards the dependence of the V.A.F. on T , it is easy to prove that, with m and a constant, the above expression has a maximum value when $T = \sqrt{a}$, and the V.A.F. then $= \frac{mT}{2} = \frac{m\sqrt{a}}{2}$.

The values of R_p and r_s found in practice, and the resulting ratio a are such that greatest amplification occurs when T is not more than 5 or 6, and so furnish a theoretical reason for the statements of para. 587.

The practical audio-frequency transformer differs from the ideal case so far considered. We may still assume that the leakage reactance and resistance are negligible, but the magnetising current can only be small if the no-load reactance of the primary is very large. This means that the simple circuit assumed above should be modified by having the primary reactance in parallel with R_p/T^2 .

The effect of this is to diminish the V.A.F. below the value originally obtained, although the difference is not great if the reactance is large compared with R_p/T^2 . The only limit to this is imposed by the fact that a large primary requires an even larger secondary, and beyond a certain value the leakage losses and the self-capacities of the windings would result in a decrease in amplification.

General practical conclusions are as follows :—

R_p should be as large as possible ; the reactance of the primary should be about twice the A.C. resistance of the preceding valve ; and the transformer step should be, if possible, equal to the square root of the ratio R_p/r_a , but not so high as this if the effective amplification is being reduced by reason of self-capacities of the windings and consequent low impedance paths.

In the **Radio-Frequency Amplifier** with transformer coupling there is a considerable leakage because an iron core cannot be used, and hence it cannot be assumed that the voltage across the secondary is T times that across the primary.

If we assume, for purposes of simplification, that the resistance between grid and filament of the next valve is infinite, but take into account the inductance L_1 of the primary, the current variation I_a in the anode circuit is given by

$$I_a = \frac{mV_g}{\sqrt{r_a^2 + (\omega L_1)^2}}.$$

The voltage induced across the secondary circuit is $\omega M I_a$.

If we write $M = K\sqrt{L_1 L_2}$, where K is the coupling factor, the secondary voltage is given by

$$V_s' = \frac{\omega K \sqrt{L_1 L_2} m V_g}{\sqrt{r_a^2 + (\omega L_1)^2}},$$

and hence the V.A.F. = $\frac{m \omega K \sqrt{L_1 L_2}}{\sqrt{r_a^2 + (\omega L_1)^2}}$.

If all the quantities with the exception of L_1 are kept constant, and this expression is differentiated with respect to L_1 , the maximum value of the V.A.F. is found to occur when $\omega L_1 = r_a$.

Under these conditions, it is highly probable that the best value of L_1 will be such that, in conjunction with its self-capacity, it is resonant to a frequency within the range of the amplifier ; it will then act effectively as a high resistance, and will give exceptionally good amplification at this particular frequency.

The remedy to the above is found in making L_1 small, and adding a variable condenser in parallel with it, so that this parallel circuit may always be tuned to resonance.

We may then regard the effective resistance of the circuit as being so much greater than the A.C. resistance of the valve that the whole of the voltage mV_g is applied across it. The oscillatory current flowing in L_1 is then given by $\frac{mV_g}{\omega L_1}$ and the voltage across the secondary is given by ωM times the value of this current.

$$\therefore V_s' = \frac{\omega M m V_g}{\omega L_1} = \frac{m M V_g}{L_1} = \frac{m K \sqrt{L_1 L_2}}{L_1} V_g.$$

$$\text{The V.A.F.} = mK\sqrt{\frac{L_2}{L_1}}$$

To give a large V.A.F., L_1 should be made small; there are, however, limits to this, because, the smaller L_1 is made, the smaller becomes the effective resistance $\frac{L_1}{C_1 R}$, and hence r_a cannot be neglected by comparison, as was done above.

Alternatively, we might increase L_2 , but, by so doing, its distributed self-capacity might be such as practically to short-circuit the grid-filament of the next valve, and reduce the V.A.F. very much. The effect of this self-capacity has been neglected in the above discussion, but it is obviously of importance, especially at high frequencies.

As mentioned before, it has been found best to make $L_2 = L_1$ in radio-frequency transformer amplifiers and avoid trying to increase amplification by transformer step-up.

589. The Tuner and Amplifier Complete.—Fig. 330 is inserted as an illustration of a typical radio-frequency tuner and amplifier for low frequency, with three amplifying valves and a detector valve.

The tuner has two positions, "Stand-by" and "Tune." In the "Stand-by" position, which is used when searching for signals, the aerial tuning inductance is directly across the grid and filament of the first valve, and the secondary circuit $L_2 C_7$ is cut out. This position is thus unselective.

When the required signal has been found, the switch is made to the "Tune" position. This brings the secondary circuit into use and increases the selectivity.

The central stud of the Tune—Stand-by switch is earthed to prevent the switch from acting as a capacitive coupling between aerial and secondary circuits.

It will be seen that when the tuner is in the "Stand-by" position, the grid-filament capacity of the first valve is in parallel with L_1 . In the "Tune" position, this capacity is in parallel with the secondary tuning condenser C_7 . To preserve the aerial tuning, it is therefore necessary to insert, in parallel with L_1 , a condenser whose capacity is the same as the grid-filament capacity of the valve. This is called the Valve Equivalent Condenser (V.E.C.). A_1 and A_2 are lightning arresters to protect the receiver from damage. The aerial condenser has two positions, series and parallel, according to the frequency which is being received.

V_1 , V_2 and V_3 act as radio-frequency amplifiers, V_4 as the detector.

The interval coupling is by means of the transformers T_1 , T_2 , T_3 , whose secondaries are tuned by the condensers C_1 , C_2 and C_3 .

The grid potentials of the second and third valves can be made more or less positive by means of the potentiometer R_3 , which is connected across the filament battery terminals. So far the

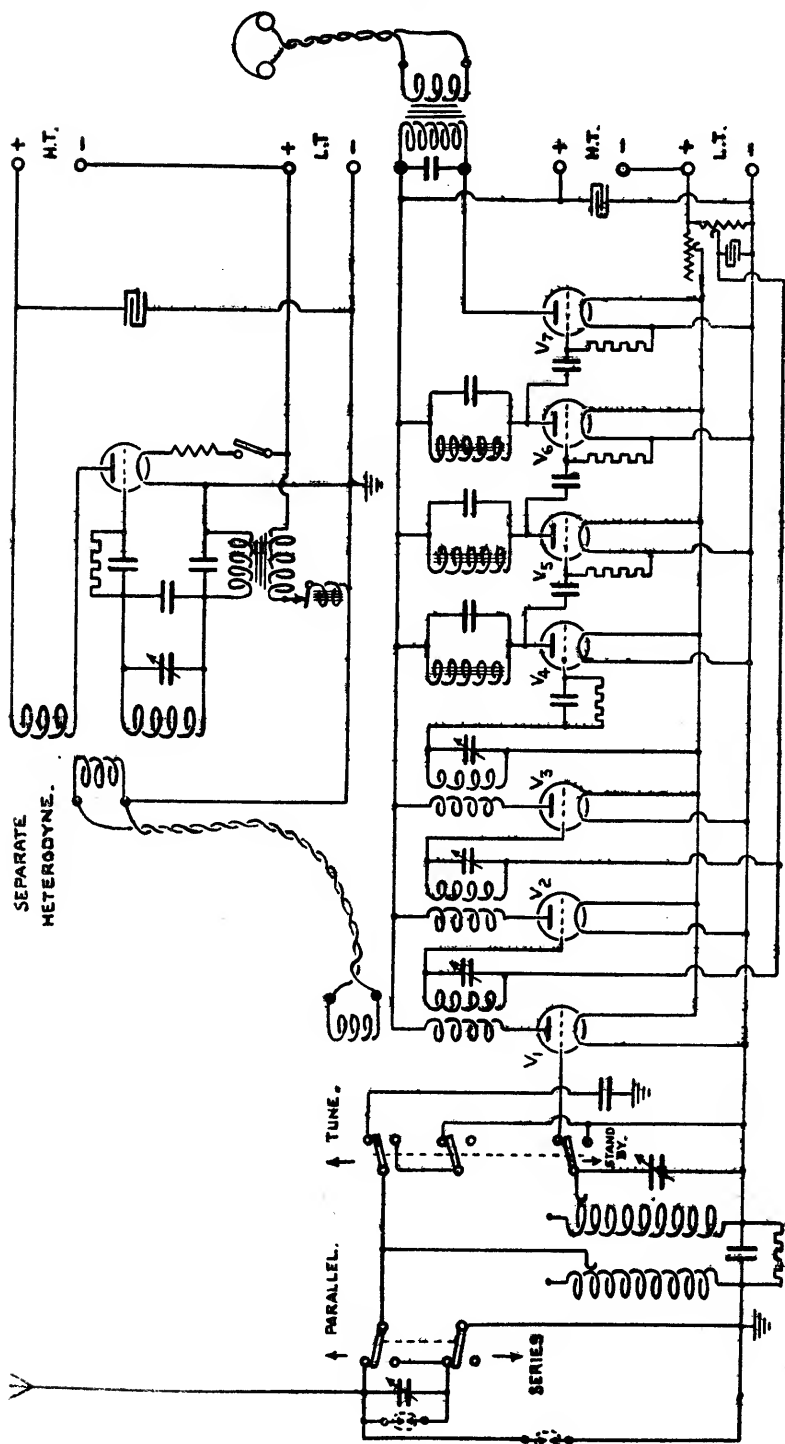


FIG. 332.

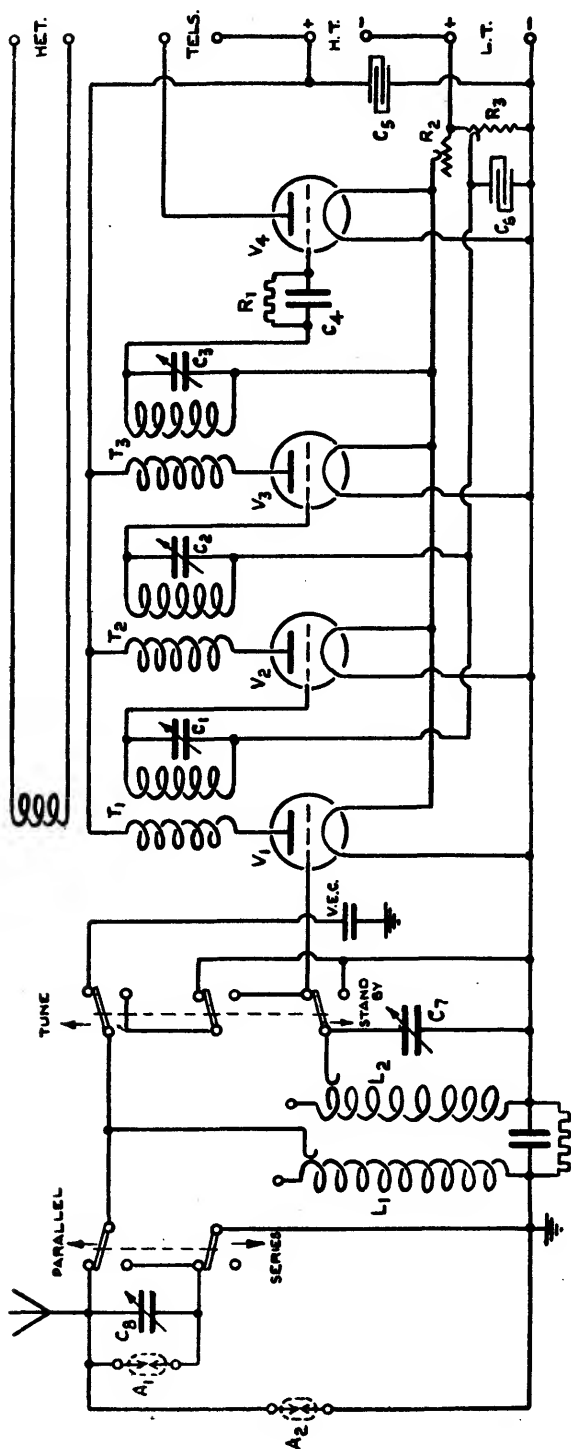


FIG. 330.

necessity of having the grid negative for distortionless amplification and efficiency has been emphasised, but we shall see in dealing with self-oscillation that the grid may be given positive bias, with consequent loss of efficiency, as a means of preventing self-oscillation.

The rheostat R_2 controls the filament current of all the valves in parallel.

In order to provide for cumulative grid detection, the condenser C_4 and leak R_1 are connected to the grid of valve V_4 , which is given positive bias by connecting its input circuit to the positive L.T. lead.

The condensers C_5 and C_6 (capacity $2\mu F$) are put in shunt with the amplifier batteries and the grid potentiometer respectively, to carry both radio and audio-frequency variations, and prevent self-oscillation, which might arise as a result of coupling between different parts of the amplifier through the resistances of the batteries.

590. The Note Magnifier.—A complete note magnifier would be constructed on the same general principles as the amplifier, with common H.T. and L.T. batteries to all the valves, and corresponding shunting condensers, &c.

In earlier Service receivers, note magnifiers are generally separate instruments; in commercial receiving sets, audio- and radio-frequency amplification are generally incorporated in the same model, and this is also the case in the later Service sets.

It is important to notice that, in separate note magnifiers, the first stage inside the box is intended to be the intervalve coupling between the detecting valve and the first valve of the note magnifier; as such, this coupling should be connected direct between the anode of the detecting valve and the positive of the anode battery. The introduction of a telephone transformer or condenser in the anode circuit of the detecting valve may reduce the efficiency of the note magnifier.

Service note magnifiers designed to share common batteries with other high frequency amplifiers have only one input terminal, which is connected to the anode of the detecting valve, and the circuit is completed through the common anode battery.

591. Power Amplification.—So far we have been concerned with output circuits designed to give as high a voltage amplification factor as possible, so that the maximum voltage may be applied to the next valve in the amplifier.

In the case of the last valve of a note magnifier, however, in the anode circuit of which is connected as output circuit the receiver (telephones or loud speaker), it is important to obtain maximum power in the output circuit as distinct from maximum voltage variation across it. This is necessary because considerable power expenditure is incurred in moving the diaphragm of the receiver. Hence we want current variation as well as voltage variation, and

the best condition is when the product of the oscillatory current in the anode circuit and the oscillatory voltage set up across the output circuit is a maximum. It is shown in the following paragraphs that this is the case, when the impedance in the external circuit is equal to the A.C. resistance (r_a) of the valve, provided maximum power dissipation alone is considered; if distortion of the power output is to be avoided, the external impedance should be about twice the A.C. resistance of the valve.

Valves used for power have, generally, A.C. resistances of from 1,000 to 5,000 ohms, and are designed to be used with higher anode voltages than usual. Their amplification factor is small (from 2 to 10), and their mutual characteristics are therefore straight over a large range of negative grid voltage, so that considerable negative grid bias may be applied and distortionless amplification be secured for large values of grid-filament voltage variation.

***592.** Let us suppose that the resistance of the output stage is given by R .

As before, $I_a = \frac{mV_g}{r_a + R}$, and the output voltage across R is $\frac{mR V_g}{r_a + R}$.

The product gives the output power $= \frac{Rm^2V_g^2}{(r_a + R)^2}$. We want R to be such that this is a maximum.

Hence, differentiating $\frac{R}{(R + r_a)^2}$ with respect to R , and equating to zero, we obtain:—

$$(R + r_a)^2 - 2R(R + r_a) = 0$$

or

$$R + r_a = 2R,$$

$$\therefore R = r_a.$$

Therefore, for maximum power output, the external resistance should equal the impedance of the valve.

The impedance of a receiver cannot, of course, be regarded as being purely a resistance, but the above result leads to the practical case, in which it can be arranged that the A.C. resistance of the last valve is about equal to the impedance of the receiving instrument.

***593. Maximum Power with Distortionless Amplification.**—In the above argument, no account is taken of the fact that when the grid swing extends to the limits of the dynamic characteristic, grid current will flow during part of the positive half-cycle of grid voltage and the wave form of the output will be distorted. For distortionless amplification, the optimum value of the external impedance is twice the A.C. resistance of the valve, as will now be proved.

Fig. 331 represents the working conditions. V_0 is the H.T. voltage on the anode under static conditions, and the corresponding static characteristic is shown. The curve labelled $V_a = V_1$ is the static characteristic for which flow of grid current commences at the same grid voltage as lower-bend curvature. V_1 is found, of course, by inspection of the family of mutual characteristics.

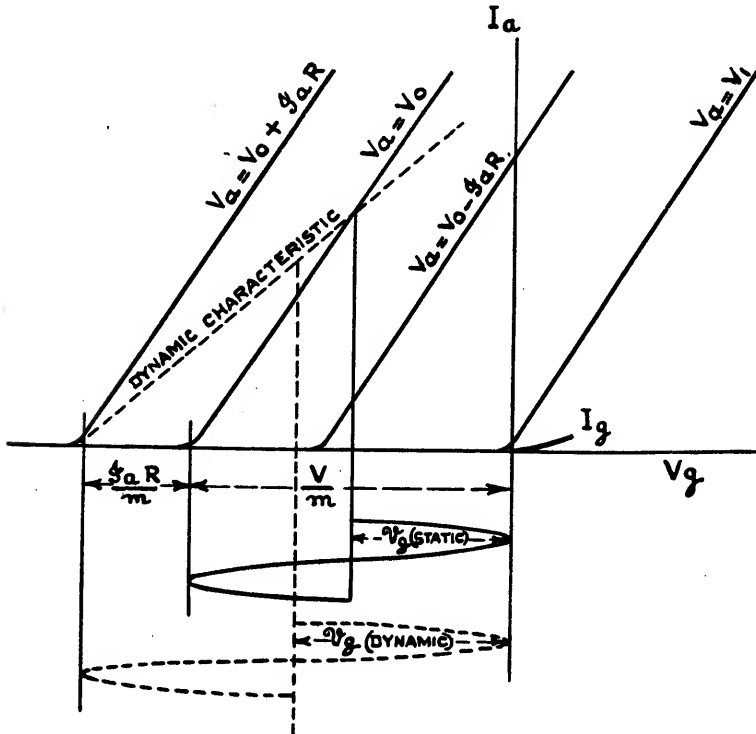


FIG. 331.

If $V_0 - V_1 = V$, the corresponding range of grid voltage over which no distortion occurs under static conditions is $\frac{V}{m}$, where m is the amplification factor of the valve.

If the external impedance has an effective resistance R , then, with an oscillatory current of amplitude J_a , the variation of anode voltage is from $V_0 - R J_a$ to $V_0 + R J_a$. The static characteristics corresponding to these voltages are shown and also the dynamic characteristic on which the valve is operating. It can easily be seen that compared with static conditions the range of grid voltage for distortionless working has been increased by an amount equal to $\frac{R J_a}{m}$.

The total permissible grid swing is therefore $\frac{V + Rj_s}{m}$. The optimum grid bias and the permissible peak grid voltage are both half of this, i.e., $\frac{V + Rj_s}{2m}$.

The slope of the dynamic characteristic is $\frac{m}{R + r_s}$,

$$\therefore j_s = \frac{m}{R + r_s} \eta_s^y = \frac{m(V + Rj_s)}{2m(R + r_s)} = \frac{V + Rj_s}{2(R + r_s)},$$

$$\therefore j_s = \frac{V}{2r_s + R}.$$

The power (P) dissipated in the external impedance is given by

$$P = \frac{Rj_s^2}{2} = \frac{RV^2}{2(2r_s + R)^2}.$$

As in the previous paragraph, the value of R for maximum power is obtained by differentiating this expression and equating the result to zero. This gives

$$(2r_s + R)^2 - 2R(2r_s + R) = 0,$$

$$\therefore 2r_s + R - 2R = 0,$$

$$\therefore R = 2r_s.$$

The expression for the best negative grid bias in terms of known quantities, and assuming that grid current starts to flow at zero grid volts, is given by

$$\frac{V + Rj_s}{2m} = \frac{V + \frac{RV}{2r_s + R}}{2m} = \frac{(r_s + R)V}{m(2r_s + R)}.$$

When $R = 2r_s$ this becomes $\frac{3V}{4m}$.

The modification required if grid current starts to flow at some other grid voltage is obvious.

594. The complete receiver, of which the radio-frequency stages were shown in Fig. 330, is illustrated in Fig. 332. There are three stages of note magnification, utilising iron-cored choke-capacity coupling. The output circuit of the last valve (the power stage) is the telephone transformer and its associated circuit. It will be observed that the telephone condenser is across the primary of the transformer.

595. Push-Pull Amplification.—The arrangement of valves in "push-pull" has many applications in wireless circuits. At present it will only be considered as an amplifier stage. Its other applications will be discussed as they arise.

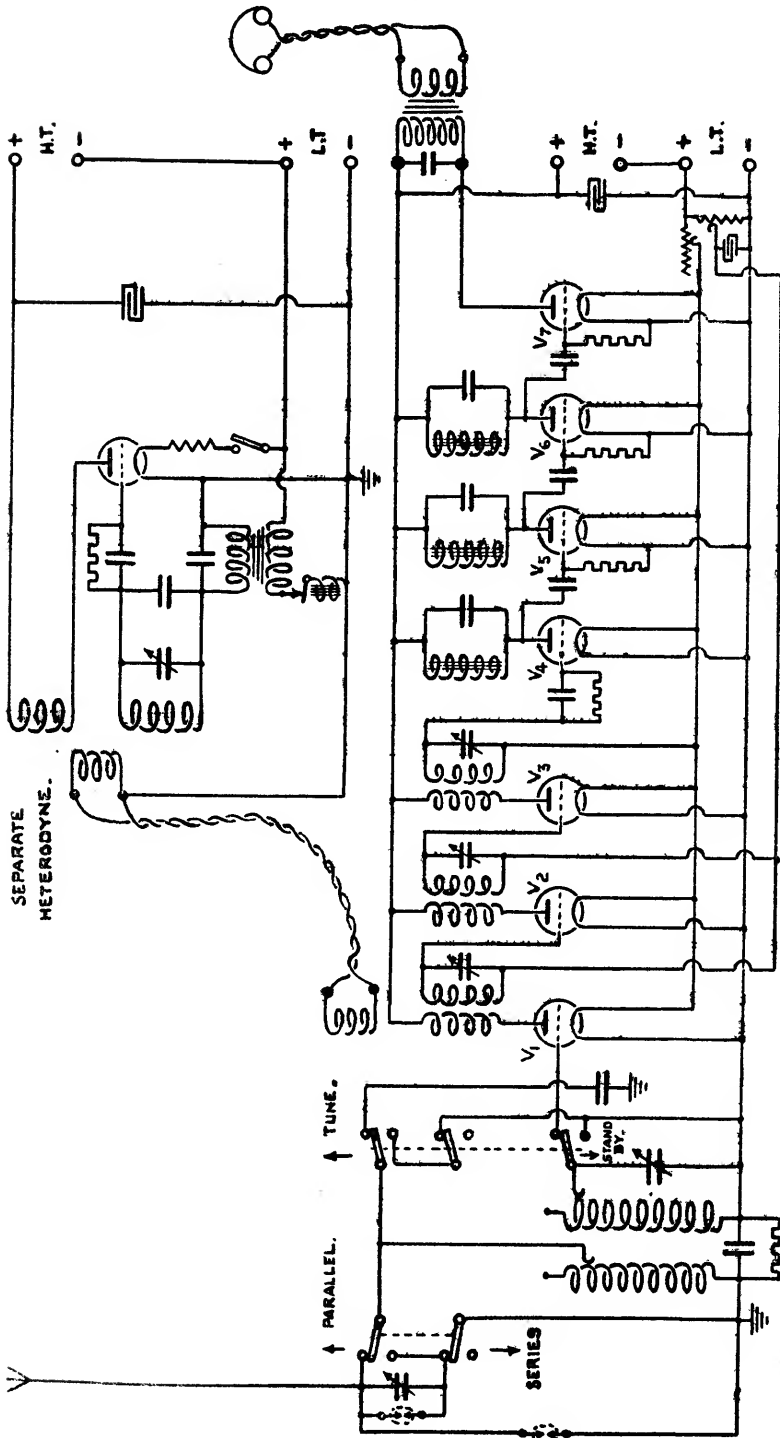
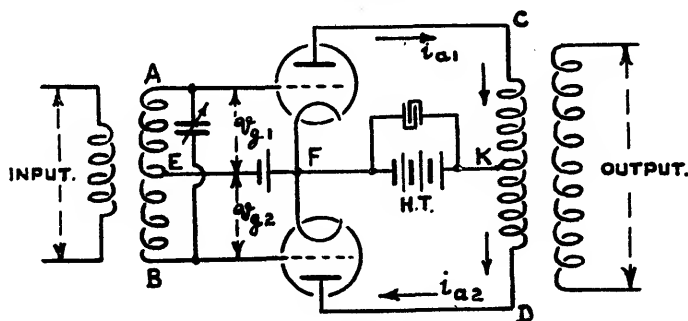


FIG. 332.

A simple push-pull circuit is shown in Fig. 333. The input circuit is, of course, the output circuit of the previous stage of amplification, and is shown as tuned-transformer coupled. The secondary coil AB, instead of being across grid and filament of one valve, as in cascade amplification, is connected across the grids of two valves, as shown. The electrical mid-point E of AB is common via the grid bias battery with the filaments of the two valves. The



Push-Pull Arrangement of Valves.

FIG. 333.

output circuit illustrated is of the same type as the input, and is connected across the anodes of the two valves. The H.T. battery is inserted between the electrical mid-point of the output primary and the common filament connection. It will thus be seen that the arrangement is a very symmetrical one, if the valves are matched so as to have the same characteristic curves.

An oscillatory voltage is produced across AB by the action of the incoming signal on the preceding stages of amplification. Thus with respect to E, whenever A is at a certain positive potential, B is at an equal negative potential and *vice versa*; i.e., the oscillatory P.D.s from A to E and from B to E are always equal and opposite. These are the grid-filament P.D.s applied to the two valves. Hence V_{g1} and V_{g2} are equal in amplitude, but 180° out of phase with each other.

Now consider the output circuit. When no signal is being received, a steady electron current is flowing from filament to anode of each valve, its value depending on the steady anode and grid voltages. The currents in the two valves are obviously equal, if the valves are perfectly matched and the H.T. battery positive tap is at the electrical mid-point of the output circuit. As regards the output coil, however, these two equal currents flow in opposite directions through the two halves of the coil. The steady fluxes they produce in the coil are thus equal and opposite, and there is no resultant flux and magnetisation of the core. The advantages of this in preventing distortion of the wave form will be evident from Chapter VI (para. 334).

When a signal is being received, equal grid voltages in antiphase are applied to the two valves. If i_{a1} and i_{a2} are the oscillatory anode currents produced in consequence, these two currents will also be equal and opposite, provided that the grid swings do not extend beyond the straight portions of the dynamic characteristics of the valves, and that grid current is not allowed to flow. The resulting oscillatory P.D.s. produced across the two halves of the output coil from C to K and D to K are therefore also equal and opposite (180° out of phase) and so the P.D.s. produced in the coil

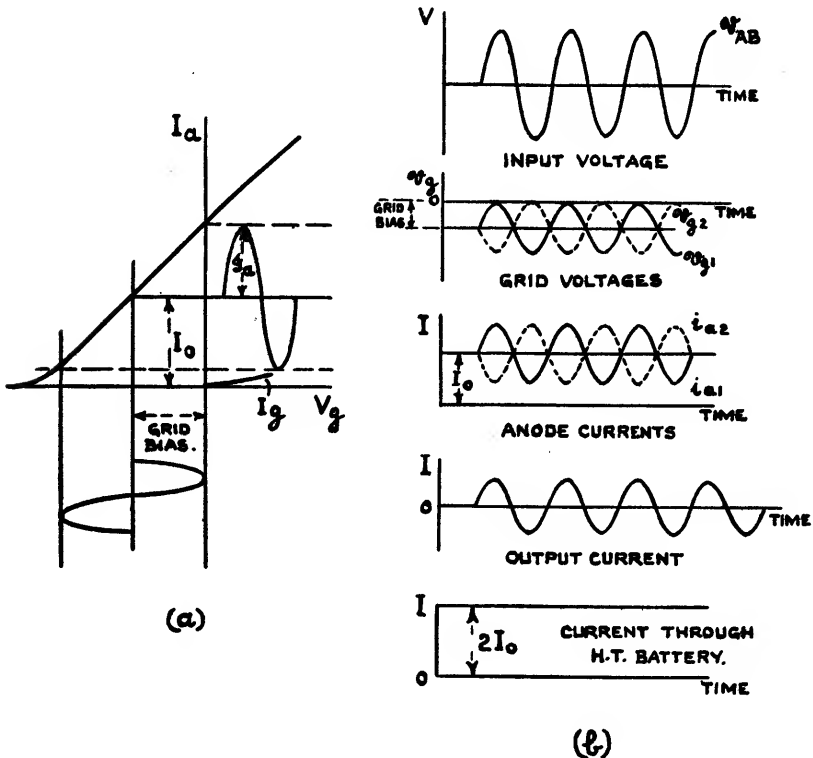


FIG. 334.

from C to K and K to D are equal and **in the same direction (in phase)**. In other words, the oscillatory current in the two halves of the coil is the same both as regards amplitude and phase, which is equivalent to one oscillatory current of the same value throughout the transformer winding.

On the other hand, i_{a1} and i_{a2} , flowing through the H.T. battery and common lead, are additive, and being equal and opposite cancel each other out, *i.e.*, no resultant oscillatory current flows in this lead, and the possibility of back coupling from this stage to earlier stages through the common H.T. battery and anode lead is avoided, with beneficial results on the stability of the amplifier.

In the case discussed above, the best grid bias for distortionless amplification is the voltage half-way between the lower bend of the dynamic characteristic and the voltage at which grid current starts to flow.

As only half the total input voltage is applied between grid and filament of each valve, the push-pull arrangement will give distortionless amplification of twice the oscillatory input voltage that either valve would deal with singly as a stage in a cascade amplifier, and so is particularly useful in the later stages of amplification where the input voltage is large.

596. The advantages for power amplification of a push-pull arrangement with the above grid bias may be seen by comparing it with two other systems which may be used for large power output in the last stage of a receiver :—

- (a) A super-power valve.
- (b) Two power valves in parallel.

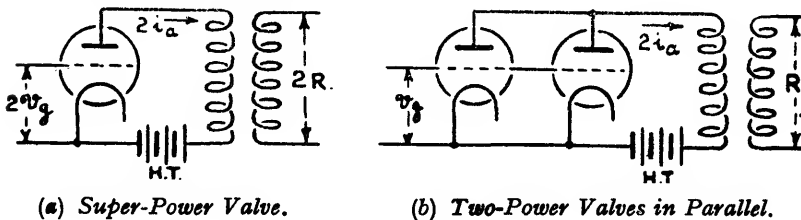


FIG. 335.

These two systems are shown in Fig. 335 (a) and (b), which should be compared with the push-pull arrangement shown in Fig. 333. For this reason the output circuits are drawn as shown, and not in the standard method used in other figures.

The super-power valve is a valve which we may assume in this case to be designed to deal with twice the permissible grid swing of an ordinary power valve without distortion from characteristic curvature or flow of grid current, but having the same constants. It will require about twice the H.T. voltage and filament emission of an ordinary power valve. One of its mutual characteristics, compared with that of the ordinary valves used in the parallel and push-pull arrangements, is shown in Fig. 336.

The output circuit will be arranged in each case to give maximum power under the given conditions. The internal A.C. resistance of the two valves in parallel is half the A.C. resistance of either valve, and so the effective external resistance $\left(\frac{\text{power}}{(\text{current})^2} \right)$ in the anode circuit is half that in the super-power valve circuit. They are labelled R and $2R$ respectively in Fig. 335. Further, the two valves in parallel can only be allowed the same grid swing (V_g) without

distortion, as either valve singly, *i.e.*, half the permissible grid swing of the super-power valve ($2 V_g$).

In the push-pull arrangement, the A.C. circuit is through the two valves in series and the external anode circuit. The total internal A.C. resistance is thus twice that of either valve alone, and so the external effective resistance should be four times that of the parallel combination, or twice that of the super-power valve, *i.e.*, $4R$. The push-pull arrangement can cope without distortion with the same total input voltage as the super-power valve ($2 V_g$).

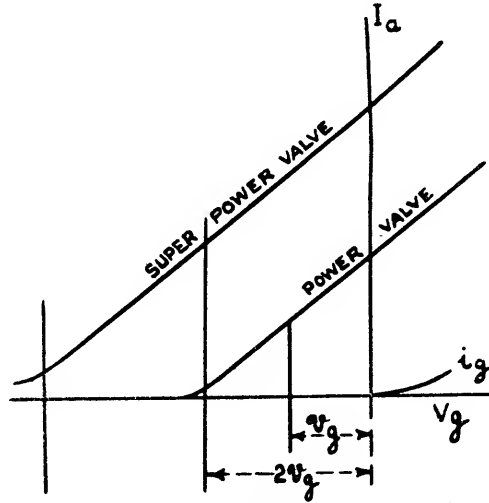


FIG. 336.

Under the above operating conditions, we may consider that the dynamic characteristics in the three cases, like the static ones, have the same slope (g_m'); thus we may say that for an oscillatory input voltage of R.M.S. value V_g , the R.M.S. oscillatory anode current produced is I_a in each valve ($= g_m' V_g$).

- (a) **Super-Power Valve.**—The input voltage (R.M.S.) is $2V_g$, and so the anode current is $2I_a$.

Hence the power obtained is $2R \times (2I_a)^2 = 8RI_a^2$.

- (b) **Two Valves in Parallel.**—The input voltage is V_g . The anode current is I_a from each valve, *i.e.*, a current of R.M.S. value $2I_a$ flows in the external circuit.

Hence the power obtained is $R \times (2I_a)^2 = 4RI_a^2$.

- (c) **Two Valves in Push-Pull.**—The input voltage to each valve is V_g . It will be readily seen from para. 595 that the current in the external circuit is I_a in this case, produced respectively in the two halves of the transformer winding by the two valves.

The power obtained is thus $4R \times (I_a)^2 = 4RI_a^2$.

597. This super-power valve would thus give in the external circuit under best conditions twice as much power for the same input voltage as the push-pull arrangement, without distortion as far as the valve characteristics are concerned. It suffers, however, from the following grave disadvantages :—

- (a) A large steady current flows in the output primary coil, or choke. With a winding similar to that which could be used in the push-pull arrangement, the core would probably be saturated. In any case, the effective inductance of the coil would be greatly reduced, and in addition, the grid voltage would have to be much smaller to avoid distortion (para. 384).

The assumption above that the same grid voltage could be applied in both cases would thus be unjustified, and the power obtained without distortion might be less than in the push-pull arrangement. Alternatively, if the coil is wound so as to avoid saturation, it becomes unduly bulky, and further, in the case of transformer coupling, the coupling coefficient K between primary and secondary will be lowered, thus producing a decrease in the power transferred to the secondary load.

- (b) The oscillatory current flows through the H.T. battery, giving rise to back-coupling and instability.
- (c) It requires much higher H.T. voltage than the push-pull arrangement.

Two valves in parallel, giving the same power output as the same valves in push-pull, also suffer from the disadvantages (a) and (b) above, i.e., difficulty of designing an efficient output circuit and back coupling from the large oscillatory current flowing in the H.T. battery. They require only the same H.T. as the push-pull valves. It would seem a point in favour of this arrangement that the same distortionless power output is obtained for half the input voltage, but in practice this is unimportant. Any ordinary receiver, in which these arrangements would be considered, could easily overload the output stage.

The above discussion of various types of power stage should also help to emphasise the importance of having the impedance of the output circuit in the correct ratio to the A.C. resistance of the valve system. This is not always easy to arrange, but the process is considerably simplified if transformer coupling to the output stage is used. The effective output impedance then depends on the transformation ratio, thus introducing another adjustable factor.

598. Curvature Biasing.—The input voltage to a push-pull arrangement can be approximately doubled, without distorting the output, by increasing the grid bias until each valve is operating on the mid-point of the bottom bend of its mutual characteristic. Either valve singly would then act as a rectifier, for during the

negative half-cycle of grid voltage practically no current would flow. But in the push-pull arrangement, the negative half-cycle on one grid is the positive half-cycle on the other, since the grid voltages are in antiphase. The result is that during a complete cycle of input voltage, one valve amplifies the positive half-cycle without distortion, and the other valve does the same for the negative half-cycle. This action is illustrated in Fig. 337. The two valves must have identical characteristics if distortion is to be prevented. The characteristics of the two valves are shown in opposite directions to take account of the fact that positive oscillatory grid voltages on one valve correspond to negative ones on the other. It will be

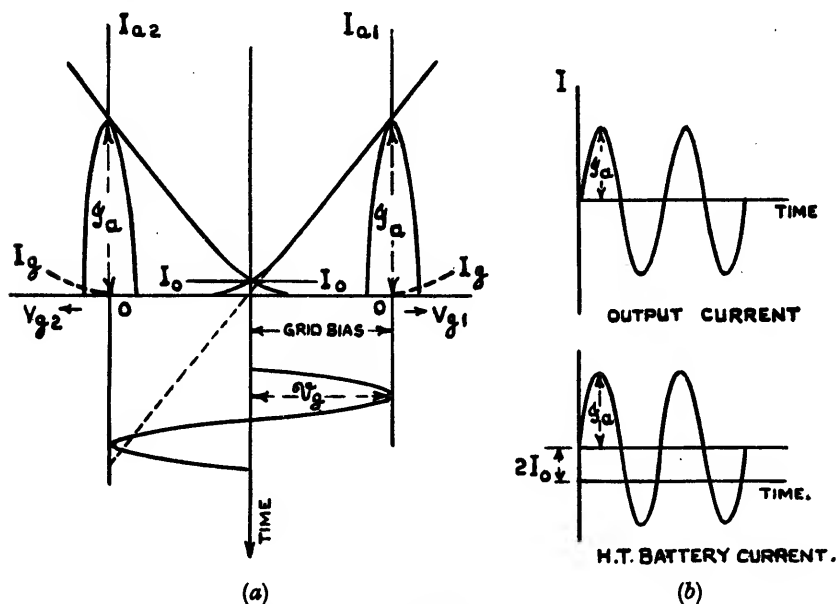


FIG. 337.

seen from the figure that the curvatures at the bottom of each characteristic cancel each other out as regards incoming signals, and the net effect is to give a straight characteristic with zero flow of grid current over more than twice the range of either valve independently. This is indicated by the dotted prolongation of the right-hand characteristic.

This was the original method employed in push-pull circuits, but it suffers from the following disadvantages compared with the "mid-point biasing" scheme:—

- (a) During the negative half-cycle for either valve, its A.C. resistance is extremely large. The positive half-wave of oscillatory anode current produced in one valve thus flows through half the external circuit, and then returns

to filament via the H.T. battery. During the next half-cycle the same process occurs with the other valve (Fig. 337 (b)). Thus, although the steady current through the H.T. battery is smaller with this method, the oscillatory current through it is large, and the advantages possessed by the push-pull arrangement for preventing back coupling through the H.T. battery are sacrificed.

- (b) The grid bias is critical, and demands a careful study of the characteristics to find the correct operating points. In addition, the valves must be much more carefully matched than in the mid-point biasing system, particularly as regards the bottom bends of their characteristics, and this is the point where deviations from uniformity in the same type of valve are most pronounced. Further, with mid-point biasing, a difference in the slope of the mutual characteristics of the two valves will not produce distortion. The alternating flux-linkage with the output coil is proportional to the product of the number of turns and the alternating current. Though these are different for the two halves of the coil, each half still produces flux-linkage whose wave form is a faithful reproduction of the input voltage wave over a complete cycle, and the only effect is that the amplitude of resulting flux-linkage is the mean of the flux-linkages produced in the two halves of the coil, and not exactly equal to either. Different slopes, however, will mean that the oscillatory currents flowing through the battery from the two halves of the coil are not equal in amplitude, so that there will be a resultant oscillatory current through the battery, with possibilities of back coupling. Hence only slight differences in characteristic slope should be permitted and the H.T. battery should always be by-passed by a condenser.

With bottom bend biasing it is easily seen that different slopes of the characteristics will mean that the amplitude of the positive half-cycle of flux-linkage is different from the amplitude of the negative half-cycle, and so distortion is produced.

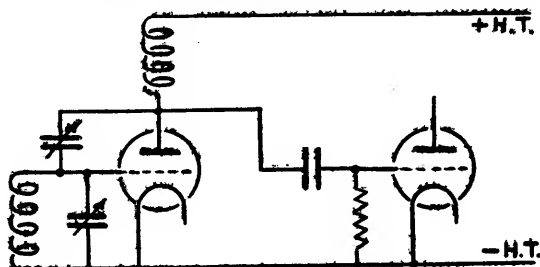
- (c) Although the grid swing is doubled, the power output is not proportionally increased—in fact, it is probably about the same in both cases. This is due to the fact that each half-cycle of current flows only in one half of the output winding. The flux produced is thus the same as when a current of half the amplitude flows in the whole winding, which is the case with mid-point biasing.

590. Use of Controllable Reaction in Amplifiers.—In all amplifiers, especially radio-frequency amplifiers, reaction, or back coupling, is present to a certain extent, either intentionally or unintentionally.

In dealing with regenerative amplification in the last chapter, we had an illustration of the intentional use of inductive reaction to diminish the damping of the input circuit of a receiver, and hence to increase the strength of signal heard in the telephones.

Various other circuit arrangements can be used to give effectively the same result.

Instead of an inductance, a capacity may be used to give reaction. A circuit which illustrates this method is given in Fig. 338.



Capacitive Reaction.

FIG. 338.

The radio frequency anode current through the first valve has, in this case, an alternative path back to filament through the variable condenser and grid-filament tuned circuit. The proportion of current which takes this path depends on the reactance of the condenser, and so is controllable. The energy fed into the grid circuit in this way may either assist or oppose the original input current according to the phase of the feed-back current. A full discussion of this point is complicated, and the result only can be given here. For practical purposes it may be taken that **unless the output circuit has a capacitive reactance, the energy feed-back is such as to assist the original oscillation in the input circuit.** With a capacitive output, however, the feed-back damps the original oscillation.

The importance of this type of reaction is very great, as will be seen below when unintentional reaction is discussed.

Both these methods are similar in that they are instances of reaction being deliberately applied so as to reduce the damping on the input side of the amplifier, and hence increase the amplitude of the oscillations.

Theoretically, of course, the damping may be reduced to zero, and hence the amplitude of the oscillations increased to infinity; but in practice it is necessary to leave a margin of stability, so that the amplifier may not be too near the border line at which it generates self-oscillations.

It should also be noticed that the decrease in damping of the input circuit increases its selectivity.

Controllable reaction will be further discussed in Chapter XVI.

600. Self-Oscillation in Amplifiers.—Especially at high frequencies and with multi-valve amplifiers, it is more usually the case that sufficient reaction, inductive, capacitive or resistive, is inherent in the amplifier to give a tendency to self-oscillation without the use of intentional reaction.

Reaction coupling of these various types enables a portion of the oscillatory energy in circuits near the output side of the amplifier, where this oscillatory energy assumes large dimensions, to be handed back to circuits near the input side of the amplifier, tending to nullify their positive resistance.

Inductive coupling may occur if inductances used in the amplifier are not carefully screened from one another.

Capacitive Coupling may occur by means of stray capacities between leads.

A much more important reason for the occurrence of capacitive coupling between circuits is, however, the presence of capacity between the different electrodes of the valves used in the amplifier.

The most important inter-electrode capacity, from this point of view, is that between grid and anode, usually denoted by C_{ga} .

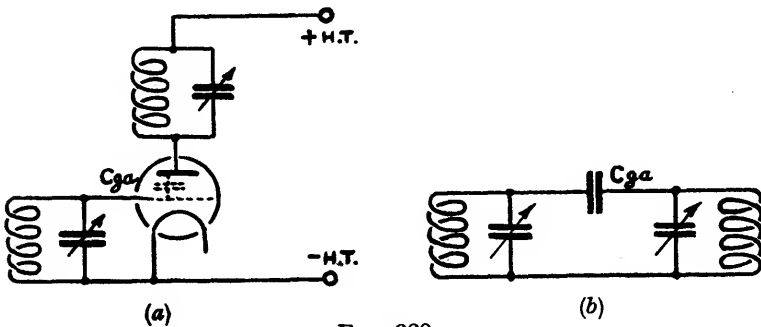


FIG. 339.

If we take the simple case of a tuned circuit between grid and filament and a tuned anode output circuit, as illustrated in Fig. 339, it is easy to see that they are coupled together by virtue of C_{ga} , and therefore energy can be fed from the tuned anode circuit back to the input circuit. In fact, this type of coupling, in this case undesirable, is exactly the same as the deliberate capacitive coupling illustrated in Fig. 338, and if the output circuit is suitable, as is generally the case in amplifier stages, energy is fed back to the input circuit.

The greater the value of C_{ga} , the higher is the coefficient of coupling between the circuits, and the greater the amount of energy fed back.

Part of this inter-electrode capacity is, of course, between the lead-in wires to the electrodes, and this can be arranged to be small by keeping the wires as far away from each other as possible.

Resistive Coupling may occur by circuits being coupled through the resistances of the batteries and common leads to these. Oscillations in the anode circuit of one valve cause high frequency voltage variations across these resistances, which are transferred to the anode circuits of other valves fed from the same source.

These various types of accidental reaction coupling may lead to self-oscillation where self-oscillation is not wanted, *e.g.*, when spark, I.C.W., or R/T are being received, since these are already modulated at the transmitter.

If the amplifier is designed to receive C.W. as well, it is necessary to have some arrangement by which a local oscillation is produced, either by separate heterodyne or by sufficient reaction being available and **controllable**, so that the amplifier may act as an autodyne when necessary.

It is the problem of unwanted oscillation which sets a definite limit to the amplification obtainable with any model, the difficulty of preventing it increasing with the number of valves used, the V.A.F. which results, and the frequency which is being amplified.

The fact that, with the usual output circuits, the tendency to self-oscillation increases with the number of valves, and the amplification produced, can easily be shown as an extension of the case of capacitive coupling by anode-grid capacity illustrated in the last paragraph.

If we take the case of a multivalve amplifier instead of a single valve amplifier, the output circuit of each valve being tuned, the following figure represents a simplified arrangement of the circuits and the capacities which couple them together.

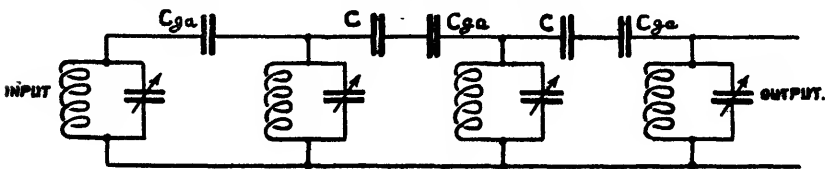


FIG. 340.

The coupling capacities are the inter-electrode capacities C_{ga} and the insulating condensers in the grid circuits of the valves.

Now with several valves in cascade, as in this case, the last output circuit is coupled to the input circuit of the first valve by a capacity which varies inversely as the number of valves.

Neglecting the insulating condensers C , which are large compared with C_{ga} , and therefore do not materially affect the equivalent capacity between each pair of circuits, we have in the above figure a number of capacities C_{ga} in series.

If there are n stages of amplification, the equivalent value of all of these is $\frac{C_{eq}}{n}$, which, as stated above, varies inversely as n .

But the amplification increases in geometric progression with the number of stages, and so the amplitude of oscillatory energy in the last tuned circuit increases correspondingly.

The amount of energy feed-back is proportional to the energy in the output circuit multiplied by the value of the coupling capacity, and as the output energy increases much faster than the coupling capacity decreases, their product must also increase when the number of steps is increased.

Hence any amplifier will generate self-oscillations if the number of stages is increased sufficiently, because the total energy fed back is always increasing, and must finally be greater than the damping losses in the input circuit.

601. Prevention of Self-oscillation in Amplifiers.—Various methods are adopted for dealing with the tendency of amplifiers to oscillate, and these will now be described in detail. They may be classified as follows :—

(1) Simple precautionary measures, designed to prevent inductive and capacitive coupling between different portions of the circuit.

Such are :—

Use of short leads, well spaced apart, crossing at right angles.

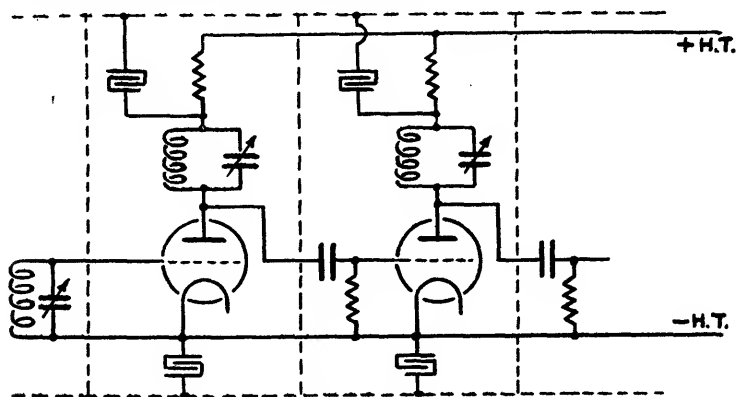
Separation of the stages of the amplifier from each other by earthed metal screens. This leads to a loss of energy, through eddy currents induced in the screens.

(2) By-pass condensers across batteries and potentiometers to avoid resistive coupling.

Large condensers shunted across batteries or potentiometers present to R.F. currents only a small reactance. Without these, the resistance of the batteries or potentiometers would cause considerable oscillatory voltage variations across them. With common H.T. supply, this P.D. would be a source of energy to every stage, whilst in the case of a potentiometer every valve whose grid was connected to it would be affected. In the case of batteries, the effect would be increased as they were used up, owing to their increasing internal resistance.

With efficient amplifier stages, it is found that this precaution is not enough to prevent feed-back, since the coupling due to the common lead from the H.T. battery to the anodes of the various valves, and to the common filament leads, is sufficient to produce self-oscillation. It is necessary in modern amplifiers to conduct the radio frequency anode current by the shortest possible path to filament once it has passed through the output impedance. This is achieved by the use of de-coupling condensers and resistances, as

illustrated in Fig. 341. The oscillatory anode current, after passing through the tuned anode output circuit, has two paths in parallel open to it, one through the de-coupling condenser and the other through the resistance in the anode lead. The reactance of the condenser is much less than the resistance, and so the anode current is almost completely by-passed to the screen, to which each filament is also connected by a large condenser.



Decoupling.

FIG. 341.

(3) Reduction of filament current or H.T. voltage.

These reduce the V.A.F. of any stage by reducing the amplification factor of the valve, and so the energy fed back is also diminished.

(4) Reaction applied deliberately "the wrong way."

A reaction coil or condenser may be connected so as to prevent the generation of oscillations instead of assisting it. This will be the case if an ordinary reaction coil has its connections reversed.

In some amplifiers the reaction coil or condenser can be connected to various anodes in the amplifier. It can be shown that if, with the connection made to the odd anodes, oscillations tend to be maintained, the joining up of the connection to an even-numbered anode will tend to stop oscillations.

This arises from the fact that in any one valve of the amplifier, the oscillatory grid and anode voltages are more than 90° out of phase, *e.g.*, with resistance capacity coupling they are 180° out of phase, since the anode current and grid voltage are in phase while the anode current and anode voltage are 180° out of phase. The anode of one valve is connected to the grid of the next through the coupling condenser. It follows therefore that the grid voltages of two succeeding valves in an amplifier are more than 90° out of phase, and similarly for the anode voltages.

Though this always holds for two successive valves in an amplifier, caution should be exercised in applying it, for instance,

to the first and last stages of an amplifier. Due to various reasons, the anti-phase relationship is not strictly preserved. It can easily be seen, for example, that in a choke capacity-coupled amplifier, where even in successive valves corresponding voltages are not in exact anti-phase, the rule given above for applying "positive" or "negative" reaction is not justified.

Negative reaction obviously reduces amplification.

(5) Increase of resistance in the input circuit.

One method of doing this is simply to increase the ohmic resistance.

This obviously acts as a deterrent to oscillation, making the amount of energy fed back less capable of overcoming the damping losses.

Another method is to bias the grid positively so that grid current flows.

This is equivalent to saying that the A.C. resistance between grid and filament is decreased from its theoretically infinite value (when the grid is negative) to some finite value, and the condenser of the input circuit has therefore a finite resistance connected across it. By the same theory as that used in Chapter VII, such a resistance is equivalent to a series resistance in the oscillatory circuit itself, which is greater the less the value of the shunt resistance. Hence the total effective resistance of the input circuit is increased, and the energy fed back is less capable of overcoming the damping losses.

It should be noted that if the positive grid bias is so large that it causes saturation grid current to flow, the grid-filament A.C. resistance again becomes infinite, and produces no damping of the input circuit.

In practice the grid is biased positively by taking the filament connection from the input circuit to the sliding contact of a potentiometer across the filament battery (*cf.* para. 589 and Fig. 330).

It is usually only necessary to maintain the first grid at a positive potential.

A certain amount of rectification effect and consequent distortion of the signal will occur.

Obviously, as with previous methods, the increased damping of the input circuit results in a loss in amplification.

(6) Flattening the input and output circuits.

It is found that flattening either of these circuits increases the stability of an amplifier.

The V.A.F. is, of course, decreased if the output circuit is flattened ($\frac{L}{CR}$ becomes less), and, while flattening the grid circuit does not affect the V.A.F., it does mean a diminution in input voltage across the condenser of the tuned input circuit for a given voltage

induced into it from the aerial, and therefore results in less total amplification from aerial to output.

602. Methods (1) and (2) deal with specific causes of reaction ; the others make it more difficult for reaction present in the amplifier to produce undesired effects.

The above methods give a reduction of amplification as a necessary accompaniment to greater stability.

The following measures, which need not reduce the amplification, are therefore more efficient.

(7) **Neutralising Circuits.**—Neutralising consists in the insertion of a condenser or condensers to produce an equal and opposite reaction effect to that inherent in the amplifier by reason of the inter-electrode capacity between grid and anode.

There are many ways of joining up such neutralising condensers. Two typical circuits are illustrated below.

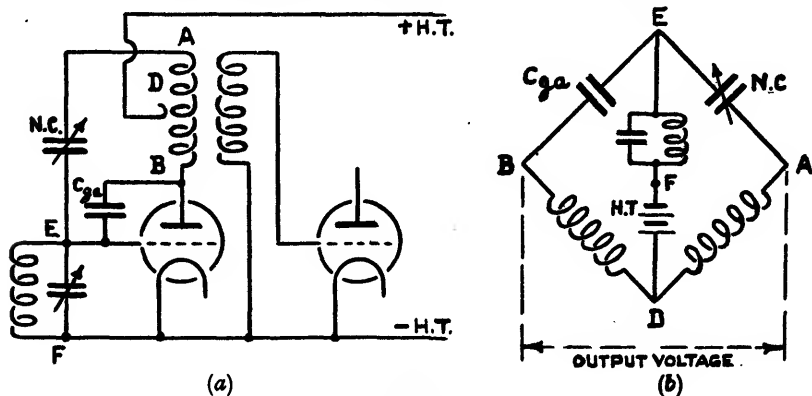


FIG. 342.

In Fig. 342 (a) the H.T. is fed to the valve at the mid-point of the inductance in the anode circuit, which may be a choke coupling, or the inductance arm of a tuned-anode coupling, or the primary of a transformer.

From the other end of the inductance a neutralising condenser (N.C.) is joined up as shown.

The energy conveyed through C_{ga} and N.C. from the output circuit is fed into the grid circuit from opposite ends of the output coil, the mid-point of which is at approximately the same oscillatory potential as the filament. The P.D.s across the two halves of the output coil are therefore 180° out of phase as far as the input circuit is concerned, and so the supplies of oscillatory energy through the two coupling condensers C_{ga} and N.C. are in opposite phases, and tend to neutralise each other. Neutralisation of the inter-electrode capacity feed-back is therefore obtained by adjusting the variable neutralising condenser until the feed-back through it is exactly equal to that through C_{ga} .

The circuit can be redrawn as an A.C. bridge system. The theory of balancing such a bridge for direct currents was explained in para. 72. The same condition holds for impedances in the A.C. case as for resistances in the D.C. case.

The neutralising circuit in this form is shown in Fig. 342 (b). As regards feed-back, the source of energy is the output circuit, which therefore corresponds to the battery in the D.C. case. It will be seen that, if the bridge is balanced, no current due to the output voltage can flow through the input circuit. The operation of neutralising thus consists of adjusting the variable condenser N.C. until this condition is satisfied. In the circuit as drawn, this would be the case if the capacity of N.C. were equal to C_{ga} , since D is supposed to be at the electrical mid-point of AB. In practice, owing to the mutual flux-linkage between the two halves of AB and various stray capacities, it is impossible to arrange the tapping point D so as to fulfil accurately the above condition.

The chief practical result is that the correct setting of the neutralising condenser varies with the frequency received.

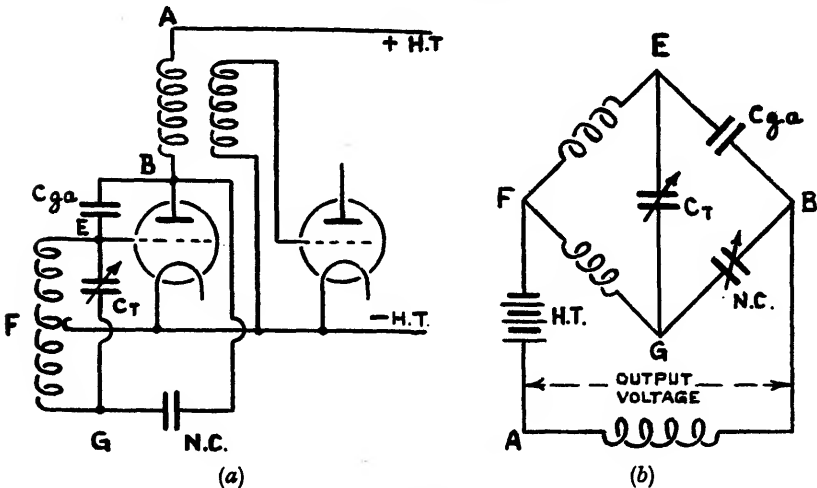


FIG. 343.

Another common neutralising circuit and its equivalent bridge are shown in Fig. 343. In this case the centre of the input coil is tapped to filament, and so only half the input voltage is applied between grid and filament. The behaviour of this circuit will be obvious from the figure and the explanation of the previous neutralising circuit. The capacity of N.C. is adjusted until the currents flowing in the bridge arms due to the output voltage produce the same oscillatory P.D.s across N.C. and C_{ga} . No energy can then be fed into the input circuit.

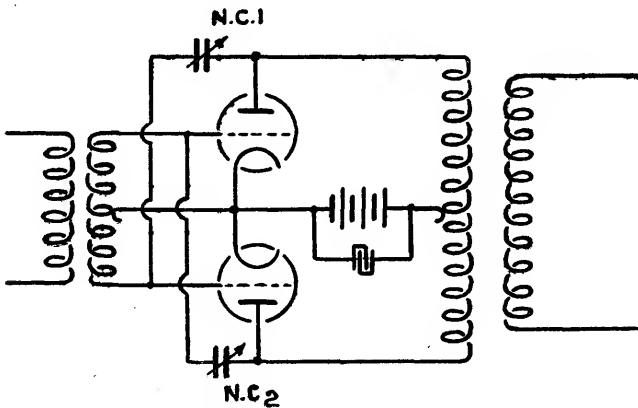
It may be remarked that the variable condenser N.C. in the above circuits, instead of being adjusted strictly for neutralisation, may be

used to control the amount of energy fed back through C_{gs} , i.e., the amount of energy fed back through N.C. may be less than the regenerative feed-back through C_{gs} , provided the former is large enough to prevent self-oscillation. In this case N.C. may be regarded as a negative reaction capacitive coupling.

If more energy is fed back through N.C. than through C_{gs} , the damping of the input circuit will actually be increased beyond that due to its own losses. This will tend to occur as the capacity of N.C. increases above C_{gs} .

It will be seen, however, that N.C. (and half the input inductance in series with it) is in parallel across anode and filament with the output circuit. Considerable increase of N.C. may thus eventually result in this total output circuit having a nett capacitive reactance. With a capacitive output circuit, the feed-back through C_{gs} damps the input circuit, while that through N.C. is regenerative, and there is therefore a possibility of self-oscillation.

A neutralising condenser is never likely to be large enough for this to happen, but damping of the input circuit by feed-back through C_{gs} often occurs in detector valves. The output circuit is designed, for example, to have an inductive reactance at audio frequencies, but its self-capacity may be such that at radio frequencies the nett reactance of the coil is capacitive. Thus the feed-back of radio frequency energy to the input through C_{gs} is in such a direction as to damp the input oscillation and reduce the efficiency of the receiver.



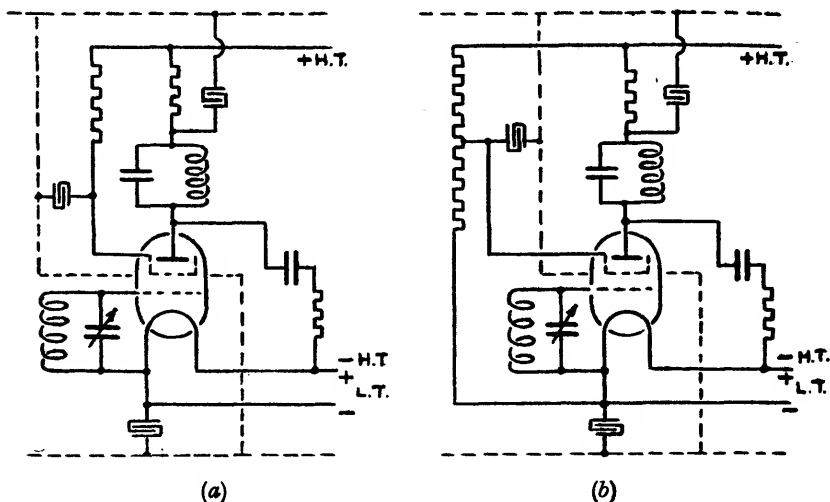
Neutralised Push-Pull Circuit.

FIG. 344.

(8) **Push-Pull Circuits.**—It has already been pointed out (para. 595) that with properly-matched valves in a push-pull arrangement and mid-point grid bias, the oscillatory current flowing through the H.T. battery and common leads may be reduced to an exceedingly

small amplitude and the risk of resistive coupling due to this cause thereby diminished. Owing to its symmetrical nature, the push-pull arrangement also lends itself readily to the use of neutralising condensers for balancing inter-electrode capacitive coupling. Such a neutralised circuit is shown in Fig. 344. The neutralising condensers are connected between the anode of one valve and the grid of the other. It will be seen that this system combines the advantages of both the previous neutralising circuits given for one valve, both input and output being centrally tapped.

(9) **Four-Electrode (Screen Grid) Valve or Tetrode.**—The characteristics of this valve were fully discussed in Chapter XI, and it was pointed out that the inter-electrode capacity C_{gs} between grid and anode could be reduced to an exceedingly small value. Thus the use of such valves provides a powerful method of cutting down the feed-back from output to input through this capacity and hence of preventing instability.



Screen Grid Stage in Amplifier.

FIG. 345.

To take full advantage of this very loose internal capacitive coupling, it is essential that external coupling between output and input should also be reduced to a minimum, *i.e.*, the external screening between grid and anode must also be as complete as possible. To ensure this, the valve as fitted for use in an amplifier stage is passed through a hole in an earthed metal screen in the same plane as the internal screen electrode. The method of fitting is shown diagrammatically in Fig. 345, the external screening being indicated by dashed lines.

Fig. 345 (a) and (b) illustrate respectively two methods of obtaining the screen voltage from the H.T. supply:—

- (a) A resistance is inserted between H.T. positive and the screen terminal. The screen current flowing through this resistance thus produces a P.D., and the screen potential is less than that of H.T. positive by the amount of this P.D. Thus, to quote some usual figures, the screen current might be 0.5 mA with a 50,000 Ω resistance. The P.D. across the resistance would then be 25 volts, and, with 120 volts H.T., the screen potential would be 95 volts.
- (b) A tapping is taken to the screen from a high resistance potentiometer.

With an oscillatory input there will, of course, be an oscillatory screen current, just as there is an oscillatory anode current. With external impedance in the screen grid circuit this oscillatory current would set up oscillatory potentials on the screen and the correct operating screen voltage as obtained above would be altered. To prevent this the screen lead is connected to the earthed screen by a large condenser. This has negligible impedance and so the oscillatory variation of screen voltage is also negligible. Thus as regards **oscillatory** potential, the screen and the filament may be considered common, although there is, of course, a large **constant** P.D. between them.

603. Tetrode Amplification.—It was seen in Chapter XI that the amplification factor m and A.C. resistance r_a of the screen grid valve were very high as compared with those of the triode. This is not primarily because higher values of these constants could not be obtained with triodes, but because no advantage could be taken of the higher amplification possible owing to the instability it would certainly produce through inter-electrode capacity coupling. The very small amount of the latter in the tetrode allows a stable amplification stage to be used while giving a much higher V.A.F.

The theoretical calculation of the V.A.F. is the same as for the triode. Apart from its effect on the constants, the extra electrode does not affect the algebra of the valve. Thus with an external resistance R in the anode lead, the V.A.F., as before, is

$$\text{V.A.F.} = \frac{mR}{R + r_a}.$$

The disadvantage of resistance-capacity coupling, from the point of view of the extra H.T. voltage necessary, has already been discussed for triodes. In tetrodes, the much larger value of r_a requires a corresponding increase in the external resistance to take advantage of its amplifying properties, and so this disadvantage is still more pronounced. In general, therefore, a tuned anode output circuit will be employed, acting effectively at its resonant frequency as a very high resistance.

The superiority of screen grid valve amplification may be best seen by a numerical illustration, taking from the average values quoted in Chapter XI the following constants for triode and tetrode :—

—	g_m .	r_a .	m .
Triode	mA/volt. 0.75	Ohms. 20,000	15
Tetrode	0.5	200,000	100

With an effective external resistance of 100,000 Ω in each case, the V.A.F.s are

(a) Triode :—

$$\frac{15 \times 100,000}{120,000} = 12.5$$

(b) Tetrode :—

$$\frac{100 \times 100,000}{300,000} = 33.3$$

The advantage of the tetrode is obvious, and further, with the figures quoted, the tendency to instability would be much less with the tetrode than with the triode. It can also be shown that tetrode amplification is more selective.

The tetrode has the further advantage, as first valve in a radio frequency amplifier, that by preventing feed-back from the output circuit, re-radiation from the aerial is diminished. It is unsuitable for audio frequency stages where large input voltages are applied between grid and filament. The reason for this is explained below.

604. Pentode Amplification.—In the discussion on dynamic characteristics in para. 552 it was shown that the slope of the operating characteristic of a triode with a load in the anode circuit is much less than the static slope. This is because a rise in anode current, brought about by a change in grid voltage, will produce an increased drop in volts across the impedance of the external anode circuit, with a corresponding fall in the actual potential on the anode. This fall in anode volts is responsible for a reduction of anode current, so that the latter does not rise to the extent which the static characteristic would indicate as a result of the given change in grid volts. Now an examination of the anode characteristics of the screen grid valve, Fig. 302, shows that for a wide range of anode voltage above the screen potential the curve is nearly horizontal. In other words, fairly large changes of anode voltage have little effect on the anode current, and therefore the slope of a dynamic mutual characteristic is not much less than that of the static curve.

Thus this type of valve practically retains its high amplification factor and mutual conductance when in operation. This property is very desirable for audio frequency amplification, but the screen grid valve is limited for this purpose because the dip or kink in the anode characteristics, due to secondary emission effects, limits the available grid swing to half a volt or so. The introduction of the earthed grid in the pentode suppresses the effects of secondary emission, and extends the range of grid swing, without distortion of anode current, to 30 volts or more, making the construction an excellent one for the power output stage of an amplifier.

Owing to the difficulty of designing suitable output circuits, the methods of radio frequency amplification described in this chapter (except reaction methods) become inefficient at high frequencies, and different methods must be employed. These will be dealt with in Chapter XVI.

605. Handling of Amplifiers.—In order to get full value out of an amplifier, an operator must use his intelligence over each adjustment he makes. An amplifier is not given to him so that he can read signals with his telephones on the table. The minimum possible amplification should be used, so as to avoid waste of valves, telephones, and battery power, and to preserve the sensitivity of the ear for weak signals.

In order to reduce interference, all receiving circuit adjustments should be set to the most selective condition possible.

The use of batteries which have nearly run down, is a frequent cause of poor results.

The battery voltage should be occasionally checked by the voltmeter provided.

606. Amplifier Noises.—If the process of amplification is carried too far, or if an amplifier is badly designed, it will be found that many interfering noises will be heard in the telephones.

These noises may be classified as follows :—

Symptom.	Cause.
(1) Valve Noises.	
(a) Frying noises.	Soft valve.
(b) Intermittent noises like atmospherics	Bad filament.
(c) Clicks.	Loose contacts
(2) Bad Design.	
(d) External noises easily picked up.	Unscreened transformers.

Symptom.**Cause.****(3) Howling, etc.**

- (e) Howling in case of a note magnifier. Continuous oscillation of an audio-frequency circuit due to cross-coupling.
- (f) Howling in case of a radio-frequency amplifier. Generation of two radio-frequency oscillations giving an audible beat.
- (g) A steady succession of clicks. The result of using a grid leak of too high resistance.
A self-oscillation piles up a negative charge on an insulated grid faster than the leak can drain it away; the oscillation then stops momentarily and starts again as soon as the negative charge has drained off (para. 706).

(4) Battery Trouble.

Crackling noises.

Loose contact, or defective cells in H.T. battery.

(5) Induction from Mains.

Continuous hum.

Leads to battery or charge too near leads from battery in use.

607. Filter Circuits.—The term "Filter Circuit" is used loosely to cover any type of circuit used in conjunction with an amplifier to remove undesired frequencies and thus give selectivity.

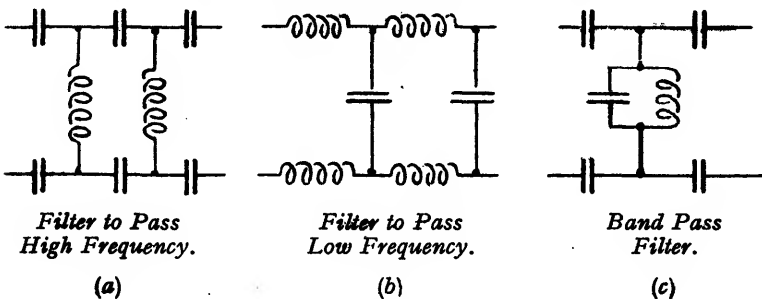


FIG. 346.

If it is desired to eliminate audio frequency interference the circuit shown in Fig. 346 (a) may be used. Here radio frequency currents will pass readily through the condensers, while audio frequency currents will find the condensers a path of high reactance, and will prefer the short-circuit path through the inductances.

Fig. 346 (b) indicates the opposite type of circuit, namely, one designed to pass audio frequency or direct current, and impede the passage of radio frequency current.

If designed to pass direct current only, as when joined up in the D.C. source of supply to a valve receiver, the inductances will have iron cores, and the condensers will have values of the order of microfarads.

The circuit shown in Fig. 346 (c) includes a rejector circuit tuned to a particular frequency. Current at this and adjacent frequencies will therefore be unable to pass through it, and must pass along the line as required.

Currents at other frequencies will find an easy short-circuit path through the rejector.

It is called a "band-pass filter," as it separates a band of frequencies in the neighbourhood of its resonant frequency from other frequencies which were originally present. Its frequency response curve may be compared with that of two tuned circuits mutually coupled.

The filter must be terminated by the correct impedance, which is determined by the ratio of the capacities and inductances used.

608. Note Filter Circuit.—A "Note Filter" or "Note Selector" circuit is used to add a further degree of selectivity, by allowing **audio-frequency** currents of one particular frequency only to pass through the telephones.

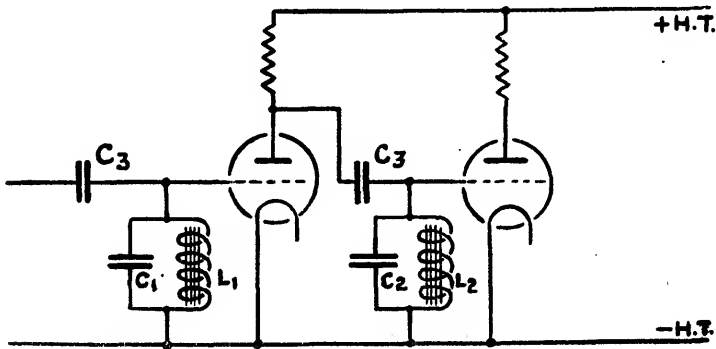


FIG. 347.

This application is of great importance. It makes possible the operation of several different channels of communication on the same radio frequency, using I.C.W. with different modulating frequencies.

Fig. 347 illustrates a typical circuit. It represents a combination of a note magnifier circuit, with rejector circuits L_1C_1 , and L_2C_2 .

The heterodyne in the radio frequency stage of the receiving gear is adjusted to give a note (in combination with the required

C.W. signal) of the same frequency as that to which the rejector circuits are tuned.

This signal will therefore suffer no loss in the rejectors, and will be magnified and passed through the telephones in the usual manner.

C.W. signals of other frequencies, however, giving other notes in combination with the heterodyne oscillation, will find a short-circuit path through the rejector circuits, L_1C_1 and L_2C_2 , and should not reach the telephones at all.

CHAPTER XIV.

THE VALVE AS GENERATOR OF OSCILLATIONS.

609. The Valve as a Generator.—It has already been shown in Chapter VII that oscillations set up at its natural frequency in an oscillatory circuit die away at a rate determined by the damping factor of the circuit, which in turn depends on the resistance losses in the circuit.

The generation of continuous oscillations by means of the Poulsen arc is possible because there is added to the circuit in series an effectively "negative" A.C. resistance, due to the peculiar shape of the characteristic curve of voltage against current across the arc. The damping is thus reduced to zero when the "negative" resistance is numerically equal to the positive resistance due to ohmic and radiation losses.

In the case of a valve and an associated circuit which is to produce continuous oscillations, the same result is achieved if, by means of suitable reaction, sufficient energy is introduced into the oscillatory circuit to compensate for that lost in damping.

We have already investigated a case of this nature in Chapter XII, where continuous oscillatory action is maintained in an LC circuit connected between grid and filament of a valve because an extra electro-motive force, in phase with the oscillatory current already present in the circuit, is introduced by the reaction coil in the anode lead.

Also, in para. 600, Chapter XIII, it has been indicated that capacitive reaction may be sufficient to give the same result.

The common characteristic of the circuits already mentioned is that the LC circuit is connected between grid and filament, and reaction is introduced from the anode circuit.

A well-designed circuit of this type is capable of maintaining quite a large amplitude of oscillatory current, but in practice its use is mainly confined to such local oscillators as have already been described. For transmitters—*i.e.*, generators designed to convey electro-magnetic energy to considerable distances—a different procedure is usually adopted. The tuned LC circuit is connected in the anode lead, and reaction is introduced from the grid circuit.

In general terms, the action can be described in a similar way to that of the circuit of Chapter XII. If any electrical shock is given to the oscillatory circuit by a slight change in current passing through it (*e.g.*, by switching on the H.T. voltage when the filament is alight), an oscillatory current is set up in the circuit. This oscillatory current, by means of the mutual induction present

between the inductance of the oscillatory circuit and that of a reaction coil in the grid circuit, sets up oscillatory variations of grid-filament voltage at the same frequency. These, in turn, cause oscillatory variations in the anode current to be superimposed on its steady value. These variations in anode current pass through the oscillatory circuit, and supply power to it so as to tend to maintain the initial oscillation.

Two requirements must obviously be fulfilled for the oscillatory action to be maintained at its initial amplitude, or increased up to a point where limiting conditions supervene:—

- (a) The phase relationships must be such that the variations in anode current assist, and do not oppose, the initial oscillation.
- (b) The coupling must be sufficient to allow the grid-filament voltage variations and the resulting anode current variations to supply enough power to the oscillatory circuit to make good the damping losses.

This type of circuit will now be examined in greater detail as regards phase relationships and the minimum coupling necessary.

610. Phasing in the Oscillatory Circuit.—The circuit to be considered is that of Fig. 348.

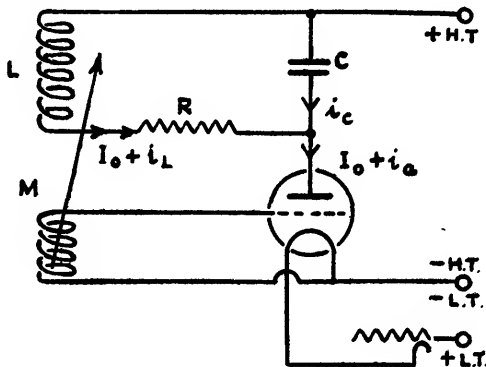


FIG. 348.

If oscillations are set up in the LC circuit, the currents flowing in the different parts of the circuit are as represented in the figure. The total current through the valve is made up of a steady component I_0 plus an oscillating component, whose instantaneous value is represented by i_a . The current in the inductance L is similarly composed of I_0 plus an oscillating component i_L , and the current in the capacitive arm of the parallel circuit consists only of an oscillatory component, instantaneous value i_c .

It follows that $i_a = i_L + i_c$.

It should be observed that, as the variations in current occur at the natural frequency of the LC circuit, which is practically equal to its resonant frequency, regarded as a rejector circuit to variations of current flowing through the valve, the relative phase relationships of I_L , I_0 and I_a are the same as those applicable to the make-up current and the circulating current in a rejector circuit (I_L , etc., represent the R.M.S. values of the currents i_L , etc.).

Therefore I_a leads I_L by approximately 90° , and I_a lags on I_0 by approximately 90° , as indicated in Fig. 349.

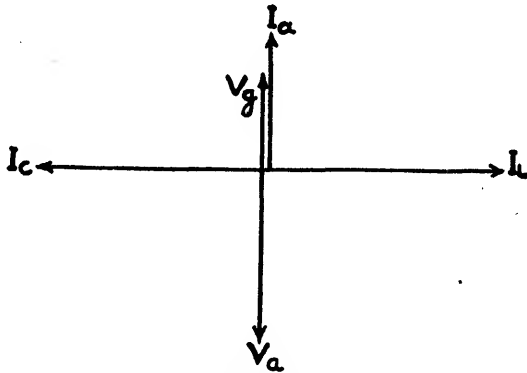


FIG. 349.

There is no oscillatory applied E.M.F. in the circuit (the supply being D.C.), and so the total oscillatory P.D. round the circuit is zero. The oscillatory P.D. (V_R) across the rejector circuit is in phase with the make-up current I_a . Hence the oscillatory anode-filament P.D. (V_a) of the valve must be equal and opposite to that across the rejector circuit, *i.e.*, $V_a = -V_R$, and so V_a is 180° out of phase with I_a .

The variations in anode current are caused by grid-filament variations in voltage, and are in phase with these when the anode external circuit behaves like a pure resistance. Therefore the grid-filament voltage variations are represented by V_g , in anti-phase to the anode voltage variations V_a .

Finally, we see from the complete vector diagram that V_g and I_L are 90° out of phase; as V_g represents the voltage induced by mutual induction due to I_L flowing in the inductance arm, this result is to be expected. V_g leads I_L by 90° , *i.e.*, the mutual inductance M between anode and grid circuits must be negative. If M is made positive, *i.e.*, if the connections of the coil in the grid lead are reversed, then V_g will be in anti-phase to I_a , the coupling will be in the wrong direction and the damping of the oscillatory circuit will be increased instead of decreased.

The vector diagram of Fig. 349 gives a simplified version of the actual phase relationships; in particular, I_a is not exhibited in it

as the sum of I_L and I_0 . The more accurate diagram is shown in Fig. 354.

The values of V_0 and V_a in terms of I_L are given by $V_0 = \omega M I_L$ and $V_a = \omega L I_L$, this latter value neglecting the voltage drop across the resistance R .

611. Power in the Oscillatory Circuit.—With the phase relationship as given above, it is possible to show that power is being supplied to the oscillatory circuit.

The total anode current at any time, $I_0 + i_a$, may be written in the form $I_0 + J_a \sin \omega t$. The voltage between anode and filament can be written $V_0 + \mathcal{V}_a \sin (\omega t + \pi)$, since its oscillatory component is 180° out of phase with that of the anode current.

The average power supplied by the battery is given by $V_0 I_0$ i.e., the steady voltage of the battery multiplied by the mean value of current.

The power dissipated at the anode of the valve in the form of heat is given by the product of anode voltage and current through the valve, and is therefore at any instant

$$\begin{aligned} & (I_0 + J_a \sin \omega t) \{V_0 + \mathcal{V}_a \sin (\omega t + \pi)\} \\ &= (I_0 + J_a \sin \omega t) (V_0 - \mathcal{V}_a \sin \omega t) \\ &= I_0 V_0 + V_0 J_a \sin \omega t - I_0 \mathcal{V}_a \sin \omega t - J_a \mathcal{V}_a \sin^2 \omega t \\ &= I_0 V_0 + J_a V_0 \sin \omega t - I_0 \mathcal{V}_a \sin \omega t - \frac{J_a \mathcal{V}_a}{2} + \frac{J_a \mathcal{V}_a}{2} \cos 2\omega t. \end{aligned}$$

The expression above contains two constant terms, and three oscillatory terms whose mean value over a whole cycle is zero.

Therefore the mean power dissipation at the anode is given by

$$I_0 V_0 - \frac{J_a \mathcal{V}_a}{2}. \text{ This is less than the power supplied by the H.T.}$$

battery by an amount $\frac{J_a \mathcal{V}_a}{2}$, which is therefore the power available for the maintenance of oscillations in the LC circuit.

612. Maximum Power Conditions.—Maximum power is available for the LC circuit when the product $\frac{J_a \mathcal{V}_a}{2}$ above is a maximum.

- (i) J_a . The amplitude of the oscillatory component of anode current cannot be indefinitely increased because of saturation conditions. If the oscillatory variation is **to remain sinoidal in form**, it is obvious that the maximum value of this amplitude is $\frac{I_s}{2}$, where I_s is the saturation current; and, for this to be possible, the steady component of anode current must also be equal to $\frac{I_s}{2}$.

- (ii) \mathcal{V}_a .—The maximum amplitude of oscillatory voltage is given by V_0 , the voltage of the H.T. battery.

If the amplitude were greater than this, then the voltage on the anode would become negative during a portion of the cycle, which, on account of the anti-phase relationship between oscillatory anode current and voltage, is exactly the time at which the maximum current flow through the valve is necessary. Hence the anode current, instead of attaining this maximum value then, would fall to zero, and its wave form would depart greatly from the sinoidal shape. This would lead to the generation of unwanted harmonics, and to a current amplitude at the fundamental frequency of less amount than the maximum possible.

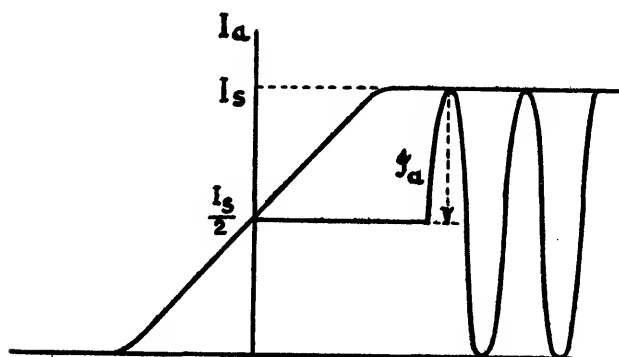


FIG. 350.

In actual practice the oscillatory amplitude of anode voltage can never be made quite equal to the D.C. H.T. voltage, because it is impossible to get saturation value of anode current unless, in addition to the high grid potential existing at that instant, some residual voltage is left on the anode. Another important limitation of this condition in practice is that the anode-filament P.D. must never be less than the maximum positive grid-filament P.D., otherwise the grid robs the anode of current, and instead of the anode current rising to a maximum at the point of minimum anode-filament P.D. it will fall to a very low value. This produces unwanted harmonics, and is detrimental to the efficiency. Further, the heavy grid current may lead to large secondary emission from the grid as the latter acquires a more and more positive potential. Owing to this the grid current increases and likewise the secondary emission. The amplitude of oscillations dies off, and the anode will then become positive to the grid. The large grid secondary emission then causes a rush of current to the anode, resulting in a greater production of heat at that electrode than the valve is constructed to dissipate, and hence damaging the valve. As minimum anode filament P.D. occurs at the same instant of a cycle as maximum

positive grid filament P.D., it follows that the oscillatory anode voltage must be less than the H.T. supply by at least the amount of grid voltage variation. However, if for simplicity these two conditions may be assumed to occur simultaneously, then the power available for the LC circuit is given by the product

$$\frac{I_a V_a}{2} = \frac{1}{2} \times \frac{I_s}{2} \times V_0 = \frac{I_s V_0}{4}.$$

613. Theoretical Efficiency.—With these conditions, the input power from the H.T. battery is given by

$$I_0 V_0 = \frac{I_s}{2} \times V_0 = \frac{I_s V_0}{2}.$$

Therefore the theoretical efficiency of a valve as a generator is

$$\frac{\frac{I_s V_0}{4}}{\frac{I_s V_0}{2}} \times 100 \text{ per cent.} = 50 \text{ per cent.,}$$

neglecting the power dissipated in the filament.

By means of special adjustments in the grid circuit, resulting in the mean grid potential being kept at a point which corresponds to a smaller value of anode current than half the saturation current, the efficiency can be raised to about 80 per cent., but the resulting wave form of oscillatory current is not sinoidal, and harmonics of the fundamental frequency are generated. In practice an efficiency of about 75 per cent. has been achieved in low frequency Service transmitters. This adjustment will be referred to later.

614. Anode Tapping Point.—In para. 612 it was stated that maximum power is delivered to the LC circuit when the amplitude of oscillatory anode current is $\frac{I_s}{2}$ and the amplitude of oscillatory anode voltage is V_0 .

Now these two amplitudes of current and voltage are interdependent.

For, writing them as I_a and V_a respectively, we know that $V_a = I_a \times \frac{L}{CR}$, $\frac{L}{CR}$ being the effective resistance of the parallel circuit at resonance. Hence, substituting their most efficient values,

$$V_0 = \frac{I_s}{2} \times \frac{L}{CR}$$

or $\frac{L}{C} = \frac{2RV_0}{I_s}.$

The product of L and C (the LC value of the oscillatory circuit) is determined by the frequency it is desired to transmit. In addition, the result just obtained shows that a definite ratio of L to

C is necessary if maximum power is to be developed in the oscillatory circuit. The practical problem is therefore to alter the ratio of inductance to capacity without affecting the tuning of the circuit. This is solved by including only part of the inductance in one side of the parallel circuit, and the remainder in series with the capacity in the other side.

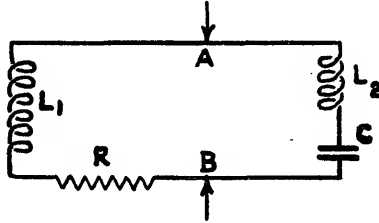


FIG. 351.

If this is done, as in Fig. 351, the natural frequency of the circuit remains unaltered provided the damping losses are negligible, but the combination of L_2 and C acts effectively as a condenser of increased value, *i.e.*, the total reactance of L_2 and C in series is capacitive, but less than that of C alone.

If this equivalent capacity is written C_1 , then

$$\frac{1}{\omega C_1} = \frac{1}{\omega C} - \omega L_2 = \frac{1 - \omega^2 L_2 C}{\omega C}$$

The natural frequency of the circuit is given by $f = \frac{\omega}{2\pi}$, where

$$\omega^2 = \frac{1}{L_1 C_1} = \frac{1 - \omega^2 L_2 C}{L_1 C},$$

$$\text{i.e., } \omega^2 (L_1 C + L_2 C) = 1, \text{ or } \omega^2 = \frac{1}{(L_1 + L_2) C} = \frac{1}{LC},$$

which shows that the natural frequency is unaltered.

Also
$$C_1 = \frac{1}{\omega^2 L_1} = \frac{LC}{L_1}.$$

The ratio of inductance to capacity in this circuit is therefore $\frac{L_1}{C_1} = \frac{L_1^2}{LC}$, and the effective resistance of the circuit is $\frac{L_1}{C_1 R} = \frac{L_1^2}{LCR}$. By this device the effective inductance of the circuit has been decreased, the effective capacity increased, and the effective resistance decreased.

It was found above that under the best conditions (50 per cent. efficiency), $V_o = \frac{I_o}{2} \times \text{effective resistance of the parallel circuit}$. Substituting the value just found for the latter,

$$V_o = \frac{I_o}{2} \times \frac{L_1^2}{LCR}.$$

$$\therefore L_1^2 = \frac{2 LCRV_0}{I_s}, \quad \text{or } L_1^2 = \frac{2 RV_0}{\omega^2 I_s}$$

$$\text{i.e., } L_1 = \sqrt{\frac{2 LCRV_0}{I_s}} \quad \text{or } L_1 = \frac{1}{\omega} \sqrt{\frac{2 RV_0}{I_s}}$$

This formula gives the amount of inductance which should be included in one arm of the parallel circuit to give maximum efficiency. As would be expected, it is inversely proportional to the frequency of the tuned circuit.

The point A in Fig. 351, at which contact is made with the inductance from the anode lead, is known as the **Anode Tapping Point**, and the formula above gives the value of the inductance to be included between the A.T.P. and the filament lead for maximum efficiency. In other words, the formula determines **the best position of the Anode Tapping Point under sinoidal conditions**.

The R.M.S. value of the oscillatory current in the LC circuit is then given by :—

$$I^2 R = \text{Power expended in the LC circuit.}$$

$$= \frac{I_s V_0}{4}$$

$$\therefore I = \sqrt{\frac{I_s V_0}{4R}}$$

The amplitude is $\sqrt{2}$ times this, i.e., $\sqrt{\frac{I_s V_0}{2R}}$.

It should be noted that an efficiency of 50 per cent. simply means that an equal amount of power is dissipated in the valve and in the external circuit, so that the external impedance is then equal to the A.C. resistance of the valve. We had exactly similar conditions in para. 592, when dealing with maximum power amplification in an amplifier. In fact, a valve transmitter working on the straight part of the dynamic characteristic may be regarded as an amplifier in which the grid-filament voltage variations are supplied by the apparatus itself, instead of from an outside source, and the greatest amount of oscillatory output power is therefore available with the same conditions of equivalence between output impedance and valve A.C. resistance.

Applying this criterion, the condition for 50 per cent. efficiency becomes

$$\frac{L_1^2}{LCR} = r_s$$

$$\text{or } \omega^2 L_1^2 = R r_s$$

$$\text{or } \omega L_1 = \sqrt{R r_s}$$

$$\therefore L_1 = \frac{\sqrt{R r_s}}{\omega} = \sqrt{R r_s LC}$$

This is another expression for the correct position of the anode tapping point. It is equivalent to that previously derived because, under the conditions given, the maximum voltage variation across the valve is V_a , and this corresponds to a maximum current variation of $\frac{I_a}{2}$.

Hence $\frac{2 V_a}{I_a} = \text{A.C. resistance } r_a \text{ of valve.}$

Hence $L_1 = \sqrt{R r_a LC} = \sqrt{\frac{2 RLCV_a}{I_a}}$ as before.

615. Coupling necessary for Maintenance of Oscillations.—So far we have investigated the phase relations necessary for self-oscillation, viz., that V_a and I_a are in phase and V_g is 180° out of phase with either. We have also found the amplitude of oscillatory current in the circuit, and the power dissipated in the circuit when the valve acts efficiently as an oscillator.

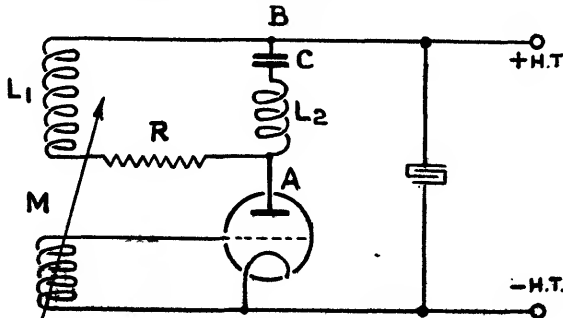


FIG. 352.

It is obvious, on the analogy of previous cases, that a certain minimum coupling is necessary for self-oscillation to take place, so that, with an oscillatory action started in the circuit, the resulting anode current variation, regarded as a make-up current through the parallel circuit, is of sufficient amplitude to maintain the circulating, or oscillatory current, at a constant amplitude.

Let us take the case of a circuit with anode tap such that the inductive branch between A and B has inductance L_1 .

Let the oscillatory current (R.M.S. value) flowing in this inductance be I , the oscillatory component of anode current be I_a , and of anode voltage be V_a . Then, under oscillatory conditions, the grid-filament voltage is varied, resulting in the variation I_a above. This current variation is, however, dependent on another factor, the change in anode voltage, which it itself sets up. The general formula for I_a must therefore be used:—

$$I_a = g_m V_g + \frac{1}{r_a} V_a$$

Now, in the circuit shown,

$$V_g = \omega MI$$

$$V_a = -\omega L_1 I$$

(V_a and I_a are in phase, V_g in anti-phase, hence the negative sign).

$$\therefore I_a = g_m \omega MI - \frac{1}{r_a} \omega L_1 I$$

The power given to the LC circuit to maintain oscillations

$$= \frac{I_a V_g}{2} \text{ (para. 611)}$$

$$= \frac{I_a}{\sqrt{2}} \times \frac{V_g}{\sqrt{2}} = I_a V_a$$

Therefore, to maintain oscillations, the product $I_a V_a$ (numerically) must be at least equivalent to the power dissipated in the circuit, $I^2 R$.

$$\text{Hence } \left(g_m \omega MI - \frac{1}{r_a} \omega L_1 I \right) \omega L_1 I \geq I^2 R.$$

$$\omega^2 \left(M g_m - \frac{L_1}{r_a} \right) L_1 \geq R$$

$$\text{Now } \omega^2 = \frac{1}{LC}, \text{ where } L = L_1 + L_2.$$

$$\therefore \left(M g_m - \frac{L_1}{r_a} \right) L_1 \geq RLC$$

$$\therefore M \geq \frac{CRLr_a + L_1^2}{L_1 g_m r_a} \geq \frac{LCRr_a}{m L_1} + \frac{L_1}{m}$$

In the case where all the inductance is on one side of the circuit, and therefore $L_1 = L$, this reduces to

$$M \geq \frac{RCr_a}{m} + \frac{L}{m} \geq \frac{L + CRr_a}{m} \geq \frac{CR}{g_m} \left(1 + \frac{L}{CRr_a} \right).$$

If M is equal to the above expression, oscillations are just maintained; if greater, oscillations build up; if less, oscillations set up in the oscillatory circuit die away more slowly than if no reaction were employed.

616. Amplitude of Oscillation Reached.—As in the case of the tuned grid circuit, with reaction coil in the anode lead, treated in Chapter XII, we can investigate the above formula with a view to determining conditions which prevent the amplitude of the oscillation from continually increasing. It has already been shown that there are limits imposed by the fact that the power available to maintain oscillations is limited by the saturation value of current flowing and the voltage of the H.T. supply. It is proposed to study here in more detail how these limits are reached.

Let us assume that the coupling M has been adjusted to some value slightly greater than the minimum required to maintain oscillations. We shall also assume the steady grid and anode voltages to be such that the valve is working at a point on the straight part of the (I_a, V_a) static characteristic about half-way up.

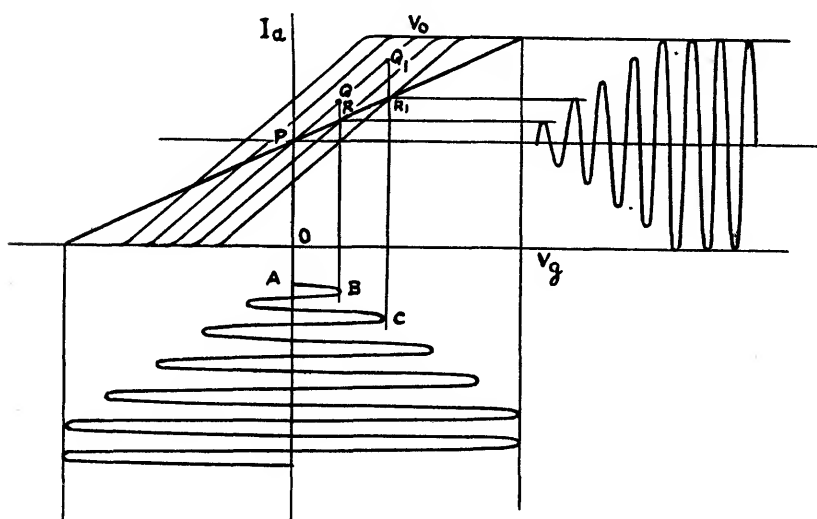


FIG. 353.

An oscillation started in the LC circuit varies V_a , and consequently the anode current. Due, however, to the load in the external anode circuit, this increase in anode current is accompanied by a decrease in anode voltage, so that a positive grid swing AB does not cause the anode current to rise from P to Q , but to some value R on a different static characteristic curve drawn for the reduced anode voltage.

The next and bigger grid swing, to C , causes an increase of anode current to R_1 only instead of to Q_1 , R_1 being on a curve corresponding to a further reduced anode potential. In this way, the amplitude of anode current keeps increasing, but more slowly than if the external impedance were neglected.

Finally, the anode current builds up until it reaches the top or bottom bends of the **dynamic characteristic**. It should be evident that P , R , R_1 , etc., are points on the dynamic characteristic derived from the static by taking the impedance of the external circuit into account, and so long as we remain on the straight part of this characteristic, conditions are exactly the same as at the start, *i.e.*, oscillations tend to build up. Once the top or bottom bend is reached, increased variations in grid voltage no longer produce correspondingly increased variations in anode current, and the amplitude of the oscillation becomes limited.

This theory can be put into more exact form by considering the condition for building up of self-oscillations, *i.e.*, $M > \frac{L + CR r_a}{m}$. The impedance of the external circuit is an effective resistance of value $\frac{L}{CR}$, so that the dynamic characteristic, which represents the relationship between anode current and grid voltage under working conditions, has a slope g'_m which is less than g_m , and is exactly

$$\frac{g_m}{1 + \frac{L}{CR r_a}} \quad (\text{para. 574})$$

The condition obtained above for the building up of self-oscillations, *viz.*, $M > \frac{CR}{g_m} \left(1 + \frac{L}{CR r_a}\right)$, may therefore be written in the form $M > \frac{CR}{g'_m}$. Thus, the formula obtained is exactly the same as that for the maintenance of oscillations in the case of the heterodyne circuit (Chapter XII), the only difference being that in the heterodyne circuit the effect of the anode impedance on the anode voltage and current was neglected, *i.e.*, the static characteristic was used instead of the dynamic one.

Therefore, as in Chapter XII, we may conclude that oscillations will build up until the upper or lower bend of the **dynamic characteristic** is reached, and, if the coupling is sufficient to cause further increase in amplitude, a limiting point is reached because g'_m becomes effectively less as the excursion of grid voltage past the bends increases. Finally, an amplitude is reached at which the inequality above becomes an equality, and oscillations are maintained at this constant amplitude.

Another factor which tends to limit the increase of amplitude of oscillations is the power consumed in the grid circuit. So far this has been ignored, but it is obvious that, with large grid voltage swings on the positive side of zero grid volts, quite considerable currents will flow in the grid circuit and cause power absorption.

In the light of this present investigation, the previous case of 50 per cent. efficiency will be obtained if the dynamic characteristic just reaches saturation at the same time as the static characteristic for zero anode voltage (it has already been pointed out that with sinoidal conditions zero anode voltage cannot be reached in practice owing to the necessity of keeping the least anode-filament P.D. greater than the maximum positive grid-filament P.D.); and if the coupling is **just sufficient** to maintain oscillation, *i.e.*, to avoid excursions of grid voltage past the bends of the dynamic characteristic and hence a departure from sinoidal wave form in the **make-up** and oscillatory currents.

*617. Actually the dynamic characteristic is not a straight line, but an elliptic closed curve. This follows from the fact that the correct vector diagram representing phasing of currents and voltages in the circuit is rather more involved than the simplified case shown in para. 610, Fig. 349, where all the vectors are at right angles.

If we start from the oscillatory current I_L in the inductance as a basis, the voltage induced between grid and filament, V_g , leads on it by exactly 90° (since M is negative), and is equal to $\omega M I_L$.

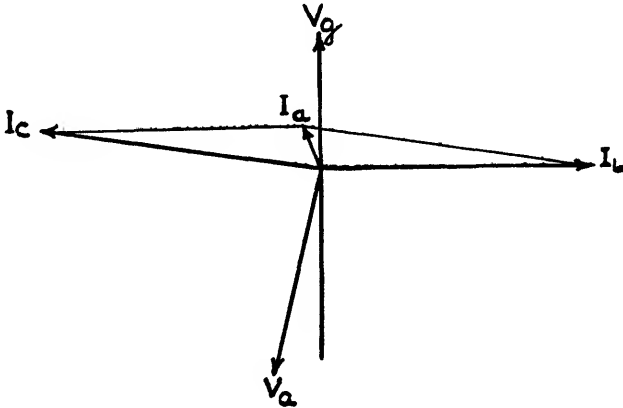


FIG. 354.

The anode voltage variation, V_a , lags by more than 90° on I_L , however, because of the fact that there is resistance as well as inductance in that arm of the parallel circuit.

$$v_a = -R i_L - L \frac{di_L}{dt}.$$

Writing i_L as $\mathcal{I}_L \sin \omega t$, we therefore obtain

$$\begin{aligned} v_a &= -R \mathcal{I}_L \sin \omega t - \omega L \mathcal{I}_L \cos \omega t \\ &= \sqrt{R^2 + \omega^2 L^2} \mathcal{I}_L \sin \left(\omega t - \frac{\pi}{2} - \theta \right), \end{aligned}$$

$$\text{where } \tan \theta = \frac{R}{\omega L}.$$

$$\therefore V_a = \sqrt{R^2 + \omega^2 L^2} I_L \text{ and lags on } I_L \text{ by } \frac{\pi}{2} + \theta.$$

$$i_c = -C \frac{dv_a}{dt}, \text{ and therefore } I_c \text{ obviously lags by } 90^\circ \text{ on } V_a.$$

Finally, the make-up current I_a , being the vector sum of I_L and I_c , leads the grid voltage V_g by a small angle, as shown in the figure. Also I_a and V_a are not exactly in antiphase. This out-of-phase relationship between V_g and I_a results in the elliptic form of the dynamic characteristic.

If, of course, R is neglected, the ellipse collapses, and gives the straight line already obtained.

618. Modifications to the Simple Circuit.—Two modifications to the simple circuit of para. 610 are shown in Fig. 355 (a):—

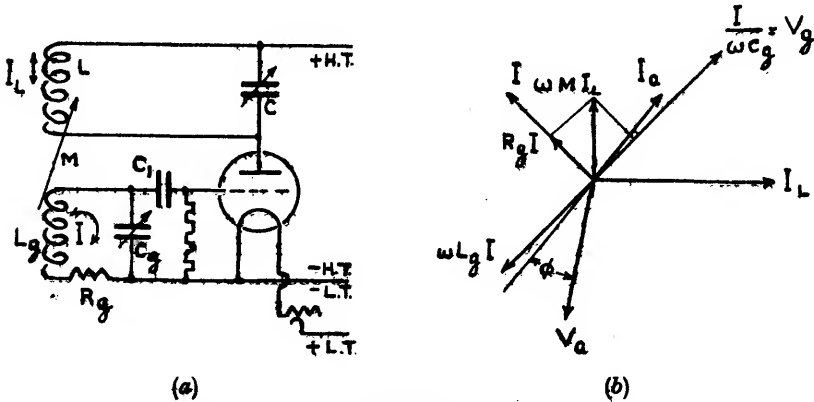


FIG. 355.

(a) Tuned Grid Circuit.

A condenser C_g is fitted in parallel with the grid coupling inductance.

(b) Grid Condenser and Leak.

The grid is insulated from the filament by a condenser C_1 , with a high resistance leak across it.

The reasons for these additions will now be considered.

619. The Tuned Grid Circuit.—With only an inductance in the grid circuit, the grid voltage is given by $\omega M \times$ current in the induction branch of the oscillatory circuit.

This voltage between grid and filament can be increased very much if a condenser is added across the inductance, and this parallel circuit tuned to resonance.

For, under these conditions, a voltage ωMI_L induced into this circuit at its resonant frequency sets up a circulating current round it given by $I = \frac{\omega MI_L}{R_g}$, where R_g is the resistance of the circuit.

The actual voltage applied between grid and filament is that across the inductance, or the capacity, and is $\frac{I}{\omega C_g}$, or $\omega MI_L \times \frac{1}{\omega C_g R_g}$. This is greater than ωMI_L if $\frac{1}{\omega C_g}$ the capacitive reactance, is greater than R_g , the resistance, which is certainly the case.

The theory above is the same as that for a tuned secondary circuit coupled to the aerial in a receiving circuit, in which the voltage across the condenser is greater than that introduced into the circuit.

The question of phase relationships, however, complicates matters in this case.

The current I is in phase with the voltage applied to the circuit, and so the voltage between grid and filament is now **90° out of phase** with the induced voltage ωMI_L ; and therefore in phase or anti-phase with I_L itself.

Thus the vector diagram of para. 610 would not hold good. V_g , as we saw, had to be 90° out of phase with I_L for I_g to be 180° out of phase with V_g , and hence to allow a supply of energy to be given to the oscillatory circuit. Therefore, if the grid circuit is tuned to the same LC value as the anode oscillatory circuit, although the resultant voltage applied to the grid is greatly increased, yet the phasing is incorrect for the maintenance of oscillations.

It is quite easy to show that if the phase angle between I_g and V_g is $(180^\circ - \phi)$, the power available for the LC circuit is $I_g V_g \cos \phi$.

In the case above, $\phi = 90^\circ$, and the power is *nil*.

The actual procedure employed, therefore, is to tune the grid circuit only partially, an intermediate step between leaving it resonant and leaving it aperiodic.

Under these conditions the big grid voltage changes required when a large LC value is used in the oscillatory circuit (with correspondingly small ω) are easily achieved through the grid circuit being partially resonant, while the angle ϕ of departure from anti-phase relationships of I_g and V_g is intermediate between zero and 90°, so that power is supplied to the LC circuit. In other words, I_g is increased by this arrangement to such an extent that the power supply, $I_g V_g \cos \phi$ is increased, although the power factor, $\cos \phi$, has decreased from its maximum value of unity since I_g and V_g are not now exactly in antiphase.

Practical values used are: for an oscillatory circuit tuned to 230 kc/s, the grid circuit is tuned to a frequency between 500 and 600 kc/s.

In addition, resistance is introduced into the grid circuit by means of lamps, to limit the value of the current in case of accidental tuning to resonance during adjustment. Without this precaution the current at resonance would damage the grid coil. The vector diagram is shown in Fig. 355 (b).

620. Grid Condenser and Leak. Efficiency with Negative Grid.

—The purpose of the introduction of the condenser and leak is to maintain the mean potential of the grid, about which oscillations are to take place, at a considerable negative value relative to the filament. The reason for this is given later in this paragraph.

When the valve is generating continuous oscillations, the grid potential varies through large values on either side of its steady value, and when it is positive to the filament, grid current flows. The condenser thus acquires pulses of negative charge, and with a high-resistance leak these slowly drain away, with the result that the mean grid potential relative to the filament is held at a negative value equal to the IR drop across the resistance. (Compare the steady negative grid bias attained by a cumulative grid detector receiving C.W.) The more powerful the oscillations, the greater is the negative potential of the grid. By designing the leak suitably the grid can always be kept **at any desired value of negative potential.**

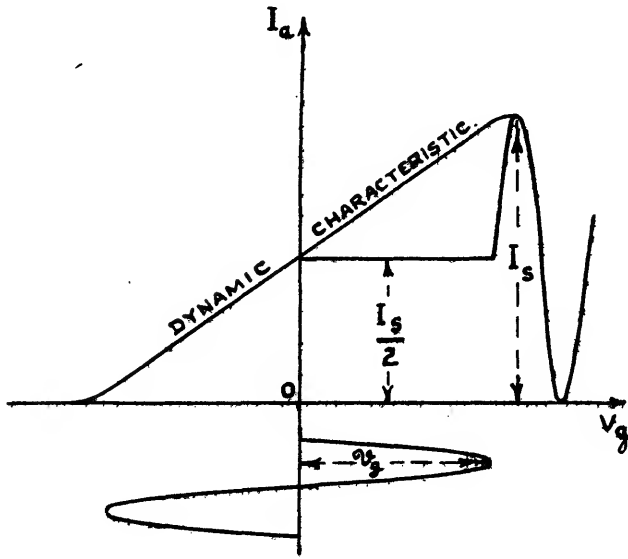
The values of the condenser and leak resistance are important. If the time constant CR of the combination is too large, the grid builds up so large a negative potential that the amount of power transferred from the valve to the oscillatory circuit is insufficient to maintain oscillations. When oscillations cease, no more grid current flows, and as the condenser discharges through the leak, the negative potential of the grid decreases until it is possible for oscillations to start again. This effect will be discussed in more detail when considering self-quenching (para. 706). It is obviously to be avoided when continuous oscillations are desired. It may be mentioned here that the rate of increase of mean negative grid potential with increase of leak resistance is only large for small values of the latter, so that the amplitude of oscillations only falls off slowly as the leak resistance is increased above its optimum value.

As will be shown below, although the use of negative grid bias increases the efficiency, it also leads to the production of harmonics of the fundamental frequency. The energy which is wasted in these harmonics is increased if the variation of mean grid potential during a radio frequency cycle is at all pronounced. This occurs if the condenser has too small a capacity. The charges which it acquires through grid current flowing during the positive half cycle, and loses through the leak resistance during the negative half cycle, produce corresponding variations of grid potential, which are larger the smaller the capacity of the condenser. Hence the condenser must be large enough to obviate pronounced fluctuations in the mean negative potential of the grid.

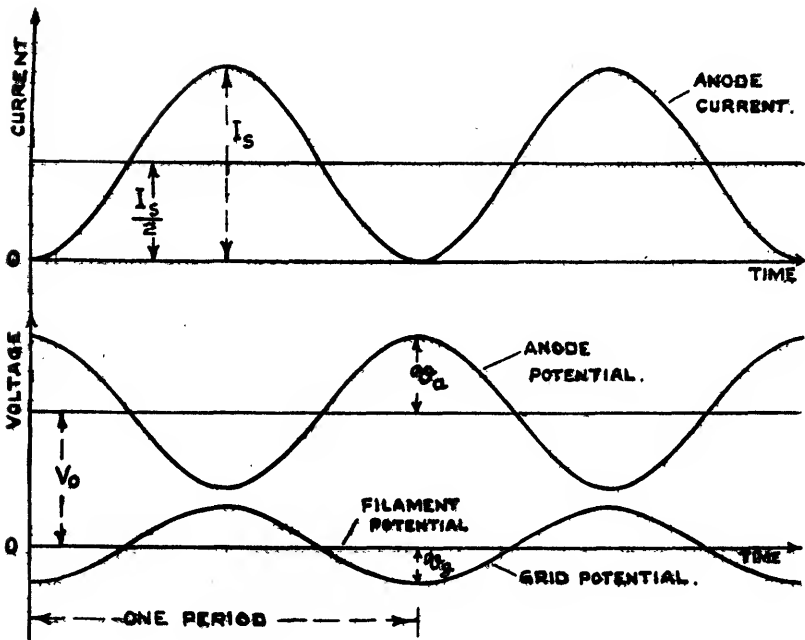
When oscillations build up with tight enough coupling for the positive variation of grid potential to sweep over the whole of the straight part of the (I_a , V_g) characteristic, the resulting anode current variations depart from the sinusoidal form obtained by working about a point half-way up the straight part of the characteristic.

The conditions when working with a mean value of anode current equal to half the saturation current, and coupling just

sufficient to maintain oscillations, can be represented graphically as follows :—



(a)



(b)

FIG. 356.

Fig. 356 (a) shows the dynamic characteristic on which the valve and its external circuit are operating.

In Fig. 356 (b) the actual variation with time of anode current, anode voltage and grid voltage is represented.

The anode current varies sinoidally between I_s and zero, and the anode voltage, provided the tapping point is suitably adjusted, has a variation about its mean value V_0 whose amplitude is less than V_0 by slightly more than the maximum positive grid voltage, and which is 180° out of phase with the anode current.

The grid voltage variation is in phase with the anode current.

As mentioned above, the introduction of negative grid bias by means of a grid condenser and leak results in an anode current waveform which is no longer a simple sine curve. In addition to the fundamental or first harmonic oscillatory current at the natural frequency of the circuit, higher harmonic currents are also produced, *i.e.*, currents at frequencies which are simple multiples of the fundamental frequency. The sum of all these currents gives the resultant current flowing through the valve.

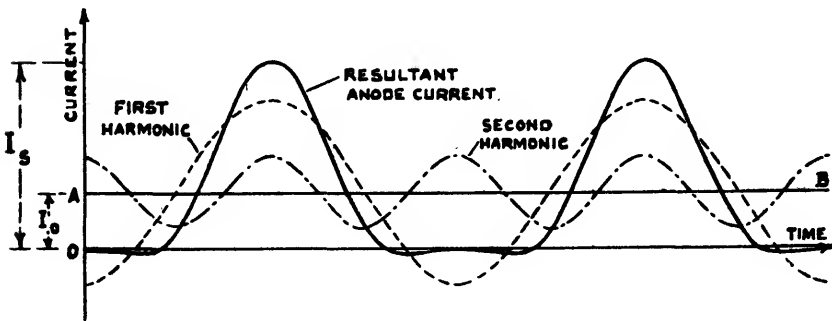


FIG. 357.

In Fig. 357 a first harmonic current wave is shown, together with a second harmonic, *i.e.*, a current at twice the frequency of the first harmonic. The amplitude of the second harmonic is taken as one-third of that of the fundamental, and it lags on the latter by one-eighth of the fundamental period. They are plotted about a common axis, AB, and the resultant waveform which is the sum of these two components is indicated by the full-line curve.

Fig. 358 (a) shows, by plotting from the dynamic characteristic, the anode current wave when the grid bias is such that the operating point is at the lower bend, and the positive half cycle of oscillatory grid voltage sweeps over the whole of the straight characteristic.

It will be seen that these two current waveforms are almost identical. Thus the analysis of Fig. 357 gives a very good indication of what actually happens under the conditions represented by Fig. 358 (a), provided the zero current line is shifted down

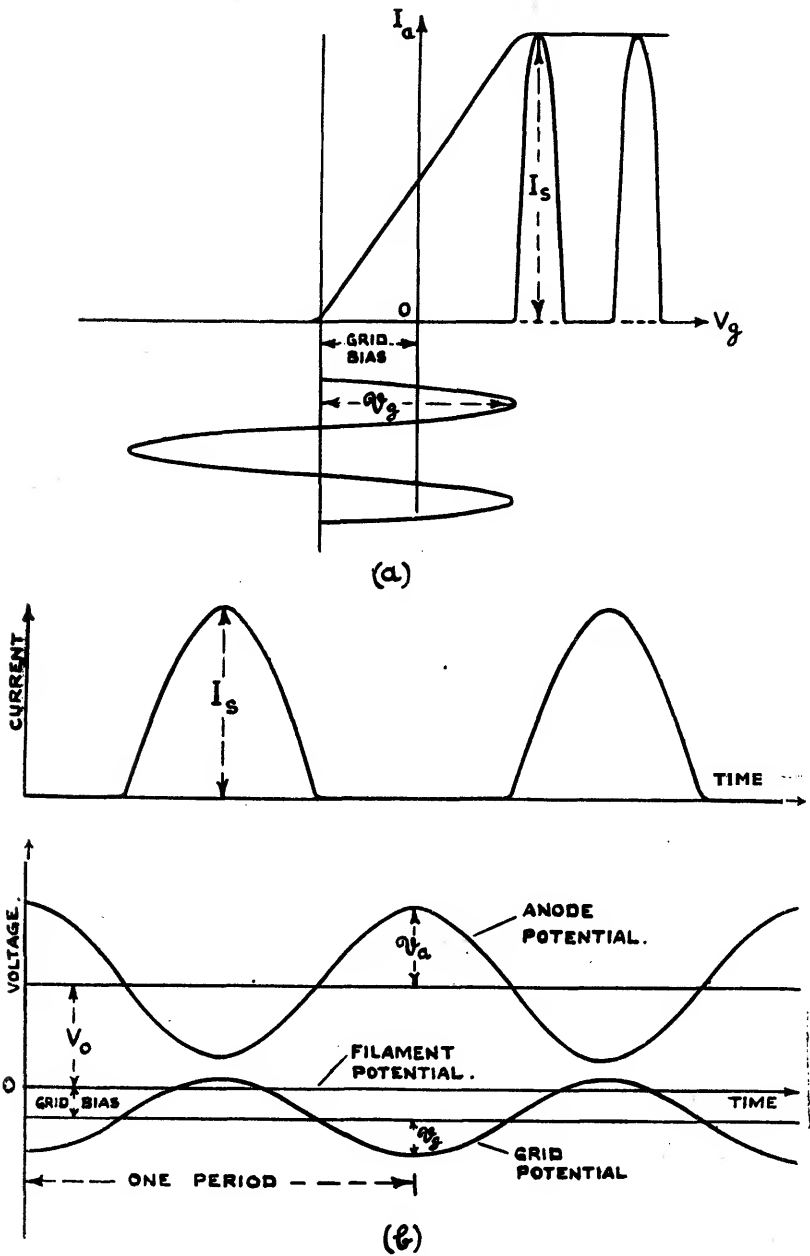


FIG. 358.

by an amount $OA (I_0)$. In other words, the anode current in Fig. 358 (b) can be considered as the resultant of

- (i) A steady current I_0 (where I_0 is less than $\frac{I_s}{2}$, *i.e.*, less than the steady current which flows under sinoidal conditions).
- (ii) A first harmonic current of amplitude $\frac{I_s}{2}$, *i.e.*, an oscillatory current identical with that produced under sinoidal conditions.
- (iii) A second harmonic current.

To compare the efficiency in this case with the optimum value of 50 per cent. which was derived for sinoidal conditions, we may assume as before that the anode tapping point is adjusted to give an amplitude \mathcal{V}_a of oscillatory anode voltage approximately equal to the H.T. voltage V_0 .

The power input from the H.T. supply is $V_0 I_0$, which is less than $\frac{V_0 I_s}{2}$, the power input in the sinoidal case, since I_0 is less than $\frac{I_s}{2}$.

The power output at the natural frequency, $\left(\frac{V_0 I_s}{4}\right)$, is the same in both cases, for the first harmonic wave is identical with the pure sine wave previously considered. Thus the same output is obtained with a lower input when negative grid bias is employed, *i.e.*, this arrangement is more efficient.

In practice, efficiencies up to 75 per cent. have been obtained by this method in some Service transmitters.

As in the sinoidal case, it is not possible to make the amplitude of oscillatory anode voltage exactly equal to the H.T. voltage, for the grid must never be allowed to have a more positive potential than the anode, and the grid potential must run positive for some part of its positive half cycle to allow grid current to flow and enable the grid to maintain a steady negative potential. Some idea of the anode voltage, grid voltage and anode current cycles is given by Fig. 358 (b).

***621.** A numerical estimate of the efficiency of a generator with negative grid bias under the best conditions may be simply obtained by making the approximations that $\mathcal{V}_a = V_0$, and that the anode current waveform may be taken as simple harmonic of amplitude I_s during the positive half cycle of grid voltage, and zero during the negative half cycle.

During the negative half cycle of grid voltage, the power input and the power dissipated at the anode are therefore both zero.

The mean power input over the positive half cycle of grid voltage is

$$\begin{aligned}\frac{2}{T} \int_0^{\frac{T}{2}} V_0 i_a dt &= \frac{2}{T} \int_0^{\frac{T}{2}} V_0 I_s \sin \omega t dt \\&= \frac{2 V_0 I_s}{T} \left[-\frac{\cos \omega t}{\omega} \right]_0^{\frac{T}{2}} = \frac{2 V_0 I_s}{\omega T} (1 + 1), \text{ since } \omega T = 2\pi \\&= \frac{2 V_0 I_s}{\pi}.\end{aligned}$$

The mean power input per cycle is half of this, i.e., $\frac{V_0 I_s}{\pi}$.

The mean power dissipated at the anode during the positive half cycle of grid volts is

$$\begin{aligned}\frac{2}{T} \int_0^{\frac{T}{2}} i_a v_a dt &= \frac{2}{T} \int_0^{\frac{T}{2}} I_s \sin \omega t \times (V_0 - V_0 \sin \omega t) dt \\&= \frac{2 I_s V_0}{T} \int_0^{\frac{T}{2}} (\sin \omega t - \sin^2 \omega t) dt \\&= \frac{2 I_s V_0}{T} \int_0^{\frac{T}{2}} \left(\sin \omega t - \frac{1}{2} + \frac{\cos 2 \omega t}{2} \right) dt \\&= \frac{2 I_s V_0}{T} \left[-\frac{\cos \omega t}{\omega} - \frac{t}{2} + \frac{\sin 2 \omega t}{4 \omega} \right]_0^{\frac{T}{2}} \\&= \frac{2 I_s V_0}{T} \left[\frac{1}{\omega} \cos 0^\circ - \frac{1}{\omega} \cos \frac{\omega T}{2} - \frac{T}{4} + \frac{1}{4 \omega} \sin \omega T - \frac{1}{4 \omega} \sin 0^\circ \right] \\&= \frac{2 I_s V_0}{T} \left[\frac{2}{\omega} - \frac{T}{4} \right], \text{ since } \omega T = 2\pi \\&= \frac{4 I_s V_0}{\omega T} - \frac{I_s V_0}{2} = \frac{2 I_s V_0}{\pi} - \frac{I_s V_0}{2}.\end{aligned}$$

The mean power dissipated at the anode per cycle is therefore

$$\frac{I_s V_0}{\pi} - \frac{I_s V_0}{4}.$$

The mean oscillatory power input to the LC circuit per cycle is the difference of these two results, i.e.,

$$\frac{I_s V_0}{\pi} - \left(\frac{I_s V_0}{\pi} - \frac{I_s V_0}{4} \right) = \frac{I_s V_0}{4}.$$

The efficiency is therefore

$$\frac{I_s V_0}{4} \div \frac{I_s V_0}{\pi} = \frac{\pi}{4}, \text{ or } 78.5 \text{ per cent.}$$

It must be remembered, however, that all the oscillatory power transferred to the LC circuit, as calculated above, is not at the first harmonic frequency, and so does not represent useful oscillatory energy for radiation at the desired frequency.

622. The following points should be noted in connection with this adjustment of the grid to a negative potential :—

- (1) The coupling necessary for self-oscillations to build up until the amplitude of grid voltage variation covers the whole of the straight part of the (I_a, V_g) dynamic characteristic must be greater than when the operating point is half-way up the characteristic. For M , by ordinary theory, must equal at least $\frac{RC}{g'_m}$ where g'_m represents the total change in anode current divided by the total change in grid volts (Fig. 358 (a)), and this is much smaller than the slope of the dynamic characteristic, which formed the denominator of the corresponding fraction when working half-way up.

Thus the best value of M may easily be two or three times as great as that previously deduced for sinoidal conditions.

- (2) If the coupling is increased beyond this value, the amplitude of oscillatory grid voltage will increase, but the larger grid current flowing will then cause the mean potential of the grid to become more negative, so that the peak of grid voltage during the positive half cycle returns to approximately the same potential as before. Anode current, however, now flows for less than half a complete cycle, and the power input to the oscillatory circuit falls off. With increasing coupling beyond the best value, the amplitude of the oscillations thus decreases, and the grid potential may become so negative that sufficient power is not available to maintain self-oscillatory conditions. When oscillations cease, the grid potential rises towards zero as the condenser charge leaks away through the resistance, and oscillations will start to build up again, *i.e.*, continuous oscillations cease to be generated.
- (3) The production of harmonics in the anode current waveform leads to the presence of similar harmonics in the circulating current round the oscillatory circuit, and therefore to the radiation of energy at undesirable frequencies.
- (4) The formula obtained for the correct position of the anode tapping point under sinoidal conditions is no longer applicable, but it is evident that the amplitudes of

oscillatory anode current and anode voltage are still interdependent, and that adjustment of the impedance of the external anode circuit without altering its frequency is still necessary to obtain the most efficient power output. In other words, the necessity for an anode tapping point is unaffected, but its position at any particular frequency differs from that which would be calculated from the formula previously given.

- (5) It is preferable to insert the grid condenser and leak between the grid tuned circuit and the grid itself. This prevents the tuned circuit from being at the large steady negative potential of the grid.

VALVE TRANSMITTER CIRCUITS.

623. In the above paragraphs, a simple circuit of a type which will generate oscillations has been examined as regards phase relationships between currents and voltages, amplitude of oscillation possible, minimum coupling necessary, efficiency under different conditions, and so on.

Any valve circuit used for the generation of self-oscillations must conform to these general principles. For instance, the oscillatory grid voltage and anode voltage must always be more than 90° out of phase if oscillations are to be possible, and the supply of energy to the oscillatory circuit in this correct phase must be sufficient to overcome the damping losses. A multifarious variety of circuits has, however, been devised within these wide limits, and it is out of the question to deal individually with even a small fraction of the valve transmitting circuits that may be encountered in practice. An endeavour has therefore been made in the following pages to classify under a few convenient heads the major differences in the details of self-oscillatory circuits, more particularly with reference to Service practice.

624. Position of Tuned Circuit.—The most general distinction can probably be drawn with respect to the two electrodes, of the possible three in a triode, between which the tuned circuit is connected :—

- (a) The **tuned circuit** may be between **grid and filament**. This is generally the case in the local oscillators used, for example, in heterodyne receivers, but is not common in transmitting circuits.
- (b) The **tuned circuit** may be between **anode and filament**. An example of this is the self-oscillatory circuit already discussed in this chapter as a typical transmitting circuit.

- (c) The **tuned circuit** may be between **anode and grid**. This type of circuit is illustrated in Fig. 359 and discussed in para. 626.
- (d) Combinations of the above, employing two tuned circuits, are also found. A good example is the tuned-grid, tuned-anode type of amplifier stage whose self-oscillatory tendencies due to inter-electrode capacitive coupling were discussed in Chapter XIII. Further examples will be found below, when multi-valve transmitters, *e.g.*, with a push-pull valve arrangement, are considered.

625. Grid Excitation.—A second point of difference arises in the nature of the grid excitation, *i.e.*, the method by which there is obtained an oscillatory P.D. between grid and filament in the correct phase and of large enough amplitude for the maintenance of self-oscillations. The variety of methods of grid coupling for this purpose follows the classification of types of coupling considered in Chapter V. The coupling is either **direct** or **mutual**, and under both these heads it may be subdivided into **inductive**, **capacitive** and **resistive** coupling.

Thus the grid excitation in the transmitter already discussed is obtained by mutual inductive coupling. The inter-electrode coupling to the grid in the tuned-anode, tuned-grid circuit is **direct capacitive**.

626. A transmitter with the tuned circuit between anode and grid, and with direct inductive grid excitation, is shown in Fig. 359. An oscillation set up in the tuned circuit produces an oscillatory voltage across the portion of the inductance from G to F, *i.e.*, between grid and filament. The condition for self-oscillations is that this voltage should be in antiphase to the anode-filament oscillatory P.D., *i.e.*, the P.D. across the portion AF of the tuned circuit inductance. This is secured by connecting the filament to a point on the inductance intermediate between the points connected to anode and grid. The point F is obviously always at an intermediate potential between the potentials of A and G. If A is positive to F at any instant, G must be negative to F at that instant, and so the P.D.s from A to F and G to F are always in opposite directions. These are respectively the anode-filament and grid-filament P.D.s, and so V_a and V_g are in antiphase.

An important point to note in this circuit is that the anode-grid inter-electrode capacity is in parallel with the tuned circuit condenser, and so does not behave as an uncontrolled coupling capacity.

As before, the anode tapping point A should be adjustable so that the impedance of the tuned circuit between A and F may be altered to give maximum efficiency without altering the frequency.

Similarly, to obtain the correct value of the grid excitation, the grid tap G should be adjustable, assuming the filament tap F to be fixed.

Under sinoidal conditions, *i.e.*, without a grid condenser and leak, the minimum value of inductance between grid and filament for

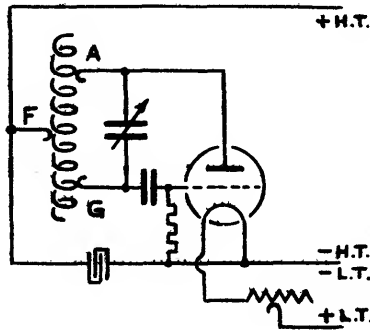


FIG. 359.

maintenance of oscillations may be simply deduced from the formula obtained for mutual inductive coupling. Calling this inductance L_{GF} , the total inductance L , and the inductance between anode and filament L_{AF} , it is obvious that the only difference in the argument of para. 615 is that L_{GF} takes the place of M , and so the formula becomes

$$L_{GF} = \frac{LCR_s}{mL_{AF}} + \frac{L_{AF}}{m}$$

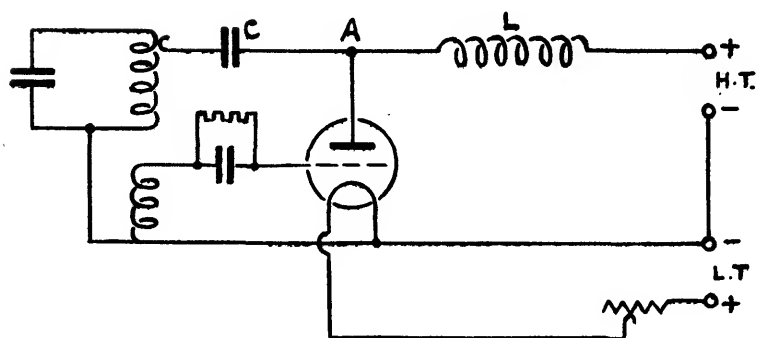
The best value of L_{GF} for efficiency, when a grid leak and condenser are inserted, may be two or three times as great as this.

The methods of applying H.T. voltage to the anode in this circuit, and the modifications these produce, are discussed below under the heading of "feed."

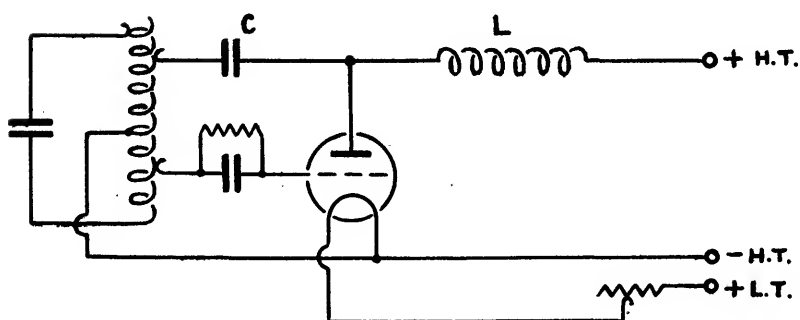
627. Feed.—This is the name given to the method of applying the necessary steady H.T. voltage between anode and filament of the oscillating valve. It is not to be confused with the methods whereby the large H.T. voltages required for transmitting valves are produced. This will be dealt with under rectifying valves.

Two types of feed are commonly used in valve transmitting circuits:—

- (1) **Series Feed.**—The H.T. supply, the valve and the tuned circuit are in series. Examples are the transmitters shown in Figs. 348 and 359.
- (2) **Parallel Feed.**—The valve and the tuned circuit are in parallel across the H.T. supply. Fig. 360 (a) and (b) shows the transmitters of Fig. 348 and Fig. 359 respectively adapted for parallel feed.



(a)



(b)

FIG. 360.

628. Parallel Feed.—Two additional components are rendered necessary by this type of feed, the choke L , known as the **anode choke**, and the condenser C , known as the **anode blocking condenser**.

The anode blocking condenser is necessary to prevent a short circuit of the H.T. supply through the tuned circuit inductance. It is large enough to be regarded as of negligible impedance to radio frequency currents.

The function of the anode choke is as follows :—

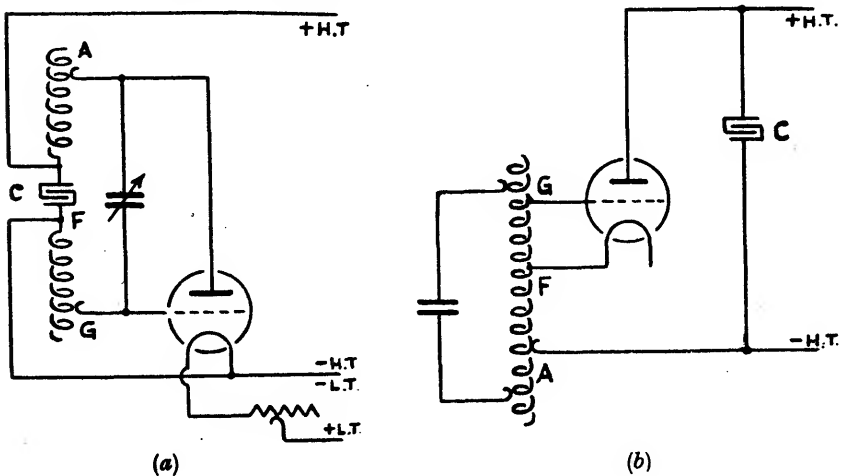
The H.T. supply is in parallel with the tuned circuit across the valve. Being of comparatively low resistance, it would constitute practically a short circuit for the tuned circuit as regards oscillatory currents, and introduce such damping losses as would effectually prohibit the maintenance of oscillations. The introduction of a large choke in series with the H.T. supply increases

the impedance to radio frequency currents of this parallel path sufficiently to render its damping effect negligible, and operates so as practically to confine the radio frequency currents to the valve and tuned circuit. There will, of course, be a small radio frequency current through the choke and H.T. supply corresponding to the oscillatory P.D. across the tuned circuit and valve. In practice, the H.T. supply will be paralleled by a large condenser to by-pass this small radio frequency current.

The choke thus enables the anode to adjust itself to its necessary radio frequency voltage changes, without robbing the tuned circuit of excessive radio frequency current.

There is also a mean anode current flowing through the valve, as has been shown by the analysis given in earlier paragraphs. To this the choke presents a negligible impedance.

629. Series Feed and Direct Inductive Grid Excitation.—In series feed it is still more important that the H.T. supply should be by-passed by a large condenser, for otherwise the whole of the oscillatory make-up current would flow through it. Two methods of inserting the feed and its by-pass condenser are shown in Fig. 361. Another method was shown in Fig. 359.



Series Feed.

FIG. 361.

In Fig. 361 (a) the inductance is in two portions separated by a large condenser. The H.T. positive terminal is connected to the anode side of this condenser, and the other plate is connected to filament. The reactance of this condenser is negligible at radio frequencies, but it effectually isolates the H.T. positive terminal from the grid, and, of course, by-passes the H.T. supply.

The function of the anode tapping point is served in this circuit by adjusting the ratio of the two portions of the inductance. This

adjustment also provides a method of varying the amount of grid excitation, *i.e.*, the oscillatory voltage between G and F, to obtain greatest efficiency.

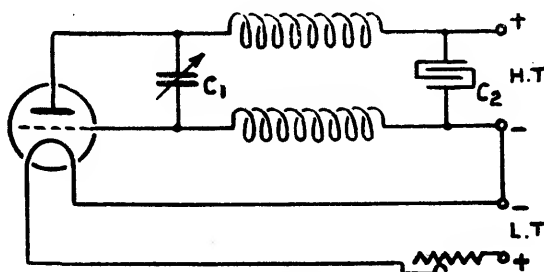


FIG. 362.

This circuit, which is of common occurrence in Service wireless practice, is usually shown in a diagrammatic manner as in Fig. 362. This arrangement of the two halves of the inductance and the two condensers conveniently "labels" the circuit, and makes it easily recognisable. It is often known as the "divided" circuit.

630. In Fig. 361 (b) the H.T. positive terminal is directly connected to the anode. As in Fig. 361 (a), the condenser C is a large by-pass condenser whose reactance at radio frequencies is negligible. In consequence, the point A and the anode may be regarded as common, as in the forms of this circuit discussed immediately above, when oscillatory potentials only are considered. The D.C. potentials of the anode and the point A are, of course, very different, since the steady potential of the anode is that of the H.T. supply, and the point A is earthed, but there can be practically no oscillatory variation of anode potential. The electrode whose potential actually undergoes the large oscillatory variation (of amplitude approaching the H.T. voltage when the transmitter is adjusted efficiently) is the filament. This type of feed accordingly leads to another important practical distinction between transmitters.

(1) Transmitters in which the **anode potential undergoes oscillatory variation of large amplitude**. This has been the case in all the previous types discussed.

(2) Transmitters in which the **large oscillatory potential variation occurs at the filament**, as in the case just considered. This involves in practice that the filament heating circuit must be well insulated from earth, a condition which is most easily fulfilled by an alternating heating current provided via the secondary of a transformer.

In both cases, of course, the high D.C. potential of the anode involves efficient insulation of that electrode.

The instantaneous potentials of the three electrodes in case (1) have already been illustrated in Fig. 356. These instantaneous

potentials in case (2) are shown in Fig. 363 (a). The corresponding anode-filament and grid-filament P.D.s for this case are exhibited in Fig. 363 (b) to impress the point that there is no difference in fundamental principle in the two cases. The derivation of Fig. 363 (b) from Fig. 363 (a) is obvious.

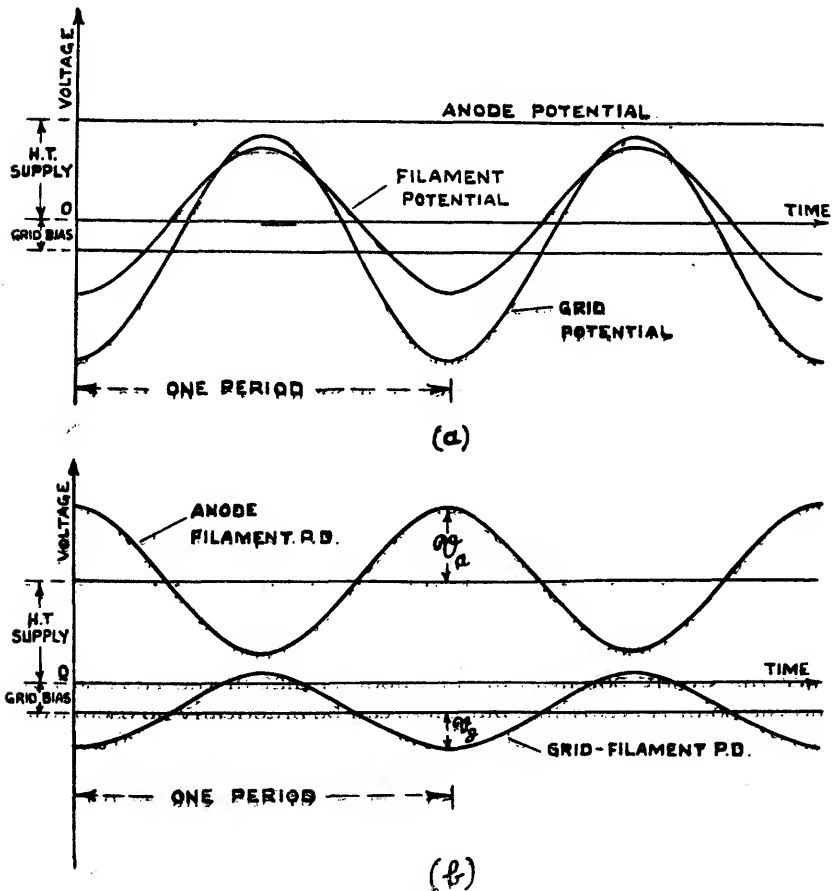


FIG. 363.

631. Excitation of Aerial Circuit.—The consideration of the tuned circuit has hitherto been confined to the question of how an undamped radio frequency current may be maintained in it. We now pass to the question of how the oscillatory energy thus obtained is to be made available for radiation as electromagnetic waves into space.

(1) The tuned circuit itself may be employed for this purpose, *i.e.*, it may be in the form of an open oscillatory circuit. The condenser then corresponds to the distributed aerial capacity to

earth, and the inductance to the natural aerial inductance and artificial inductances in series. This may be called **direct aerial excitation**.

A Service transmitting circuit of this type is shown in Fig. 364. With reference to the points of transmitter design discussed above, its description is as follows :—

- Tuned circuit between anode and grid.
- Direct inductive grid excitation.
- Series feed.
- Filament at high oscillatory potential.
- Direct inductive aerial excitation.

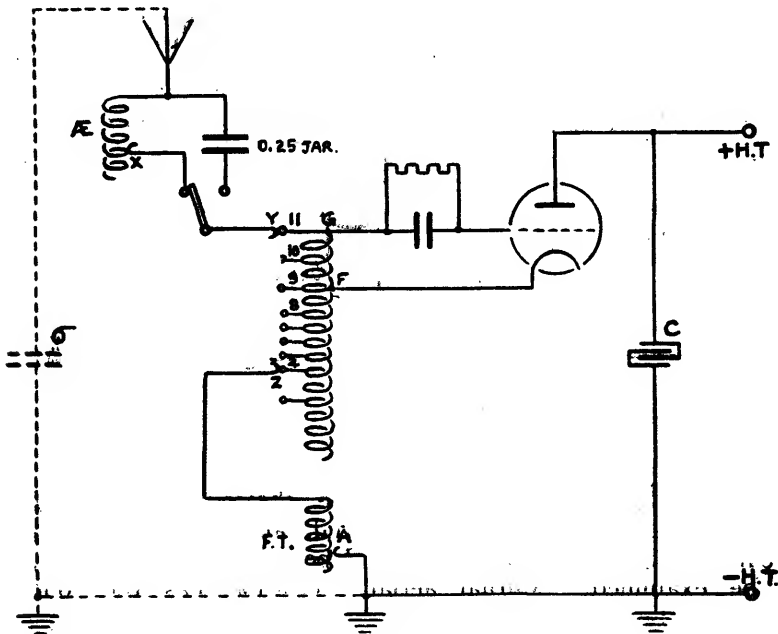


FIG. 364.

In this transmitter, efficiency has to some extent been sacrificed for simplicity of adjustments, as no provision is made for varying the grid excitation independently.

The 0.25 jar condenser in series with the aerial capacity is the ordinary series condenser for higher frequencies. The filament heating circuit has been omitted to simplify the diagram. As previously remarked, the heating circuit must be adequately insulated.

(2) The aerial circuit may be a separate open oscillatory circuit coupled to the valve tuned circuit, which is then a closed oscillator. This may be described as **mutual aerial excitation**. The coupling is usually mutual inductive.

Fig. 365 shows a transmitting circuit of this nature.

This circuit would be classified as follows :—

Tuned circuit between anode and filament.

Mutual inductive grid excitation.

Parallel feed.

Anode at high oscillatory potential.

Mutual inductive aerial excitation.

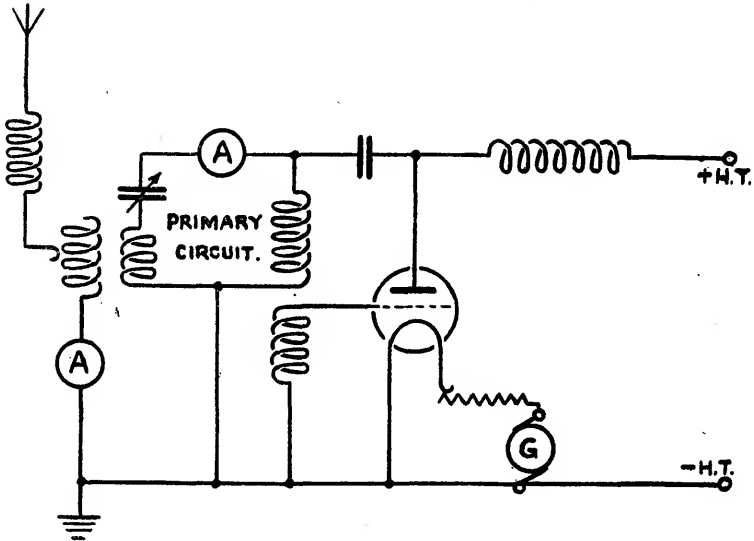


FIG. 365.

632. Mutual Aerial Excitation.—The choice between a closed oscillatory circuit with coupled aerial, and an open oscillatory circuit directly excited by the valve depends on certain factors, now to be examined. Mutual aerial excitation has the following advantages :—

- (a) Valves are constructed so as to be capable of dissipating a certain amount of power at the anode with safety. This is known as the valve rating (*see* para. 537). The insulation of the aerial (if an open oscillatory circuit is employed) may sometimes fall to a very low value. This may occur in submarines with the deck insulator "washing over," or in surface ships during heavy rain, and the load thus thrown on the valve may be so great that the aerial stops oscillating. Under these non-oscillatory conditions the whole of the power taken from the H.T. supply is dissipated at the anode of the valve, instead of a percentage of it (say 30 per cent. to 50 per cent.), and the valve may be burnt out. If, however, a mutually coupled aerial circuit is used, the closed circuit

continues to oscillate and to absorb its share of the power. The closed circuit is always trying to produce forced oscillations in the aerial, and hence tends to dry off the moisture from the aerial insulators.

- (b) The inevitable occurrence of harmonics in a self-oscillatory circuit with negative grid bias has already been emphasised. With direct aerial excitation, energy at the frequency of these harmonics is radiated without any limitations, and produces undesirable interference.

When mutual excitation is employed, the aerial circuit is naturally tuned to the first harmonic, and so presents considerably less impedance to the E.M.F. induced in it at this frequency than it does to the E.M.F.s induced by the higher harmonic currents flowing in the closed oscillatory circuit. Thus the higher harmonic currents in the aerial circuit are considerably reduced in amplitude compared with the first harmonic current, and correspondingly less energy is radiated at these unwanted frequencies.

In addition, the proportion of harmonic to fundamental current flowing in the inductive branch of the primary circuit is considerably less than the corresponding proportion in the make-up (anode) current, for the capacitive branch of the primary tuned circuit presents less reactance to these higher frequency currents than does the inductive branch.

- (c) The question of the variation of the natural frequency of self-excited oscillations will be considered in greater detail when master oscillators are discussed. It may be pointed out now, however, that one common cause of such variation arises through changes in aerial capacity. These changes may be due to the aerial swaying with the wind, or to alterations in the "earth," such as are occasioned by training a turret on board ship, or proximity in dock to moving machinery, *e.g.*, cranes and sheer legs. With direct aerial excitation these changes in capacity are directly operative in altering the LC value of the oscillatory circuit, and therefore the transmitted frequency. They also alter the $\frac{L}{C}$ ratio of the tuned circuit, and so affect the oscillatory power transferred from the valve.

With a closed oscillatory circuit and coupled aerial, variations in aerial capacity still operate indirectly in the same way, but their effect then depends also on the coupling factor, as may be seen from the discussion in

Chapter V of oscillations in coupled circuits. With loose coupling the variations in frequency due to this cause may therefore be greatly reduced. Mutual aerial excitation is particularly advantageous at high frequencies where an untuned coupled aerial is usually employed. In this case, variations in aerial capacity have a completely negligible effect. With direct aerial excitation, variations in aerial capacity would then produce the greatest actual changes in the transmitted frequency, the percentage variation remaining the same.

The obvious **disadvantage** of mutual aerial excitation is that two circuits are maintained in oscillation with a corresponding increase in damping losses. Also, the looser the coupling, the smaller is the amount of energy transferred to the aerial circuit. Nevertheless, fairly loose coupling must be employed, one reason being the advantage in minimising frequency changes as mentioned above. Of more importance, however, is the avoidance of a phenomenon known as "**frequency jump**," which occurs in valve transmitters with tightly-coupled aerial circuits.

This may be compared with the double frequency effect in coupled spark transmitters. The distinction is that a coupled valve transmitter cannot oscillate on both these frequencies at once, though it may oscillate on either, according to the total equivalent impedance of the two tuned circuits. The valve oscillates on whichever of the two frequencies corresponds to the lower equivalent impedance.

It is impossible, however, to keep the primary and aerial circuits exactly in tune. The natural frequency of the primary circuit depends to some extent on the valve constants and the potentials of the electrodes, and that of the aerial circuit varies with the aerial capacity. The difficulty of keeping all these factors constant produces the result that both the primary and aerial circuits vary to some extent about the frequency to which they are both originally tuned, and so the equivalent impedance also varies. Thus, when oscillations are taking place on one of the two possible frequencies, the variation of equivalent impedance may result in the impedance at the other possible frequency becoming less than that of the actual frequency of oscillation, although it was previously greater. The frequency of oscillation then suddenly alters to this other possible value, giving a "frequency jump" equal to the difference between the two possible frequencies. This is accompanied by sudden changes in the values of the primary and aerial currents, which enable the effect to be detected.

This phenomenon is only noticeable with tight coupling, where there is a large difference between the two possible frequencies of oscillation. It is obviously undesirable, and should be avoided by always employing loose coupling.

633. Drying-Out Circuit.—The disadvantage arising in directly-excited aerial circuits due to intermittent wetting of the aerial insulation, and the consequent risk of cessation of oscillations and damage to the valves, may be obviated by a "drying-out" circuit such as that shown in Fig. 366.

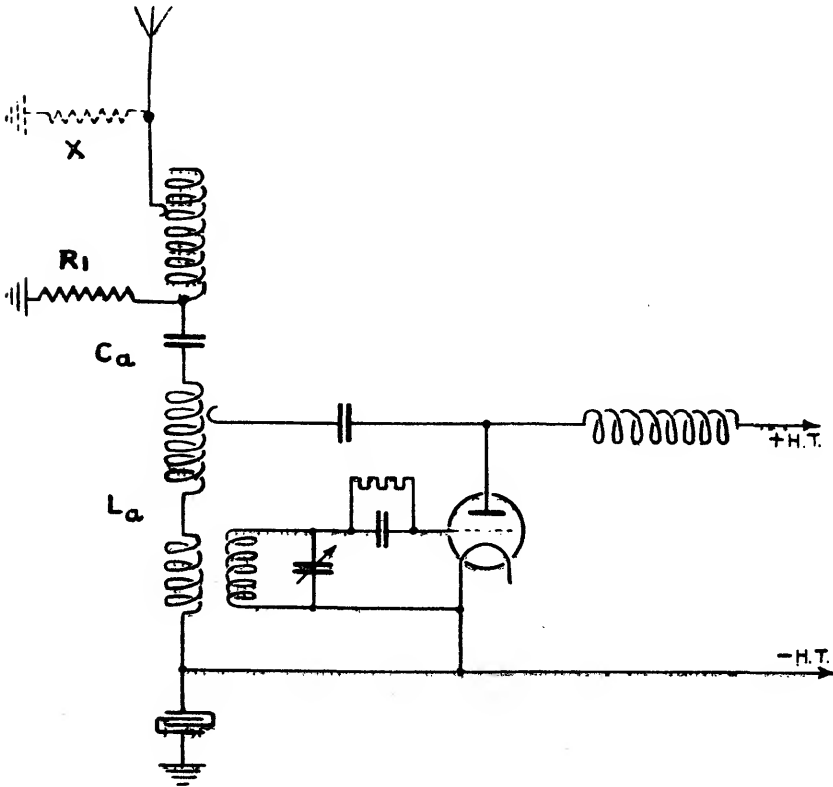


FIG. 366.

It consists of a series acceptor circuit ($L_a C_a$) introduced into the aerial circuit, and a connection to earth through a resistance (R_1) from the junction of these two circuits.

The introduction of the acceptor circuit $L_a C_a$ has provided a circuit which will oscillate even if the aerial circuit is earthed at the deck insulator.

Now the aerial circuit is resistance-coupled to this circuit by reason of the P.D. across the resistance R_1 .

If the deck insulator is earthed by salt water over its surface as indicated at X, the high frequency P.D. across it will cause a leakage current to flow through this moisture, tending to dry off the insulator. At the same time, provided the aerial and acceptor circuits are in resonance, oscillations will be forced on the aerial circuit.

As the deck insulator is a point of high potential in the aerial circuit, the moisture is rapidly dried off, once the aerial oscillations are set up.

This loss by leakage is equivalent to a damping loss R_2 introduced into the aerial.

The circuit is similar to that described in Chapter IX (para. 490). As pointed out there, the joint resistance of R_1 and R_2 is equal to $\frac{R_1 R_2}{R_1 + R_2}$, and can never be greater than R_1 , however great the aerial damping.

634. Multivalve Arrangements.—The heat capable of being dissipated at the anode of one valve is limited and, therefore, so is the amount of oscillatory power available for radiation. To increase the power in the aerial circuit, the number of valves must be increased, and this raises the question of their relative arrangement. There are three possible valve dispositions:—

- (1) In series.
- (2) In parallel.
- (3) In push-pull.

The first of these involves the necessity of providing an H.T. voltage supply as many times greater than that necessary for one valve as there are valves in series. It is, therefore, never used in practice unless the existing supply voltage is much too high for one valve, and will not be further considered. Parallel and push-pull arrangements of valves are, however, frequently encountered.

635. Valves in parallel.—An arrangement of valves in parallel is illustrated in Fig. 367.

The anodes are joined in parallel, with the same H.T. voltage applied to each. In Fig. 367 (a), parallel feed is shown, and series feed in Fig. 367 (b).

The grids are connected in parallel to the grid excitation coil. In one case they are shown with separate insulating condensers and leaks, in the other case this fitting is common. This depends on the nature of the valves used. The filaments are normally provided with separate rheostats, but this precaution is rapidly being dispensed with, owing to the improved similarity of performance of modern transmitting valves.

If two valves in parallel have the same valve constants r_a , g_m and m , it follows that:—

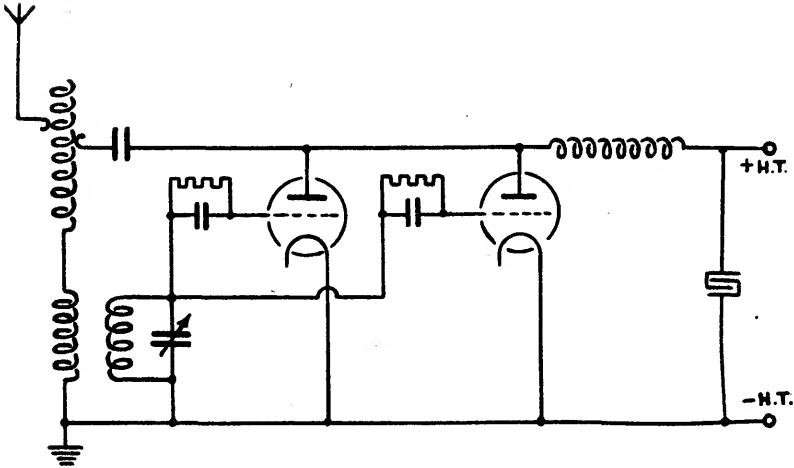
- (a) Their joint A.C. resistance is $r_a/2$.
- (b) Their joint mutual conductance is $2g_m$, for the same applied grid voltage gives twice the total anode current through the output circuit that would be obtained from either valve singly.

(c) The amplification factor of the combination

$$= r_a/2 \times 2g_m = r_a g_m = m,$$

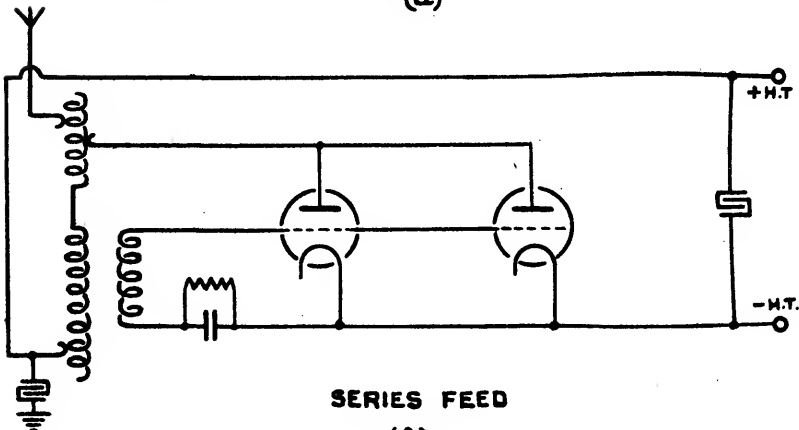
and so is the same as the amplification factors of the individual valves.

(d) The amount of heat capable of being dissipated at the anodes is doubled, *i.e.*, the anode rating of the combination is twice that of the individual valves; thus the power that can be developed safely in the oscillatory circuit is doubled.



PARALLEL FEED

(a)



SERIES FEED

(b)

Transmitting Valves in Parallel.

FIG. 367.

If the damping losses in the oscillatory circuit are equivalent to a resistance R , and the R.M.S. circulating current is I_1 with one valve, the oscillatory power transferred to the aerial when continuous oscillations are maintained is $I_1^2 R$.

If the corresponding oscillatory power available with two valves in parallel is $I_2^2 R$, then from the results deduced above,

$$I_2^2 R = 2I_1^2 R$$

$$\therefore I_2^2 = 2 I_1^2 \text{ and } I_2 = \sqrt{2} I_1$$

Thus the amplitude of oscillatory circulating current is increased $\sqrt{2}$ times.

With n similar valves in parallel, the amplitude of the circulating (aerial) current is \sqrt{n} times that obtained with one valve.

The decrease in A.C. resistance of this arrangement necessitates a corresponding decrease in the impedance of the oscillatory circuit to maintain the most efficient conditions, *i.e.*, the position of the anode tapping point must be altered. Under sinoidal conditions, the formula for the amount of inductance " L_1 " between anode and filament, *viz.*, $L_1 = \sqrt{LCR r_a}$, allows the change in L_1 to be deduced.

With two valves in parallel, r_a becomes $\frac{r_a}{2}$, and therefore " L_1 " becomes $\frac{L_1}{\sqrt{2}}$, *i.e.*, the anode tapping point must be lowered.

With n valves in parallel, L_1 would have to be reduced to $\frac{L_1}{\sqrt{n}}$ to preserve maximum efficiency.

Alternatively, it may be considered that as the circulating current has increased \sqrt{n} times, the inductance between anode and filament must be decreased \sqrt{n} times to preserve the same anode-filament P.D. ($\omega L I_L$).

It is obvious that the anode tapping point cannot be lowered indefinitely, and in practice a point is eventually reached at which the phase relationships necessary for the maintenance of oscillations cease to be preserved. The number of valves that can be used in parallel to increase the power of a transmitter is thus limited.

Another difficulty that arises with increasing number of valves is the difficulty of preventing parasitic oscillations. The inductance of the common valve leads and the valve inter-electrode capacities and other stray capacities provide oscillatory circuits of high natural frequency. H.F. parasitic oscillations may therefore be set up which rob the main oscillatory circuit of energy.

636. Push-Pull Transmitting Circuits.—A push-pull arrangement of valves, as opposed to an arrangement of valves in parallel, necessitates some changes in the relative disposition of the valves and the tuned circuit. These adjustments, however, introduce no new

principle, and follow simply from the explanations already given of push-pull amplifier stages, and of transmitting circuits with one valve. A simple push-pull transmitter is shown in Fig. 368 (a). The tuned circuit is connected between the anodes of the two valves, and the middle point of the tuned circuit inductance is common with the filaments. This is the ordinary push-pull output circuit as found in amplifiers. The condition for self-oscillations already encountered, that in each valve V_1 and V_2 must be as nearly in antiphase as possible, has also to be satisfied. Direct grid excitation is used in this transmitter, and so the grid tap and the anode tap of each valve must be on opposite sides of the common filament tap.

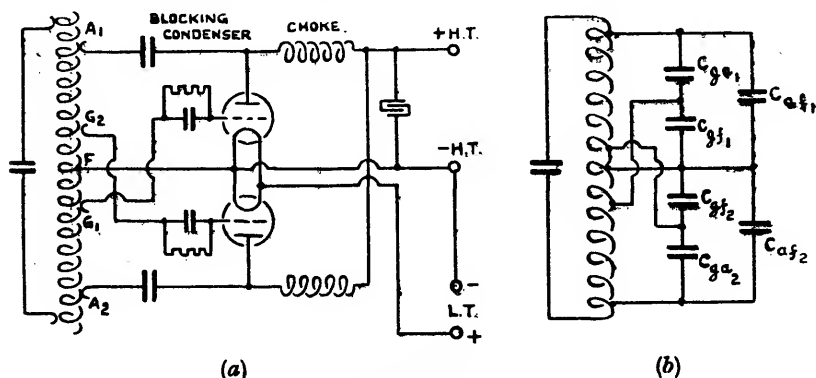


FIG. 368.

Looking at the disposition of the tuned circuit with regard to either valve, the circuit is seen to be exactly of the type described as tuned circuit between anode and grid, direct inductive grid excitation. The feed as shown is parallel, and the anode is the electrode of high oscillating potential. The oscillatory circuit, as in previous cases, may either be the aerial circuit or be mutually coupled to an aerial circuit.

A push-pull circuit of the type described as tuned circuit between anode and filament, mutual inductive grid excitation, series feed and high oscillating potential anode, is shown in Fig. 369 (a). This should be compared with Fig. 352, which shows the same type of circuit with one valve. A large condenser is inserted in the filament lead, as shown, to prevent a short circuit of the H.T. supply.

The use of a grid condenser and leak to increase the efficiency of transmitting circuits involves in the push-pull arrangement that the grid bias of each valve approximates to the condition described as "curvature biasing" in Chapter XIII. When efficient self-oscillations are being produced, the positive half-cycle of grid voltage in each valve sweeps over the whole of the straight part of the dynamic characteristic of that valve. Thus the circulating

current in the oscillatory circuit corresponds in wave form to the output oscillatory current shown in Fig. 337. This approximates very nearly to a simple sine curve, and the distortion produced in the wave form with one valve under the same conditions (Fig. 358) is avoided. In other words, the higher harmonic currents, and particularly the second harmonic, which was shown to be principally responsible for this distortion, are almost completely absent. The result is that a much purer wave is radiated, while the higher efficiency of the grid condenser and leak arrangement is preserved.

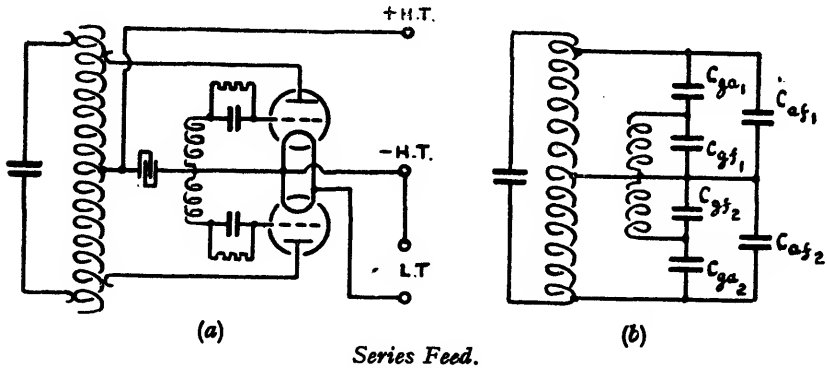


FIG. 369

Another advantage over the parallel arrangement is the diminution of the nett inter-electrode capacity. With two valves in parallel, the corresponding inter-electrode capacities of the individual valves are, of course, also in parallel, and so their effect is doubled. In the push-pull arrangement it will be seen by inspection of the equivalent tuning diagrams of Figs. 368 (b) and 369 (b) that corresponding inter-electrode capacities are halved. Changes in these capacities during transmission thus have a much smaller effect on the frequency in the push-pull arrangement. This is more important at high frequencies, where the valve capacities are a larger proportion of the total capacity of the tuned circuit.

MASTER OSCILLATORS.

637. Frequency Variation.—In the elementary treatment of the valve as a generator, given in the earlier paragraphs of this chapter, the frequency of the self-oscillations was taken as the natural frequency of the oscillatory circuit, $f = \frac{3 \times 10^4}{2\pi\sqrt{LC}}$ kc/s., where L is in mics. and C in jars.

This is only a first approximation to the truth, and more exact analysis shows that the frequency depends, in addition, on the valve constants r_a and g_m . It was pointed out in Chapter XI

that these could only be regarded as at all constant over the straight parts of the characteristics, and even then their values are slightly different for differing steady potentials of the anode and grid. Further, the inter-electrode capacities, as indicated above in discussing the push-pull arrangement of valves, must also be taken into account when considering the oscillatory circuit capacity. When the tuned circuit is between anode and grid, for example, the anode-grid inter-electrode capacity is in parallel with the tuning condenser. Now these inter-electrode capacities depend on the space charge in the valve—*i.e.*, on the P.D.s between the electrodes and on the filament emission (para. 553)—and changes in any of these quantities alter the values of the capacities.

It follows from both these causes that a constant frequency cannot be obtained unless the steady potentials of the anode and grid, and the filament emission, are kept constant. In practice some fluctuation is inevitable, and the frequency varies correspondingly.

Another cause of frequency variation is the variation in value of the tuned circuit inductance and capacity. The variation of aerial capacity has already been remarked, and in a ship, where the whole transmitting set is on a moving base, changes in the components are difficult to avoid.

Thus in all self-oscillatory sets, where the transmitted frequency depends on the electrical properties of the circuit, frequency variation is likely to take place. This was not a matter of primary importance in the early days of wireless communication, for the comparatively small number of frequencies required enabled their values to be fairly widely spaced. Under modern conditions, however, the increasing "congestion of the æther" renders it imperative that the frequency of a transmitting set should only vary within narrow limits from what it purports to be, if interference is to be avoided. For instance, it was previously mentioned that plain aerial and tightly-coupled spark transmitters had been prohibited by International Convention for this reason. It is also obvious that frequency variation renders reception more difficult; this is particularly the case for C.W. (heterodyne) reception at high frequencies.

638. Master Oscillators.—It is not possible, for the reasons given above, to obtain a frequency sufficiently constant for modern requirements with a high-powered self-oscillatory transmitter, and some other method must be devised of controlling the frequency. The general principle of the method adopted is to generate the self-oscillations in a circuit which allows of their frequency being kept constant. The oscillatory voltage produced at this frequency is then applied to the grid of a valve or valves capable of giving sufficient output power for the transmission required. The oscillations in the power output stage are thus forced oscillations, and not

free oscillations as in the self-oscillatory generators previously considered, and they take place at the frequency of the generating circuit. Alterations in the resonant frequency of the output stage then merely produce variations in the power output and do not affect the frequency. This power stage is thus essentially the same in principle as the power amplification stage in a receiving circuit, though the power output is, of course, enormously greater. The circuit in which self-oscillations are initially generated is called the **Master Oscillator**, the reason for the name being obvious. The distinction between self-oscillatory and master oscillator transmitting circuits may also be compared with that between self and separately-excited dynamos.

639. The simplest type of master oscillator is merely a rigid low power self-oscillatory circuit, in which special precautions are taken to avoid variation in the electrical quantities which affect the frequency. The use of any properly-adjusted master circuit ensures in the first place that no power is transferred from the master circuit to the aerial circuit, and *vice versa*. Thus the effects of variations in aerial capacity and inductance on the frequency are eliminated. Variations in frequency in the master circuit itself are minimised by paying particular attention to such points as smoothing of the H.T. supply voltage to the anode, choice of values of grid condenser and leak, filament supply regulation, and by using valves which need not be worked up to the limit of their rating to give the power required in the master circuit. All components should be rigid in design, and the whole circuit should be mounted in such a way as to prevent mechanical vibration of its parts. A flat oscillatory circuit, *i.e.*, one with a large ratio of capacity to inductance, ensures that residual variations of inter-electrode capacities, after the above measures have been applied, produce the least proportional effect on the frequency. This is most easily achieved, of course, at low frequencies, and in general it may be said that master circuits of this type require more careful design as the frequency increases.

The master circuit must be carefully screened from the main power stage, and the components of both circuits arranged so as to minimise stray coupling. Inter-electrode capacity coupling must also be neutralised. This point will be dealt with in more detail below.

To give efficient control by the master, the power developed in the main circuit cannot be more than about ten times as great as that in the master circuit.

A transmitting circuit of this type designed for medium frequencies is shown in Fig. 370. The master oscillatory circuit is of the type described as tuned circuit between anode and grid, direct inductive grid excitation, parallel feed, the anode being the electrode at a high oscillatory potential. The main circuit consists

of two valves in parallel, their external anode circuit being the aerial circuit. Parallel feed is also employed for the main power valves. The oscillatory P.D. developed across the portion of the master tuned circuit inductance between the grid and filament taps

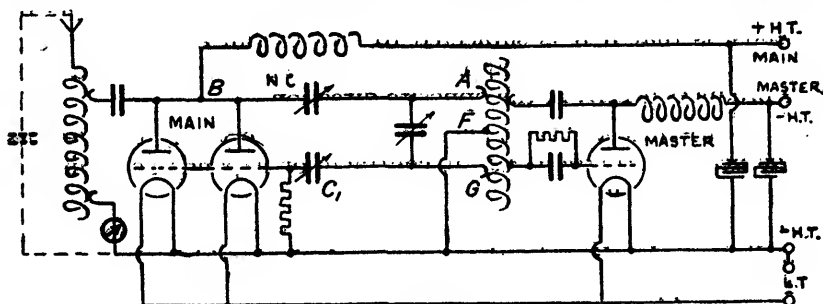
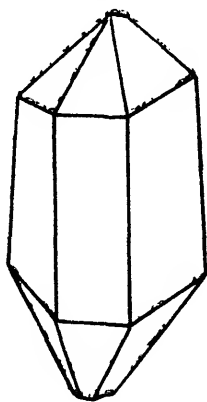


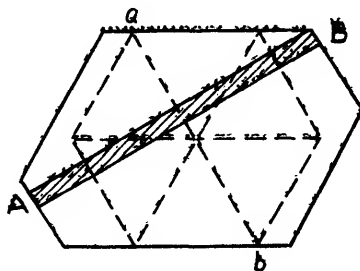
FIG. 370.

is applied to the grids of the power valves through a coupling condenser C_1 . N.C. is the neutralising condenser to balance out inter-electrode capacity coupling between main and master circuits.

640. The other type of master oscillator circuit is one in which an endeavour is made to render the frequency of the self-oscillations independent of the electrical constants of the circuit. The control



(a)



(b)

FIG. 371.

may be exercised, for example, by a tuning fork vibrating at a definite frequency. The commonest master oscillator of this nature makes use of the special properties of certain crystalline materials, notably quartz, and is therefore called "**quartz control.**"

Quartz occurs naturally in the crystalline form shown approximately in Fig. 371 (a). Chemically it is an oxide of silicon (SiO_2) and silica is merely fused quartz. The crystalline structure is of

the type technically known as "hemihedral with inclined faces." Its optical behaviour determines a particular direction in the crystal called the optic axis. This direction is that of any line in the crystal parallel to the line joining its two extremities.

The peculiar property of such a crystal, on which quartz control depends, is that if it is subjected to pressure an electrical P.D. is developed across it. The application of a tension instead of a pressure produces a P.D. in the opposite direction.

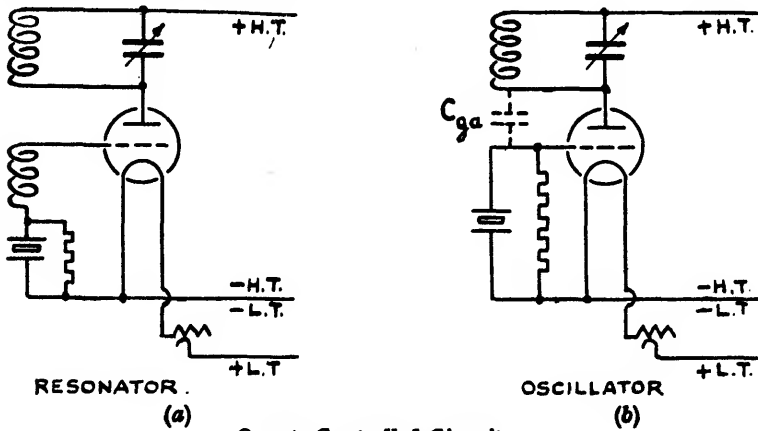
Conversely, if a P.D. is applied across the crystal, it undergoes mechanical deformation, *i.e.*, it contracts or expands according to the direction of the P.D.

These effects are said to be **piezo-electric** and their amount varies according to the direction in the crystal along which the pressure or P.D. is applied. The directions in which the effects are most marked are called the piezo-electric axes, and are shown for a quartz crystal in Fig. 371 (b), which represents a section of a crystal perpendicular to the optic axis. Thus, to obtain the greatest piezo-electric effect a bar or plate is cut perpendicular to one of the piezo-electric axes *ab*, as indicated by AB in Fig. 371 (b).

Suppose now that an oscillatory P.D. is applied across such a plate. This sets up a stress in the plate which is alternately a pressure and a tension, *i.e.*, the plate executes a mechanical vibration. As in the electrical case of an alternating E.M.F. applied to an oscillatory circuit, this vibration may be considered to consist of two components, a forced oscillation at the frequency of the applied P.D., and a free oscillation at the natural frequency of the plate. The frequency of the free oscillation depends only on the mechanical properties of the quartz plate, and not at all on that of the applied P.D., *e.g.*, its value is inversely as the thickness of the plate. The logarithmic decrement of a quartz plate is much smaller than that of any electrical oscillatory circuit, and its response to an applied stimulus is correspondingly more selective. The forced oscillation is of small amplitude, but if the frequency of the applied P.D. is anywhere near the natural frequency of the crystal, conditions approaching mechanical resonance ensue (para. 300), and the amplitude of the free oscillation reaches a large value. This free vibration sets up an oscillatory P.D. of correspondingly large amount across the crystal plate, and this P.D. may be applied between the grid and filament of a valve to maintain electrical oscillations in the valve and its associated circuit. The frequency of such oscillations is the natural mechanical frequency of vibration of the quartz plate, and is independent of the electrical properties of the valve circuit. Thus frequency variation due to changes in these properties is eliminated, and quartz control operates to give a constant transmitted frequency.

641. Two methods of applying quartz control to a transmitting circuit are shown in Fig. 372.

The circuit of Fig. 372 (a) is of the type described as tuned circuit between anode and filament, mutual inductive grid excitation and series feed, but the grid excitation is not sufficient to generate self-oscillations. A damped oscillation would be produced when H.T. was applied to the anode with the filament alight, and an oscillatory P.D. would be developed across the grid coupling coil, but the resulting oscillatory anode current flowing as make-up current to the LC circuit would not supply sufficient energy to maintain continuous oscillations, and they would be damped out. If, however, a quartz plate, cut as described above, be inserted between the grid coil and filament, the initial oscillatory impulse sets it in oscillation, and the piezo-electric P.D. produced gives sufficient grid excitation to maintain continuous oscillations, these taking place at the natural frequency of the crystal.



Quartz Controlled Circuits.

FIG. 372.

A quartz crystal used in this fashion may be considered as electrically equivalent to an acceptor circuit and is said to be used as a **resonator**. The anode oscillatory circuit should, of course, be tuned nearly to the natural frequency of the crystal to produce maximum power. (Due, in part, to the large amount of power required, it is found in practice that oscillations cease when exact resonance is attained.)

It is found in the above type of circuit that the presence of the grid coil has a slight effect on the frequency and, further, when the circuit is to be used over a wide range of frequencies, it may be impossible to avoid sufficient mutual coupling at some frequencies to generate self-oscillations without the interposition of a crystal. In these circumstances the crystal no longer controls the frequency and the circuit of Fig. 372 (b) may be adopted. The coupling between anode and grid circuits is then only that due to C_{ga} , and there is nothing in the grid circuit except the quartz crystal to

affect the frequency. The equivalent electrical circuit of the crystal in this case is a rejector, and the crystal is said to be used as an **oscillator**. The crystal as an oscillator has to supply the whole of the grid filament excitation, and so its response to an applied P.D. must be greater than when it is used as a resonator, i.e., it must be more "lively." In practice it is difficult to produce quartz plates lively enough to act as oscillators without a considerable risk of mechanical fracture in operation.

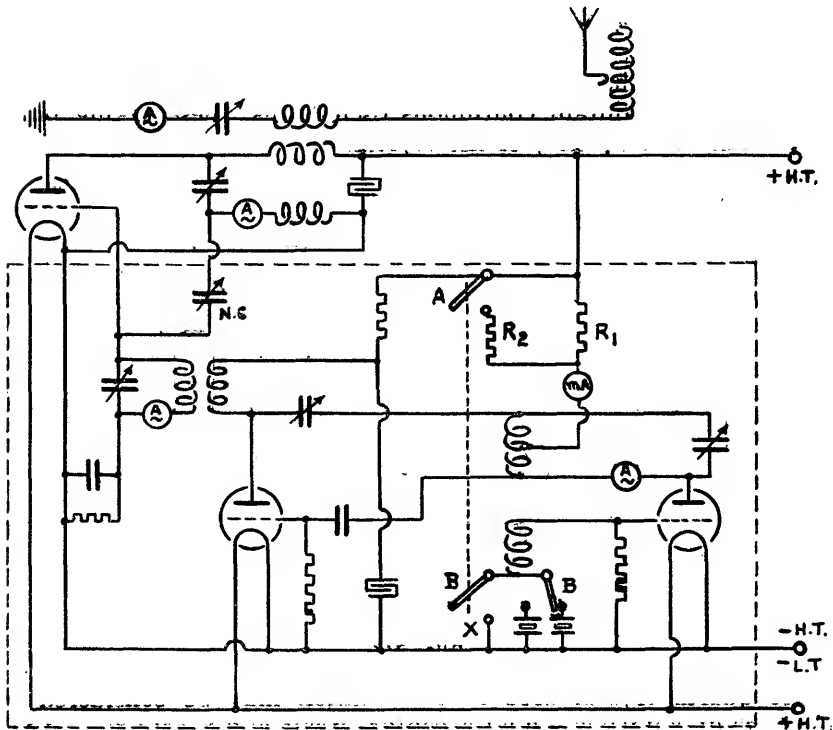


FIG. 373.

The amount of power that can be developed in a quartz-controlled oscillator is limited, and when higher power is required the quartz-controlled circuit must be coupled to a power stage, as described above for the case of the ordinary master oscillator. A circuit of this type is shown in Fig. 373, consisting of a master stage, one amplifying stage and a power stage. Two crystals will be observed, corresponding to two frequencies of transmission. In addition, the master circuit may be converted to a rigid low-power oscillatory circuit at frequencies intermediate between those of the crystals, by making switch B to X. By mechanical coupling of the switches this, at the same time, makes switch A, which is open in the other two positions of B.

It has already been seen that the slope of the valve mutual characteristics increases with the anode voltage (para. 543). With the resistance R_1 alone in the H.T. lead, the steady anode voltage is too low to allow the self-oscillatory condition $\frac{Mg_m'}{C} > R$ (para. 616) to be fulfilled with the values of M (grid coupling) and C obtaining, and the extra grid-filament P.D. provided by the quartz resonator is necessary for oscillations to be maintained. When switch A is made, however, a much smaller resistance, R_2 , is put in parallel with R_1 , and the consequent increase in steady anode voltage renders g_m' large enough for self-oscillations to take place without the necessity for a crystal.

The oscillatory P.D. across the master tuned circuit is applied between grid and filament of the amplifying valve, the voltage output of which is in turn applied by means of tuned secondary transformer coupling to the grid and filament of the power stage. The output circuit of the power stage is a tuned circuit arranged like the customary Service series feed, direct grid excitation, self-oscillatory transmitter. It is not, of course, self-oscillatory in this case. The output stage is neutralised by the tapped output method (para. 601 and Fig. 342). The balancing of the amplifying stage uses the tapped input circuit (para. 601 and Fig. 343).

642. Other points that may be mentioned in connection with quartz-controlled circuits are:—

- (1) With mutually coupled aerial circuits, it is possible to use much tighter coupling without the occurrence of frequency jump, and therefore to get greater power in the aerial. Frequency jump cannot occur until the quartz crystal loses control, and the whole circuit becomes self-oscillatory.
- (2) As harmonics of the natural crystal frequency are produced as usual in the oscillatory anode current flowing through the valve, any one of these may be picked out and amplified in the power stage, thus rendering possible the transmission of several frequencies with one crystal.
- (3) It is found that the natural frequency of a quartz crystal varies with temperature. When a quartz-controlled transmitter is operating, the quartz heats up, and so there is a "temperature drift" of frequency until the quartz reaches its steady working temperature. For very constant frequency it is therefore necessary to control the temperature of the crystal by some thermostatic device. This is particularly important when the crystal-controlled transmitter is to be used in a climate widely different from that in which it was constructed.

Mounting of Quartz Crystal.—The crystal, as indicated diagrammatically in Fig. 373, is mounted between two metal plates.

Oscillations are difficult to maintain if it is mounted vertically. A small air gap must be left to allow for the change in dimensions of the crystal, and the crystal tends to lean on one plate or the other. This gives a tapering air gap at each side, and the thickness of the gap at some point may be such as to set up air vibrations which damp the quartz vibrations. This may be avoided by mounting the crystal horizontally, when it floats between the plates in operation and gives an air gap whose thickness is the same at all points. In this position, however, the crystal readily collects a dust deposit which fills the gap and damps out the oscillations. One solution is to enclose the crystal and plates and keep them in a partial vacuum.

643. Neutralisation of Master Oscillator Circuits.—The difficulties that arise in securing adequate prevention of regenerative reaction between the main and master circuits present a complex problem, and only the elements of it can be considered here. Unless such reaction is eliminated the transmitted frequency may vary even more than would be the case with a self-oscillatory circuit. Any reaction of the main on the master circuit renders the frequency of the master dependent to some extent on the constants of the main circuit, and re-introduces in more virulent form the frequency variations which the use of a master oscillator is designed to avoid. Conditions may also occur in which both the main and master circuits are rendered self-oscillatory. As the natural frequencies of the two circuits are never likely to coincide for any period of time, the effect of self-oscillatory conditions in both circuits is that the oscillations in the main circuit are modulated at the frequency of the master. Three frequencies will then be transmitted instead of one, all three in addition being variable. (This point will be better understood after the explanation of side-bands in Chapter XV.) The necessity for careful screening has already been emphasised, and it is also necessary to neutralise the flow of energy through the coupling condenser and the grid-anode inter-electrode capacities of the power valves. As a typical instance, the neutralisation of the circuit shown in Fig. 370 will be discussed. A neutralising condenser is shown connected between the points B and A. The operation of such condensers has already been explained for amplifying circuits. Fig. 374 shows one aspect of the neutralisation of this circuit, the relevant parts of Fig. 370 being re-drawn as an A.C. bridge with respect to feed of energy from the master, and may be compared with Fig. 343 in Chapter XIII.

It is obviously of the greatest importance in the operation of such a master oscillator circuit to ensure accurate neutralisation, and for that reason the purpose to be kept in mind when adjusting the neutralising condenser will now be treated in greater detail.

Suppose that the master circuit is oscillating and that the filaments of the main power valves are alight, but that their H.T.

supply is not made ; also that the coupling condenser is set at a fixed adjustment. In Fig. 374 there is then an oscillatory P.D. between A and G, and oscillatory current will flow through the two parallel paths presented by the master inductance, AFG, and the neutralising condenser N.C., anode-grid valve capacities and coupling condenser, ABG. One aim of neutralisation is to adjust N.C. so that the points B and F are at the same oscillatory potential as far as this applied P.D. from the master is concerned. If this is ensured, there can then be no oscillatory P.D. across BF from this cause, and therefore no energy can be fed from the master circuit to the output circuit. Hence this condition for neutralisation is that the voltage drop

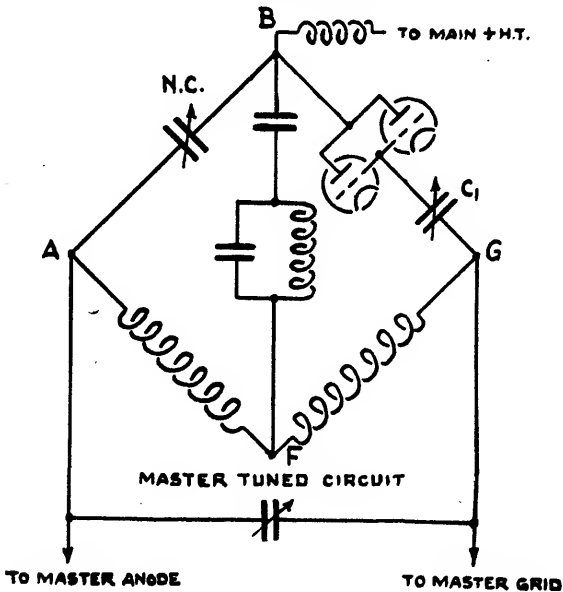


FIG. 374.

across AB should be equal to that across AF (necessitating also the equality of the voltage drops across FG and BG). The neutralising condenser must be adjusted until this condition is satisfied.

Of more importance, however, is the question of feed-back of energy from the main tuned circuit to the master inductance when the main H.T. supply is made. The reactance of the large coupling condenser C_1 is negligible, and so the main circuit (across BF) is in series with the master inductance from F to G is in parallel with the valve capacities between the anodes and grids. This constitutes a possibly self-oscillatory circuit of the type classified as tuned circuit between anode and grid and direct grid excitation, the latter being across the inductance FG. The introduction of the neutralising condenser enables an equal and opposite P.D. to be developed across the other part, AF, of the master inductance. The mutual flux

linkages of the two parts of the inductance are then equal and opposite to their self flux linkages, and the whole coil from A to G is rendered non-inductive (with respect to feed-back of energy from a P.D. across BF). It can easily be seen that the setting of the neutralising condenser to satisfy this condition is the same as that demanded by the conditions of the last paragraph. With this setting therefore, self-oscillation of the main circuit is prevented, and also any reaction of main on master circuit, and *vice versa*.

To obtain the correct adjustment, the following procedure may be adopted, the conditions being as postulated at the beginning of the discussion. When the adjustment is incorrect, there will be an oscillatory P.D. across the main circuit. This circuit is tuned until its ammeter registers maximum current. The neutralising condenser is then adjusted until the ammeter reading falls to zero.

It will be observed that the arm ABG of the bridge is in parallel with the master oscillatory inductance, and so any alteration of either the neutralising condenser or the coupling condenser alters the master tuning. Hence, when neutralising, the main circuit must be re-tuned to give maximum ammeter reading at intervals during the adjustment. When approximate neutralisation has been obtained, the final frequency adjustments of the master and main circuits can be made, and the process repeated.

A factor rendering good neutralisation difficult is the inductance of the leads in the bridge arms containing the neutralising condenser, the power valves and the coupling condenser. When this is appreciable, the setting of the neutralising condenser varies with the frequency.

A common disturbing factor is the occurrence of H.F. parasitic oscillations in the circuit formed by the inter-connection of the valve anodes and grids, as has already been mentioned when considering valves in parallel. Another circuit which may generate parasitic oscillations is provided by the master tuning condenser, and the arm ABG of the bridge in conjunction with their connecting leads. These oscillations may be damped out by inserting resistances in the arms of the bridge. To preserve phase balance in the bridge, the resistances must be adjusted symmetrically in each limb. This introduces damping into the main and master circuits.

Another method is to eliminate the grid excitation of the valves at the parasitic frequency by inserting between the grid and its leak resistance a coil and condenser in parallel, of reactance equal and opposite to the original grid filament reactance at this frequency.

There is then no grid-filament P.D. at the parasitic frequency, and so these spurious oscillations cannot be set up. The use of this method requires an alteration in the setting of the neutralising condenser whenever a change is made in the frequency it is desired to transmit.

Alternatively, it may be considered that this introduction of a coil and condenser in parallel alters the parasitic frequency so as

to make it an odd harmonic of the anode choke resonant frequency. The standing waves (para. 768) then set up in the choke have approximate voltage nodes at each end, and so there is practically no anode voltage variation at the parasitic frequency, i.e., self-oscillations cannot be maintained.

HIGH-TENSION SUPPLY FOR C.W. TRANSMITTERS.

644. In low-power L.F. sets the H.T. voltage required at the transmitting valve anodes may only be of the order of 100 or 200 volts, and can be supplied by accumulators or small D.C. generators. If, however, ranges of hundreds or thousands of miles are required, it is necessary to increase the H.T. voltage to thousands of volts. It has been seen that the power delivered to the oscillatory circuit is proportional to the product of the emission current of the filament and the voltage applied to the anode, and for a given value of the former it therefore increases proportionately with the anode voltage. The limiting factor to an indefinite increase in either or both of these quantities is provided by the anode rating of the valves employed, as with, say, a 66 per cent. efficiency circuit, half as much power as is given to the oscillating circuit is dissipated in the form of heat at the anodes.

Steady potentials of the order of 10,000 volts may be produced by :—

- (a) A motor generator giving a direct current output at very high voltage.
- (b) A motor alternator and transformer, the output from which is rectified, normally by the use of thermionic valves.

The motor generator method has advantages as regards efficiency and space occupied, but under ship conditions it is generally not as reliable as the low voltage motor alternator and transformer. In addition, the commutator sparking on a high voltage motor generator may give rise to serious interference with reception. The majority of Service transmitters use a rectified A.C. supply.

Rectifying valves have been discussed in Chapter XI. Diodes are used for rectification, although it is obvious that triodes could also be employed.

645. Half-Wave Rectifying Circuit.—Fig. 375 illustrates the simplest type of rectifying circuit.

It comprises :—

An alternator and step-up transformer, similar to that described in Chapter VIII. The primary circuit contains a switch which may be the signalling key.

A rectifying valve, generally known as a "U" valve.

A motor alternator and step-down transformer for supplying, and a rheostat for controlling, the heating current to the filament of the rectifying valve.

A smoothing condenser C, of large capacity.

Supply leads to the anode and filament of the oscillating valve.

When the switch in the step-up transformer primary circuit is made, an alternating voltage of large amplitude is applied to the anode of the rectifying valve. If the filament is alight, a current only flows through the valve when there is a positive P.D. from anode to filament, *i.e.*, during the positive half-cycle of alternator voltage. During the negative half-cycle, the anode is negative to the filament and no current can flow. When current is flowing, electrons are being transferred from the filament to the anode, *i.e.*, from one plate of condenser C to the other, and so a P.D. is established across the condenser. Assuming that the lower plate in Fig. 375 is earthed, the potential of the upper plate and the filament

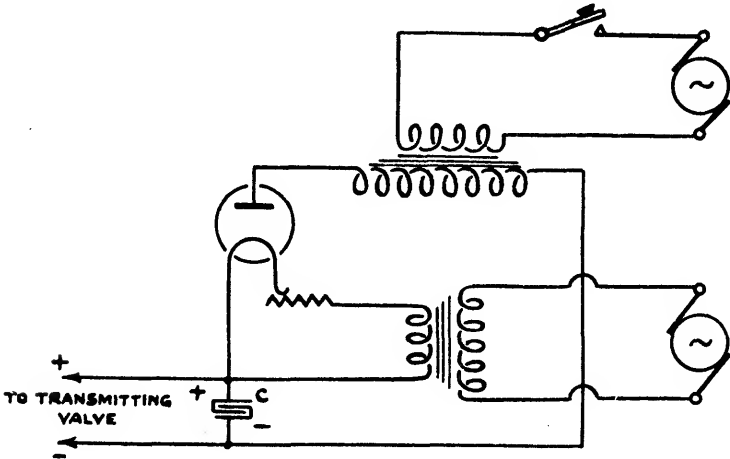


FIG. 375.

thus rises during the positive half-cycle of alternator voltage, and so diminishes the part of this half-cycle during which the anode is positive to the filament, *i.e.*, the time during which the condenser is being charged. If the transmitting circuit is in operation, current is always flowing out of the smoothing condenser to supply the steady anode current taken by the transmitting valves, the condenser being, in fact, equivalent to the H.T. battery in a receiving circuit. At the beginning of the rectifying process, the charge received by the condenser while current flows in the rectifying valve is greater than the charge it loses to the transmitting valves during one cycle of alternator voltage. As the P.D. across its plates rises, however, and the charge received by it decreases, a state of equilibrium is eventually reached when the anode of the rectifying valve only remains positive to the filament for such time as will suffice to make up for the loss of charge to the transmitting valves per alternator cycle. The P.D. across the condenser then maintains a

sensibly constant value, apart from a slight ripple due to the fact that it is charged discontinuously while its discharge through the transmitting valves is continuous.

If the mean condenser P.D. were to rise above this equilibrium value, the charge it would receive would not be sufficient to make up for the losses to the transmitting valves, and the P.D. would fall again. If it fell below the equilibrium value, the current, flowing through the rectifying valve for a longer time, would charge the condenser faster than it discharged, until the equilibrium state was again reached.

The electron path should be noticed. Starting from the positive condenser plate, electrons travel from filament to anode of the rectifying valve and through the transformer secondary to the filaments of the transmitting valves, thence through these valves back to the positive condenser plate.

In early transmitting circuits, use was made of rectifying valves of comparatively small filament emission, with the result that the rectifier current reached saturation almost as soon as the anode-filament P.D. became positive, and remained at its saturation value practically until the anode potential again fell below that of the filament. This system was obviously inefficient, for once the anode-filament P.D. was sufficient to give saturation, no increase in rectifier current was obtained during the time that the anode-filament P.D. was greater than this value. With the demand for ever increasing power in the aerial, it has become necessary to work the alternator and transformer as nearly as possible at their maximum output. Hence, the filaments of rectifying valves are now designed to give sufficient emission to prevent saturation ever being reached during the rectifying cycle, and the current through the rectifying valve increases as long as the anode-filament P.D. increases.

With the circuit shown in Fig. 375, rectified current only flows during part of the positive half-cycle. The process is therefore called half-wave rectification. The action is illustrated in Fig. 376.

The transformer secondary voltage is shown for simplicity as a simple sine wave. This is not the case in practice owing to the irregular power output during a cycle. This point is considered below (para. 648).

The current flowing through the rectifying valve when the anode is positive to the filament obeys the $3/2$ power law (para. 530), as indicated in the figure. The shaded area represents the charge acquired by the smoothing condenser during one cycle of alternator voltage. The mean condenser P.D. is shown by the dotted line, while the continuous line indicates how its P.D. actually varies during a cycle. The discharge to the transmitting valves goes on steadily, and when the condenser is not being charged, its P.D. falls uniformly according to the steady anode current in the transmitters.

During charging, the condenser P.D. rises approximately as indicated, the rate of charge being a maximum when the anode-filament P.D. of the rectifying valve is a maximum, if the lag due to the inductance of the transformer secondary is neglected. This H.T. voltage ripple is one of the factors leading to frequency variation in transmitters (para. 637). It can easily be seen that its amount

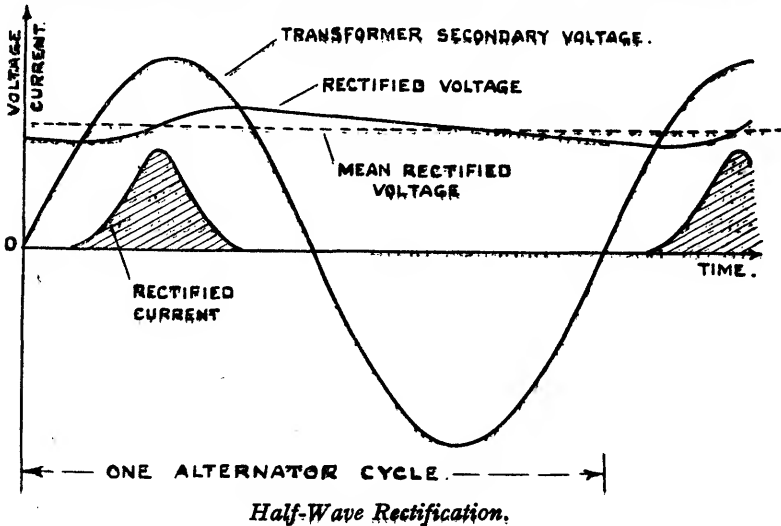


FIG. 376.

varies inversely with the capacity of the smoothing condenser, since the smaller this capacity the greater is the change in P.D. produced by a given charge ($V = \frac{Q}{C}$). The greater the capacity of the smoothing condenser, the longer does it take for steady conditions to be reached after the circuits are first switched on, and this may form a limitation to its size in high speed signalling, but normally the upper limit is fixed by considerations such as dimensions and cost, and the amount of ripple that can be allowed.

646. Full-Wave Rectification.—This is the system normally employed in practice. The circuit is shown in Fig. 377 (a). Two rectifying valves are used. Their anodes are connected to the terminals of the transformer secondary, the centre point of the secondary being earthed and therefore common with the negative plate of the smoothing condenser. Thus the alternating potential applied to the anode of each valve is half the total secondary voltage, and maximum positive potential on one anode corresponds in time to maximum negative potential on the other. Consequently, current flows in the two valves during alternate half-cycles, the rectifying action of either valve being the same as that described above. The process is illustrated in Fig. 377 (b).

The smoothing condenser receives a charge during every half-cycle of alternator voltage, instead of once per cycle as in half-wave rectification. The condenser voltage ripple is therefore less pronounced and occurs at twice the frequency of the alternator.

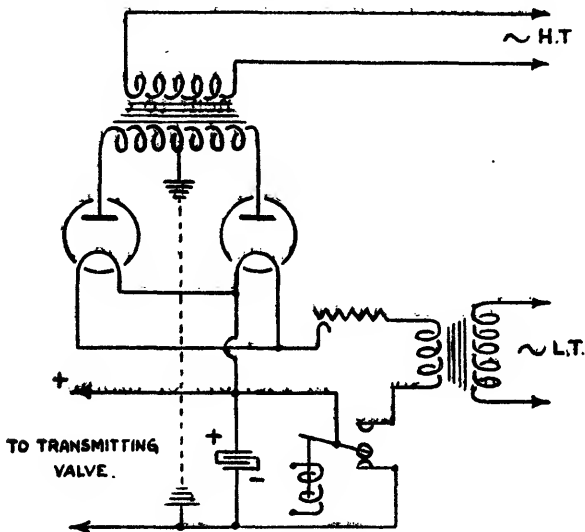
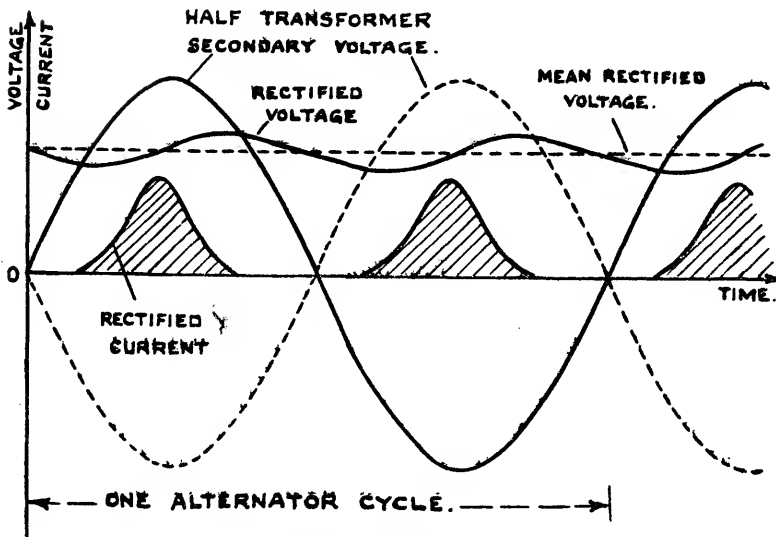


FIG. 377 (a).



Full-Wave Rectification.

FIG. 377 (b).

647. Filament Arrangements.—As the rectifier filaments are at a high potential to earth, they and their heating circuit must be carefully insulated. This is most easily accomplished by using an A.C. heating current from a motor alternator and step-down transformer, as shown in Figs. 375 and 377. It is a comparatively simple matter to provide efficient insulation for the transformer secondary winding.

In full-wave rectification it may be convenient to provide a separate rheostat or choke for each filament to control its emission, in case the characteristics of the two rectifying valves differ.

In Fig. 377 there is also shown a magnetically-operated switch, called the rectifier switch. In the position shown, this short-circuits the smoothing condenser when transmission has ceased. This is most important, as the charge remaining on the condenser may easily give a fatal shock to the operator. In the other position, the rectifier switch makes the filament heating circuit.

Equaliser Coils.—In cases where large powers are being dealt with, the arrangement shown in Fig. 377 of tapping off the H.T. rectified supply from **one side** of the two rectifier valve filaments is unsuitable.

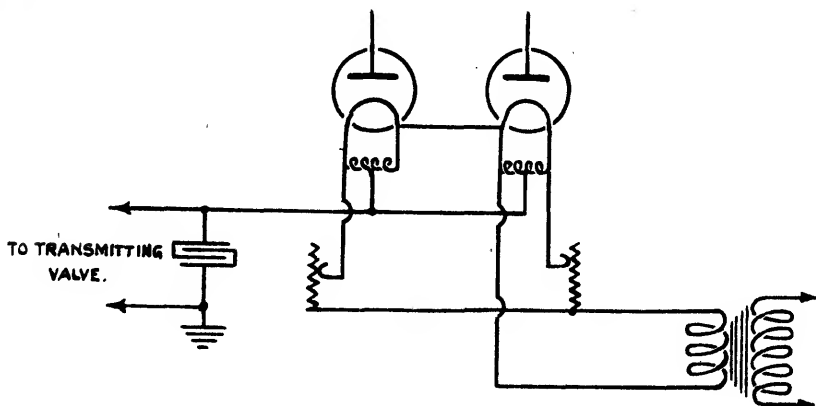


FIG. 378.

The filament is generally arranged as a loop, and if the arrangement shown in Fig. 377 were used, one side of the loop would be carrying a much greater proportion of the rectified current than the other. The extra current would raise the temperature and increase the emission from one half of the loop, and this half would wear out sooner than the other, giving the filament an unduly short life.

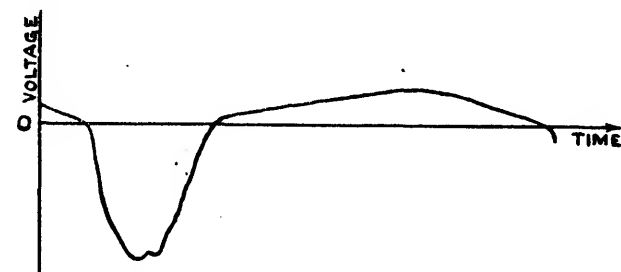
It is preferable to arrange the rectifier filament circuits as in Fig. 378, where there is an "equaliser coil" across each filament, and the high-tension lead to the transmitting valve is connected to the mid-point of each coil.

The equaliser coil has a high inductance, and consequently takes very little of the filament heating current.

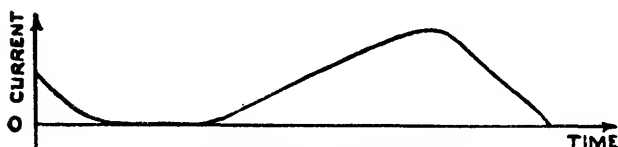
The flow of H.T. rectified current, however, is in opposite directions in the two halves of the coil, which is consequently non-inductive to it.

Thus paths of equal conductivity are provided for the rectified current down both sides of the filament simultaneously, equalising the current distribution in the two halves of the filament loop.

648. The **transformer secondary voltage** has been assumed in the above discussion to follow a simple sine curve, but the variable load presented by the rectifying valves renders it impossible for this to be the case in practice. In the analysis of half-wave rectification given above, for example, current flows through the rectifying valve



TRANSFORMER SECONDARY VOLTAGE

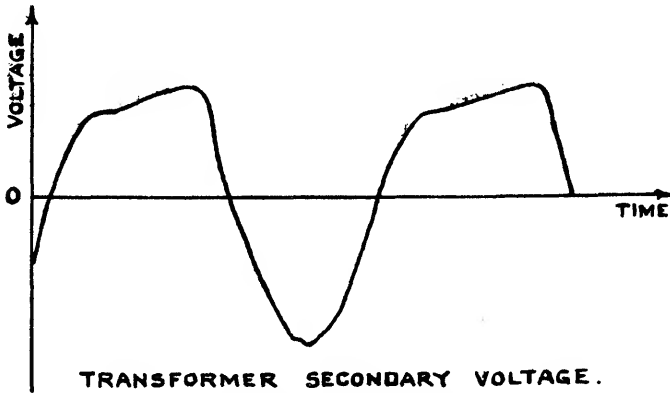


RECTIFIED CURRENT.
Half-Wave Rectification.

FIG. 379.

in pulses corresponding to parts of the positive half-cycles of secondary voltage, and no current flows during the remainder of each cycle. The effect of this variable load is to distort the secondary waveform, *i.e.*, to introduce harmonics of the alternator frequency. The rectified current flowing through the secondary winding also has a mean value in one direction, since all the pulses of current are in the same direction through the winding. This produces a steady magnetisation of the core, and therefore distorts the secondary voltage wave, as explained in para. 384. The effect in practice is to increase the time during which current flows in the rectifying valve, but to reduce the maximum value of rectified current. Experimental curves of the anode-filament P.D. and the rectified current during one cycle are shown in Fig. 379. It will be observed

that there is a considerable second harmonic (*see also* Fig. 357) in the voltage waveform. This is largely due to the transformer iron distortion, and is not nearly so pronounced in full wave rectification, as shown by the experimental curve of Fig. 380. The rectified current pulses from the two valves then flow in opposite directions through the secondary winding, and so the two half-cycles of the voltage waveform are more similar in shape, *i.e.*, even harmonics are reduced.



Full-Wave Rectification.

FIG. 380.

649. Efficiency of a Rectifier Unit.—An exact calculation of the efficiency is complex, as will be realised from the descriptive account of rectification given above. It must take into account the $\frac{3}{2}$ power law of variation of rectified current with anode-filament P.D., the distortion of the transformer secondary voltage waveform, the smoothing condenser rectified voltage ripple, and the impedance of the transmitting circuit. Even an approximate calculation presents difficulties which render it outside the scope of this book, but some approximate figures will be quoted below.

The efficiency of the rectifying system is the ratio of its power output to the transmitting circuit to its power input, which consists of the main alternator power output and the power expended in the filament in producing the emission. The principal power loss arises in the heat produced at the anodes of the rectifying valves.

The efficiency increases with the ratio of H.T. rectified voltage to transformer secondary voltage amplitude. It is therefore an advantage to rectify at as high a value of this ratio as possible. The rectifying circuit is usually designed to give a peak anode-filament P.D. of from 2,000 to 3,000 volts during the positive half-cycle. (The peak anode-filament P.D. during the negative half-cycle is, of course, much greater than this, *e.g.*, with a rectified voltage of

10,000 volts and a secondary amplitude of 13,000 volts, the peak P.D. is 23,000 volts, and the valve insulation must be capable of standing this voltage.) Decreasing this peak P.D. would increase the efficiency of rectification. The transformer secondary voltage, however, is more or less fixed for a given alternator and transformer, since altering the alternator voltage spoils the alternator regulation, although some variation may be obtained by an auto-transformer arrangement of the primary winding. On the other hand, the rectified voltage is the H.T. voltage to the transmitting valves, and so is determined by power considerations in the transmitting circuit, such as the anode rating of the valves. Hence the ratio of rectified voltage to transformer secondary voltage cannot be increased merely from considerations of rectifier efficiency alone, and a compromise must be effected to give the best over-all efficiency.

Some figures may be quoted for full wave rectification in the case of a transmitter requiring 15 kW. of rectified power at a mean H.T. voltage of 10,000 volts. Taking the maximum anode-filament P.D. across the rectifying valves during the positive half-cycle of transformer secondary voltage to be 3,000 volts, this requires a total secondary P.D. of 26,000 volts and the ratio of H.T. rectified voltage

to secondary P.D. across each valve is $\frac{10}{13}$ or 0.77. The power

loss in the rectifying valves under these conditions is 1.8 kW. per valve, making 3.6 kW. in all, and the efficiency of rectification is

$$\frac{15}{15 + 3.6} = 81 \text{ per cent.}$$

The smoothing condenser ripple is about 4 per cent. in this case, with a condenser whose capacity is about 1.5 μ F, and the rectifier filaments must be capable of an emission of 5.5 amps. without saturation conditions being approached.

650. Overheating of the rectifier anodes indicates that the rectifier filament emission is too low, and it should be increased. This increases the charge received by the smoothing condenser under the given conditions and so raises the equilibrium P.D. across the said condenser. The efficiency of rectification is therefore increased and the power dissipation at the rectifier anodes is diminished, with a corresponding diminution in anode temperature.

Overheating of the transmitting valve electrodes, if the anode tap is correctly adjusted, indicates that more power is being developed in the transmitting circuit than the anode rating of the valves will allow. The output from the rectifier unit should then be lowered by reducing the main alternator voltage. This reduces the H.T. rectified voltage proportionally and so diminishes the power supply to the transmitting circuit.

651. Metal Rectifiers are commonly used instead of valve rectifiers to produce the H.T. supply for receiving and low power

transmitting circuits when the primary electrical supply is alternating. The metal rectifier operates on the same principle as the crystal detector. The most usual form is a copper disc, one face of which is oxidised by a special heat treatment. The contact between the film of copper oxide and the metallic copper behaves like a crystal and cat-whisker or crystal couple. A voltage applied in the direction from oxide to metal gives a much greater current than the same voltage applied in the opposite direction—from metal to oxide. Alternatively the metal rectifier may be compared to a diode in which the metal is the anode and the oxide is the cathode or filament.

The flow of current during rectification develops heat in the disc, and, as the temperature of the disc increases, the difference between its conductivities in opposite directions is diminished, *i.e.*, its efficiency is decreased; further, overheating may cause chemical

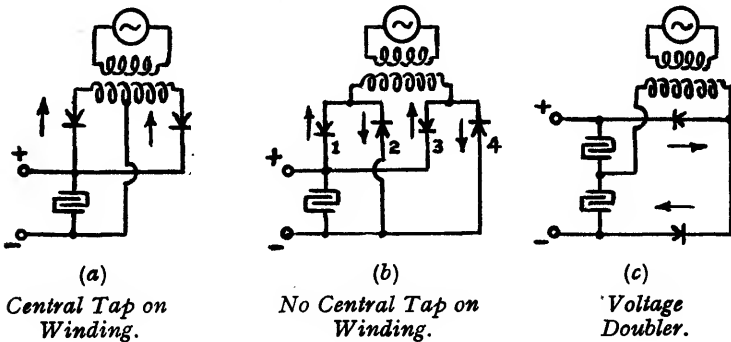


FIG. 381.

changes which permanently impair the action. There is hence an upper limit to the voltage which may be applied across any one disc, and, for higher voltages, two or more rectifiers in series must be used. To increase the flow of rectified current for a given applied voltage, the appropriate number of rectifiers may be connected in parallel.

The ordinary rectifier unit consists of a number of rectifier discs mounted in series on a screwed bolt and insulated from it by a sleeve. The discs are separated by lead washers, and conducting spacers and large metal fins are interposed alternately between discs to dissipate the heat developed.

Provided a metal rectifier unit is not overloaded, its life should be very much longer than that of a valve rectifier.

Typical full-wave metal rectifier circuits are shown in Fig. 381. The arrowhead in the "bench mark" symbol for a metal rectifier indicates that direction of an applied voltage for which it is conducting, and hence also indicates the direction of conventional current flow through it. The electronic flow is in the opposite direction,

as shown by the separate arrows. Fig. 381 (a) corresponds exactly to Fig. 377 for the valve full-wave rectifier. In Fig. 381 (b) an arrangement is shown whereby the centre tapping on the transformer secondary winding may be dispensed with. Rectifiers 1 and 4 are conducting during one half-cycle of applied alternating voltage, and rectifiers 2 and 3 during the next half-cycle. For the same transformer secondary voltage, the output voltage is twice that given by the circuit of Fig. 381 (a).

Fig. 381 (c) shows a circuit giving a rectified voltage approximately twice the R.M.S. value of the transformer secondary voltage. The two rectifiers, connected in opposite directions to the same secondary terminal, behave as half-wave rectifiers during alternate half-cycles, and produce rectified voltages in the same direction across their respective smoothing condensers. These condensers are in series, and so their voltages are additive.

652. Signalling Methods.—For telegraphic transmission the continuous oscillation produced in the above types of transmitter must be broken up into wave trains whose duration is the same as that of the dots and dashes of the morse code. A hand key suitable for signalling can only be directly inserted in low-power circuits, and the general practice is to make and break some lead in the circuits by a magnetic key, which, in turn, is operated by the hand key. Oscillations are maintained while the key is pressed, and cease when it is released. This naturally leads to transient conditions at the instants when contact is made or broken by the key, giving rise to "key clicks" in the received signal and variation in the transmitted frequency.

In Service transmitters, the key may be found in various positions, according to the power developed in the set. Thus, in high-power sets supplied by rectified A.C., it is placed in the transformer primary. In sets supplied by a D.C. generator, it is normally in the positive H.T. lead to the anode.

The key is also commonly inserted in the H.T. lead to the filament between the filament itself and the point where the grid leak is attached to the filament lead. Breaking the key then breaks the anode supply, and at the same time isolates the grid from the filament. Thus the grid is charged to a high negative potential, and oscillations are damped out more quickly. For high speed signalling, the grid may be made to go negative at a still faster rate in order to reduce the transient period; and keying may be confined to the output side of the rectifier unit so as to avoid the lag which arises when the smoothing condenser has to be charged anew for each wave train.

MODULATED C.W.

653. Types of Wave.—All the transmitters so far described in this chapter have been designed to produce a radio-frequency current of constant amplitude in the aerial circuit, except in so far

as this current is interrupted by keying for signalling purposes. The type of wave produced by such transmitters is classified as Type A1 and defined as follows :—

Type A1.—Continuous waves, unmodulated, key controlled. Continuous waves of which the amplitude and/or frequency is varied by the operation of keying in telegraphic transmission.

The keying of valve transmitters described in para. 652 reduces the amplitude from a constant value to zero, *i.e.*, it gives amplitude variation. An example of frequency variation is the marking and spacing method employed in arc transmitters.

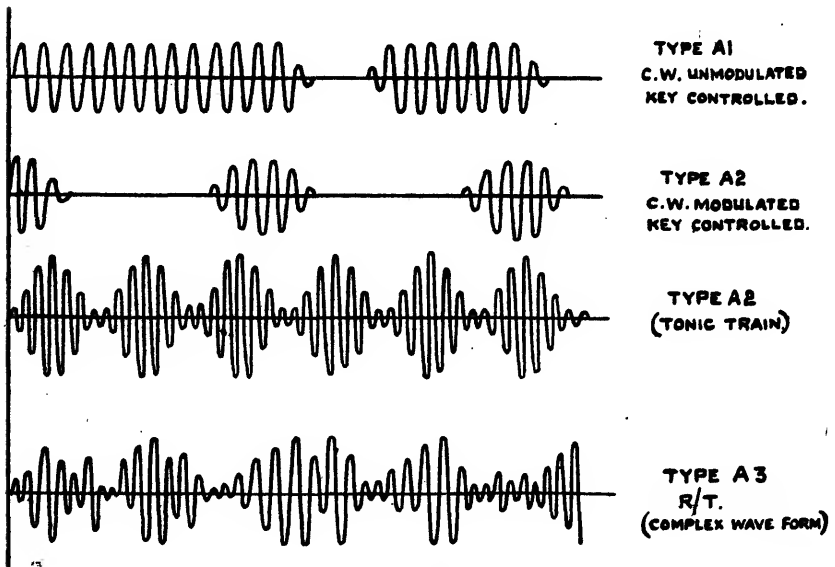


FIG. 382.

The reception of Type A1 waves necessitates the use of a local oscillator, *i.e.*, heterodyne reception, at the receiving station, or of some equivalent method of rendering the signal perceptible to the senses.

An alternative method of signalling is to impress the audio frequency variation necessary for aural reception on the amplitude or frequency of the radio frequency waveform as radiated from the transmitter. An example of this has already been seen in spark transmission. The damped waveform radiated from a spark transmitter falls under the class known as **Type B** waves, and defined as waves forming successive wave trains, in each of which the amplitude after reaching a maximum progressively decreases.

The other types of wave of this nature are classified as Type A2 and Type A3, and defined as follows :—

Type A2.—Continuous waves in which a variation of amplitude and/or frequency is made in a periodic manner at an audible frequency and key-controlled for the purposes of telegraphic communication.

Type A3.—Continuous waves in which a variation of amplitude and/or frequency is made in accord with the characteristic vibrations of speech or music.

These different waveforms are shown in Fig. 382.

Type A2 waves are usually called Interrupted Continuous Waves (I.C.W.). In the particular case where the variation of amplitude, or "modulation," is sinoidal, the kind of I.C.W. produced is called "Tonic Train."

Type A3 waves correspond to the method of communication known as Radio Telephony (R/T), which is considered in Chapter XV. The production of Type A2 waves, or I.C.W., by valve transmitters will now be dealt with.

654. The general method is to vary the anode or grid voltage (or both) of the transmitting valves at an audible frequency, the oscillatory circuits, coupling, aerial circuits, &c., being the same as those discussed in previous paragraphs. It was there seen that steady anode and grid voltages gave a radio-frequency current in the aerial of constant amplitude, this amplitude being proportional to the H.T. supply voltage to the anodes of the valves.

If this supply voltage is varied at an audible frequency, the aerial current will vary correspondingly in amplitude, and a waveform whose amplitude varies at audio frequency will be radiated.

The methods employed in the Service for producing I.C.W. are as follows :—

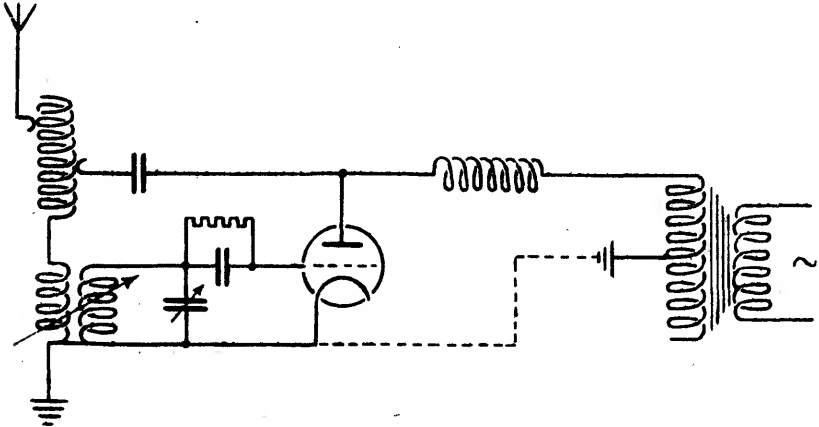
(1) Transformer secondary voltage from an alternator and transformer arrangement applied directly between anode and filament of the transmitting valves.

A circuit for this purpose is shown in Fig. 383. The transmitter is of the type described as tuned circuit between anode and filament, mutual inductive grid excitation, parallel feed, direct aerial excitation, and the H.T. supply is an alternating voltage of half the amplitude of the transformer secondary voltage.

The amplitude of the radio-frequency oscillations follows roughly the transformer secondary waveform during the positive half-cycle. This is shown as a sine curve in Fig. 384, but the remarks in para. 648 should be noted. During the negative half-cycle the anode is negative to the filament and no self-oscillations can be generated in the transmitting circuit.

As there is no smoothing condenser, the transformer secondary winding must be by-passed for radio-frequency currents, otherwise

large radio-frequency P.D.s. sufficient to damage the insulation may be developed across it (the inductive reactance of the winding is, of course, very large at radio-frequencies). The self-capacity of the winding is usually sufficient for this purpose, but small by-pass condensers may be shunted across the two halves of the secondary.



Single Pulse I.C.W. Circuit.

FIG. 383.

These are commonly fitted even in C.W. transmitters to by-pass any radio-frequency currents that may find their way to the transformer secondary by means of stray coupling to the H.T. leads. The reactance of these condensers is large enough at the alternator frequency not to affect the secondary P.D.

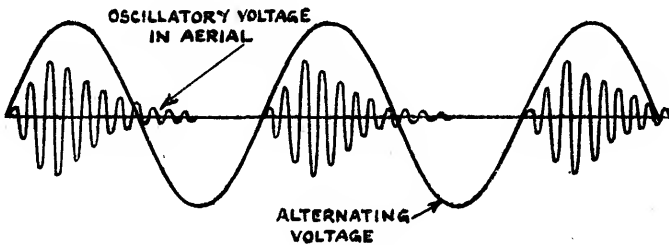


FIG. 384.

As will be seen from Fig. 384, the audio frequency variation of amplitude is by no means sinoidal, and the audible result in the receiver is more of a noise than a note. The audio frequency variation occurs at the frequency of the alternator, and the waveform radiated is called Single Pulse I.C.W.

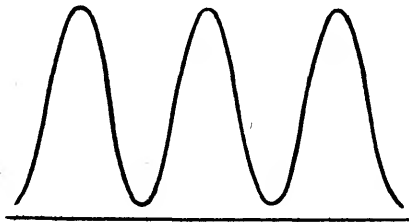
Service circuits are commonly designed to transmit either C.W. or I.C.W., so that the transformer secondary is across the anodes of the rectifying valves. The modifications required when changing

over from C.W. to I.C.W. are to break the rectifier filament heating circuit and to transfer the H.T. lead from the positive plate of the smoothing condenser to one end of the transformer secondary.

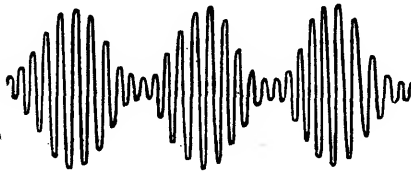
Keying for telegraphic transmission may be arranged by a key in the transformer primary circuit.

(2) Use of rectified alternating voltage with the smoothing condenser removed.

It was seen in para. 645 that the ripple, or percentage change, in the D.C. voltage across the rectifier condenser applied to the transmitting valves was inversely proportional to the capacity of the condenser. If then the condenser is cut out altogether, the ripple on the rectified supply becomes very marked, and the voltage applied to the transmitting valve will be of the form illustrated in Fig. 385.



VOLTAGE FROM RECTIFIER.



CURRENT IN OSCILLATORY CIRCUIT.

FIG. 385.

The voltage from the rectifier never quite falls to zero, on account of the capacity to earth of the filaments and the inductance of the anode choke, which tend to keep the current flowing during the intervals between the current pulses from one or other of the two rectifying valves.

It should be noticed that, if full-wave rectification is employed, the variations in the supply voltage occur at **twice** the frequency of the A.C. supply, and so this method gives a note whose frequency is twice that of the note given by the previous method.

For this reason the waveform radiated is called Double Pulse I.C.W.

Keying is effected, as before, in the primary circuit of the transformer.

If half-wave rectification is used, Single Pulse I.C.W. is produced.

(3) Mechanical methods for interrupting the generation of oscillations at a frequency corresponding to audibility.

A simple method of producing I.C.W. by this means is to make and break the grid lead of the oscillating circuit by inserting a buzzer wheel in series with the grid leak. When the buzzer wheel is bearing on a conducting segment, an oscillation will be set up in the aerial, but when it bears on an insulated segment the grid will be left insulated and will accumulate such a negative charge that the aerial oscillation will be quenched. The result will be I.C.W. of a frequency depending on the speed of revolution of the buzzer wheel.

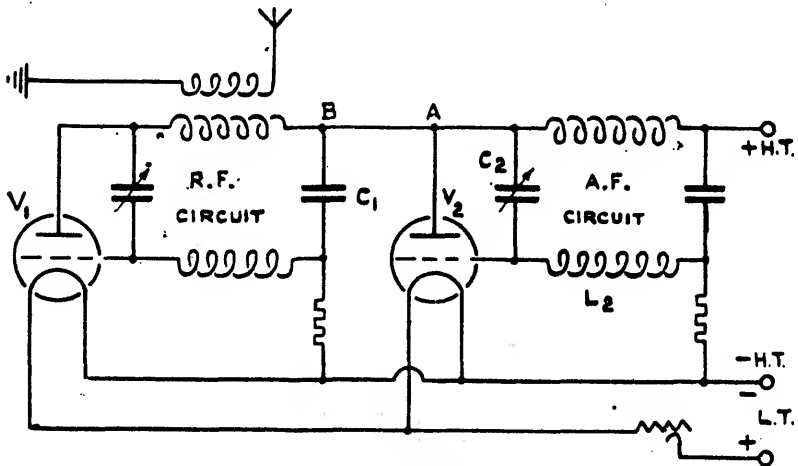


FIG. 386.

During the time that oscillations are being generated, their amplitude is sensibly constant, and the waveform corresponds to that of Fig. 382 (b). I.C.W. produced in this manner is sometimes called "chopped C.W.," a term which is explained by the waveform. This method is suitable for use with an A.C. source of supply whose frequency is too low to give a good note.

The three methods hitherto given produce variations in amplitude at audible frequency which are not sinoidal, and cannot be said, therefore, to give the special type of I.C.W. known as "Tonic Train." This latter is only given by the fourth method, now to be examined.

(4) Modulation of the H.T. supply to the radio-frequency oscillatory valve anode by a valve-maintained audio-frequency voltage.

A typical circuit for this purpose is shown in Fig. 386. A radio-frequency oscillation is maintained in V_1 and its accompanying tuned circuit, the arrangement being of the type classified as tuned

circuit between anode and grid, direct inductive grid excitation, series feed, and anode at high oscillating potential. The H.T. supply to the anode of V_1 , however, is not taken directly from a D.C. or rectified A.C. source, but is, in fact, the voltage developed across the condenser C_1 due to an audio-frequency oscillation maintained in the valve V_2 and its tuned circuit. C_1 has a reactance to audio-frequencies of the same order as C_2 , and is in parallel with C_2 and L_2 in series; it must therefore be considered as part of the audio-frequency tuned circuit, and an audio-frequency P.D., equal in magnitude to the oscillatory P.D. between the anode and filament of V_2 , is impressed across it. Thus the H.T. supply to the anode of V_1 is modulated at audio-frequency, and the R.F. oscillations vary correspondingly in amplitude. Since the P.D. across C_1 can be arranged to be very nearly sinoidal by applying the appropriate grid bias to the grid of V_2 , the R.F. amplitude modulation is likewise sinoidal, *i.e.*, tonic train is produced. The depth of modulation, *i.e.*, the ratio of the amplitude of the audio-frequency P.D. across C_1 to the amount of the steady H.T. supply, also requires careful adjustment if sinoidal modulation of the R.F. oscillations is to be achieved. This point is discussed more fully in the next chapter (para. 672).

To increase the P.D. across C_1 , a choke is sometimes added between A and B, of such a value that the path through C_1 from A to filament is partially tuned to resonance at the audio-frequency produced by V_2 . A greater audio-frequency current then flows through C_1 , and a correspondingly greater P.D. is produced across it, provided that the total oscillatory P.D. from A to filament is not too much decreased by the insertion of the choke. This, however, is liable to be the case since the impedance of the V_2 tuned circuit between anode and filament has thereby been considerably reduced. If tuned to resonance, for example, the choke and C_1 would constitute a path of very low impedance in parallel with the remainder of the V_2 tuned circuit, and there would be a large departure from the most efficient condition, *viz.*, that the said impedance should approximate in value to the A.C. resistance of V_2 .

The valve V_2 is also sometimes omitted, either C_1 or C_2 being removed, and the free end of the grid inductance of the R.F. circuit being connected to the grid inductance of V_1 . V_1 then has to maintain both the A.F. and the R.F. oscillations. It is difficult to obtain a circuit of this type which operates satisfactorily; even if oscillations can be maintained, the efficiency is poor.

(5) Self-quenching Method.—This is mentioned here for completeness, but its description will be postponed until the principles of super-regenerative receivers are discussed (Chapter XVI, para. 706).

655. It will be seen that all these methods of producing I.C.W. involve considerable variation of the H.T. voltage applied to the

anode. The result is that the radio-frequency oscillation produced is liable to vary considerably in frequency as well as in amplitude, particularly at high radio-frequencies. To minimise this frequency variation the radio-frequency may be master controlled, *e.g.*, by a quartz crystal.

656. Self-oscillatory Valve Transmitting Circuit Complete.—In Fig. 387 is illustrated a typical self-oscillatory valve transmitting circuit for producing either C.W. or I.C.W. According to the classification given earlier in this chapter, it is described as:—

Tuned circuit between anode and filament.

Mutual inductive grid excitation, the grid being partially tuned. Parallel feed.

Direct aerial excitation.

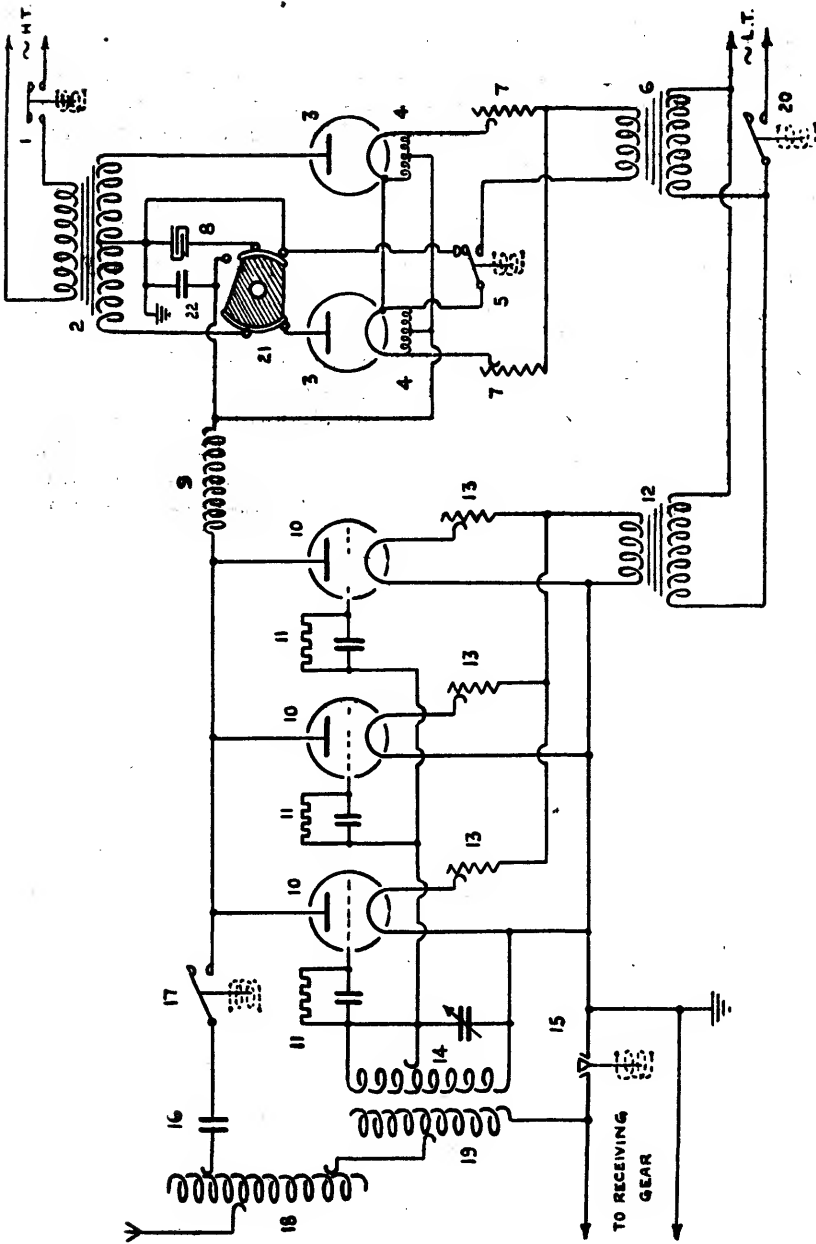
Three valves in parallel, their anodes being the high oscillating potential electrodes.

The H.T. voltage on the anodes is obtained from a rectifier unit employing full wave rectification of an A.C. supply.

When I.C.W. is required, the smoothing condenser (8) is cut out of circuit by means of the C.W.—I.C.W. switch (21).

The components indicated by numbers are as follows:—

- (1) The magnetic key to interrupt the A.C. supply for signalling purposes.
- (2) The H.T. step-up transformer.
- (3) The rectifying valves.
- (4) The equaliser coils.
- (5) The rectifier switch for completing the rectifying valve filament heating circuit, and for discharging the smoothing condenser (through the equaliser coils) when transmission is finished.
- (6) The step-down filament transformer for the rectifying valves.
- (7) The rheostats for controlling the filament circuit of the rectifying valves.
- (8) The smoothing condenser.
- (9) The anode choke.
- (10) The transmitting (or oscillating) valves.
- (11) The grid insulating condensers and leaks.
- (12) The step-down filament transformer for the transmitting valves.
- (13) The rheostats for controlling the filament current of the transmitting valves.
- (14) The partially-tuned grid excitation circuit. The idle turns of the inductance are short-circuited.



Self-oscillatory Valve Transmitting Circuit Complete.

FIG. 387.

- (15) The operating switch (across which the receiving gear is joined), which completes the earth for the aerial circuit during transmission.
- (16) The anode blocking condenser.
- (17) The anode key, which breaks the circuit between anodes and aerial when the transmitting key is not pressed, thus preventing a path to earth through (18), (16), (9) and (8) for received signals.
- (18) The aerial coil.
- (19) The aerial coupling coil.
- (20) The filament switch for breaking the primary supply to the filament transformers.
- (21) The C.W.-I.C.W. switch.
- (22) The small condenser to allow the R.F. current through the anode choke to pass directly to earth (or L.T.—) in the I.C.W. position. Since the smoothing condenser is then out of circuit, the easiest path for this current to earth would otherwise be across the windings of the filament transformer, which might therefore be damaged.

CHAPTER XV

RADIO TELEPHONY.

657. Radio Telephony (R/T) consists in the transmission of speech, music or any audible sound to a distance by means of electro-magnetic waves. It bears the same relation to wireless telegraphy as line telephony does to line telegraphy, and possesses the same relative advantages and disadvantages. More particularly with reference to Service requirements these may be summarised as follows :—

Advantages.

- (a) All the normal advantages of oral over written communication, *e.g.*, actual saving in time, and the lessening in traffic that arises by the adjustment of minor points without the necessity of voluminous correspondence.
- (b) The ability to make and read morse is not required.

Disadvantages.

- (a) Where a message has to be written down, R/T is slower than W/T.
- (b) Coded messages are difficult to pass owing to risk of phonetic errors, and plain language involves the risk of interception.
- (c) The interference caused by R/T is worse than that caused by W/T. Conversely, interference exerts greater influence on a receiver adjusted for R/T than on one adjusted for W/T, and, in addition, the effect of any interference heard is of more serious consequence.
- (d) Listening-through is not possible with R/T unless the "Single Side-band" system is employed.

658. Essential Features of R/T.—Various features characterising the production and reception of R/T will be summarised here and referred to later in more detail.

(a) Audio-frequencies cannot be transmitted direct. If an aerial were to be energised with audio-frequency oscillations, the quantity of energy radiated would be negligible. It would also be impossible to select any one transmitting station by tuning, as all transmitters would radiate similar frequencies. It is therefore necessary to superimpose the various audio-frequencies on one radio-frequency (known as the "carrier"). This process is known as "modulation." The modulated carrier can be selected by tuning in the usual manner, but subject to slight limitations on account of "side-bands."

(b) The oscillations generated by the transmitting apparatus at the carrier frequency must be C.W. (not spark or I.C.W.), and provision must be made for modulating the carrier at the required audio-frequencies. A microphone and microphone amplifier will virtually replace the morse key employed for W/T transmission.

(c) The receiving apparatus must use telephones or a loud speaker as the final indicator of the signal, since no automatic recording device giving a result which can be interpreted by the eye is possible.

(d) Heterodyne reception need not be used, since the audio-frequency variations in amplitude of the carrier produce corresponding audio-frequency currents after detection. If heterodyne reception is employed, the heterodyne frequency must be adjusted to the centre of the "dead space," or an undesired beat note will be heard. The use of heterodyne reception will always introduce some distortion, and should therefore be avoided whenever possible. It is sometimes of value in increasing sensitivity of the detector when very weak signals are being received. This method should not be confused with super-heterodyne reception (Chapter XVI), which is quite satisfactory for R/T.

(e) Various precautions must be taken to prevent distortion of the audio-frequency wave forms. Sources of distortion are dealt with in detail later, and include :—

(i) At the transmitter—

Use of inferior microphones.
Incorrect modulation.

(ii) At the receiver—

Tuning circuits which are too lightly damped (*i.e.*, excessively sharp tuning); this includes the use of too much "reaction."

Unsuitable detector adjustments.

Valves in audio-frequency amplifiers overloaded or otherwise worked under incorrect conditions.

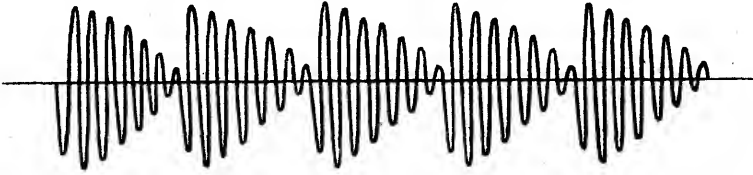
Inferior iron-cored chokes and transformers.

Inferior reproducing devices, *e.g.*, loud speakers.

659. Speech and Music.—Speech and music consist of highly complex sound waves, *i.e.*, waves containing many different frequencies and rapidly-varying amplitudes. Examples of such wave forms are shown in Figs. 388 and 389. Before proceeding to an analysis of the composition of such waves and their effect on the human ear, so that the points involved in their transmission by R/T without serious distortion may be clearly grasped, it is convenient to define the following terms associated with speech and music.

Sound intensity is a measure of loudness and depends on the amplitude of the sound waves.

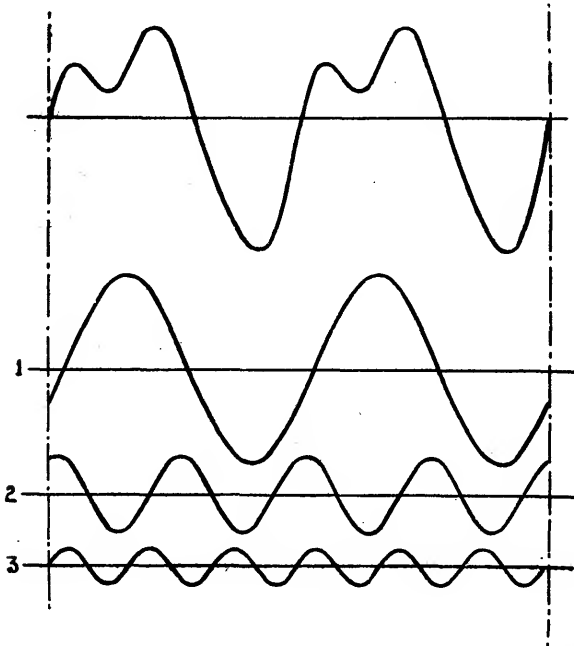
Pitch may best be described by comparing it with colour in vision. It is a physiological effect which depends on the frequencies in the speech or music. In the same way as the colours red and blue



Wave Form of Vowel Sound "Ah!"

FIG. 388.

describe light in which low and high visual frequencies respectively predominate, so the terms "low pitch" and "high pitch" describe speech or music in which low and high audio-frequencies respectively predominate.



Curve of a Violin Tone and its three Harmonic Components.

FIG. 389.

The term "pitch" is also used in another sense to denote the standard frequencies used in music. For example, in "concert pitch" the note "middle C" has a frequency of 273 cycles per second ; in French standard pitch the note "middle C" has a

frequency of 261 cycles per second, while for scientific purposes this note is defined to have a frequency of 256 cycles per second.

A **pure tone** is a pure sinusoidal wave of constant frequency and of sensibly constant amplitude. This rarely, if ever, occurs in practice, one of the nearest approaches to it being the note produced by a tuning fork.

A **complex tone** is a combination of waves of several frequencies which may be of constant or varying amplitude. A wave-form at a single frequency which varies rapidly in amplitude is also equivalent to a complex tone. Any complex tone can be analysed into component "pure tones" over a short period. The lowest component pure tone is known as the "fundamental," and the remaining tones as "overtones." The fundamental tone is usually, but not always, the strongest, and therefore predominates. The fundamental tone usually gives a complex tone its characteristic pitch (determines the note), while the overtones are responsible for the characteristic "timbre" or "tone quality" (*see below*). The component pure tones which go to form a complex tone are often called "partials."

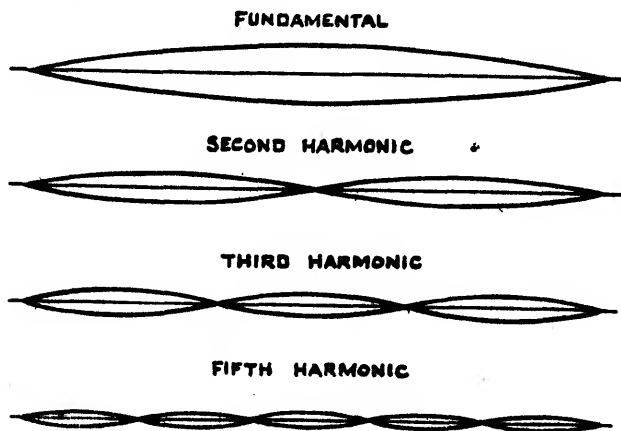


FIG. 390.

Harmonics.—Overtones whose frequencies are exact multiples of that of the fundamental tone are called "harmonics." The fundamental is also called the first harmonic. The wave whose frequency is twice that of the fundamental is known as the second harmonic. The third harmonic has a frequency three times that of the fundamental, and so on. Fig. 390 shows a stretched string vibrating freely at its fundamental frequency and also at its second, third and fifth harmonics.

The second harmonic of any fundamental frequency is an "octave" above the fundamental; the third harmonic is a "fifth" above the second, and so on.

Timbre or Tone Quality.—Notes of the same fundamental frequency produced on different instruments give rise to very different sound sensations when they impinge on the human ear. For example, if "middle C" (256 cycles/sec.) be played on the flute and the same note be played on a violin, although the note will appear the same, the tone quality is entirely different, and it is possible to tell which instrument has produced it. This is due to the fact that the overtones accompanying the fundamental frequency differ according to the instrument. The note of a tuning fork is almost entirely free from overtones, and the note sounds "pure." A flute produces a note which contains a few weak overtones. A violin produces much stronger overtones, and the note is "harder" than that of a flute. Reed instruments produce very strong overtones, so much so that in some cases it is difficult to distinguish the fundamental note.

In many cases of complex sound waves, it is possible to suppress the fundamental frequency entirely and yet to hear it. This is due to a rectifying action in the ear which produces a low frequency from the combination of two overtones of much higher frequency. This effect can be observed in portable gramophones and small horn-type loud speakers, where the lowest note of the piano (of fundamental frequency about 50 cycles per second) can be heard, although the instrument is incapable of reproducing to any extent frequencies below about 200 cycles/sec.

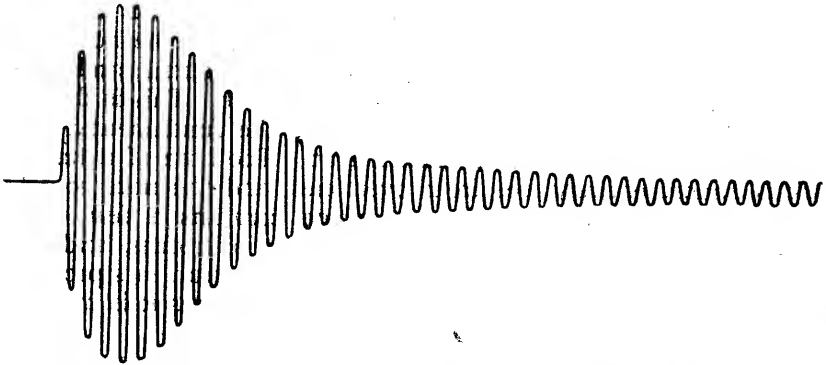
Hence it will be seen that "tone quality" lies almost entirely in the overtones, and any reproducing system must reproduce these as faithfully as the fundamental if the tone quality is to be distinguishable. Normally, for good (but not perfect) reproduction, the general tone quality of any note will be distinguishable if only the overtones up to twice the frequency of the fundamental (*i.e.*, the second harmonic) are uniformly reproduced.

660. Transients (known in musical parlance as "attack") are produced by rapid changes in sound intensity or sudden impulses of a non-periodic nature. Violent transients appear to the human ear as **noise** without any definite note (*e.g.*, pistol shots, slamming doors, &c.). This effect can be compared to that of atmospherics, which, being heavily damped, exert a shock effect on aërials tuned to almost any frequency, and consequently appear to possess no definite periodic frequency of their own.

Transient conditions occur when enunciating words with explosive initial consonants, such as *Cat*, *Boy*, *Town*, and in the sounds produced by percussion instruments such as cymbals, bells, drums, and, to a certain extent, the piano. Fig. 391 shows the transients produced at the commencement of a note played on a piano.

Other examples are the heavily-damped sound waves caused by hand-clapping and pistol shots. Angular or rectangular wave

forms also give rise to them. The wave form of any stringed instrument played with a bow is usually of an angular nature, as shown in

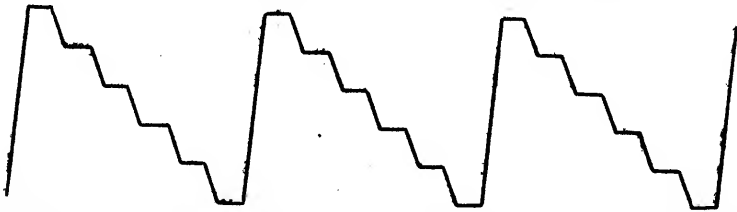


Vibration of Piano String at Commencement of Note Shewing Transient Condition and Large Initial Amplitude.

FIG. 391.

Fig. 392. Transients are among the factors which determine Tone Quality.

Transient wave-forms can be analysed (by Fourier's Theorem) into a large or infinite number of periodic sound waves, the frequencies of which vary from zero to infinity. From this it will be seen that any apparatus capable of reproducing perfectly all transient wave forms must be capable of reproducing uniformly all



Helmholtz's Diagram of the Vibrations of a Violin String under the Action of a Bow, Showing the Angular Wave-Form.

FIG. 392.

frequencies from zero to infinity. The converse is also true, namely that any apparatus which will reproduce **uniformly** all frequencies from zero to infinity will also reproduce all transient wave-forms perfectly.

The human ear, however, can only detect frequencies lying between 20 and 20,000 cycles per second. Consequently, any apparatus which will reproduce uniformly all frequencies within this range will also reproduce all transients as perfectly as can be detected by the human ear.

Any resonances in a reproducing system will cause imperfect reproduction of transients. The shock effect of the latter will give rise to disproportionately loud reproduction at any resonant frequency in the audible range, and the transient wave form will therefore be "coloured" by that frequency.

661. The Human Ear.—A simple explanation of the action of the human ear is to consider it as containing a very large number of tightly-stretched "strings," each of which is tuned to a certain frequency. The resonant frequencies of these "strings" are separated by very narrow intervals, and are spread over a frequency range of about 20 cycles per second to 20,000 cycles per second. Each "string" is connected by a nerve to the brain. When any particular "string," *e.g.*, that tuned to 1,000 cycles per second, is set into vibration, the consequent nervous impulse reaching the brain causes a sensation which the brain has learnt to associate with a 1,000-cycle note.

If a pure tone impinges on the human ear, only one "string" (theoretically) is set into vibration. If, however, a complex tone reaches the ear, the complex tone is automatically analysed into its component pure tones (*viz.*, the fundamental tone and the over-tones). Each of these component pure tones sets in vibration the corresponding "string" in the ear, and the overall nervous impulse reaching the brain gives rise to the sensation which the brain has learnt to associate with that particular complex tone.

The human ear is not affected by the relative phases of the component frequencies of a complex tone, as would be expected from the above theory. Therefore any reproducing apparatus which does not reproduce all audio frequencies in their correct relative phases may still give distortionless reproduction.

The effect of transients on the ear is to shock some or all of the "strings" in the ear into vibration for a very small space of time. This effect is analogous to that produced by slamming the lid of a piano when the loud pedal is pressed, which causes all the piano strings to vibrate. The effect on the brain of the resultant jumble of nervous impulses is that which it has learnt to associate with a "noise" which has no definite tone or frequency.

662. Frequency Limits for Reproduction of Speech and Music.—For perfect reproduction, all the frequencies which are audible to the human ear should be uniformly reproduced. In practice, however, it will usually be satisfactory to work with a considerably smaller band of frequencies.

If all frequencies below 40 cycles per second be removed, only the very deepest pedal notes of an organ and the fundamental tones of the double bass, bass drum, &c., will be affected. At the present time it is not generally practicable to reproduce frequencies lower than about 40 cycles per second at anything approaching their true relative intensity.

If all frequencies below 300 cycles per second be removed, the general **clarity** of speech or music will not be seriously impaired, but the reproduction will be thin and lacking in body.

If all frequencies above 10,000 cycles per second be removed, only the very finest shades of tone will be affected, and this defect in reproduction will normally be detectable only by those with musically-trained ears. The effect on speech will not be noticeable.

If all frequencies above 5,000 cycles per second be removed, the natural quality of speech and music will be slightly impaired. With a large orchestra, it will not be so easy to pick out the individual instruments, and the finer differences between similar types of instrument will be lost. In the case of speech it will not be so easy to recognise the voice of the operator.

If all frequencies above 3,000 cycles per second be removed, speech will still be intelligible, but will have lost its natural quality. The consonants will be weak, especially the sibilants. Music will be "drummy," and will have lost most of its natural quality.

Thus the frequency bands which should be retained in practice for radio telephony are roughly as follows :—

For intelligible speech	300 to 3,000 cycles/sec.
For speech where special clarity is desired (e.g., for important orders delivered by R/T) ..	200 to 5,000 ..
For music where realistic quality is not essential	150 to 5,000 ..
For music where realistic quality is required	50 to 8,000 ..

The latter frequency band (50—8,000) is that covered approximately by the British Broadcasting Company's transmitters.

663. Requirements for R/T and Sound Reproducing Apparatus.

—In order that speech or music may be satisfactorily transmitted or received by any apparatus, such apparatus must be designed to fulfil the following requirements :—

- (a) All frequencies within the limits specified in the above paragraph must be uniformly reproduced. This will necessitate the system being free from resonances within these limits, but it is also necessary to avoid resonances at any audible frequency that is capable of being reproduced by the apparatus.
- (b) The relative intensity of the reproduction and the original must be constant for all sound intensities.

Failure to effect (a) is known as "frequency distortion"; failure to effect (b) is known as "amplitude distortion."

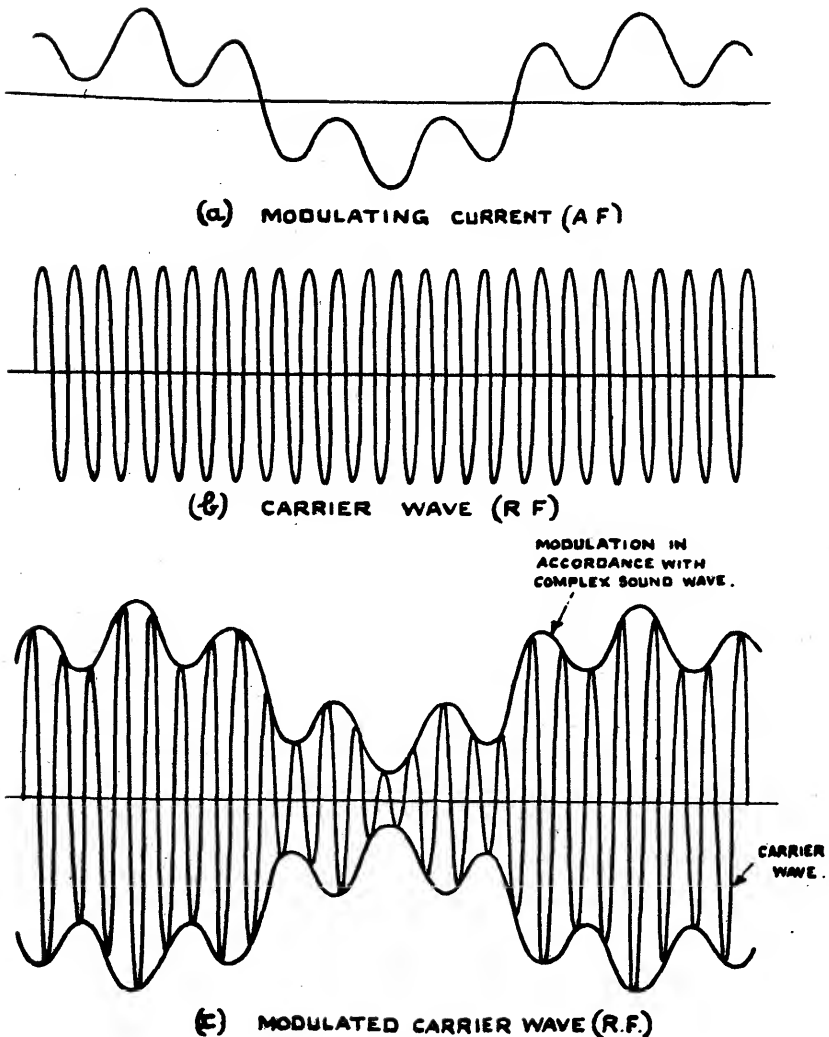


FIG. 393.

664. Essential Processes in Radio Telephony.—There are four processes which must always be employed in transmission and reception of speech or music. These are :—

- (a) Conversion of sound vibrations in the air into corresponding audio-frequency oscillatory currents and voltages. This is effected by a microphone.
- (b) "Modulation" or the superimposition of the audio-frequency oscillations on a radio-frequency "carrier."
- (c) "Detection," which is the process of abstracting the audio-frequency oscillations from the modulated carrier at the receiver.

- (d) "Reproduction," which is the process of converting the audio-frequency electrical oscillations back into sound vibrations in the air.

It will also be necessary to employ audio-frequency and radio-frequency amplification at intervals throughout the system, and to make use of the various methods of generating, radiating and receiving the radio-frequency carrier already explained for W/T transmission and reception.

It will be observed that operations (a) and (d) above are exactly opposite in principle, as also are (b) and (c). In line telephony, operations (a) and (d) alone are necessary.

These transformations are illustrated in Figs. 393 (a)-(c). Fig. 393 (a) shows the wave form of the original and reproduced sound waves, and of the audio-frequency current. Fig. 393 (b) shows the carrier in an unmodulated state, and in Fig. 393 (c) the carrier is shown modulated by the A.F. wave form of Fig. 393 (a).

It should be noted that, after modulation and prior to detection, the audio-frequency currents do not exist as such; they are only represented by the variations in amplitude of the carrier. A modulated carrier wave is a radio-frequency oscillation, and corresponding circuits must therefore be employed for all operations after modulation and before detection.

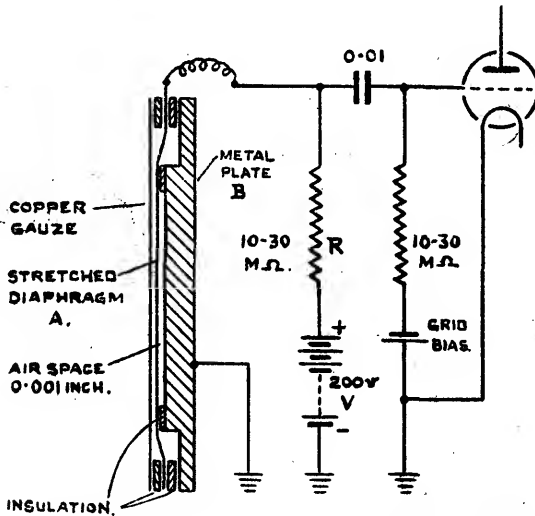
The most common methods by which the above four operations may be effected will now be considered in more detail. The question of amplification at various stages is dealt with later (paras. 679-682).

665. Microphones and Associated Circuits.—A microphone is an instrument for converting sound vibrations into corresponding audio-frequency electrical oscillations.

A microphone is nearly always followed by a valve amplifier; it is the function of the microphone and its immediate associated circuit to impress between the grid and filament of the first valve of the amplifier an oscillatory voltage whose wave form corresponds to that of the original sound vibrations.

The two commonest types of microphone in use at the present time are the "Condenser Microphone" and the "Carbon Microphone." The condenser microphone is used in most cases where the highest quality is required, as in broadcasting and talking picture studios. It possesses an extremely uniform response over the major portion of the audio-frequency band, and is entirely free from "background hiss." It is, however, comparatively insensitive. The carbon microphone is more useful as a general purpose instrument. It is much cheaper, and about a hundred times as sensitive as a condenser microphone. The response is not so uniform at all frequencies as that of a condenser microphone, and a certain amount of background hiss is usually present.

666. The Condenser Microphone.—A typical condenser microphone is shown in Fig. 394. It consists essentially of a thin tightly-stretched diaphragm (A) of aluminium alloy, secured around its edge so that it lies very close to, but well insulated from, a solid metal plate (B). These form the electrodes of the microphone. A steady D.C. polarising P.D. (V), of about 200 volts, is applied between the electrodes through a high resistance R, which may have a value of 20 MΩ or more. When sound waves impinge on the diaphragm (A), they cause it to vibrate, and so to vary the capacity



Condenser Microphone and Associated Circuit.

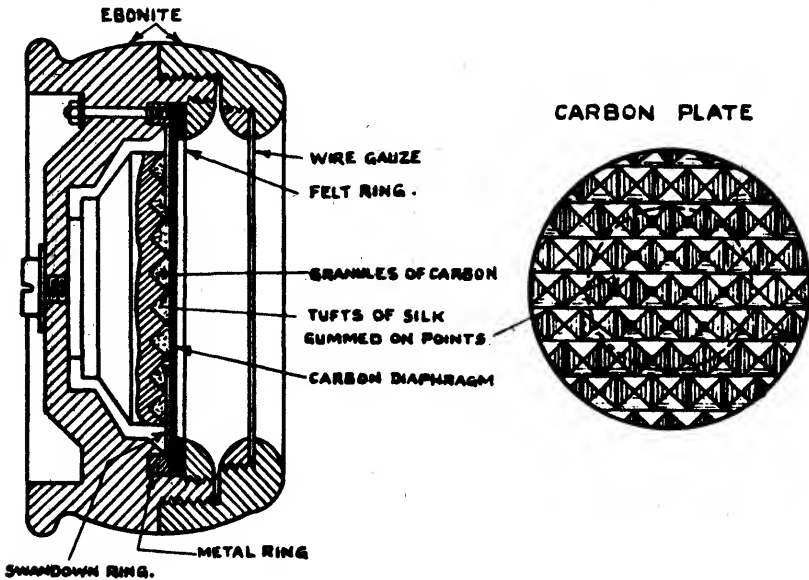
FIG. 394.

between the electrodes. The resistance R is sufficiently large to prevent any change in the quantity of electricity (Q) stored in the capacity of the microphone, except at a very slow rate. Consequently, it will be seen from the definition of capacity, $C = \frac{Q}{V}$,

that since Q is constant, any changes in the capacity (C), such as are caused by impinging sound waves, will cause a corresponding change in V. These voltage variations will be equal and opposite across the microphone and the resistance, and the grid-filament circuit of the first valve of the microphone amplifier may therefore be connected across either.

667. The Carbon Microphone.—A carbon microphone is illustrated in Fig. 395. A thin carbon diaphragm, or a rubber diaphragm coated on one side with carbon dust, is secured around its edge so that it lies close to a carbon disc. These form the two electrodes of the microphone, connections being taken from them to

the microphone terminals. The space between the electrodes is filled with carbon granules, which, under normal steady conditions, will be under slight pressure. When sound waves impinge on the diaphragm, they cause it to vibrate and so to vary the pressure exerted on the carbon granules. It is a property of carbon that the resistance between two carbon surfaces varies approximately in an inverse ratio to the pressure between them. Consequently the resistance of the microphone will be varied in accordance with the impinging sound vibrations.



Carbon Microphone.

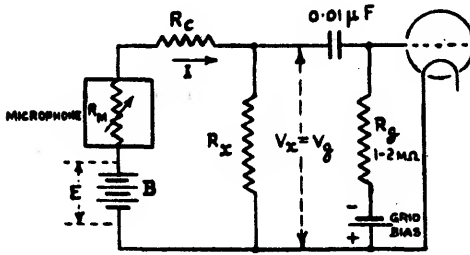
FIG. 395.

These variations in microphone resistance have to produce corresponding oscillatory voltages between the grid and filament of the first valve of the amplifier, and so it is necessary to employ a constant external E.M.F., and a resistance or inductance in series with the microphone, as shown in Figs. 396 (a) and 396 (b). The inductance may be replaced by the primary winding of a microphone transformer, as shown in Fig. 396 (c), in order to take advantage of the voltage step-up provided by transformer coupling. The action of these circuits is dealt with in the next paragraph. The practical considerations affecting them are :—

- (a) The resistance R_z should normally have about two or three times the value of the resistance of the microphone.
- (b) The impedance of the choke (L) in Fig. 396 (b), or of the primary of the transformer (T) in Fig. 396 (c), at the lowest frequency which it is required to reproduce

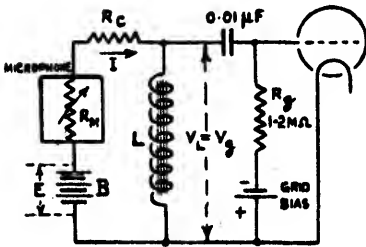
efficiently, must be greater than the resistance of the microphone.

- (c) For maximum sensitivity the voltage of the battery (B) should be as large as possible, this being limited by the maximum current which the microphone can be allowed to carry without overheating and "packing" of the carbon granules.



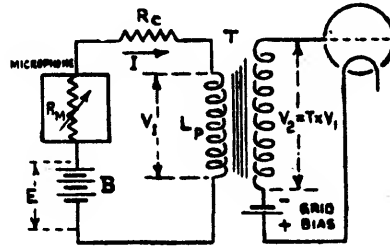
CIRCUIT EMPLOYING MICROPHONE RESISTANCE.

(a)



CIRCUIT EMPLOYING MICROPHONE CHOKE.

(b)



CIRCUIT EMPLOYING MICROPHONE TRANSFORMER.

(c)

Circuits for Use with Carbon Microphone.

FIG. 396.

The transformer method is much the most popular, since the oscillatory voltages developed across the secondary winding may be up to fifty times as great as those developed across a choke or resistance. There is no necessity, nowadays, for a microphone transformer to introduce any appreciable distortion. The use of a transformer also removes the necessity for a grid insulating condenser (Figs. 396 (a) and 396 (b)) and the consequent grid leak. These must be inserted when a choke or resistance is used, since the grid bias required on the first valve of the amplifier is usually quite different from the steady voltage developed across the resistance or choke.

*688. Operation of Carbon Microphone.

(1) **Resistance in External Circuit.**—The circuit is shown in Fig. 396 (a).

Let R_m = steady microphone resistance.

R_e = external resistance.

R_s = remaining resistance of circuit, including that of battery and leads.

E = EM.F. of battery.

Assume that a pure sinoidal sound wave of frequency $p/2\pi$ is impressed on the microphone diaphragm, and let the intensity of the sound wave be such that the resultant variation of microphone resistance is of the form $R'_m \sin(pt + \theta)$, where θ is an arbitrary constant.

The total microphone resistance at any instant " t " is then

$$R_m + R'_m \sin(pt + \theta),$$

and the resistance of the whole circuit is

$$R_s + R_e + R_m + R'_m \sin(pt + \theta),$$

$$= R + R'_m \sin(pt + \theta),$$

where $R (= R_s + R_e + R_m)$ is the total steady resistance of the circuit.

The instantaneous current at time " t " is

$$i = \frac{E}{R + R'_m \sin(pt + \theta)},$$

and the consequent voltage developed across R_e at the same instant is

$$v_e = \frac{ER_e}{R + R'_m \sin(pt + \theta)}.$$

v_e can be more conveniently expressed by multiplying numerator and denominator by

$$R - R'_m \sin(pt + \theta).$$

This gives

$$v_e = E \frac{R_e [R - R'_m \sin(pt + \theta)]}{R^2 - (R'_m)^2 \sin^2(pt + \theta)}$$

R'_m is always small compared with R_m , and hence with R , and so the term $(R'_m)^2 \sin^2(pt + \theta)$ may be neglected without introducing appreciable error.

The expression for v_e may therefore be re-written as

$$\begin{aligned} v_e &= \frac{E R_e R}{R^2} - \frac{E R_e R'_m \sin(pt + \theta)}{R^2} \\ &= \frac{E R_e}{R} - \frac{E R_e R'_m}{R^2} \sin(pt + \theta) \end{aligned}$$

The first term of this equation denotes the steady voltage developed across R_e when the microphone diaphragm is at rest.

The second term denotes the superimposed oscillatory voltage due to the vibrations of the microphone diaphragm. The latter component is the audio frequency voltage applied through the condenser to the grid of the first valve of the microphone amplifier.

An examination of the expression for this component shows that:—

- (a) It varies directly as R_m' , *i.e.*, as the intensity of the incoming sound wave, and the sensitivity of the microphone.
- (b) For any given value of R , it varies directly as the battery voltage E .
- (c) For any given battery voltage E , it is proportional to the term $\frac{R_s}{R^2}$, which, for given values of R_s and R_m , is a maximum when $R_s = R_s + R_m$. Hence for any given battery voltage E , the value of R_s should be equal to the total remaining resistance of the circuit if maximum sensitivity is to be obtained.

When it is desired to obtain the maximum possible output from a given microphone, the limiting factor is usually the current that can be allowed to pass through it. In such a case it is permissible to employ a value of R_s several times that of R_m , and to increase the value of E until the maximum permissible current is flowing through the microphone. This procedure will enable a somewhat greater voltage output to be obtained than in the case when R_s is made equal to $R_s + R_m$. The reason for this will be clear from an examination of the last term of the expression for v_s , which may be written as

$$\frac{E}{R} \cdot \frac{R_s}{R} \cdot R'_m \sin (pt + \theta)$$

If $\frac{E}{R}$ (= steady current in microphone) is maintained constant, it will be seen that the greater the value of R_s , the greater will be the voltage developed across R_s . Since, however, R_s is included in R , the factor $\frac{R_s}{R}$ must always be less than unity. The value of $\frac{R_s}{R}$ approaches unity as R_s approaches infinity. Hence there is little to be gained by increasing R_s to a value greater than, say, four-fifths of the total resistance R , *i.e.*, four times the remaining resistance ($R_m + R_s$).

(2) **Choke in External Circuit.**—The external resistance R_s may be replaced with advantage by an iron-cored choke L , as shown in Fig. 396 (b). Such a choke will have a low D.C. resistance, but a comparatively high impedance at audio frequencies. If the impedance be considerably greater than the total resistance of the circuit, (a condition which should always prevail), the effect of the choke

will be to keep the microphone current practically constant, irrespective of any changes in the resistance of the microphone. Hence any change in microphone resistance (caused by an impinging sound wave) will cause corresponding changes in the P.D. across the microphone (since $E = I \times R$, I being kept constant in this case by the choke L). Such voltage variations will be balanced by equal and opposite variations in the P.D. across the choke. It is therefore immaterial whether the voltages for the grid-filament circuit of the first valve of the microphone amplifier are tapped off across the choke (as in Fig. 396 (b)) or across the microphone.

Let L = inductance of choke.

R = total resistance of circuit.

$R'_{\infty} \sin pt$ = variation of microphone resistance due to impressed waves.

E = steady E.M.F. of battery.

Then the steady current flowing in the circuit is $\frac{E}{R}$.

If the reactance pL of the choke be large compared with R , the current will not be appreciably affected by the resistance variation $R'_{\infty} \sin pt$. Therefore this resistance variation will cause a corresponding variation of P.D. across the resistance of the circuit, and an equal and opposite variation of P.D. across the impedance pL . With this assumption, the value of these variations of P.D. is

$$v_z = \frac{E}{R} \cdot R'_{\infty} \sin pt.$$

From this it will be seen that the amplitude of the voltage variations depends upon two factors:—

(a) The sensitivity of the microphone.

(b) The value, $\frac{E}{R}$, of the steady current flowing in the circuit.

As before, this current is limited by the maximum current-carrying capacity of the microphone.

(3) **Transformer in External Circuit** (Fig. 396 (c)).—If the choke be replaced by the primary winding of a microphone transformer, the inductance of this primary winding will be subject to exactly the same influence as the choke. In this case, however, the magnetic flux cuts the much larger number of turns composing the secondary winding, and so induces a large secondary E.M.F.

Let v_2 = voltage developed across the secondary,

v_1 = voltage developed across the primary,

$$T = \frac{\text{number of turns in secondary winding}}{\text{number of turns in primary winding}}$$

Then $v_2 = v_1 \times T$.

But, as in the case of the choke, $v_1 = \frac{E}{R} \cdot R'_m \sin pt$, again assuming that the impedance pL of the transformer primary is large compared with the total resistance of the microphone circuit.

$$\text{Hence } v_2 = \frac{E}{R} \cdot T \cdot R'_m \sin pt.$$

The advantage of using a microphone transformer with a turn ratio of 20/1 or more will be at once apparent.

669. Modulation.—Modulation is the process whereby the audio frequency electrical oscillations produced by the microphone circuit are superimposed on the radio frequency C.W. oscillation called the carrier; the resultant waveform is known as a "modulated wave." The audio frequencies no longer exist physically; their intensity at any instant is represented by the difference between the actual amplitude of the carrier at that instant and its unmodulated amplitude. A modulated wave is entirely a radio frequency oscillation, and prior to detection it exhibits no physical audio frequency properties. This has already been illustrated in Fig. 393.

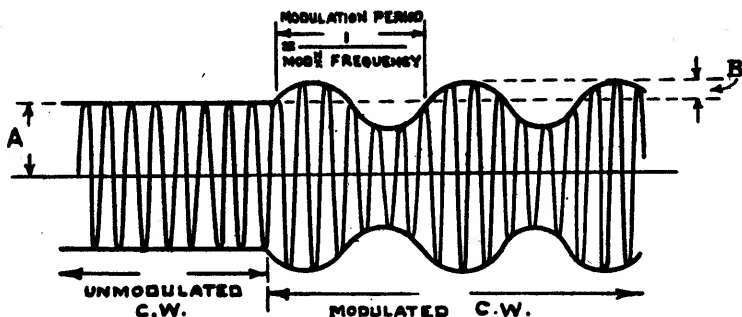


FIG. 397.

Fig. 397 shows a carrier wave of unmodulated amplitude A , modulated by a pure tone, *i.e.*, the modulated wave is of the type called Tonic Train. The variation of the carrier amplitude is sinoidal, and takes place between the limits $A + B$ and $A - B$. B is called the "Depth of Modulation."

The ratio $\frac{B}{A}$ gives a measure of the degree of modulation applied to the carrier amplitude. Expressed as a percentage, *i.e.*, $\frac{100B}{A}$, it is known as the "Percentage Modulation."

For 100 per cent. modulation, ($B = A$), the amplitude of the modulated carrier varies from zero to double its unmodulated amplitude. In R/T, the percentage modulation should never be allowed to exceed about 80 per cent., for reasons to be discussed below.

670. Sidebands.—Instead of regarding the modulated carrier wave as an R.F. oscillation of constant frequency and varying amplitude, it is often more convenient to consider it as the resultant of a number of superimposed R.F. oscillations which differ both in amplitude and frequency from each other, but each of which is of constant amplitude and frequency, *i.e.*, possesses a C.W. waveform. The sinoidally modulated carrier wave of Fig. 397, for instance, may be considered as the resultant of three C.W. oscillations whose frequencies are the frequency of the carrier, and the frequency of the carrier plus and minus the modulating frequency respectively.

This result may be derived as follows. The unmodulated carrier, of frequency $\frac{\omega}{2\pi}$, and amplitude A , is represented by the expression

$A \sin \omega t$. With a modulating frequency $\frac{p}{2\pi}$, and a depth of modulation B , the amplitude of the modulated carrier (the envelope of the oscillation of Fig. 397) may be expressed as $A + B \sin pt$, and so the complete expression for the modulated carrier becomes $(A + B \sin pt) \sin \omega t$.

$$\begin{aligned} \text{On expansion this becomes } & A \sin \omega t + B \sin pt \sin \omega t \\ &= A \sin \omega t + \frac{B}{2} \cos (\omega - p) t - \frac{B}{2} \cos (\omega + p) t \\ &= A \sin \omega t + \frac{B}{2} \sin \left[(\omega - p) t + \frac{\pi}{2} \right] + \frac{B}{2} \sin \left[(\omega + p) t - \frac{\pi}{2} \right] \end{aligned}$$

The modulated carrier is thus equivalent to three C.W. oscillations:—

(1) $A \sin \omega t$, whose amplitude and frequency are those of the unmodulated carrier.

(2) $\frac{B}{2} \sin \left[(\omega - p) t + \frac{\pi}{2} \right]$, whose frequency is **less** by the modulation frequency than that of the unmodulated carrier, and whose amplitude is half the depth of modulation. It is known as a "**Lower Sideband**" oscillation.

(3) $\frac{B}{2} \sin \left[(\omega + p) t - \frac{\pi}{2} \right]$, whose frequency is **greater** by the modulation frequency than that of the unmodulated carrier, and whose amplitude is half the depth of modulation. It is known as an "**Upper Sideband**" oscillation.

It is to be observed that all three components are R.F. oscillations.

It has been seen that the complex vibrations of speech and music can be resolved into a number of pure tones similar to that represented by $B \sin pt$. Hence, when a carrier wave is modulated by the audio frequency electrical oscillations corresponding to these complex vibrations, the modulation of the carrier by each component pure tone is equivalent to the production of three C.W.

radio frequency oscillations. The wave radiated may therefore be analysed into a C.W. oscillation at the carrier frequency, and a large number of C.W. oscillations whose frequencies are greater and less than that of the carrier by amounts equal to the frequencies of the original component pure tones in the speech or music. The band of such frequencies above the carrier frequency is known as the "Upper Sideband," and that below the carrier frequency as the "Lower Sideband," *e.g.*, if the carrier frequency is 1,000 kc/s. and the limiting frequencies of the speech or music are 50 and 8,000 cycles per second, the frequency range of the upper sideband is from 1,000,050 to 1,008,000 cycles per second, and that of the lower sideband is from 999,950 to 992,000 cycles per second.

A modulated wave thus covers a considerable band of frequencies, the width of the band being twice the highest modulation frequency (16,000 cycles per second in the example above). A receiving

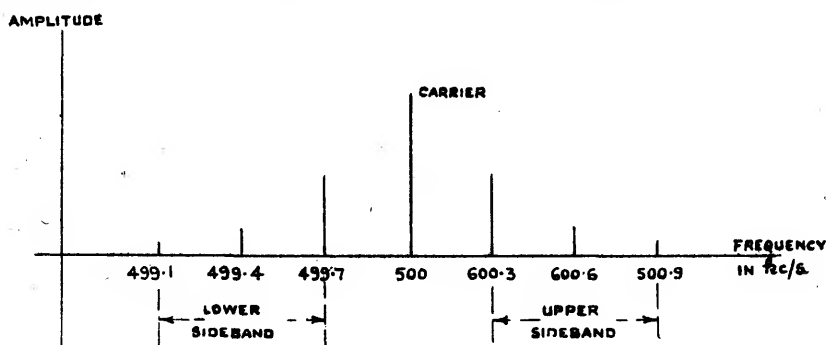


FIG. 398.

circuit for such waves must therefore be adjusted so that all frequencies in the band are equally well received, and to prevent interference in reception the difference between the carrier frequencies of any two R/T transmitters must be more than the sum of their highest modulation frequencies. The greater the number of R/T transmitters that have to be accommodated in a given range of frequencies, the less is the highest modulation frequency that can be allowed if interference is to be avoided. In commercial broadcasting, the highest normal modulation frequency is 8,000 cycles per second. The difference between carrier frequencies should then be about 20 kc/s. for stations likely to interfere with each other. This is not at present the case in practice.

The upper and lower sideband frequencies may also beat with each other and produce still higher frequencies, which is undesirable on account of the greater tendency to interference and the distortion introduced. This effect, however, can be made negligible by limiting the percentage modulation to about 80 per cent.

The sidebands produced when a carrier wave (500 kc/s.) is modulated by the waveform of Fig. 389 are shown in Fig. 398.

The first harmonic of the violin note is taken as 300 cycles per second.

It will be seen that each component of the modulating frequency produces its own sideband oscillation, and that the amplitudes of the sideband oscillations are proportional to the amplitudes of the component frequencies producing them. Thus, in Fig. 398 the sideband amplitude of the fundamental modulation frequency is greater than the sideband amplitudes of the weaker harmonics.

671. Methods of Modulation.—It was seen in Chapter XIV that the power developed in the tuned circuit of a valve generator, and hence the amplitude of the oscillatory current, depends on several different factors, including the following:—

- (a) The H.T. voltage applied to the anode.
- (b) The steady potential of the grid.
- (c) The relation between the impedance of the tuned circuit and the A.C. resistance of the valve.

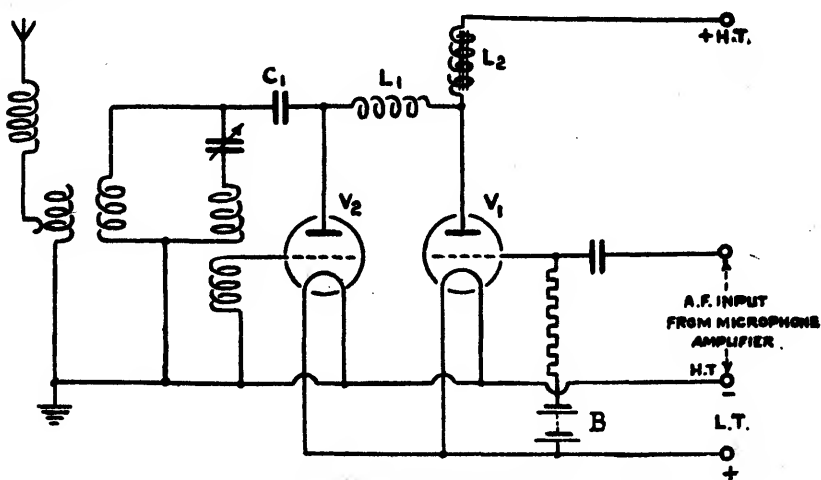
An alteration of any one of these factors produces a change in the amplitude of the oscillatory current. If the alteration is an audio frequency variation whose amplitude corresponds to that of a musical or speech vibration, the R.F. oscillation will therefore be modulated in amplitude in a similar manner.

The methods of modulation in common use depend on the three factors enumerated above, the most usual being variation of the H.T. supply to the anode. Various circuits have been devised for this purpose. The most popular and the only one that will be considered here is known as the "Choke Control," or "Constant Current" method. The methods depending on the other two factors, which are both found in Service R/T transmitters, are known respectively as grid modulation and modulation by absorption.

672. "Choke Control" Modulation.—A typical circuit for this system is shown in Fig. 399. V_2 is the R.F. oscillator valve, coupled in the usual manner to an oscillatory circuit, which is tuned to the required "carrier" frequency. L_1 is a radio frequency choke which serves to confine the R.F. oscillations to the valve V_2 and its oscillatory circuit. V_1 is the modulator valve and L_2 is an audio frequency choke in the common high tension supply lead to both valves. A grid bias battery B (or other source of E.M.F.) serves to maintain the grid of the modulator valve at the correct mean negative potential required by a power amplifier valve (para. 593). The audio frequency modulating voltages from the microphone amplifier (through amplifying stages if necessary) are applied between the grid and filament of the modulating valve V_1 . Considered as part of the output circuit of V_1 , the choke L_2 is in parallel with the oscillator valve, and its impedance at audio frequency is much greater than the A.C. resistance r_a of the oscillator. This also applies to the

blocking condenser C_1 , and the impedance of L_1 (in series with r_a) is negligible at audio frequencies. The output circuit of the modulator valve may thus be taken to be the A.C. resistance r_a of the oscillator valve. The modulator valve is essentially an audio frequency power amplifier, and the audio frequency voltage developed across its output, the oscillator valve, appears as a modulation of the anode voltage of the latter.

When the input voltage to the modulator valve is steady, the mean anode voltage of the oscillator, and hence the amplitude of the carrier, is also steady. The application of an A.F. input to the modulator valve produces a corresponding variation in the mean anode voltage of the oscillator, and therefore modulates the amplitude of the carrier in accordance with the original microphone diaphragm vibration.



Choke Control Method of Modulating Anode Voltage.

FIG. 399.

Certain precautions are necessary in order to prevent distortion during this process. It was seen in para. 598 that, with the correct circuit conditions, the output resistance of a power amplifier stage should be twice the A.C. resistance of the valve to give the maximum undistorted output. Thus the A.C. resistance of the oscillator valve should be twice the A.C. resistance of the modulator valve. This involves the use of a valve of low A.C. resistance as modulator, or of two modulator valves in parallel. Alternatively, the choke may be replaced by a transformer with a suitable transformation ratio T . The effective resistance of the oscillator valve then becomes r_a/T^2 , and so can be adjusted to the correct value.

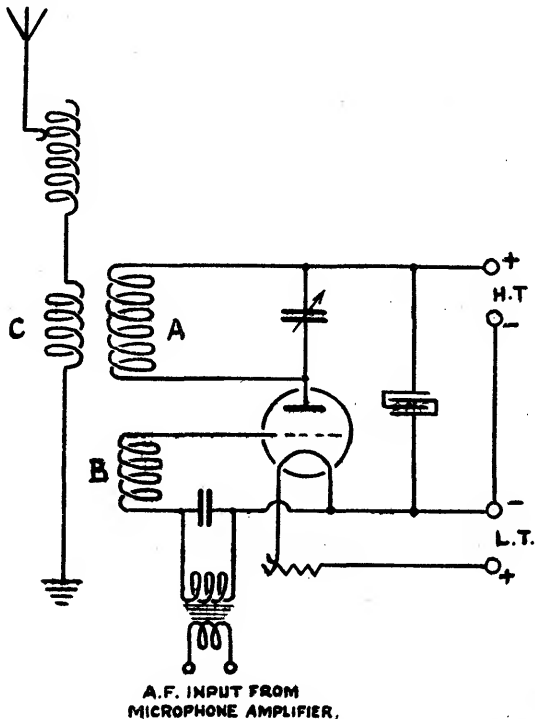
The impedance of the choke L_2 , or transformer, must be considerably larger than the A.C. resistance of the oscillator valve at the lowest audio frequency required. This choke must also be capable

of carrying, without distortion, the direct component of the H.T. current, and superimposed on it a small A.F. component of the modulator output current. By using a transformer the direct currents to the modulator and oscillator valves may be arranged to magnetise the core in opposite directions, and so to diminish its resultant steady magnetisation.

If it is desired to increase the depth of modulation beyond the amount that can be obtained in the above circumstances, the modulator valve may be supplied with a higher H.T. voltage than the oscillator valve.

It has been mentioned that a variation in the supply voltage to the anode of a transmitting valve is liable to give rise to frequency modulation. This may be avoided by carrying out the whole process in a low-power circuit and controlling the frequency of the R.F. oscillations by a quartz plate or similar method. The resulting modulated oscillation is then transferred to the aerial circuit through suitable amplifying stages.

673. Grid Modulation.—A circuit for this purpose is shown in Fig. 400. The A.F. output from the microphone amplifier is applied between grid and filament of the oscillator valve by means of



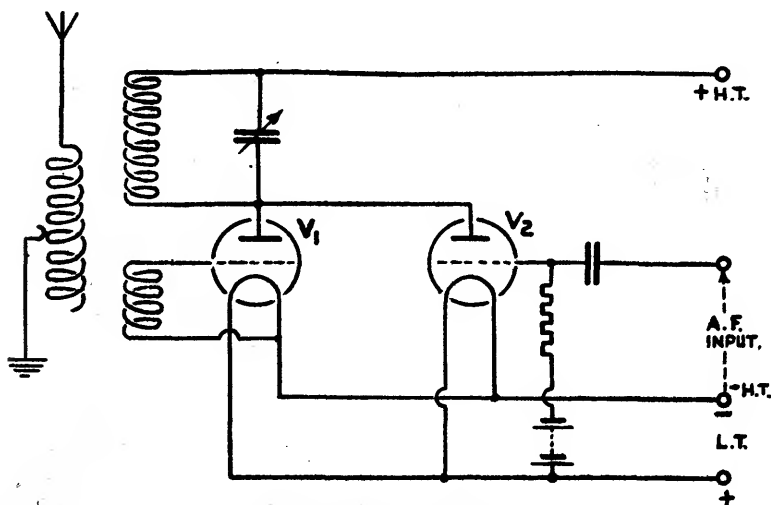
Grid Modulation.

FIG. 400.

transformer coupling. The mean grid voltage is thus varied at audio frequency, and the amplitude of the carrier is modulated correspondingly. The condenser across the terminals of the transformer secondary acts as a by-pass for R.F., and so ensures that practically all the radio frequency E.M.F. induced into coil B from the anode circuit is applied between grid and filament. Along with the transformer secondary, it also provides the grid insulating condenser and leak resistance.

The chief advantage of this system is the smaller power required for the same depth of modulation as in the choke control method, since the modulating voltage is applied to the grid of the oscillator.

674. Modulation by Absorption.—The typical circuit is shown in Fig. 401. V_1 is the oscillator valve and V_2 the modulator. The R.F. tuned circuit and V_2 form the external circuit of V_1 , and are in parallel with each other. Thus any variation in the A.C. resistance of the modulator valve alters the impedance of this external circuit.



Modulation by Absorption.

FIG. 401.

This alters the ratio of the oscillator valve A.C. resistance to the impedance of the external circuit, and so varies the power developed in the external circuit. The A.F. output from the microphone amplifier is applied between grid and filament of the modulator valve, and the grid bias is adjusted so that the operating point is on the bend of the anode characteristic. Since the grid voltage varies according to the A.F. input, the A.C. resistance of the modulator valve also varies, and therefore varies the impedance of the external circuit of the oscillator valve. The power developed in this circuit therefore varies correspondingly. In addition, the

audio frequency variation of the modulator valve A.C. resistance relative to the impedance of the oscillatory circuit varies at audio frequency the proportion of R.F. current from the oscillator valve flowing as make-up current to the oscillatory circuit. For both these reasons, the R.F. current in the oscillatory circuit is modulated at audio frequency, and a modulated wave is radiated.

It is difficult to avoid distortion when using this method.

RECEPTION.

675. The general procedure for R/T reception is the same as that for W/T reception. The incoming signal must be collected by a suitable aerial circuit. It will then normally pass through R.F. amplification stages, a detector stage and A.F. amplification stages, the final audio frequency output being converted to sound vibrations in telephones or a loud speaker. In W/T reception, the main requirement is to produce an audible vibration of sufficient amplitude to enable the signal to be read, *i.e.*, the sensitivity of the receiver is the primary consideration. An R/T receiver, however, is not likely to be operated under conditions where sensitivity is a primary requisite, and the emphasis lies rather on the quality of the reception, *i.e.* on ensuring that the audible output of the receiver is as faithful a reproduction as possible of the original sound vibrations which produced the modulation of the carrier.

It is comparatively simple to minimise distortion in aerial and R.F. amplifier circuits, and the first source of appreciable distortion is likely to be the rectifying stage. The input voltage to this stage largely determines the design of the preceding stages, and so the question of distortionless rectification will first be dealt with.

676. Detection. — The general principles of anode and cumulative grid rectification, as applied to W/T reception, were described in Chapter XII. No account was there taken of the fidelity with which the rectified current reproduced the input voltage modulation. It was pointed out, however, that with weak signals, the rectified current was proportional to the square of the audio frequency variation of the incoming voltage. For distortionless rectification, of course, an essential condition is that the rectified current should be proportional to the incoming voltage and not to its square. The rectified current produced by a weak input voltage is thus considerably distorted.

Before a completely undistorted rectified current could be obtained, the characteristic of a detector valve would require to be as shown in Fig. 402 (*a*). This consists of two straight lines, one of them being the zero current line. If the operating voltage were adjusted to the point O, where the sloping part of the characteristic meets the horizontal part, the amplitude variation of the rectified current would be a replica of that of the input voltage for all values of the latter,

The characteristic shown may be considered either as the dynamic mutual or grid current-grid voltage characteristic, according as anode or cumulative grid detection is being employed. In practice, of course, no valve characteristic has a discontinuous slope of this kind, but apart from the curvature at the bends, the dynamic characteristics can be arranged to have a fairly constant slope. If the modulated input voltage amplitude never falls below the straight part of the characteristic, as shown in Fig. 402 (b), where V is the least permissible input peak voltage, nearly distortionless rectification may be obtained.

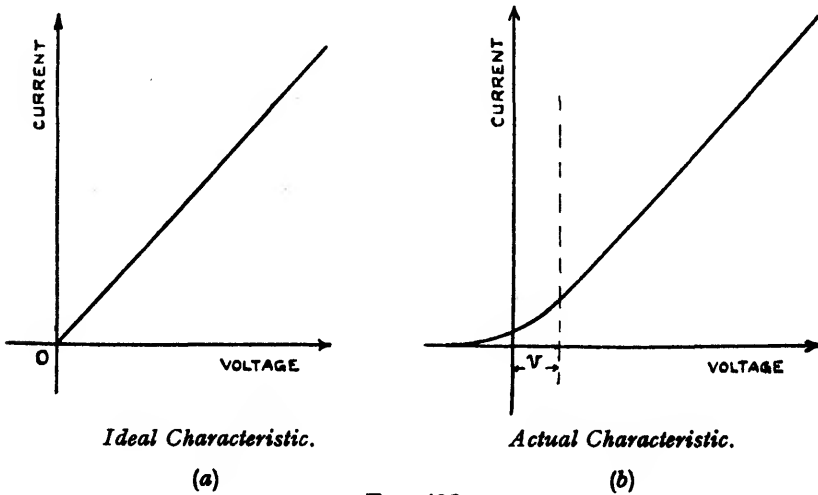


FIG. 402.

The feasibility of this depends to a large extent on the maximum depth of modulation of the carrier. With 100 per cent. modulation, the amplitude varies between twice the amplitude of the unmodulated waveform and zero, and so it is impossible to avoid the curved part of the characteristic. The greater the depth of modulation, the greater must be the mean amplitude of the input voltage in order to ensure that the lowest input amplitude reaches the straight part of the characteristic. It is, of course, also necessary with large input voltages to ensure that the maximum amplitude of the modulated input does not extend beyond the straight part of the characteristic, or into the region when grid current flows, in the case of anode rectification.

Both the general methods of rectification are used for R/T. Owing to the use of much larger input voltages than are common in W/T reception, they are known as power anode detection and power grid detection respectively. In general, and provided an indirectly heated cathode valve is employed, power grid detection leads to less amplitude distortion in the rectified output than power

anode detection. The grid current-grid voltage dynamic characteristic becomes nearly straight at a fairly small input voltage, and has a straighter slope for higher inputs, than the mutual characteristic, the lower bend of which is longer and more gradual. At the present time, power grid detection is the standard method in the highest quality receivers.

Frequency distortion, *i.e.*, the production of different amplitudes of rectified output voltage for the same amplitude of R.F. input voltage at different modulation frequencies, must also be guarded against. Its occurrence is mainly due to a badly-chosen output circuit.

677. Power Anode Rectification.—It was seen in Chapter XII that the output current of an anode rectifier is still a radio frequency oscillation, whose mean value, however, is no longer zero, and varies at audio frequency; thus the rectified waveform can be considered as the sum of two components, one at radio frequency and one at audio frequency, and each of mean value zero. (There is also a D.C. component, but this is unimportant for our present purpose.) If no distortion has arisen owing to characteristic curvature, the audio frequency component reproduces the modulation of the carrier. It is still necessary to separate the two components before the audio frequency oscillation can be converted to the corresponding sound vibration. In the W/T detection circuit this was accomplished by means of a by-pass condenser across the telephones, or across the input to the first note magnifier stage. The same procedure is adopted in R/T, but if frequency distortion is to be avoided, care must be exercised in choosing the values of the detector audio frequency output impedance and its by-pass condenser C. It will be assumed that the audio frequency output impedance is a resistance R.

The first consideration is the efficient separation of the audio frequency and radio frequency components of the output. This requires that the reactance of the condenser be negligible compared with the resistance at radio frequencies. On the other hand, it must be large compared with the resistance at the highest audio frequency dealt with, and this limits the value of the capacity. If C is too large, attenuation of the higher audio frequencies will be the result.

From another point of view the value of the product " CR ," the time constant of the output combination, must be small enough to enable the audio frequency collection of charge on the condenser to follow the modulation of the carrier. If " CR " is too large, the highest audio frequencies—*i.e.*, those vibrations whose amplitude changes most rapidly—will be distorted. It is necessary, however, for the resistance to be high compared with the working A.C. resistance of the valve, or the dynamic characteristic will not be straight enough to prevent distortion due to characteristic curvature.

Finally, the question of R.F. feed-back from the output circuit to the input through the anode-grid inter-electrode capacity must be considered. The R.F. output circuit has a net capacitive reactance, and so the feed-back is in such a phase as to damp the input circuit (para. 601). The feed-back increases with the reactance of the output condenser, *i.e.*, it varies inversely as the capacity, and from this point of view the capacity should be the largest possible.

It should now be obvious that general rules cannot be laid down for the best values of C and R , and these must be chosen with respect to the individual circuit, the type of valve employed, and so on. As an instance, the figures in a modern broadcast receiver, whose detector valve constants were $m = 25$, and $r_a = 25,000 \Omega$, were $R = 250,000 \Omega$, and $C = 0.0003 \mu F$.

It may be added that the anode voltage should be high enough to prevent flow of grid current for the largest depth of carrier modulation dealt with.

678. Power Grid Rectification.—It is preferable when discussing the quality of the rectification given by this method, to consider the function of the grid insulating condenser and leak from a point of view slightly different from that adopted in Chapter XII. The rectified current is now grid current, and returns to filament in the external grid circuit. The condenser and leak resistance thus play similar parts to the by-pass condenser and output resistance in the anode rectification circuit, and their values will be dictated by the same kind of consideration as those discussed in that case. In particular, to prevent attenuation of the higher audio frequencies, the time constant " CR " must not be too high. The static I_g - V_g characteristic has less curvature than the static mutual characteristic, and so the ratio of the leak resistance to the grid-filament working A.C. resistance need not be so high to obtain a straight dynamic characteristic as for the corresponding quantities in anode rectification. This allows the use of a larger relative value of capacity. In view of the larger grid input voltages, the H.T. voltage on the anode must also be sufficiently high to prevent the greatest input amplitude from reaching the lower bend of the mutual characteristic, otherwise distortion of the anode output current will be produced.

The steady anode current flowing is usually too great to be passed through the primary winding of a transformer coupling, and so the output circuit must be resistance-capacity, choke-capacity, resistance-capacity-transformer, or choke-capacity-transformer coupled to the next stage. Resistance-capacity coupling gives the best results, but requires a high H.T. voltage to give a high enough anode voltage after the drop in the resistance has been taken into account.

In order to emphasise the difference in the operating conditions between power grid detection and ordinary cumulative grid

detection, as used in W/T practice, the following comparative table of values is given :—

—	Anode Voltage.	Condenser.	Resistance.
Cumulative grid ..	50	$\mu\text{F.}$ 0·0001	$\text{M}\Omega$ 2·0
Power grid ..	150	0·0003	0·2

The most serious disadvantage of power grid detection is the damping it produces on the input circuit, which is much greater than in the case of the power anode detector. It arises through two causes :—

- (1) The flow of grid current necessary for rectification involves a diminution in the working A.C. grid-filament resistance. This is in parallel with the input circuit, and so its damping effect is inversely proportional to its actual value.
- (2) The operating part of the mutual characteristic is the straight part, where the amplification factor for the R.F. input has its highest value. The effective amplification factor on the lower bend, which is the operating point for the anode rectifier, is considerably smaller. The amount of feed-back obviously depends on the voltage developed across the output circuit, and so is proportional to the amplification factor for given values of inter-electrode capacity and R.F. output impedance. Thus the damping due to this feed-back is greater in power grid detectors than in power anode detectors.

The higher amplification given by power grid detection is more than sufficient to compensate for this increased damping of the input as compared with power anode detection, but a decrease in selectivity is inevitable. The damping due to feed-back may be considerably reduced in both anode and grid rectification by using two valves in the detector stage, their grids being in push-pull and their anodes in parallel.

The use of a screen grid valve for detection would minimise the feed-back in both systems, as has already been seen when discussing amplifier stability (para. 602). The ordinary screen grid valve characteristics, however, are unsuitable, though a push-pull screen grid valve detection stage has been successfully used. Regenerative reaction can also be employed, or the circuit may be balanced by a neutralising condenser. Probably the best solution is to use an acceptor circuit for the radio-frequency oscillation instead of a by-pass condenser in the anode output circuit, but this involves an extra tuning control.

679. Aerial and R.F. Amplifier Circuits.—In W/T reception, provided audibility is ensured, the chief consideration in the design of the aerial circuit and the R.F. amplification stages is the provision of sufficient selectivity to enable the desired signal to be received with a minimum of interference. It has been seen that this may be accomplished by the use of an aerial secondary circuit, with loose coupling and sharply-tuned circuits in the R.F. stages. Selectivity is obviously desirable in an R/T receiver also, but its attainment conflicts with another important quality, namely, that the response of the receiver should be the same for all frequencies of modulation of the carrier wave. If the circuit is too selective, this will not be the case, and the higher audio frequencies in the final output will be relatively weaker than the lower audio frequencies, thus giving rise to distortion.

This effect may be considered from two points of view. It was shown in para. 428 that when an alternating E.M.F. is applied to a circuit, the current does not assume immediately its final value obtained by dividing the E.M.F. by the impedance, but that a transient damped oscillation is also set up; a similar effect occurs when the E.M.F. is removed and, in general, whenever the amplitude of the E.M.F. is altered. This oscillation is damped out to negligible proportions in a comparatively small number of cycles, and when the time during which an E.M.F. of constant amplitude is applied is long compared with the period of a cycle, as in a C.W. signal, the transient effect is unimportant. In an R/T signal, however, the amplitude of the carrier is continually altering, and the transient effect must then be taken into account, for as long as it remains appreciable the current is not an accurate reproduction of the applied E.M.F. The higher the modulation frequency, the more quickly is the carrier amplitude changing, and so the higher audio frequencies are more distorted than the lower ones. The rate of decay of the transient oscillation is proportional to the ratio of resistance to inductance in the circuit. (It was seen in para. 428

that the damping factor $\alpha = \frac{R}{2L}$.) Hence the higher the inductance, the slower is the rate of decay and the greater the distortion. The selectivity of the circuit, however, is directly proportional to the ratio of inductance to capacity, and inversely as the resistance (para. 302), i.e., at any given carrier frequency, the greater the ratio of inductance to resistance the higher is the selectivity. Thus, the more selective the circuit, the greater is the distortion.

The effect may also be explained in terms of sidebands. It has been shown in para. 670 that a modulated carrier wave is equivalent to a number of C.W. waves of constant amplitude, whose frequency range is twice the highest modulation frequency. For undistorted reception, the response of the receiver should therefore be uniform over this band of frequencies. This is the reverse of the response of a sharply-tuned circuit, in which maximum sensitivity is obtained

at one selected frequency, and the response falls off rapidly as this frequency is departed from (Fig. 403 (a)). It is obvious that if the circuit is tuned to the carrier, the distortion is greater the more the sideband frequency differs from the carrier frequency. In other words, attenuation of the higher audio frequencies is the price of good selectivity. A compromise has therefore to be made.

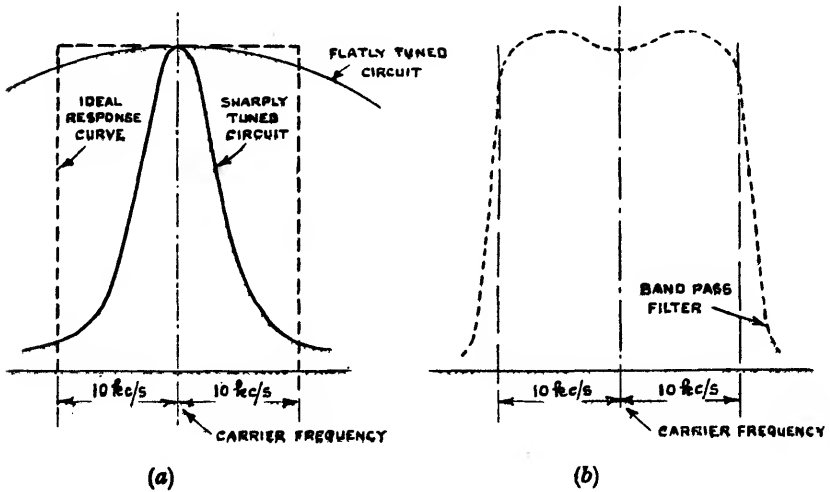


FIG. 403.

680. The ideal R.F. response curve for an R/T receiver is shown in Fig. 403 (a), for comparison with the response curves of a sharply and flatly-tuned circuit respectively. It is possible to approximate to this ideal curve by making use of the double-frequency effect produced in two circuits tuned to the same frequency and coupled to each other (para. 336). It has been seen that two peaks in the response curve are then produced, their distance apart increasing as the coupling is made tighter.

The primary and secondary circuits are separately tuned to the carrier frequency and the coupling is arranged so that the resonance peaks are each about 5 kc/s. from the carrier frequency, giving a response curve as shown in Fig. 403 (b). When used in this way, the coupled circuits form a **band-pass filter**, since they respond nearly uniformly to a band of frequencies, and hardly at all to frequencies outside the band. Provided the interfering frequency is more than about 10 kc/s. removed from the carrier frequency, sufficient selectivity as well as a fairly uniform response to the modulated carrier is obtained. With a constant coupling factor, the separation of the peaks is always the same percentage of the carrier frequency, and so increases with that frequency. But the band-pass filter is required to give a constant peak separation over a range of carrier frequencies. This may be obtained by several

varieties of mixed inductive and capacitive coupling between the two tuned circuits.

An alternative method is to use very sharply-tuned circuits in the R.F. stages to cut out interference and to compensate for the resulting sideband attenuation by means of a **tone-corrector** in the A.F. stages. The A.F. output circuits are designed so that they give an amplification increasing in the correct proportion with frequency to balance the decreasing response of the R.F. stages as the difference between the carrier and sideband frequency increases. This rising characteristic is followed by a sharp cut-off at about 8 kc/s., and so gives the same result as a band-pass filter in the R.F. stages. An extreme example of this is found in the "Stenode Radiostat," where very sharply-tuned R.F. circuits are followed by a correspondingly generous tone-corrector.

681. The number of R.F. stages required depends largely on the type of detection employed, a power anode detector requiring a higher input voltage than a power grid detector. It is important to keep this voltage at the best value for distortionless rectification, and sufficient R.F. amplification must be available for this purpose with the most distant signals it is desired to receive, some form of volume control being incorporated in the first stage to prevent overloading with stronger signals. This commonly takes the form of a high resistance potentiometer across the aerial secondary condenser. This is preferable to the use of reaction, as it has less effect on the selectivity. It will be remembered that the damping effect of a shunting resistance is inversely proportional to its actual value, and if the potentiometer is of high enough resistance, its damping effect is negligible. Reaction sharpens the tuning, and its use is liable to give rise to distortion. Potentiometer volume control requires a greater number of R.F. amplification stages than the use of variable reaction.

Screen grid valves may be used to obtain high amplification, while preserving stability and avoiding the neutralisation necessary with triodes; and the other precautions against back coupling, described in Chapter XIII, are even more essential in R/T than in W/T amplification stages. Even if the stray coupling is not sufficient to give rise to self-oscillation, it is liable to produce distortion. Screen grid valve dynamic characteristics are not so straight as those of triodes, and distortion may arise either due to partial rectification of the wanted signal, or to rectification of an interfering signal, which then modulates the carrier of the wanted signal. This latter effect is known as "cross modulation," and is minimised by the use of a band-pass filter in the aerial circuit and potentiometer volume control.

Quite recently there has been developed a special type of tetrode, known as the "variable- μ " valve, which seems likely to provide a solution for most of these difficulties. Its chief feature is the

irregular mesh constituting the control grid, which lessens considerably the grid control at large negative grid voltages, and so gives to the characteristic a long "tail" of small and nearly constant slope instead of the usual sharply-curved lower bend. A suitable arrangement of feed resistances to the various electrodes enables volume control to be obtained by an automatic variation of grid bias according to the strength of the incoming signal, thus giving a nearly constant output for a large range of input voltages.

682. A.F. Amplification.—The requirement in A.F. amplifying stages is that the voltage developed across the output circuit should be a replica on a larger scale of the input voltage applied between grid and filament. The V.A.F. of the stage, in fact, must be independent both of the frequency and the amplitude of the input voltage over the required range of frequencies and amplitudes.

Amplitude distortion, *i.e.*, variation of V.A.F. with the amplitude of the input voltage, can arise from various causes. One of the commonest is the application of an input voltage of too large amplitude (overloading the grid). The grid swing is necessarily larger in A.F. than R.F. stages, and it becomes a matter of increasing difficulty to get a valve dynamic characteristic which is straight over a large enough range of grid voltages. The higher the ratio of the external anode impedance to the valve A.C. resistance, the more likely is this condition to be satisfied. In addition, the flow of grid current will cause amplitude distortion, and so high anode voltages are necessary to obtain a sufficiently large amount of the straight part of the characteristic at negative grid voltages. The appropriate negative grid bias must also be employed.

Amplitude distortion may also arise in a badly-designed or overloaded output stage, *e.g.*, if iron core transformers are employed and the primary has to carry a large steady anode current (para. 384). This will normally be accompanied by the generation of harmonics, which alter the tone quality. It can be considerably reduced by applying the H.T. to the anode through a separate choke, which also carries the steady anode current, as in the parallel feed system for transmitting circuits.

Frequency distortion, or variation of the stage V.A.F. according to the frequency of the input voltage, will be produced unless the output impedance is independent of frequency. From this point of view the best inter-valve coupling would therefore be resistance-capacity coupling. The resistance must be small enough for any stray, self or inter-electrode capacities in parallel with it to have a much higher reactance at the highest audio frequency dealt with, or the higher frequencies will be attenuated. Similarly, the coupling condenser must be large enough for its reactance to be negligible compared with the resistance of the grid leak, or lower frequencies are attenuated; but the time constant of the grid coupling condenser and leak must be kept as small as possible consistent with this:

otherwise overloading of the grid and the consequent flow of grid current may increase the negative grid bias so much that the grid swing reaches the bottom bend of the characteristic, and amplitude distortion is produced.

The above limitation in the size of the output resistance requires that a valve of correspondingly low A.C. resistance be used if a reasonably straight dynamic characteristic is to be obtained, and amplitude distortion avoided. An approximate figure for the external resistance is 25,000 ohms, and, with this value, the valve A.C. resistance should not exceed 7,000 ohms. This combination will give nearly uniform amplification from 30 to 10,000 cycles per second.

Transformer coupling is also often employed in A.F. stages, particularly with a push-pull valve arrangement. In general, care must be taken that the transformer windings do not tune with their self or stray capacities anywhere near the audible range, otherwise frequency distortion will be produced. Sometimes, however, the coupling is arranged to be resonant at a particular frequency to counterbalance a loss in amplification at that frequency in another stage of the receiver (*cf.* the tone-corrector).

To give a V.A.F. independent of frequency, the reactance of the primary at the lowest audio frequency must be large compared with the A.C. resistance of the valve. Since the dimensions of the output stage are limited, this leads to the use of transformers with only a small step-up ratio. In addition, the smaller the secondary the less is the effect of its self-capacity in producing attenuation of the higher audio frequencies.

The advantages of a push-pull valve arrangement in amplifying large input voltages without distortion were dealt with in para. 597.

683. Reproduction.—This is the process whereby the A.F. oscillatory currents are made to produce sound waves of corresponding frequency and amplitude. The principle has already been described for the case of telephones (para. 495) and is essentially the same for the other reproducing devices known as loud speakers. The alternating magnetic field of the current gives rise to an alternating mechanical force on a diaphragm and the to-and-fro motion of the diaphragm produces alternate compression and rarefaction of the air in its vicinity, *i.e.*, a sound wave.

For perfect reproduction the intensity of the sound, or the "acoustic output," must bear the same proportion to the electrical power input at all input frequencies and amplitudes, but at the present time there is no reproducing device which satisfies these ideal requirements even approximately.

684. Loud Speakers.—These may be divided into two main categories :—

- (a) **Horn type.**—In this type a large horn is used to provide a resonating column of air so that a small diaphragm may be able to produce a large volume of sound. This has

the advantage that a correspondingly small electrical input is required to agitate the diaphragm. The shape of the horn must follow an "exponential" curve, *i.e.*, cross-sections at equal axial intervals from the throat of the horn are such that the ratio of the area of any one to the succeeding one is constant. If this is not the case, the sound waves reflected from the inside of the horn produce distortion in the resultant sound issuing from the horn.

For good quality, a large horn is also essential. The smaller the horn, the higher is the frequency below which the volume of sound from the loud speaker falls off from the proportionate volume at higher frequencies. This frequency, known as the "Lower Cut-off Frequency," is as high as 300 cycles per second in an ordinary small horn-type loud speaker. In talking picture installations, horns about 15 feet long and 4 feet across at the wide end are used. These bring the lower cut-off frequency down to about 70 cycles per second.

- (b) **Cone diaphragm type.**—The diaphragm in this type is conical in shape and comparatively large, so that it acts directly on a large volume of air, no horn being employed. In order to obtain efficient reproduction of low frequencies it is also necessary to separate the air behind the diaphragm from that in front of it by means of a large plane board (generally made of wood), called a baffle; otherwise the air in front of the diaphragm, when compressed, can return to normal pressure by escaping to the back of the diaphragm, instead of compressing the air exterior to itself and so propagating a sound wave. This is obviously most likely to be pronounced at low frequencies where the time between successive compressions is greatest. In other words, the smaller the baffle, the higher is the Lower Cut-off Frequency.

Diaphragms are usually made from paper, linen, balsa wood or aluminium alloy. The last is the most efficient.

685. Driving Systems.—The arrangement in a loud speaker whereby an audio-frequency alternating current is made to produce an audio-frequency mechanical vibration of the diaphragm is usually known as the "drive." Driving systems are either of the moving coil or moving iron type, and the principles involved are the same as in the measuring instruments of these types described in Chapters III and VI. The loud speaker commonly known as the moving coil loud speaker employs a cone diaphragm and a moving coil drive, and is the only type that will be described in any detail here.

686. Moving Coil Loud Speaker.—Fig. 404 (b) is a diagrammatic representation of this instrument. A coil of wire wound on a light cylindrical former—the moving coil—is suspended so that it can travel backwards and forwards in the direction of its axis through a distance of about 0.25 to 0.5 inch, but is not capable of movement

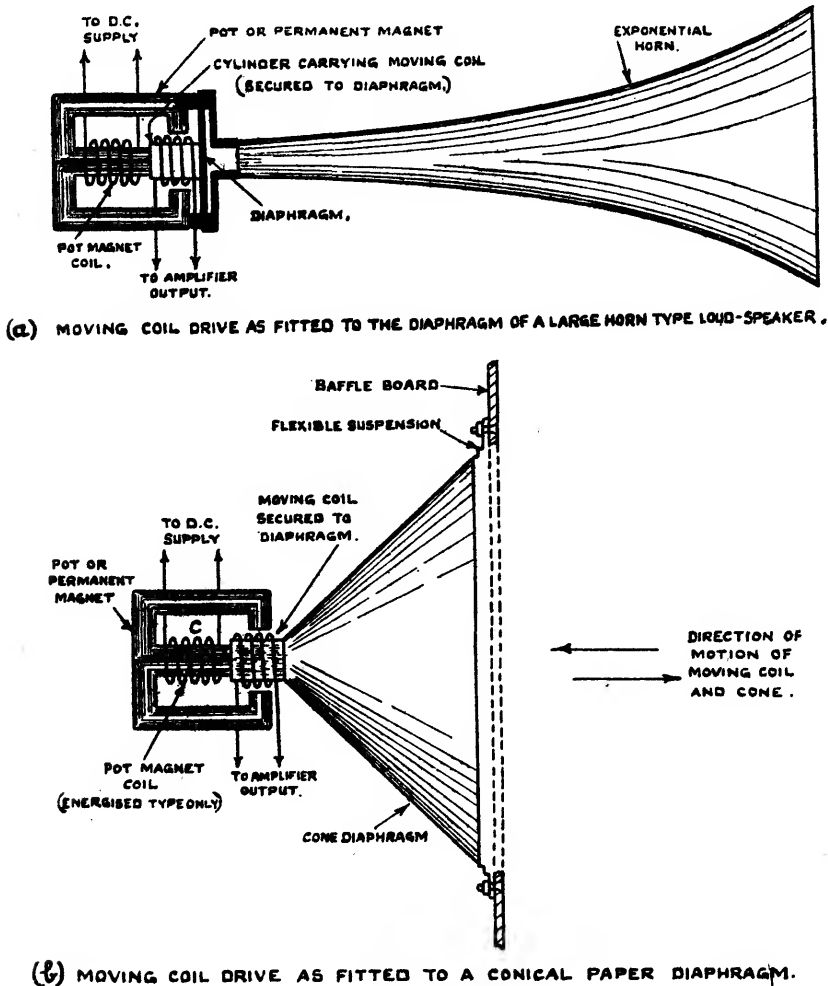


FIG. 404.

in any other direction. It lies in the field of a permanent or electro-magnet, of the shape illustrated in the figure, and called a pot magnet. This field is radial and therefore at right angles to the coil of wire at every point. The coil, when carrying current, is thus acted on by an electrodynamic force (para. 129), which sets it in motion at right angles to the plane of its own current and the

direction of the field. According to the direction of the current, the coil therefore moves in the direction of its axis to right or left, as determined by Fleming's Left Hand Rule. When an alternating current flows in the coil, the direction of motion is reversed every half-cycle, *i.e.*, the coil undergoes a vibratory motion. The coil is rigidly secured to the diaphragm, and so the latter partakes of the same motion and sets up an air vibration or sound wave of corresponding amplitude and frequency.

The diaphragm and coil are usually suspended by a ring of silk, leather, or other flexible substance round the circumference of the diaphragm. A "spider" in the centre ensures that only axial movement is possible.

The chief advantage of the moving coil drive is the relatively large movement permitted by the free suspension. This has the effect in practice of giving better reproduction at low frequencies than is commonly the case with other types of drive, where the necessities of design lead to a much smaller maximum permissible movement of the diaphragm.

687. To prevent frequency distortion the impedance of the output valve circuit, including the loud speaker impedance, should be independent of frequency in the ideal case. The coil, however, inevitably possesses a certain amount of self-inductance. In addition, the movement of the coil in the field of the pot magnet sets up in it an induced E.M.F. This E.M.F. is actually in the opposite direction to the back E.M.F. of self-induction, and so has the same effect on the impedance as if the coil possessed a certain amount of capacitive reactance. The corresponding capacity is called the "motional capacity."

The increase of inductive reactance with frequency, and the consequent increase in impedance, has the effect of reducing the current for a given voltage input to the power stage as the frequency increases. This would lead to attenuation of the higher frequencies, but it seems to be compensated for to a large extent by the fact that the conical diaphragm, in addition to its axial vibration as a whole, undergoes elastic vibrations of its material at these frequencies. The number of possible vibrations of this type is very large, and the complicated mechanical resonances produced combine to give a fairly uniform output at high frequencies.

Moving coils are either wound with many turns of fine wire, in which case they are said to be "high impedance" coils, or with a few turns of thicker wire which can carry a larger current. The latter are known as "low impedance" coils. For reference purposes an average effective impedance is usually quoted, although, as pointed out above, the actual impedance varies with the frequency. This average impedance is about twice the D.C. resistance of low impedance coils, and four times the D.C. resistance of high impedance coils.

It was shown in para. 593 that for maximum undistorted power in the output impedance the latter should be twice the A.C. resistance of the associated valve, if a triode. It is thus of great importance to ensure that the valve and loud speaker are correctly matched. Apart from the loss of power, higher frequencies are attenuated if the loud speaker impedance is too large, for its varying impedance with frequency then produces a greater proportional effect in varying the total impedance of the stage. The value of transformer coupling in altering the effective impedance of the output so as to give the correct ratio was pointed out in para. 597. This type of coupling also carries out the necessary function of preventing the steady anode current from flowing through the moving coil, and, if it is not employed, a choke filter circuit must be inserted in its stead.

When a pentode output valve is used, the impedance of the loud speaker will generally be considerably less than that of the valve, and so a very uniform frequency response may be expected. If the loud speaker impedance is too low, the power output will be inefficient, and if too high the voltage developed may damage the valve and frequency distortion may also arise. The manufacturer's instructions in any particular case should therefore be carefully followed.

688. Moving Iron Loud Speakers.—These may employ either a "balanced armature," "reed" or "inductor" drive.

The principle of each of these drives is the same. A small, light, soft iron armature is suspended in the magnetic field of a permanent magnet, and attached to the loud speaker diaphragm. A coil of wire (or two coils in series) is wound round the yoke of the magnet, the ends being connected to the loud speaker terminals.

The audio frequency (speech) currents from the amplifier pass through the loud speaker winding, and so give rise to corresponding variations in the flux from the magnet, which, in turn, vary the force exerted by the magnetic field on the iron armature, thus causing it to vibrate together with the loud speaker diaphragm in approximate accordance with the speech current oscillations.

In the "reed" drive, a metal reed is securely fixed at one end so that it lies close to but just not touching the poles of the loud speaker magnet. The diaphragm is attached to the free end of the reed. Variations in the magnetic flux density cause the reed to bend to a greater or less extent, and so to vibrate about its fixed end. With this type of movement, care must be taken that if any direct current is allowed to flow through the windings it flows in the direction which increases the flux of the permanent magnet. This is usually indicated by the terminals of the loud speaker being marked + and -.

In the "balanced armature" type of drive, the armature is suspended in the centre, and the magnets are so arranged that

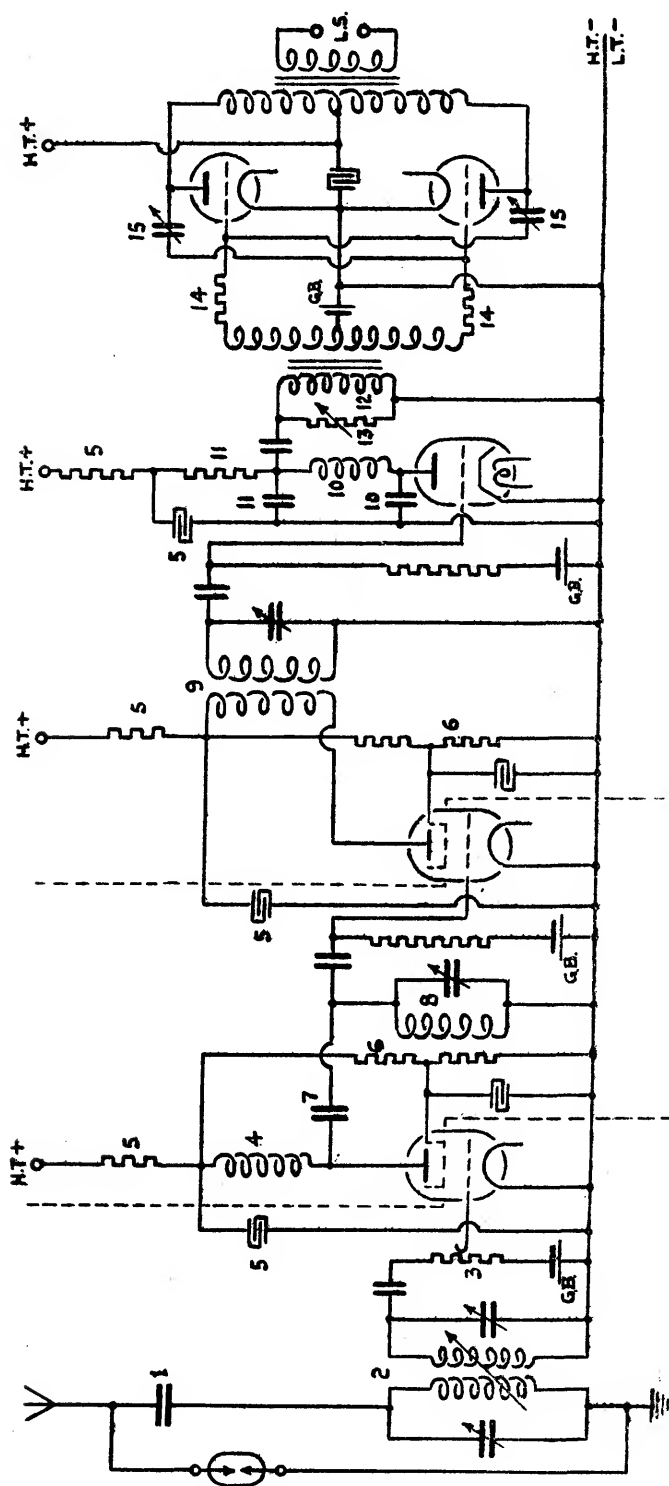


Fig. 405.

their initial pull on the armature is practically eliminated, the latter being affected only by the variations of the magnetic flux due to the oscillatory speech currents. This method enables the armature to be more freely suspended than in the "reed" type, resulting in more uniform reproduction and a greater response to the lower audio frequencies.

On the other hand, any direct current flowing through the loud speaker windings will have a greater tendency to pull the armature over on to the magnet poles. Hence, with this type of loud speaker it is always advisable to use a choke-filter or transformer coupling between the amplifier and the loud speaker.

In the "inductor" type of loud speaker, the iron armature is supported in a manner which allows considerably greater freedom of movement than is possible with either the reed or balanced armature type of loud speaker. Instead of the armature moving towards and away from the poles of the magnets, it moves like a piston between two pairs of magnetic poles.

The chief advantage of this type of speaker is that the freedom of suspension, combined with the large permissible movement of the armature, provides a better frequency response characteristic than that of other types of moving iron speaker. This type of loud speaker compares favourably with an average cheap moving coil loud speaker, both as regards sensitivity and quality of output, and has the advantages of lightness and cheapness, and also that no field energising current is required. As in the balanced armature and moving coil types, no direct current should be allowed to flow through the loud speaker windings.

689. A typical R/T receiving circuit is shown in Fig. 405. It contains two R.F. amplification stages employing screen-grid valves, an indirectly-heated valve power grid detection stage, and one neutralised push-pull stage of A.F. amplification. The purpose of the more important components is as follows :—

- (1) Small series condenser ($0.0001 \mu\text{F.}$) in the aerial circuit, to minimise the effect of variations of aerial capacity upon the tuning.
- (2) Band-pass filter to give the correct frequency response curve for R T reception.
- (3) High resistance potentiometer volume control to adjust the input to the detector valve. It is fitted in the first stage so that the R.F. stages will not be overloaded. The coupling condenser is necessary to prevent the grid bias battery from discharging through the tuning inductance.
- (4) An R.F. choke for parallel feed of the H.T. supply to the first stage. Any A.F. currents produced by back coupling, etc., in this stage pass through the coupling condenser (7) and the output impedance (8) to filament.

The impedance of (8) is negligible compared with that of (7) at audio frequencies, and so practically no audio frequency P.D. is developed across the input of the next stage.

- 5) De-coupling condensers and resistances.
- (6) Potentiometers for supplying screen voltages.
- (7) Coupling condenser whose purpose is explained under (4).
It also prevents a short circuit of the H.T. supply through the inductance of the output impedance (8).
- (8) Tuned anode output impedance of first R.F. stage.
- (9) Transformer coupled output of second R.F. stage.
- (10) Choke and by-pass condenser providing a preliminary separation of the R.F. and A.F. components of the anode current from the detector valve.
- (11) Resistance-capacity coupled detector output circuit to give uniform amplification at all audible frequencies.
- (12) Primary of input transformer coupling to push-pull stage.
Along with the blocking condenser to prevent short circuit of the H.T. supply, this primary is in parallel with the output resistance of the detector stage, and so has the A.F. variations of potential impressed across it. The amplitude of the input voltage to the push-pull stage is controlled by
- (13) A variable resistance across the input primary.
- (14) Resistances to damp out any parasitic oscillations that may tend to be set up in the push-pull stage. They also act as R.F. stoppers (para. 698).
- (15) Neutralising condensers. The residual feed-back through inter-electrode capacity is not likely to cause self-oscillation in this stage, but these capacities act as a coupling to the output circuit (loud speaker), and hence any variations in the impedance of the latter will alter the load across the secondary of the input transformer and affect its frequency response. The neutralising condensers prevent this.

CHAPTER XVI.

HIGH-FREQUENCY RECEPTION AND TRANSMISSION.

690. The frequencies used for wireless transmission have been divided into several bands whose limits are determined mainly by the nature of their propagation through space. (This subject is treated in Chapter XVII.) The nomenclature and limits adopted for these frequency bands are as follows:—

Below 100 kc/s.	..	Low Frequencies	..	L.F.
100–1,500 kc/s.	..	Medium Frequencies	..	M.F.
1,500–6,000 kc/s.	..	Intermediate Frequencies	..	I.F.
6,000–30,000 kc/s.	..	High Frequencies	..	H.F.
Above 30,000 kc/s.	..	Very High Frequencies	..	V.H.F.

It is convenient, however, in this chapter to use the term high frequency in a general sense to cover the whole band of intermediate, high and very high frequencies.

Special difficulties are encountered at these frequencies, particularly in reception, which render necessary the use of different methods from those already considered. It is proposed in this chapter to give a short description of these difficulties, and to explain how they are overcome or avoided.

HIGH-FREQUENCY RECEPTION.

691. The general methods of amplification discussed in Chapter XIII, (with the exception of reaction), depend for their success on the possibility of designing an output impedance which is comparable with and, if possible, greater than, the A.C. resistance of the valve. This is comparatively simple for low and medium frequencies and the lower intermediate frequencies, but is rendered impossible at higher frequencies by the presence of unavoidable capacities, which by-pass the high-frequency currents.

The self-capacity of the coil or resistance used as output impedance, the anode-filament capacity of the valve and the grid-filament capacity of the next valve, are all in parallel across the output of any amplifier stage, and so the total output impedance cannot be made greater than the combined reactance of these three capacities. An average value for this reactance at 20,000 kc/s., say, would be about 5,000 ohms, which is, of course, much less than the A.C. resistance of any valve used for radio-frequency amplification. This difficulty in obtaining efficient amplification applies equally both to triodes and tetrodes, as may be illustrated by calculating the amplification obtained with an output impedance of 5,000 ohms for the two valves considered in para. **603.**

The V.A.F.'s are :—

$$(a) \text{ Triode } \dots \text{ V.A.F. } = \frac{15 \times 5,000}{25,000} = 3$$

$$(b) \text{ Tetrode } \dots \text{ V.A.F. } = \frac{100 \times 5,000}{205,000} = 2.44.$$

The amplification obtained is thus very small in both cases. Further, in the case of a triode amplifier its external impedance is likely to have a net capacitive reactance on the higher frequencies which it is designed to receive. When this occurs, the feed-back from the output to the input circuit through C_{oa} is such as to damp the input and reduce the total amplification still more. On the other hand, if the output reactance is not capacitive, the tendency to self-oscillation will be strong owing to the large regenerative feed-back through C_{oa} , the reactance of which decreases as the frequency increases.

692. Of the methods of radio frequency amplification already discussed, there remains only one, *viz.*, amplification by the use of reaction, which is suitable for H.F. reception. In addition, there are available two general types of receiver which have not yet been mentioned. These are :—

- (i) Super-heterodyne receivers.
- (ii) Super-regenerative receivers.

The ordinary methods of note magnification after detection are obviously still available.

693. Before proceeding to the discussion of methods of amplification, some general points about high frequency receivers may be noted.

(1) Untuned aerials are generally used. High frequency receivers, as will be seen below, normally contain self-oscillatory circuits, and, unless special precautions are taken, enough radiation may take place with a tuned aerial to cause interference with other sets in the neighbourhood. This radiated energy also increases the damping of the receiver, and is likely to be variable, as the unavoidable small changes in the natural capacity of the aerial vary its natural frequency. This renders the operation of the receiver spasmodic, and may, on occasion, damp the self-oscillatory stage so much that oscillations cease altogether.

(2) When building a receiver to work over a considerable range of frequencies, it is not always a simple matter to design an aerial whose natural frequency lies outside the required range. For this reason the coupling between the aerial circuit and the first valve circuit is made as loose as possible, the necessary amplification being obtained in later stages. This may be achieved by resistance coupling or the use of a coupling condenser of small capacity. Another common method, which may be used in conjunction with the above,

coil in the anode lead. The valve in Fig. 406 will normally be the detector valve, since earlier amplifying stages are ineffective, and it will be seen that there are two parallel paths from anode to filament in the output circuit, one through the reaction circuit and the other through a choke. The current through the detector valve contains both radio and audio frequency components; the choke presents a relatively low impedance to audio frequency oscillations, but a high impedance to radio frequencies; the reaction circuit acts in exactly the opposite manner, and so the radio and audio frequency output components are separated. The radio frequency component returns to filament via the reaction circuit. It is thus all usefully employed, and, further, is kept out of the audio frequency stages. The audio frequency component passes on through the choke to the telephones or first note magnifier stage, as the case may be.

It is generally found that reaction receivers give the best results if the reaction is increased to the point where self-oscillation takes place, even when receiving I.C.W. This has, of course, the disadvantage that any local disturbances—such as irregularities in filament emission, filament vibration and “battery noises”—are also magnified. There is thus a background of extraneous noise in the telephones, known as “mush,” in addition to the note produced by the incoming signal.

695. Threshold Howl in Reaction Receivers.—When the detector valve of a reaction receiver employing cumulative grid detection is followed by a transformer-coupled stage of note magnification, it may happen that as the reaction is increased to the point where self-oscillations commence, a low frequency oscillation is also produced, which is evident in the telephones as a continuous note. This phenomenon is known as “threshold howl.” The note rises in pitch as the reaction is increased. As mentioned above, greatest sensitivity is obtained in reaction receivers at the oscillating point, so that the occurrence of threshold howl is particularly undesirable.

The production of this howl is due to the audio-frequency change in anode current on which detection depends. In cumulative grid detection this change is a decrease (para. 563), and the better the conditions for detection, the greater will it be. The anode current flows, of course, through the external inductance in the anode lead and so a back E.M.F. is set up in this inductance. When the anode current is decreasing, this back E.M.F. is in such a direction as to assist the H.T. battery, and therefore increases the voltage applied to the anode. As the amplitude of the high-frequency self-oscillations is proportional to the anode voltage, it follows that these oscillations build up to a larger amplitude than would be the case if the external inductance were absent. When the anode current reaches its lowest point and is momentarily steady, the back E.M.F. disappears and so the amplitude of the self-oscillation decreases.

As the anode current rises again to its initial value, the back E.M.F. will be in the reverse direction, *i.e.*, opposing the H.T. battery. Thus the anode voltage is still further decreased and the amplitude of the self-oscillations falls correspondingly. When the anode current steadies and the anode voltage is simply that due to the H.T. battery, the amplitude of self-oscillation rises again and the cycle of events is repeated. The result is that the amplitude of the high-frequency oscillations is **modulated** at an audible frequency and a howl is heard in the telephones.

A similar effect would tend to be produced in an anode bend detector with a condenser in the high frequency output circuit, as in this case both the audio frequency change in anode current and the back E.M.F. across the condenser are in the opposite direction.

In many cases this audio frequency swing is rapidly damped out, and the effect does not attain the dimensions of a "howl." In a circuit of this kind, however, transient damped oscillations, similar to the type described as "mush" above, tend to be accentuated and to increase the background of noise in the telephones. The tendency towards audio frequency self-oscillations by back coupling with the later stages of note magnification is also encouraged.

The tendency to howl is generally only pronounced when the output choke is of large inductance, and there is a large audio frequency drop in anode current. For instance, the choke used to separate the R.F. and A.F. components in the circuit shown in Fig. 406 may be large enough to carry out its function without necessarily setting up threshold howl.

The rise in pitch of the note with increasing reaction is due to the decrease in the time constant $\frac{L}{R}$ of the circuit, where L is the output inductance and R is the total series resistance through which the audio frequency current flows. This resistance, since it includes the valve A.C. resistance, increases as the mean grid potential runs more negative, which will be the case with increasing reaction because of the increased amplitude of the oscillatory grid voltage. Hence $\frac{L}{R}$ decreases as the reaction increases, and the pitch of the note rises.

The simplest method of obviating threshold howl in a reaction receiver is to modify the nature of the output impedance. In the case considered above, a purely inductive output produces audio frequency variations of anode voltage in the correct phase to the anode current variations to maintain self-oscillation, and this phasing is fairly critical. There is no possibility of threshold howl with an output circuit employing resistance-capacity coupling to the next stage.

It is possible to preserve to some extent the greater amplification of transformer coupling while preventing threshold howl by shunting the transformer primary by a resistance. This alters the phase of

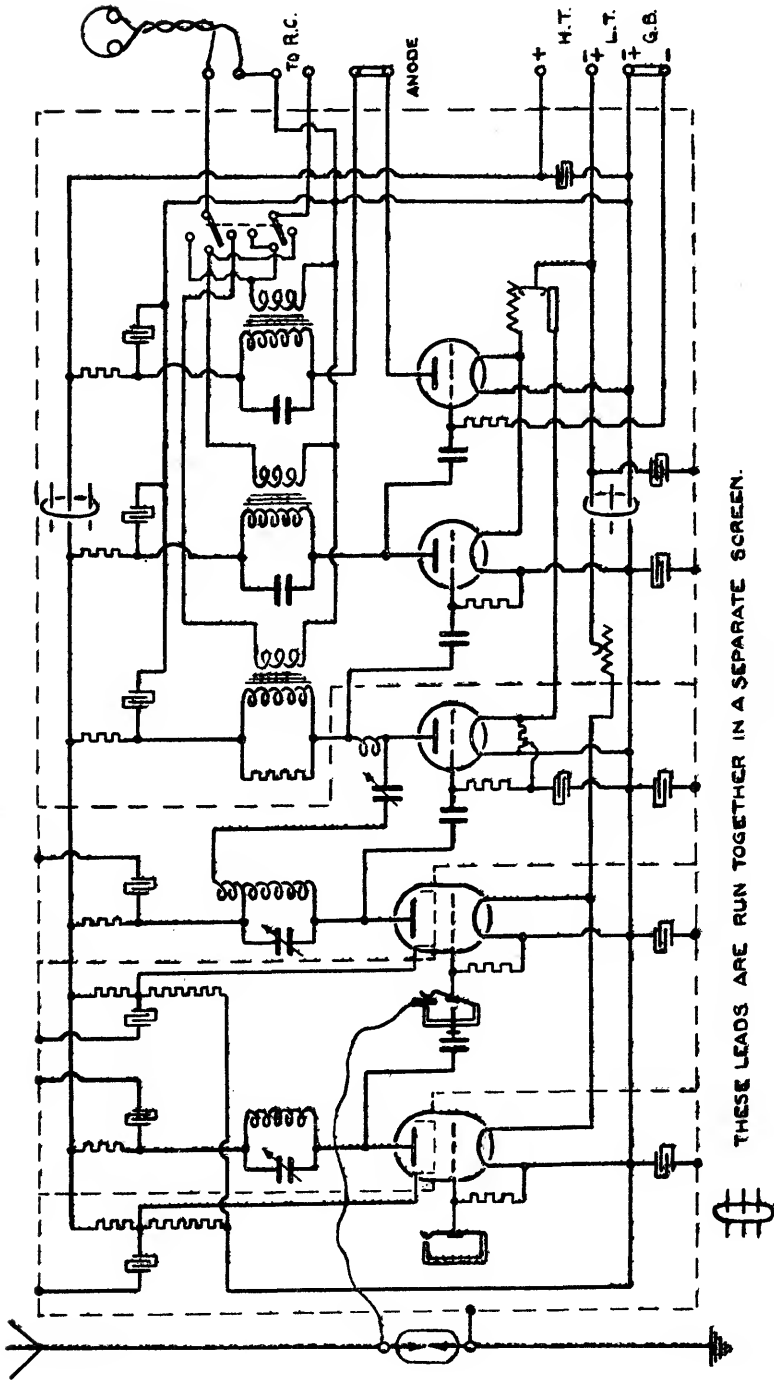


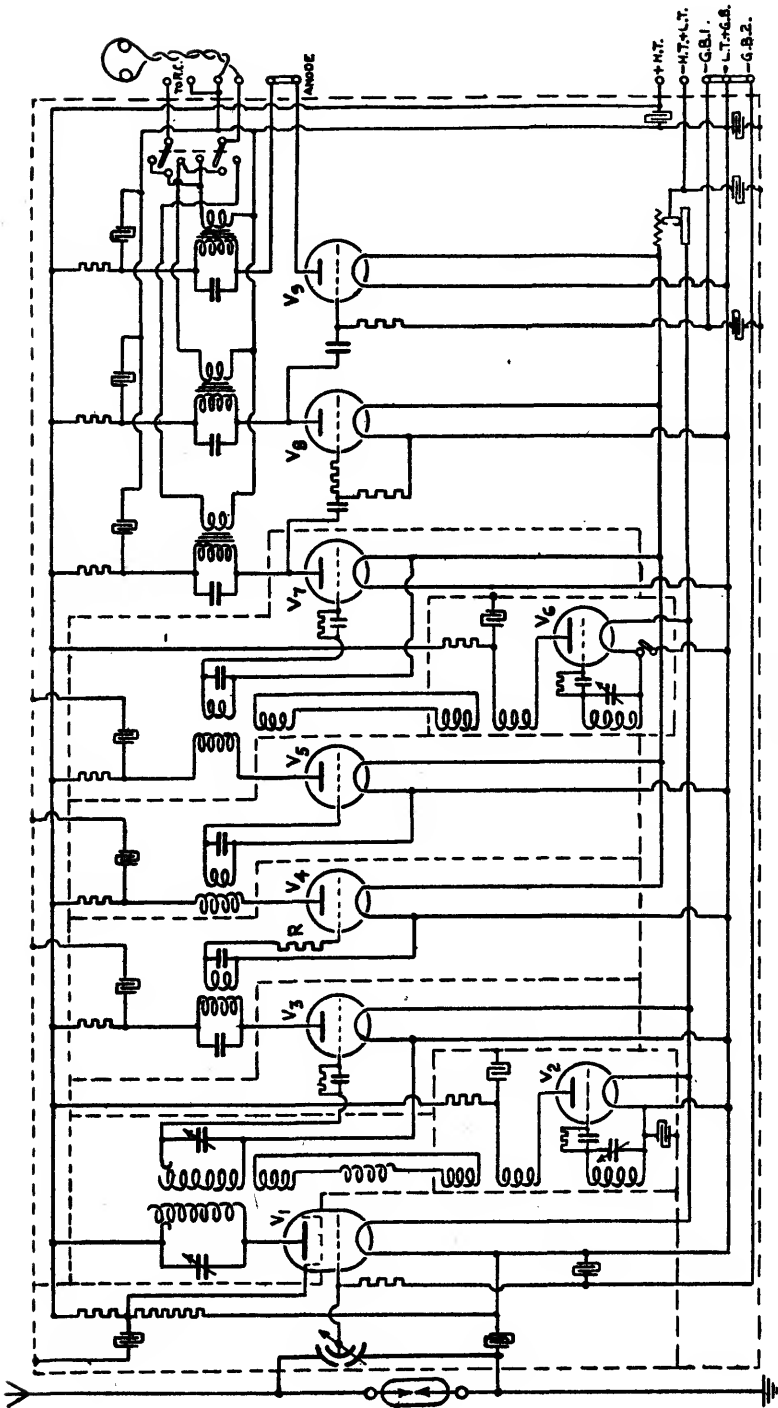
Fig. 407.

the audio frequency P.D. developed across the output, and therefore the anode voltage variation, sufficiently to prevent self-oscillation. The value of the resistance can be kept high enough, however, to give sufficient audio frequency current through the transformer primary to obtain greater amplification than would be the case if a pure resistance-capacity coupling were used.

The grid bias in a cumulative grid detector may also be altered from its best value for detection. Obviously this also reduces efficiency, but the smaller audio frequency decrease in anode current lessens the tendency to oscillation.

696. A modern Service receiver employing reaction and designed to cover the I.F. and H.F. bands is shown in Fig. 407. The aerial is untuned, and is loosely coupled to the first stage by resistance coupling. Two stages of tuned anode radio frequency amplification using screen grid valves are provided, but only give efficient amplification on the lower frequencies. One or both stages may be employed, and a plug and jack fitting is provided on the grids for this purpose. At higher frequencies the first stage is not used, and the second stage is mainly useful as an isolating valve. The third valve is a cumulative grid detector, the appropriate grid bias being obtained by a tapping from a potentiometer across the filament terminals. Capacity-controlled magnetic reaction is employed in this stage. One, two or three stages of note magnification may be used as shown. The output choke in the first audio frequency stage gave rise to a tendency to threshold howl, which was counteracted by adjusting the grid bias of the detector, as mentioned above, and by shunting the choke with a large resistance. The careful screening and the de-coupling condensers and resistances should also be noted.

697. Super-heterodyne Receivers.—The general principle of these receivers is to heterodyne the incoming H.F. signal with a locally-generated H.F. oscillation, the frequency difference between the two oscillations being arranged to lie well above the audible limit, *i.e.*, to be supersonic and of low radio frequency. The resultant oscillation is rectified, and the low frequency component can then be efficiently amplified by the ordinary low frequency methods of Chapter XIII before final detection and note magnification. The advantages of radio frequency amplification are thus preserved, while avoiding the difficulties incidental to its application at the frequency of the incoming signal. This use of amplification at a frequency intermediate between that of the incoming signal and an audible frequency gives this circuit its name of super-heterodyne, or supersonic heterodyne receiver. The method is convenient and efficient, and is widely used, even at those lower frequencies where the ordinary methods of amplification can also be employed. The amplification at a fixed frequency in the intermediate valves implies tuning adjustments considerably.



698. The principle will now be further discussed with reference to a modern Service super-heterodyne receiver shown in Fig. 408. The aerial, as before, is untuned and the first stage of screen grid valve tuned-anode amplification is mainly designed as an isolating stage. The aerial is coupled to the first stage by a "differential condenser." When a powerful transmitter is operating in the vicinity the use of an ordinary small coupling condenser and grid leak may render it difficult to hear weak distant signals, since the condenser may acquire such a negative potential that the grid is choked. The differential condenser consists essentially of two condensers in series. Rotation of the movable plate alters the ratio of the capacities of these condensers. In this receiver one condenser is in series with the other condenser and grid leak in parallel, and so a wider range of adjustments of the grid mean potential, and of the input oscillatory voltage for various strengths of incoming signal, is made possible.

V_1 is the first local oscillator, and is coupled to the grid circuit of V_2 by an untuned mutual link circuit, all these precautions being designed to minimise coupling to the aerial as much as possible. The combined oscillations due to the incoming signal and local oscillator are applied between the grid and filament of V_2 , which is a cumulative grid detector. The beat frequency is 30 kc/s. in this model. The high frequency component of the output is mainly by-passed to the screen, and the 30 kc/s. oscillation, which is of constant amplitude if C.W. is being received, but is modulated at audio frequency in the case of an I.C.W. signal, is passed on to the grid-filament circuit of V_3 by the transformer coupling shown. The resistance R in the grid lead of V_3 is known as an R.F. stopper. Any residual R.F. potential difference developed at the terminals of the transformer secondary is applied across R and the grid-filament capacity C_g , in series. At high radio frequencies the resistance of R is much greater than the reactance of C_g , so that only a small proportion of the total P.D. is applied between grid and filament. At supersonic frequencies the reverse is the case, and so R acts as a device for cutting down R.F. input voltages without sensibly affecting S.F. input voltages. V_4 and V_5 provide two stages of amplification at 30 kc/s., using tuned transformer coupling. The amplified oscillation is detected in V_6 , the method of cumulative grid rectification again being employed. V_7 and V_8 are stages of note magnification. The resistance in the grid lead of V_8 performs a similar function to that in the grid lead of V_3 , filtering S.F. and A.F. potential differences. When I.C.W. is being received, the incoming signal is modulated at audio frequency, and so an audio frequency note is obtained by detection in V_7 . In the case of C.W., however, provision must be made for heterodyning the 30 kc/s. oscillation so as to produce an audio frequency note. This is accomplished by means of another separate local oscillator V_9 , as shown.

It will be seen that the only tuning adjustments required are in the output circuit of V_1 , the first local oscillator V_2 , and the grid

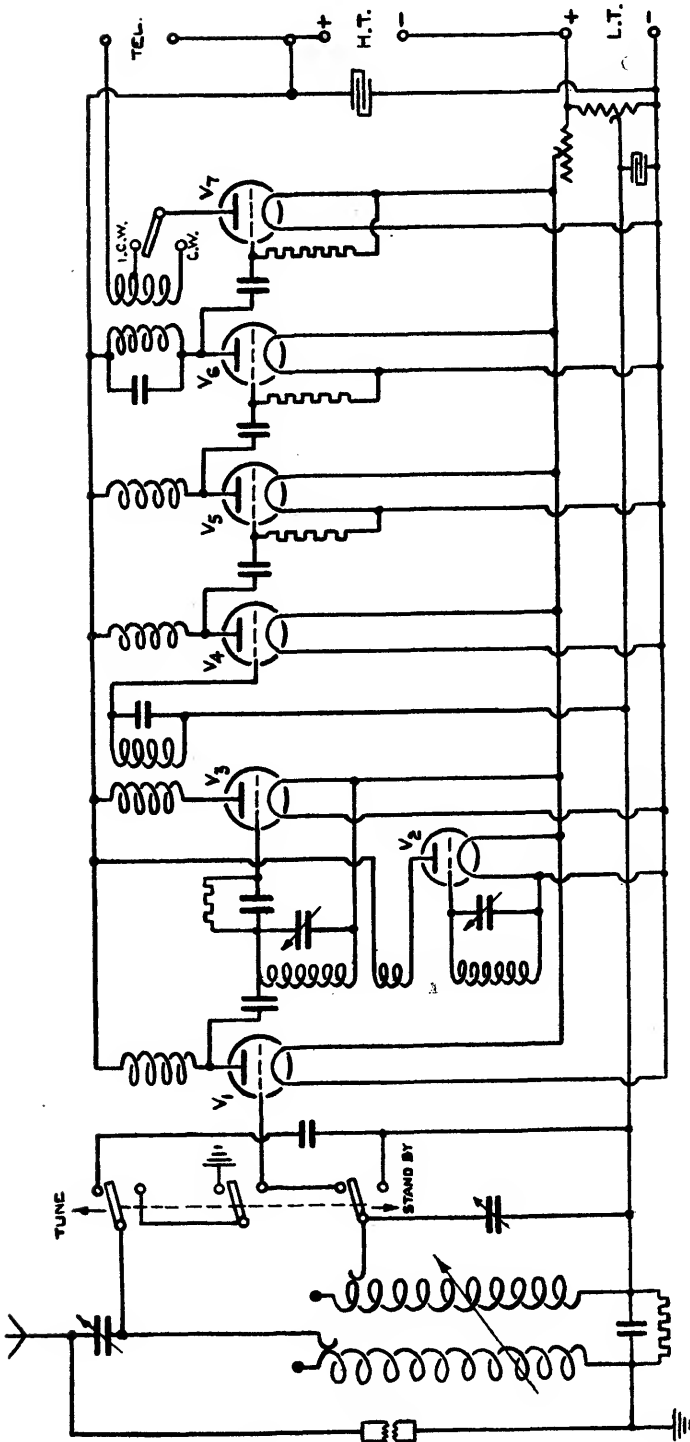


Fig. 409.

circuit of V_3 , so that in spite of its apparent complexity, the circuit is simple to operate.

The super-heterodyne circuit is also very selective, and this is one of the chief reasons for extending its use down to the low frequency range, where the less elaborate circuits of Chapter XIII can give efficient amplification. Consider, for example, the reception of a 1,000 kc/s. signal. An interfering frequency differing by 2 per cent., *i.e.*, at 1,020 or 980 kc/s., will then give a beat frequency of 50 or 10 kc/s. with the first local oscillation. The percentage difference of these from the frequency at which the supersonic amplification takes place (30 kc/s.) is 67 per cent., and so the interfering signal is very much weakened, compared with the wanted signal, by the time it reaches the audio frequency detector valve. In the case of a C.W. signal, it can then be further avoided by the ordinary methods described under heterodyne reception in Chapter X.

It is important, however, to note that although the selectivity of these receivers is very high for normal interference, there are two incoming frequencies at which any setting of the first local oscillator will give the same beat frequency. In the above receiver, for example, if the local oscillator is set for 1,030 kc/s., so as to give a beat frequency of 30 kc/s. with the desired 1,000 kc/s. signal, it will give the same beat frequency with a signal on 1,060 kc/s. Thus, if strong unwanted signals on 1,060 kc/s. and 940 kc/s. were present, the receiver would be unselective when set to receive signals at 1,000 kc/s. This type of interference can only be avoided by a careful choice of the supersonic beat frequency, having regard to the interfering frequencies most likely to be encountered.

699. An earlier Service receiver of the super-heterodyne type is shown in Fig. 409. The tuned aerial, the use of a triode in the isolating stage, and the lack of screening and de-coupling devices, should be compared with the improvements in these respects in the later model. V_1 is the isolating valve, V_2 the local oscillator, and V_3 the first rectifier. V_4 , V_5 and V_6 are the supersonic frequency stages of amplification. The output circuits are shown as chokes, but they actually tune with their self-capacities at the supersonic frequency employed. V_7 is the final detector valve. In receiving I.C.W., the supersonic frequency in this model is 41.6 kc/s., and the local oscillator is set to give a beat frequency of this amount with the incoming signal.

No separate local oscillator is provided to give an audio frequency beat note in the last stage when receiving C.W., and so the detector valve circuit must be made self-oscillatory. The output circuit of this valve contains a coil mutually coupled to the input. In receiving I.C.W., this merely gives regenerative amplification, but when the switch shown is made to the C.W. position, the energy fed back is sufficient to produce self-oscillation, and the circuit can operate as an autodyne receiver.

The natural frequency of this self-oscillatory circuit is also fixed at 41.6 kc/s., so that the supersonic frequency must be altered to differ from 41.6 kc/s. by an audible amount. This is accomplished by adjusting the frequency of the separate heterodyne oscillator V_s . To give eventually a 1,000 cycle note, for instance, the separate heterodyne is arranged to give a supersonic beat frequency of 40.6 kc/s. or 42.6 kc/s. with the incoming signal. The supersonic frequency amplification stages must thus be designed to give good amplification over this range of frequencies, and the output chokes actually possess considerable resistance in order to flatten the tuning sufficiently for this purpose.

700. Super-regenerative Receivers.—The use of ordinary regeneration or reaction as a method of amplification was shown in Chapter XII to be equivalent to reducing the effective resistance of the input circuit. In the simple regenerative circuit it was proved

that the resistance was decreased by an amount $\frac{Mg_m'}{C}$, the effective

resistance then being $R - \frac{Mg_m'}{C}$ (para. 567). When the reaction

receiver is brought to the oscillating point, the effective resistance is zero, and the generation of self-oscillations is possible. With further increase of reaction, self-oscillations can build up to a larger amplitude, and while they are building up the effective resistance is negative, *i.e.*, $\frac{Mg_m'}{C} > R$. In all these cases, however,

whatever may be its value, the effective resistance remains approximately constant during the operation of the receiver. The special feature of receivers using the principle of super-regeneration is that the effective resistance of the oscillatory circuit is deliberately varied at some chosen frequency, so that its value is alternately positive and negative, *i.e.*, free oscillations are periodically allowed to build up and to be damped out at a definite rate.

There are obviously two ways in which this may be accomplished:—

- (1) The energy fed into the oscillatory circuit by reaction (the negative resistance) may be kept constant, and the positive resistance of the circuit varied at the chosen frequency so that it is alternately greater or less than the negative resistance.

The most elementary way of achieving this would be to have an additional resistance which could be inserted in, and cut out of, the oscillatory circuit at the frequency of variation desired, by some device such as a buzzer wheel. The constant resistance in the circuit would be less in value than the "negative" resistance, and the extra resistance would be sufficient to increase the total beyond the value of the "negative" resistance, so that

the effective resistance would be alternately positive and negative.

Another possible method would be to connect across the tuned circuit a separate valve, the A.C. resistance of which was varied at the desired frequency so as to constitute a variable damping device across the tuned circuit, and thus to cause it to alternate between a self-oscillatory and a non-oscillatory condition. This would be similar to the "Modulation by Absorption" method used in R/T (para. 674).

- (2) The energy fed into the oscillatory circuit by reaction may be caused to vary periodically in value, so that it is alternately greater and less than the amount required to overcome the damping losses of the circuit, *i.e.*, the negative resistance becomes alternately greater and less than the positive resistance. This is the method employed in Service receivers. The actual ways of accomplishing it will be dealt with after the nature of the results to be expected have been considered.

701. Service super-regenerative receivers may be divided into two main categories :—

- (a) Receivers in which the periodic variation of effective resistance is produced by means of a circuit other than that in which self-oscillations are alternately allowed to build up and die away. These are known as **quench receivers**.
- (b) Receivers in which the effective resistance of the oscillatory circuit and its associated valve is arranged to be automatically self-adjusting, so that self-oscillations alternately build up and are damped out without the necessity of a separate circuit to vary the reaction. These are called **self-quenching receivers** or **squeggers**.

702. Quench Receivers.—In discussing the operation of a quench receiver it will be useful to recall to mind the nature of the current produced by the application of an alternating E.M.F. of amplitude \mathcal{E} to an acceptor circuit (*i.e.*, an oscillatory circuit tuned to the applied E.M.F.), whose resistance is so small that the difference between its natural and resonant frequencies may be neglected.

The magnitude of the current at any time t after the E.M.F. is applied was found in para. 428 to be

$$i = \frac{\mathcal{E}}{R} (1 - e^{-\frac{R}{2L}t}) \sin \omega t.$$

The nature of the current is best grasped by considering the two terms of this result separately. It consists of

- (1) A forced oscillation $\frac{\mathcal{E}}{R} \sin \omega t$, of constant amplitude if \mathcal{E} and R are constant.

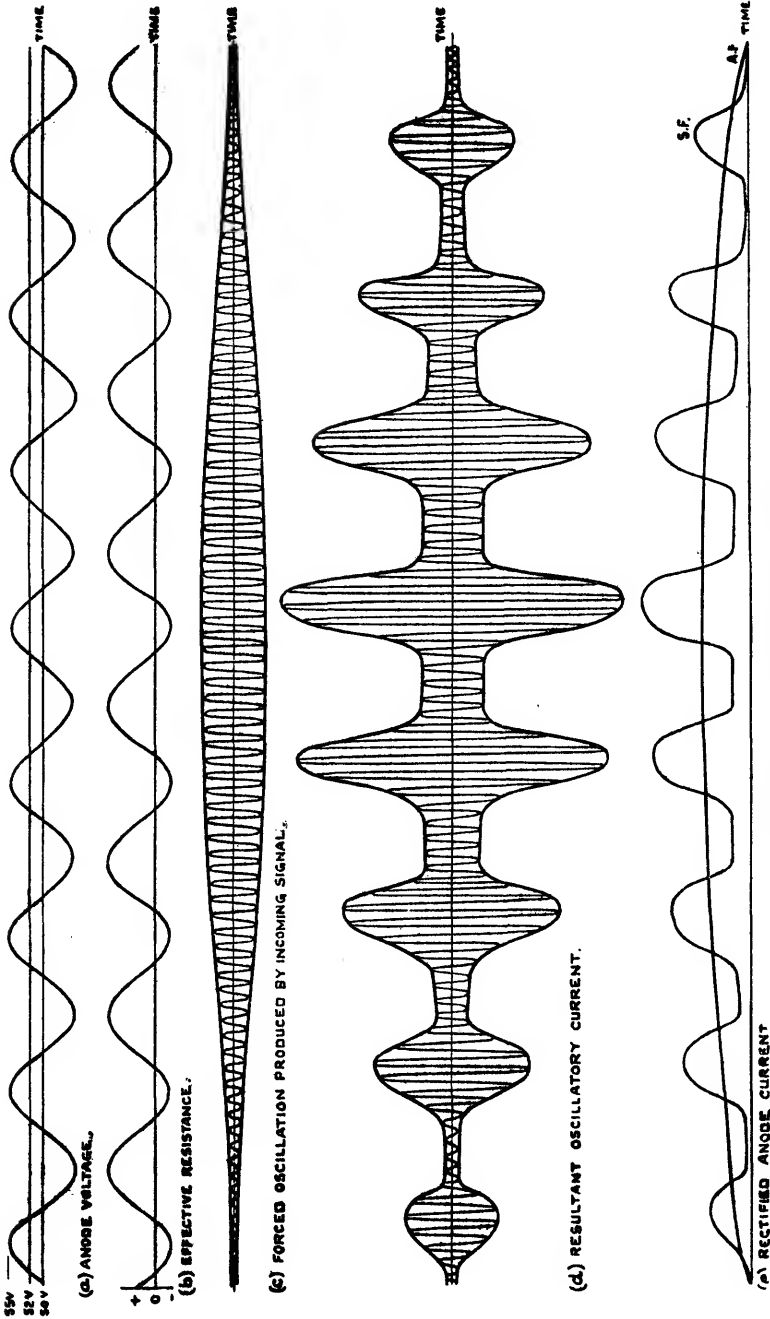
- (2) A **free** oscillation of amplitude $\frac{C}{R} e^{-\frac{R}{2L}t}$. The initial amplitude (at $t = 0$) of the free oscillation is $\frac{C}{R}$, the same as that of the forced oscillation.

If the effective resistance of the circuit is positive, the free oscillation rapidly decreases in amplitude. This is the normal case in the receiving circuits so far considered, even in circuits with regenerative amplification nearly up to the oscillating point. The forced oscillation alone is then of importance, and regenerative amplification may be considered merely as a means of reducing the value of R so as to increase the amplitude $\frac{C}{R}$ of this oscillation.

When the effective resistance is negative, however, the result is very different. The free oscillation then builds up as quickly as it was damped out in the case of positive resistance, and it is the forced oscillation which may soon be neglected by comparison.

This building up of free oscillations has already been considered at length in the chapter on the valve as generator. They would theoretically build up to an infinite amplitude, given an infinite time. In practice the amplitude is limited in self-oscillatory valve circuits by the decrease of the negative resistance as the amplitude of oscillations increases, until eventually any further increase in amplitude makes the effective resistance again positive; oscillations are then maintained at a constant amplitude. But before these limiting conditions supervene, the following points are to be noted concerning the free oscillations:—

- (a) They must be initiated by some momentary electrical disturbance of the oscillatory circuit, corresponding to the application of an E.M.F., considered above. An incoming signal will perform this function, but the ordinary disturbances in a valve circuit will have a similar effect.
- (b) Once they begin, they will build up as long as the resistance remains negative, whether the original stimulus is removed or not. If the latter is continued, as in the case of an incoming signal, the forced oscillation it produces is negligible in comparison.
- (c) The initial amplitude of the free oscillation is the same as that of the forced oscillation $\left(\frac{C}{R}\right)$, *i.e.*, proportional to the applied electrical disturbance, and while the oscillation is building up its amplitude retains this proportionality, for after any time t from the commencement of free oscillations it is $\frac{C}{R} e^{-\frac{R}{2L}t}$.



Quench Reception of I.C.W. Signal.

FIG. 411.

the mean value of the H.T. voltage by the amplitude of the supersonic induced E.M.F. in the anode lead.

It will be remembered that although, for simplicity, we normally consider the characteristics to be parallel, the mutual conductance (g_m) of a triode actually increases as the anode voltage is increased, and it is on this fact that the possibility of varying the effective resistance of the V_1 circuit in this receiver depends. The slope of the dynamic characteristic g_m' is, of course, proportional to g_m , and therefore to the anode voltage. The "negative resistance" or energy fed into the oscillatory circuit is proportional to g_m' , and so, ultimately, to the anode voltage. Thus the supersonic variation of anode voltage can be arranged to produce a supersonic variation of negative resistance above and below the positive resistance of the circuit, and therefore to make the effective resistance alternately positive and negative at the supersonic frequency. Approximate values in this receiver are as follows: the steady anode voltage is 50 volts; the 20 kc/s. induced E.M.F. in coil L has an amplitude of 5 volts. When the amplitude of anode voltage is above about 52 volts, free oscillations build up, *i.e.*, the nett resistance is negative; below this value of anode voltage, free oscillations are damped out. The variations of anode voltage and effective resistance are shown in Fig. 411 (a) and (b). Fig. 411 (c) shows the forced oscillation produced by the incoming I.C.W. signal voltage over one cycle of its audio frequency modulation. In Fig. 411 (d) is shown the alternate building up and damping out or "quenching" of the free oscillations. The points to notice are:—

- (1) The final amplitude which the free oscillation reaches during any one period of negative resistance is proportional to the amplitude of the forced oscillation, *i.e.*, of the incoming signal at the commencement of that period. This depends on the fact that the free oscillation is quenched before its amplitude nears the condition when the valve is saturated.
- (2) Since the negative effective resistance is not constant over this period, but rises from zero to a maximum and falls again to zero, the free oscillations do not build up exactly according to an exponential law. Their rate of growth is largest when the effective resistance of the oscillatory circuit has its maximum negative value, and is lowest at the beginning and end of the self-oscillatory period, but the final amplitude of the free oscillation in every such period always bears the same ratio to the amplitude of the forced oscillation at the beginning of the period.
- (3) The **free oscillation must be completely quenched during the period of positive resistance.** If there is a residual free oscillation when the effective resistance again becomes negative, the initial amplitude from which free oscillations build up is increased.

tions build up is not that of the forced oscillation alone, but is the resultant of the forced oscillation and the remaining free oscillation. In consequence, the final amplitude of the free oscillation during a period of negative resistance is not proportional to the amplitude of the forced oscillation at its commencement, *i.e.*, is not proportional to the amplitude of the incoming signal at that instant. The mean audio frequency variation of the current in the oscillatory circuit, as shown in Fig. 411 (e), would then no longer necessarily bear any relationship to the modulation of the incoming signal.

- (4) The measure of the amplification produced is obviously the ratio of the final amplitude of the free oscillation to that of the forced oscillation at the beginning of the period of effective negative resistance, and this can be made very large. It should be observed, however, that any atmospheric, interfering signal, or local disturbance such as that caused by irregular filament emission, will likewise initiate free oscillations if it occurs during such a period. The forced oscillation due to such disturbances (except atmospherics) will normally be of smaller amplitude than that due to the wanted signal, but the ratio of the two is preserved, and a background of "mush" will be heard in the telephones. The selectivity of a super-regenerative receiver is thus not outstanding. The weaker the signal, the noisier is the background if the signal is amplified to the same degree of audibility in each case.

704. The above considerations allow some simple deductions to be made as to the value of the quenching frequency, *i.e.*, the frequency with which the effective resistance of the receiving circuit varies between a positive and a negative value. The higher this frequency, the more often do oscillations build up and die away during one incoming wave train, and therefore the more faithful is the reproduction of the signal waveform. The limits of the ratio of quenching to incoming frequency are fixed by the amount of amplification required, the avoidance of saturation current, and the necessity that each set of free oscillations in the receiver should be damped out before the next set commences.

The higher the quenching frequency, the less time there is for free oscillations to build up and die away, and so the smaller is the final amplitude reached in each burst of self-oscillation, *i.e.*, the less is the amplification. For instance, in receiving a wave modulated at 1,000 cycles per second, when the quenching frequency is 20 kc/s., self-oscillations build up and are damped out twenty times per audio frequency cycle. If the quenching frequency were doubled, the corresponding number would be forty times, *i.e.*, the time of building

up self-oscillations and the time in which they are damped out would both be halved.

For a given quenching frequency the amplitude of the free oscillations increases with $\frac{R}{L}$ and likewise the rate at which they die away. The larger this ratio can be made, the higher can be the quenching frequency. To a first approximation $\frac{R}{L}$ increases with the radio frequency of the incoming signal (to which the receiving circuit is tuned), and for a given signal frequency it increases with the stiffness of the circuit. Thus high amplification and faithful reproduction are more easily obtained the higher the incoming frequency, and the stiffer the tuned circuit. As the incoming radio frequency decreases, the amplification is reduced. Reduction of the quenching frequency may compensate for this up to a point, and at the expense of quality, but a limit is soon reached, for the quenching frequency must in practice remain supersonic. The amount of mush produced even at a supersonic quenching frequency has already been remarked, and when quenching takes place at an audible frequency, a continuous noise is produced in the telephones sufficient to drown any incoming signal. In addition, as the incoming radio frequency decreases, the selectivity of the receiver falls off considerably. The lowest frequency for which this method is of any value lies in the I.F. band.

Various other methods of applying the super-regenerative principle to the reception of I.C.W. have been devised; one method, for instance, is to employ supersonic modulation of the I.C.W. at a frequency which gives an audio frequency beat with the quenching frequency.

***705. Quench Reception of C.W. Signals.**—The ordinary method of introducing an audio frequency variation of amplitude into a C.W. signal is to employ a separate heterodyne or an autodyne circuit. It is possible to use the quench receiver in this way, provided that the free oscillation is not completely quenched during the period of positive resistance, but still retains an amplitude comparable with that of the forced oscillation every time the resistance becomes negative. Both the free and the forced oscillations thus continue for the whole duration of the forced oscillation, and the maximum amplitude attained by the free oscillations during a quenching cycle is proportional to the sum of the amplitudes of the free and forced oscillations at the beginning of the negative resistance period. Provided that the free and forced oscillations are of the same order of magnitude at such instants, their relative phase then is the main factor in determining the amplitude to which free oscillations build up. For this to be true it is also necessary that the free oscillations should not reach a value which saturates the valve, just as in the I.C.W. case. The free and forced oscillations thus give a maximum

resultant amplitude when they are in phase, *i.e.*, at the instants where beats occur; at the beginning of intermediate quenching cycles they are not in phase, and the smaller resultant amplitude at these instants gives a smaller final amplitude of free oscillation. Hence the envelope of free oscillations over a number of quenching cycles is modulated at the beat frequency of the free and forced oscillation, and an audible note is heard when detection takes place.

The chief difficulty of the ordinary heterodyne or autodyne method of reception at high radio frequencies is that a small percentage change in the frequency of the incoming signal gives a large actual change in frequency, *e.g.*, at 10 Mc/s. a change of 0.1 per cent. corresponds to 10,000 cycles per second. Even if the frequency of the local oscillator remained constant (which is unlikely, especially in an autodyne circuit), the beat note would change by 10,000 cycles per second. It is easily seen that under practical conditions the beat frequency often becomes supersonic, and therefore inaudible, and continual searching is necessary to keep the signal audible. The peculiar feature of the quench receiver, when adjusted so that the free oscillation is never completely quenched, is that it is still possible to get an audible note on detection, although the beat frequency between the incoming signal (the forced oscillation) and the local free oscillation is itself well above the audible range. Thus sudden changes in the signal or local frequency do not involve loss of the signal, but merely a change in the pitch of the note heard in the telephones.

This phenomenon may be explained as follows. Suppose that f_1 is the incoming signal frequency and f_2 is the frequency of the free oscillation in the quench receiver. The beat frequency is then $f_1 - f_2$, and, as pointed out above, this is often inaudible. If ϕ is the quenching frequency, the time of one quenching cycle is $\frac{1}{\phi}$ and the number of beats per quenching cycle is therefore

$$(f_1 - f_2) \times \frac{1}{\phi} = \frac{f_1 - f_2}{\phi}$$

If this is a whole number, the phase difference between the dying free oscillation and the forced oscillation at the beginning of a period of negative resistance is always the same; for instance, if they were in phase at the beginning of one such period, they would again be in phase at the beginning of the next period and so on. After a few quenching cycles the initial amplitude from which free oscillations build up would be such that their final amplitude would saturate the valve. Each succeeding burst of self-oscillation would similarly saturate the valve as long as the forced oscillation persisted, and, since the quenching frequency is supersonic, no audible result would be produced.

The possibility of $\frac{f_1 - f_2}{\phi}$ being a whole number, however, is remote, as will easily be recognised, and so the phase difference between the free and forced oscillations is different at the beginning of consecutive periods of negative resistance. Hence the amplitude reached by the free oscillations during consecutive quenching cycles differs correspondingly; it is a maximum when the free and forced oscillations are in phase at the beginning of a period of negative resistance, and a minimum when they are in anti-phase at such an instant.

Nevertheless, over a number of quenching cycles, say n , the number of beats is $n \frac{(f_1 - f_2)}{\phi}$, and for some value of n this is bound to be a whole number. If, for simplicity, we assume that the free and forced oscillations are in phase at the beginning of the forced oscillation, they will again be in phase at the end of the " n "th quenching cycle. At the beginning of intermediate quenching cycles between these limits they are not in phase, and so the final amplitude of the free oscillations during these cycles is smaller. Hence the amplitude of the free oscillations reaches a maximum value every n quenching cycles, *i.e.*, it is modulated at a frequency equal to $\frac{\phi}{n}$. This frequency will in general be audible, and so an audible note is heard in the telephones on detection. Alteration in f_1 or f_2 will alter the value of n which makes $n \frac{(f_1 - f_2)}{\phi}$ a whole number, and so will alter the pitch of the note heard, but it will remain audible over a large range of values of f_1 and f_2 . The process is illustrated for constant values of f_1 and f_2 in Fig. 412.

This kind of super-regeneration is sometimes said to be "stroboscopic." The actual phenomenon is probably more complicated than is indicated by the simplified explanation given above, but it remains of the same general nature. The essential conditions for its production are that the valve should not be saturated by the final amplitude of the free oscillations, and that these should only be damped to an amplitude of the same order as that of the incoming signal during the quenching period. This may be attained in practice by increasing the part of the quenching cycle during which the resistance is negative, and decreasing, if necessary to prevent saturation, the amplitude of variation of effective resistance about its zero value.

The advantage of this method of reception is that, even with large frequency variations in transmitter and receiver, the signal will still be heard. This involves, of course, that any interfering signals in the same range of frequencies are received with nearly equal intensity, and so the method is extremely unselective.

706. Self-quenching Receiver.—In this receiver self-oscillations are built up and quenched periodically as in the quench receiver,

but no separate self-oscillatory circuit is used to produce the necessary variation of negative resistance above and below the positive resistance of the receiving circuit. This is accomplished automatically by appropriate coupling between the anode and grid circuits and the use of a grid insulating condenser and leak.

The circuit of a self-quenching receiver is the same as that already discussed under regenerative amplification and autodyne reception (Chapter XII), and is reproduced in Fig. 413. It will be remembered that with loose reaction coupling, amplification is produced, and, as the coupling is made tighter, a point is reached at which self-oscillations are generated. The building up of continuous self-oscillations and the amplitude they acquire in a circuit with negative grid bias was discussed in Chapter XIV. In the

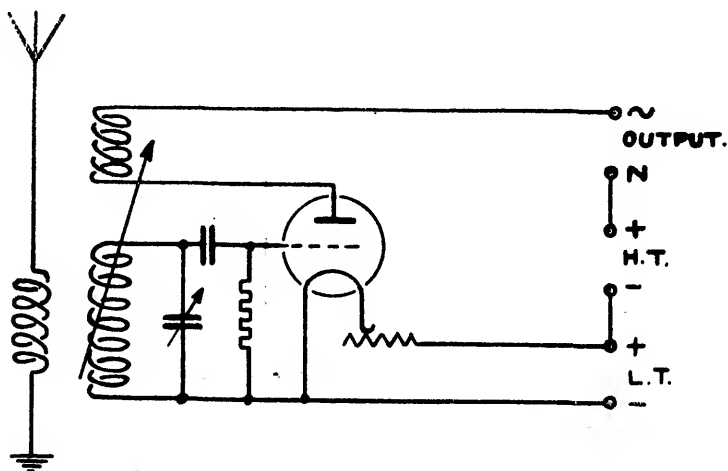


FIG. 413.

oscillator then considered, the tuned circuit was between anode and filament, but the principle is the same when the tuned circuit is between grid and filament. It was shown that under favourable conditions the positive half-cycle of grid oscillatory voltage covered the whole straight part of the dynamic characteristic, and that the mean anode current then flowing was much greater than that corresponding to the same grid bias under non-oscillatory conditions. The maintenance of continuous oscillations depends on keeping the grid bias steady at the appropriate value.

Suppose now that the coupling between anode and grid circuits is made still tighter. The immediate effect is to increase the amplitude of oscillatory grid voltage. This, however, causes an increase in the grid current flowing during the positive half-cycle; the steady potential of the grid therefore becomes more negative, and the positive peak of oscillatory grid voltage returns to approximately the same potential as before. The negative half-cycle of

grid swing then extends further into the region where no anode current flows, and the result is that the slope of the effective dynamic characteristic, *i.e.*, the ratio of the total change in anode current to the total change in grid voltage, is decreased, the nett resistance of the circuit becomes positive, and oscillations are damped out. No grid current then flows, and, as the grid condenser charge leaks away through the resistance, the grid rises to a less negative potential and oscillations can be built up again. Thus, when the coupling between anode and grid circuits is made sufficiently tight, a circuit of this type automatically allows self-oscillations to build up and die away. It will be seen that the kind of oscillation produced is that described as I.C.W., and if the self-quenching frequency is an audible one, the circuit may be used as an **I.C.W. transmitter**, the wave radiated from which will give an audible note in a receiver without the necessity of heterodyne reception. When used in this manner, the tuned circuit is generally between anode and filament, as in the ordinary C.W. transmitter.

707. The use of the circuit as a receiver will now be considered. In this case the tuned circuit is between grid and filament, as shown in Fig. 413. The possibility of reception depends on the fact that **the negative grid bias at which self-oscillations start to build up is appreciably less than that at which they are damped out or "quenched,"** *e.g.*, the values found in an experimental determination were $V = -14$ volts for the quenching, and $V = -3$ volts

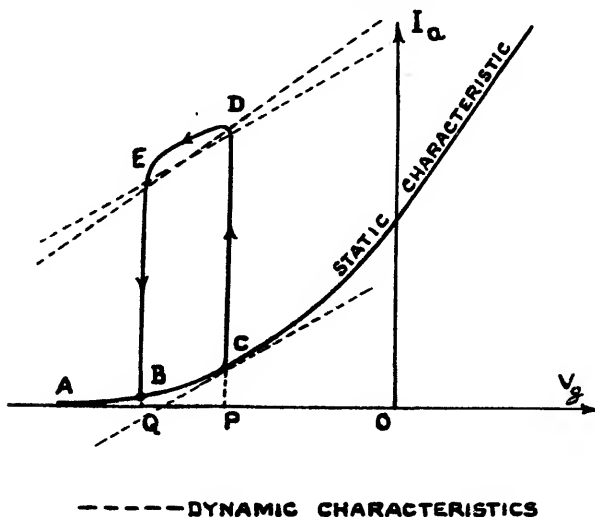


FIG. 414.

for the building up, of self-oscillations. The reason for this appears to be as follows. When oscillations are just beginning to build up, their amplitude is, of course, extremely minute. The oscillatory

P.D. in the external anode circuit is correspondingly minute, and the anode voltage remains very nearly the same as under static conditions; thus the slope of the dynamic characteristic is practically the same as that of the static characteristic, and as the actual grid bias gives a working point beyond the lower bend of the characteristic, this slope is very small.

As the oscillations build up, the oscillatory grid voltage extends into the straight part of the characteristic, and the mean anode current jumps to the much higher value associated with self-oscillations of large amplitude. For a steady grid bias of this order the conditions are shown in Fig. 414. CP is the steady anode current for negative grid bias OP when oscillations are just beginning. DP is the mean anode current for oscillations of large amplitude. The mean dynamic characteristic corresponding to DP is shown by the dotted line through D (it is actually a loop). Its slope is greater than that of the static characteristic at C, which is approximately the g_m at the beginning of self-oscillations. As the negative grid bias is increased, the mean anode current decreases along DE, and oscillations decrease in amplitude. In the neighbourhood of E, the oscillations fall to such an amplitude that the slope of the dynamic characteristic through E is not sufficient to make $\frac{Mg_m'}{C} > R$. Oscillations are damped out, and the mean anode

current falls from EQ to the much smaller value BQ, which corresponds to static conditions with a negative grid bias OQ. Thus if the negative grid bias is greater than OP, oscillations will not build up, but if they have built up they will be maintained until the grid bias reaches the larger negative value OQ. (A mathematical explanation of this phenomenon has been obtained by representing the valve characteristic by an equation of the fifth degree.)

When the grid bias is made self-adjusting by an insulating condenser and leak, the actual cycle of mean anode current values obtained is not CDEBC, for the jump of mean anode current from C to D is associated with the largest oscillatory grid voltages, and these cause grid current to flow and increase the negative grid bias. The curve connecting mean grid bias and mean anode current over one self-quenching cycle is thus as illustrated in Fig. 415 by the loop CGHKB. The mean grid potential becomes more negative during the part CGHK of the loop. After K, oscillations have almost died out, and the loss of charge through the leak resistance causes the grid bias to decrease, as shown by KBC. At C, oscillations again start to build up, and the cycle is repeated. The part CGH of the loop is traversed much more quickly than the remainder, and the part KBC, when the condenser is discharging, occupies much the greatest proportion of the time taken for one self-quenching cycle.

The initial process on making the H.T. supply is also shown in Fig. 415. The grid bias is then zero, and oscillations build up more

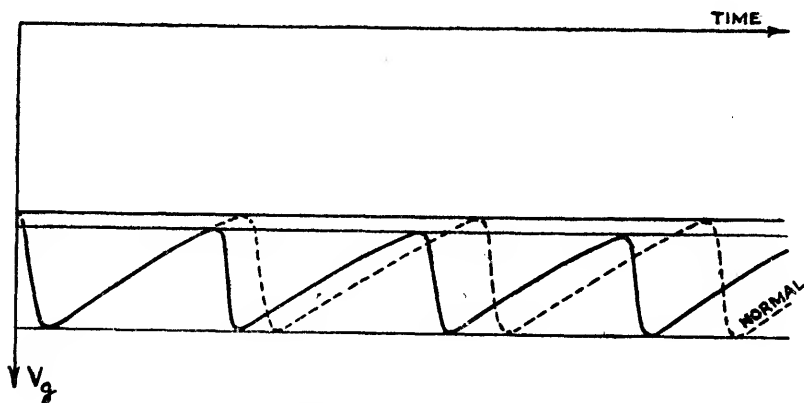
audible or supersonic as desired. Alteration of either C or R is equally effective in varying this frequency. The frequency of the R.F. oscillations is, of course, determined chiefly by the values of the inductance and condenser in the grid tuned circuit as in the case of continuous oscillations.

Thus, if "CR" is such as to make the self-quenching frequency audible, a continuous note will be heard in the telephones; but if the self-quenching frequency is supersonic no such note will be heard. In both cases, however, there is always the strong background of mush that has already been referred to under Quench Reception.

708. Effect of Incoming Signal.—An incoming signal produces a forced oscillation in the grid tuned circuit. The effect of this is negligible while the circuit is in its self-oscillatory condition, just as in quench reception; and the quenching of free oscillations occurs at sensibly the same mean grid voltage, whether the forced oscillation is present or not. The forced oscillation, however, produces an appreciable change in the mean grid voltage at which free oscillations begin to build up, *i.e.*, the voltage OP in Fig. 415. As the condenser is discharging along BC, an oscillatory voltage of finite amplitude, due to the forced oscillation, is superimposed on the mean grid voltage, with the result that the slope of the dynamic characteristic increases sufficiently to make the effective resistance negative before the mean grid bias reaches OP. As this part of the self-quenching cycle takes place relatively slowly, the time between the instant when free oscillations are quenched and the instant when they start again is reduced by an appreciable amount. In other words, the period of the self-quenching cycles is decreased, and so their frequency is increased. If the self-quenching frequency is audible before the incoming signal arrives, the effect of this increase in frequency is to raise the pitch of the note heard in the telephones; if the self-quenching frequency is originally supersonic, it will rise to a higher supersonic frequency.

709. Reception of C.W. Signals.—The above effect is sketched for a C.W. signal in Fig. 416, where only the mean grid potential variation with time is shown. The difference in grid bias for the generation of free oscillations with and without a forced oscillation may be considered roughly proportional to the amplitude of the forced oscillation. For a C.W. signal this is constant, and so the effect is as indicated.

In order to receive C.W. signals, the normal self-quenching frequency of the receiver must lie in the audible range. A note of definite pitch is then continuously audible in the telephones. When the incoming C.W. signal arrives, the pitch of the note rises and the note remains at a higher pitch for the duration of the signal, which can therefore be read. This type of self-quenching receiver is also called the "howling squegger."



Reception of C.W. Signal.

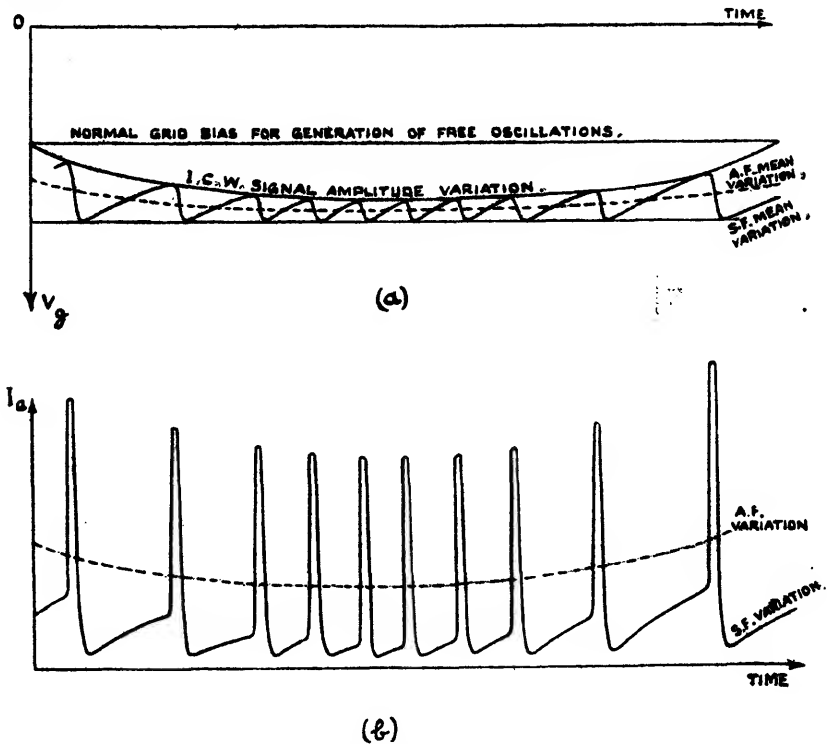
FIG. 416.

710. Reception of I.C.W. Signals.—In this case the incoming signal possesses an audio frequency variation in amplitude, and so an audio frequency self-quenching cycle is not desirable. The I.C.W. waveform can be reproduced by the envelope of free oscillations if the self-quenching frequency is supersonic. One I.C.W. cycle at, say, 1,000 cycles per second, covers the same period of time, for instance, as twenty self-quenching cycles at a quenching frequency of 20 kc/s., and during these twenty cycles the amplitude of the forced oscillation (the I.C.W. signal) at the beginning of each self-quenching cycle rises from a minimum to a maximum and falls to a minimum again.

The decrease in the period of a self-quenching cycle is proportional to the amplitude of the forced oscillation at the beginning of that cycle. With an audible quench frequency considerably higher than the I.C.W. modulation, this would be made evident by the pitch of the note heard in the telephones rising to a maximum when the I.C.W. amplitude was a maximum, and falling at the end of the I.C.W. cycle to its value at the beginning. With a supersonic quench the effect is of the same kind; self-quenching cycles are completed most rapidly in the middle of the I.C.W. cycle, and so there are most positive pulses of anode current then. The result is that the mean anode current over a few quenching cycles is greater in the middle of the I.C.W. cycle than it is at the beginning and end, *i.e.*, there is an **audio-frequency increase** of mean anode current due to this cause.

Another effect of the I.C.W. signal amplitude variation, however, is that the grid bias at which oscillations start is most negative in the middle of the I.C.W. cycle. Over such a cycle the mean grid potential therefore has an audio-frequency variation which is in the negative direction and so gives an **audio-frequency decrease** of mean anode current.

These two opposed effects occur simultaneously and the net audio-frequency variation of anode current, which is the measure of the amplification produced, is proportional to their difference. By a suitable choice of circuit constants it is possible to cause the one effect to outweigh the other sufficiently to give efficient amplification.



Reception of I.C.W. Signal.

FIG. 417.

Fig. 417 is an attempt to illustrate the reception of I.C.W. signals on a supersonic self-quenching receiver. Fig. 417 (a) shows the S.F. cycles of grid voltage for part of an audio-frequency cycle of the forced oscillation. The envelope of the latter is greatly exaggerated in order to show the effect. It should, of course, be of much smaller amplitude than the mean audio-frequency variation of grid voltage produced by the generation and quenching of free oscillations, and shown by the dashed line. In Fig. 417 (b) are sketched the corresponding supersonic and audio-frequency variations of anode current. The effect of the increasing negative grid bias is taken to be greater than that of the higher frequency of positive anode current pulses and so the audio-frequency variation is shown as a decrease.

HIGH-FREQUENCY TRANSMISSION.

711. There is no difference in principle between the methods adopted for the generation of high and low radio frequencies, as should already be evident from the section on H.F. receivers, which usually contain self-oscillatory circuits generating H.F. oscillations. Thus the classification of transmitting circuits applies to all frequencies. The differences that arise are found mainly in the design of components, transmitting valves and so on, in order to reach the small LC values required for the tuned circuits at high frequencies. The inter-electrode capacities of a transmitting valve become of increasing importance as the frequency becomes higher. The anode-grid capacity, in particular, is often used as the tuned circuit condenser, tuning being effected by varying the inductance. A variation in this capacity produces, therefore, a greater percentage variation in the transmitted frequency than in L.F. circuits, and as the same percentage variation involves an increasing actual change in frequency as the frequency rises, a self-oscillatory H.F. transmitter usually produces a wave whose frequency varies between rather wide limits. The difficulties arising from this in reception have already been dealt with. As most Service H.F. transmitters produce I.C.W., the frequency variation may be pronounced, and the later models use master control for the H.F. oscillatory circuit.

The generation of spurious oscillations also occurs more readily at high frequencies, and provision may need to be made for their suppression. Another important point, when using parallel feed circuits, is the design of the anode choke. The oscillatory frequency at some point of the range may either be the same as the resonant frequency of the choke and its self-capacity, or a harmonic of it, and standing waves (para. 768) may be set up in the choke, of sufficient amplitude to damage its insulation. Further, according as the end of the choke attached to the anode is a node or antinode of potential, *i.e.*, according as the frequency is an even or odd harmonic of the fundamental choke frequency, the oscillatory circuit may either be virtually short-circuited, or a higher P.D. than its rating will allow may be applied between anode and filament of the valve; or, when use is made of parallel feed to the anodes of the output valves of a master-controlled transmitter, the occurrence of choke-resonance producing an antinode of potential at the anodes may render the output stage self-oscillatory. In many cases Service H.F. transmitters are attachments to existing L.F. transmitters, and use to some extent the same components. This choke-resonance effect necessitates the use of a different choke for parallel feed H.F. and L.F. circuits, and, in some cases, the insertion of a large resistance in series with the H.F. choke in order to diminish the H.F. current when resonance cannot be avoided. Alternatively, provision may be made for short-circuiting portions of the choke, and hence altering its resonances according to the frequency in use.

712. A typical Service transmitter is shown in Fig. 418 (a). The main tuned circuit capacity is the anode-grid inter-electrode capacity of the valves, and a variable condenser is inserted, by means of a switch, either in series or in parallel with this capacity. The two arrangements are shown in Fig. 418 (b) and (c) respectively. The series position corresponds to the highest frequencies since the total capacity of two condensers in series is less than that of either.

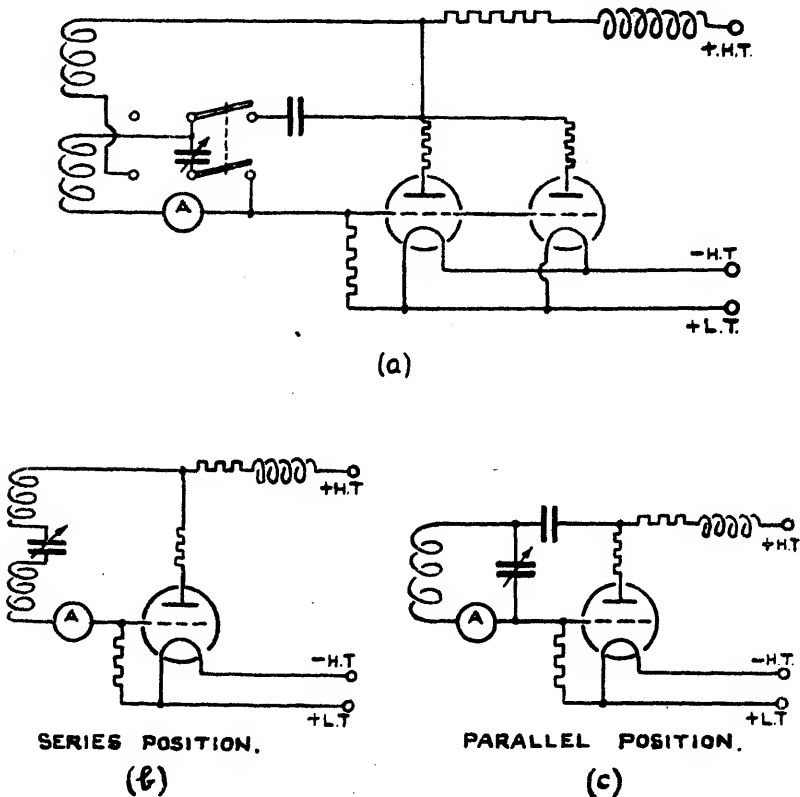


FIG. 418.

The resistance in series with the anode choke to prevent choke resonance effects should be noted, and also the resistance in the anode lead to damp out spurious oscillations. In the parallel position, these tend to be set up in the circuit comprising the inter-electrode capacity, the blocking condenser, the variable condenser and their leads; and in both positions they are likely to occur round the circuit consisting of the anode-grid inter-electrode capacities of the valves and their common leads.

The circuit is of the type classified as tuned circuit between anode and grid, and parallel feed, the anode being the valve electrode

at a high oscillating potential. There appears, however, to be no grid excitation, since the connection normally made from the filament to a point on the tuned circuit inductance intermediate between the grid and anode connections is absent. This apparent discrepancy, however, is simply explained by taking account of the valve inter-electrode capacities. At high frequencies these have values of the same order as the tuning condenser; in fact, as pointed out above, C_{ga} is the only tuned circuit capacitive path in the series position. The other inter-electrode capacities, C_{af} and C_{gf} , also possess comparable values and carry a fair proportion of the oscillatory current. The complete tuned circuit, taking account of these

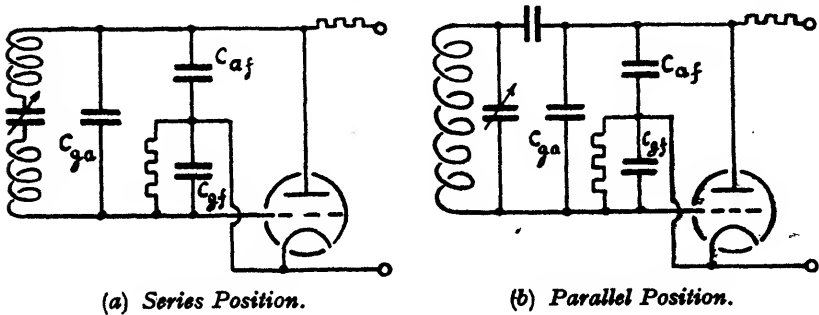


FIG. 419.

capacities, is shown for the series and parallel positions of the variable condenser in Fig. 419 (a) and (b). The grid leak resistance is also shown, but for H.F. oscillations it is by-passed by C_{gf} . A grid insulating condenser and leak are, of course, still required to give the mean negative grid bias necessary for efficiency. Inspection of these circuit diagrams shows that in both cases the grid excitation may be considered as direct capacitive. The filament connection to the tuned circuit occupies a position in the capacitive branch intermediate between the grid and anode connections. When an oscillatory voltage is developed across the tuned circuit, the filament potential is thus intermediate between the grid and anode potentials, and so V_g and V_a are in antiphase, which is the phasing condition for maintenance of oscillations.

713. An interesting example of the effect of inter-electrode capacity in a transmitting circuit is provided by the Service transmitter already shown in Fig. 364 (Chapter XIV), and reproduced in Fig. 420, which was designed to cover the L.F. and lower M.F. band of frequencies. At the lowest frequencies tap Y is kept fixed, and tuning is effected by varying taps X and Z. As the frequency increases, the aerial tuning coil is cut out altogether, and the operations of tuning and adjusting the anode tapping point for most efficient conditions are carried out by manipulating the taps Y and Z. In Fig. 421 the equivalent circuits are shown for three positions

of tap Y over this range of frequencies. The H.T. supply and smoothing condenser have been omitted as they do not affect R.F. conditions, and so tap A is shown with a direct connection to the

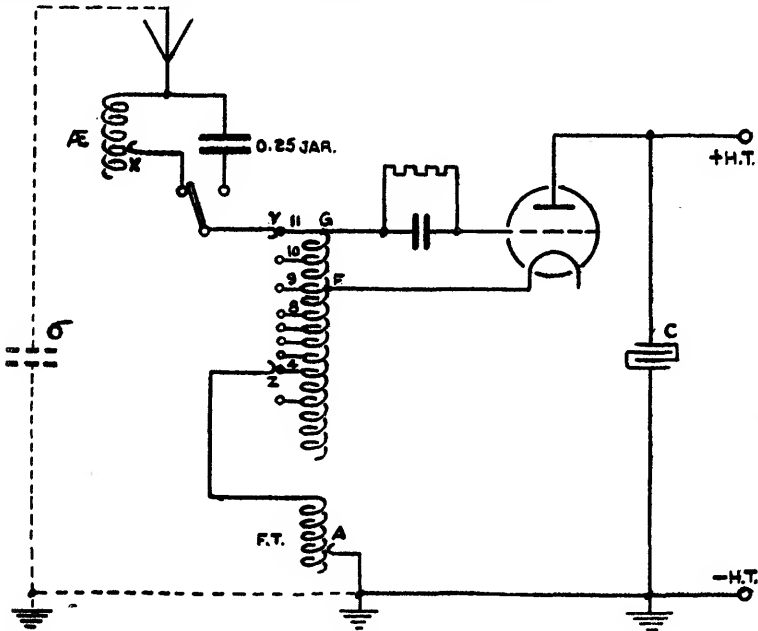


FIG. 420.

anode. This enables the circuits to be exhibited in the simplest way as tuned circuit between anode and grid, direct inductive grid excitation circuits (*cf.* Fig. 359). The lettering exactly corresponds

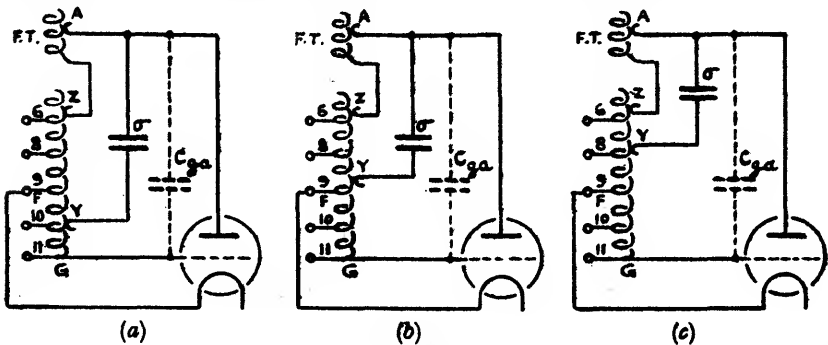


FIG. 421.

to that in Fig. 420, and should enable the relation between the actual and equivalent circuits to be easily followed. For convenience tap Z is shown at position 6 on the tuned circuit inductance in each case, but it may occupy any lower-numbered position than

tap Y. It will be seen that the current flowing through C_{ga} becomes of increasing importance as the frequency increases; C_{ga} in fact, forms the capacitive branch of the tuned circuit in Fig. 421 (b) and (c), and enables the grid excitation (the oscillatory P.D. from 9 to 11 of the inductance) to remain in the correct phase for maintenance of oscillations; the parallel circuit consisting of σ , and the part of the oscillatory circuit inductance from A to Y, is resonant at a higher frequency than that on which the whole circuit actually oscillates, and so has a net inductive reactance at the oscillatory frequency. Even at the lower frequency corresponding to Fig. 421 (a), C_{ga} plays an important part in supplying the grid excitation, although, as the filament connection is then between the anode tap A and tap Y, grid excitation in the correct phase is provided by the P.D. from 9 to 10 of the oscillatory inductance, while still considering the aerial capacity σ as part of the capacitive branch of the tuned circuit.

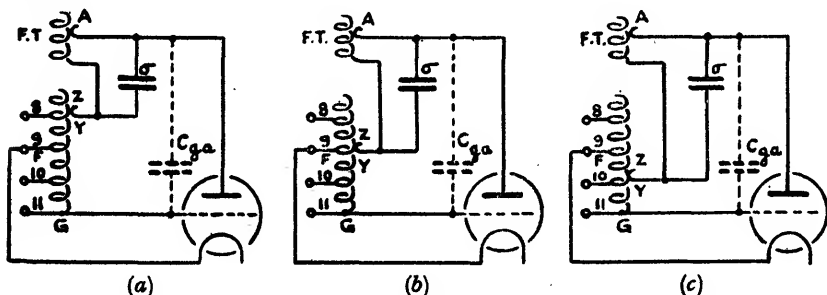


FIG. 422.

714. It was found that considerably higher frequencies could be obtained with this transmitter by combining taps Y and Z at a common point on the oscillatory inductance. The equivalent circuits for three such positions are shown in Fig. 422 (a), (b) and (c). With the same amount of fine-tuning inductance (F.T.) in circuit, the frequency increases as the combined tap ZY is moved from 8 to 10 on the oscillatory inductance. In each case, the aerial capacity, (which includes the H.F. condenser, 0.25 jar), and the fine-tuning inductance form a parallel circuit in the inductive branch of the main tuned circuit; this parallel circuit always tunes at a higher frequency than the whole circuit, and so possesses inductive reactance at the actual frequency of oscillations, *i.e.*, it can effectively be replaced by an inductance. In Fig. 422 (a), oscillations can still be obtained with the whole of the fine tuning coil out of circuit, since the branch of the main tuned circuit from A to F, through the oscillatory inductance from 8 to 9, possesses inductive reactance (σ is short-circuited). This is not the case in the circuits of Fig. 422 (b) and (c), and part of the fine tuning coil must then be included in the circuit for oscillations to be possible.

In all these cases there is a considerable departure from maximum efficiency, and the amount of power transferred from the valve to the oscillatory circuit is only a small proportion of the power input. The power dissipation at the valve anode is correspondingly increased, and care must be taken to prevent this from rising above the anode rating, and hence damaging the valve. This is particularly necessary for the adjustment shown in Fig. 422 (c), where the only part of the grid excitation in the correct phase is the P.D. from points 10 to 11 of the oscillatory inductance. The whole anode (make-up) current flows through the inductance from 9 to 10, and so produces a P.D. which is 90° out of phase with that from 10 to 11. V_a is thus by no means 180° out of phase with V_g at this adjustment, and the efficiency is low. In some cases it is not possible for oscillations to be generated at all at this setting, one determining factor being the value of the aerial capacity.

715. The circuit diagram of a Service V.H.F. transmitter producing Tonic Train is shown in Fig. 423. The R.F. oscillatory circuit is enclosed by the dotted line. The oscillating valves are arranged in push-pull, the tuned circuit being connected as usual between their anodes. With respect to either valve, however, the tuned circuit is between anode and filament, the H.T. supply lead being the filament connection. Series feed is employed and the anodes are the high oscillating potential electrodes. The aerial coupling is mutual inductive.

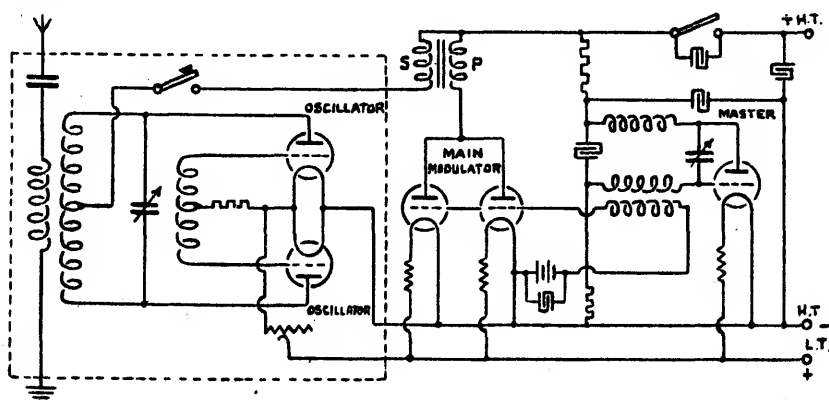


FIG. 423.

There is no inductive coupling between the grid coil and the tuned circuit inductance, the grid excitation being obtained by direct capacitive coupling through the anode-grid inter-electrode capacities of the two valves. It was seen in Chapter XIII that an amplifier stage with a tuned grid circuit and an output choke was liable to generate self-oscillations; this tendency is, of course, unaffected if the positions of the tuned circuit and the untuned

choke are reversed, and at very high frequencies is sufficiently pronounced to give a satisfactory transmitting circuit.

The modulation system employed is that variety of anode voltage modulation known as "Choke Control" and described in para. 672. A high impedance 1 : 1 transformer is used instead of a choke. This has the advantage that the direct currents to the oscillator and main modulator valves can be arranged to flow in opposite directions through the two windings, and so produce steady fluxes in opposite directions in the core. The resultant magnetisation of the core is the difference of these two effects, and so the distortion produced by a large steady magnetisation of the transformer core is avoided.

The A.F. oscillations are generated in a circuit of the familiar type classified as tuned circuit between anode and grid, direct inductive grid excitation and series feed. The H.T. supply to the anode is cut down to an appropriate amount by a resistance in the anode lead. The A.F. oscillation is amplified by two main modulator valves in parallel before being applied to the anode of the oscillator valve.

In order to produce tonic train modulation, the same precautions against distortion must be taken as those already mentioned in para. 672.

716. Master-controlled H.F. Transmitters.—Reference has been made to the greater frequency variation to be expected in H.F. oscillatory circuits, particularly when I.C.W. is being produced. As a self-oscillatory receiver is normally used at these frequencies, frequency variation of an incoming C.W. signal gives a note of constantly-changing pitch, and frequency variation of an I.C.W. signal will give at least three notes, two rising and one falling in pitch as the carrier frequency varies. Modern self-oscillatory H.F. transmitters are capable of producing C.W. with very little frequency modulation, but I.C.W. is commonly used for H.F. transmission, since it is equivalent to three C.W. oscillations at neighbouring frequencies, and is therefore of some assistance in overcoming fading (para. 739). To keep the carrier frequency constant, some form of master control is desirable.

Fig. 424 shows a high frequency I.C.W. transmitter, in which the master circuit can either be used as a rigid low-power self-oscillatory circuit, or quartz control can be employed.

The circuit is merely the self-oscillatory I.C.W. oscillator of Fig. 386. with the frequency of the R/F oscillations controlled by the master circuit shown in Fig. 373, and will easily be followed by comparison with these circuits.

A master controlled circuit using a push-pull output stage is shown in Fig. 425.

The master is a rigid low power self-oscillatory circuit of the type classified as tuned circuit between anode and grid, direct inductive

grid excitation and series feed. The output across the master tuned circuit inductance is taken to the grids of the power valves via two coupling condensers. Parallel feed is used for the H.T. supply to

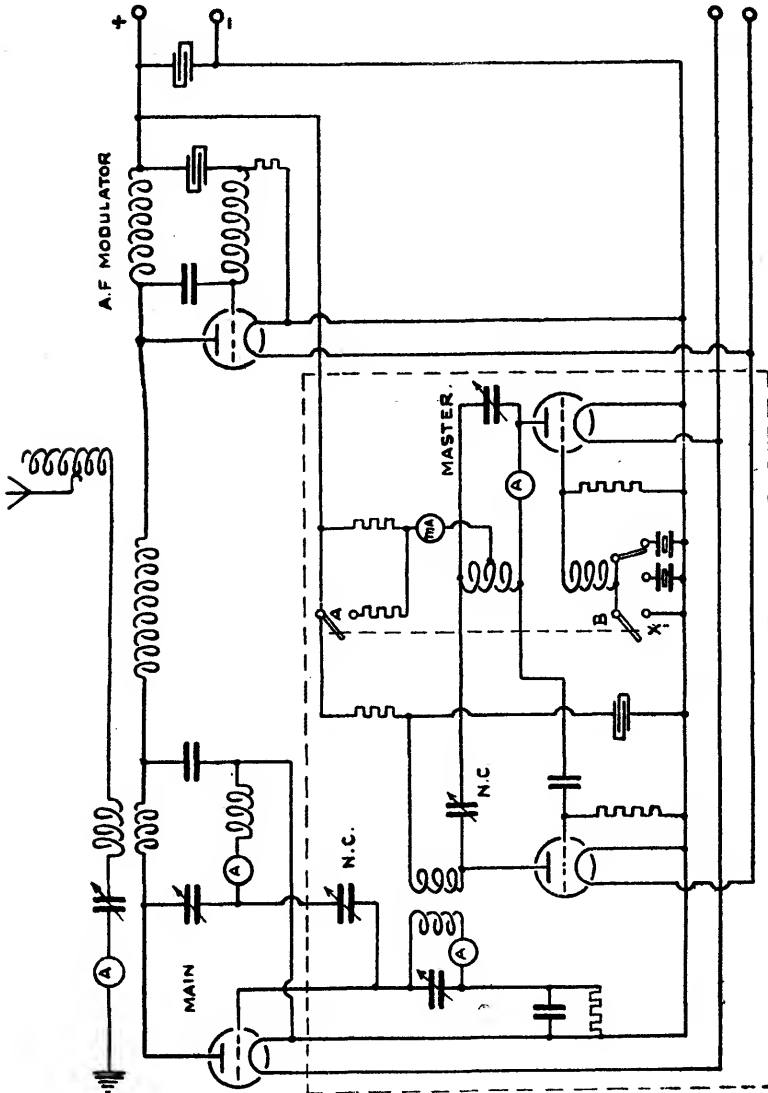


FIG. 424.

the anodes in the power stage, and provision is made for avoiding choke-resonances (para. 711) by short-circuiting part of the anode chokes according to the frequency in use.

This circuit provides an interesting example of the principles of neutralisation. Neutralising condensers were connected between

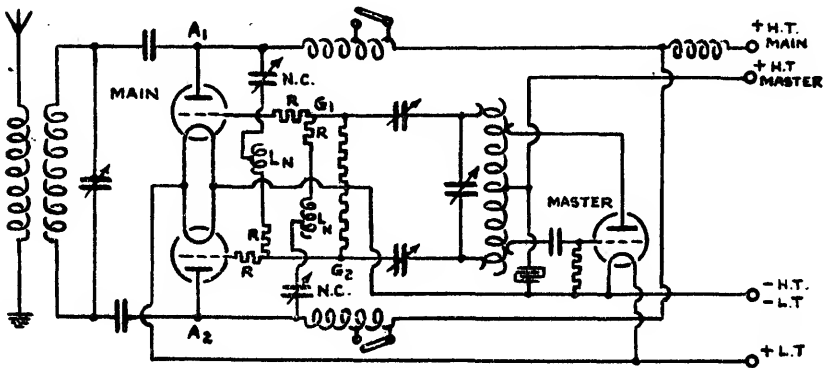


FIG. 425.

the anode of each valve and the grid of the other, and their introduction tended to give rise to parasitic oscillations round the neutralising bridge circuit (Fig. 426). These were damped out by the symmetrical insertion of resistance in the four arms of the bridge, and the bridge was then readily balanced for frequencies at which the

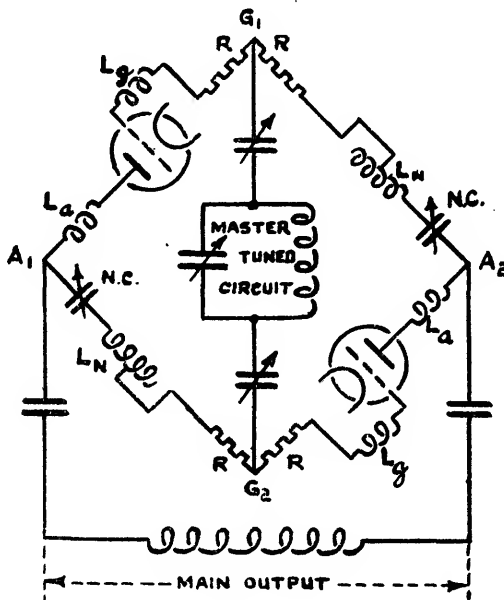


FIG. 426.

inductive reactances of connecting leads could be neglected, *i.e.*, an applied P.D. between A_1 and A_2 gave rise to no P.D. across $G_1 G_2$, the master circuit.

Although such a balance obviates reaction on the master from the main circuit, it does not necessarily prevent self-oscillation of

the latter. The filaments of the valves are earthed, and, at any moment, the anodes A_1 and A_2 have equal and opposite potentials to earth. Thus, although G_1 and G_2 must be at the same potential when the bridge is balanced, they are not necessarily at earth potential, *i.e.*, there may be a grid-filament P.D. to earth in each valve. Inspection of Fig. 426 shows that this grid excitation is in phase with the anode-filament P.D. in one valve, but in antiphase with it in the other. This causes one valve to absorb energy, but the other will tend to set up self-oscillations in the main circuit if the grid excitation is sufficient. Hence it is further necessary to balance the bridge in such a way that G_1 and G_2 are at earth potential, *i.e.*, midway between the potentials of A_1 and A_2 . Assuming the valves to be matched, so that their inter-electrode capacities are equal, this further condition is satisfied if the bridge is balanced with equal settings of the two neutralising condensers, and for this purpose the condensers are ganged.

Above a certain frequency, the inductive reactance of connecting leads is large enough to make a balanced bridge impossible unless the inductance is symmetrically disposed in the four arms. In Fig. 426 the inductances of the leads from the anodes and grids of the valves are shown as L_a and L_g respectively. The condition for a balance which is independent of frequency, and brings G_1 and G_2 to earth potential, is that the capacity, inductance and resistance of each arm of the bridge should be the same. The low frequency balance ensures that the capacities are equal, and the anti-parasitic resistances are symmetrically inserted, but it may be necessary to introduce small coils into certain of the bridge arms to give a symmetrical distribution of inductance. This is indicated by the variable coils L_N in Figs. 425 and 426.

Neutralisation is then obtained by a series of successive approximations, as follows :—

- (a) Set each inductance L_N by calculation to a value approximately equal to $L_a + L_g$.
- (b) Energise the circuit at the lowest frequency to be transmitted, and adjust the ganged neutralising condensers until a balance is obtained. (If the neutralising condensers are not ganged, they must be adjusted equally.)
- (c) Energise the circuit at the highest frequency to be transmitted, and re-neutralise by equal adjustments of the inductances L_N .
- (d) Repeat operation (b).

This process can be carried out when the transmitter is first assembled, and should give approximately correct neutralisation over the required frequency range; but if the settings are found to be critical, slight adjustments may be necessary whenever the frequency is altered.

CHAPTER XVII.

ELECTROMAGNETIC WAVES.

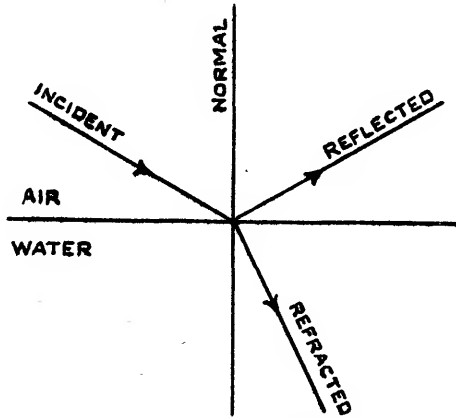
717. The conception of wave motion as a means of transferring energy from one point to another was explained in Chapter I, and it was pointed out that wireless radiation was of the same nature as heat and light radiation, the only difference being in the frequency of the waves. The energy transmitted by such wave motion is known as electromagnetic energy, since its production is associated with the presence of electric and magnetic fields. These fields may be due to the electrical structure of the atom or molecule, as in the case of light and heat rays, or of the large scale type produced in electrical oscillatory circuits when wireless waves are radiated. Waves which transmit electromagnetic energy are called electromagnetic waves. Other types of wave motion capable of transmitting energy, such as sound and water waves, are dependent on a material medium for their propagation, but electromagnetic waves can be propagated in a vacuum. The difficulty of conceiving a wave motion without a medium in which it is passed on from point to point has led to the postulation of a non-material medium called the æther, which must be assumed to permeate all space. It should be emphasised that properties similar to those of material substances cannot be attributed to the æther, and the attempt to do so produces absurd results. The conception of the æther is merely another way of saying that space empty of all material substances still possesses the property of allowing electromagnetic waves to travel through it.

When it is considered as the vehicle of transmission of electromagnetic vibrations, empty space is generally known as the free æther. All electromagnetic waves, no matter what may be their frequency, travel at the same speed in the free æther. This velocity, as determined experimentally, is very nearly 3×10^8 metres per second, this figure being generally employed in wireless calculations. The more accurate figure is 2.9982×10^8 metres per second.

When electromagnetic waves are travelling through a material medium, *e.g.*, a brick wall, it must still be considered that the actual medium in which they are propagated is the æther. It is, however, no longer free æther, but is modified by the presence of the material occupying the same space. As a result, the transmission of the electromagnetic waves is also modified by the material. The effects produced are most easily realised by considering the case of light waves, since they are then visible.

718. Reflection.—It is hardly necessary to explain the meaning of reflection. When a billiard ball strikes a cushion its direction of motion is reversed. In the same way an electromagnetic wave

trying to pass from one material medium to another may be turned back at the common surface of the two media. In the case of a light wave arriving obliquely at a point on the surface of a mirror, the reflected wave comes off at the same angle on the other side of the line through the point perpendicular to the mirror as the arriving



Reflection and Refraction.

FIG. 427.

or "incident" ray, as shown in Fig. 427. Because of this the reflected image of an object appears to be at the same distance behind the reflecting surface as the object is in front of it. Generally the wave is only partly reflected at the surface, and a part of its energy passes on into the second medium.

719. Refraction.—If a walking-stick is partly immersed in water, it no longer appears to be straight. The part under water appears to run in a different direction from that in the air. The light waves, by which the stick is made visible, alter their direction of propagation when they pass from air to water and *vice versa*, and so the stick appears to be bent. The reason for this is that the waves do not travel with the same velocity in the two media. In this case they travel more slowly in water than in air, (or rather, more slowly in æther as modified by water than in æther as modified by air), and the effect is that the direction of propagation in water is bent in towards the normal, or line drawn at right angles to the air-water surface through the point where the stick enters the water. The waves are said to be refracted, and the ratio of the velocity of the waves in free æther to the velocity of the waves in a material medium is known as the refractive index of the medium. Since the waves travel more slowly in water than in air, the refractive index of water is greater than that of air. The actual figures are 1·33 for water and 1·00029 for air (at normal pressure and temperature).

$$\text{Thus } n \text{ (refractive index)} = \frac{3 \times 10^8 \text{ metres per sec.}}{\text{velocity in medium}},$$

$$\text{or velocity in medium} = \frac{3 \times 10^8 \text{ metres per sec.}}{n}.$$

From the nature of electromagnetic waves we should expect that their velocities in various media would depend on the electrical and magnetic properties of the media, and it can be shown that if K is the dielectric constant of a medium and μ is its permeability, the velocity of electromagnetic waves in the medium is $\frac{3 \times 10^8}{\sqrt{\mu K}}$ metres per second. Since μ and K are both taken as unity for a vacuum, this gives the velocity of electromagnetic waves in free æther as 3×10^8 metres per second, as previously stated.

The permeability, μ , is approximately unity except in the case of ferromagnetic metals, and so the velocity in a given medium may be taken to be $\frac{3 \times 10^8}{\sqrt{K}}$ metres per second.

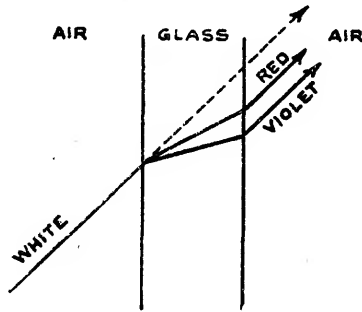
It follows from this that the refractive index of a medium is equal to the square root of its dielectric constant,

$$\text{i.e., } n = \sqrt{K}.$$

This formula is only strictly correct for a perfect insulator. For poor insulators and conductors it must be modified to take account of their conductivity.

720. Dispersion.—The dielectric “constant” of an insulator is not constant, but varies with the frequency of the alternating P.D. applied across it. Thus a material medium has a different K for electromagnetic waves of different frequencies, and therefore a different refractive index. The dielectric constant of distilled water, for example, as deduced from steady potential measurements, is about 80. For alternating potentials of the frequency of light waves it is in the neighbourhood of 2, ($n^2 = 1.7$). In other words, the velocity of an electromagnetic wave in any medium except the free æther depends on its frequency. This can be readily seen in the case of light waves by examining the light from the sun through a glass plate. Sunlight consists of a very large number of light vibrations of different frequencies. These all travel with the same velocity through free space, and with sensibly the same velocity through air, owing to its small refractive index, and so combine to give the impression of white light. The differences in velocity of the component vibrations of different frequencies are, however, appreciable in the case of glass. In consequence, these vibrations are refracted through different angles in passing through the glass, and re-appear at different points on the other side of the plate, as shown in Fig. 428. Differences in frequency in the visible range appear to the eye as differences in colour, and so a

series of differently-coloured images of the sun will be seen side by side. The white light is said to be resolved into a spectrum. In practice these images will largely overlap each other. To produce a good spectrum, a ray of sunlight entering a dark room through



Dispersion.

FIG. 428.

a very narrow slit should be allowed to fall on a glass prism. A series of images of the slit will then be seen, appearing as parallel differently coloured bands of light. The red band is least, and the violet band most, deviated from the direction of the original white light.

721. Total Reflection.—It has been seen that a light wave in passing from air to water is refracted towards the normal (Fig. 427). Conversely, a wave from water to air is refracted away from the normal. As the direction of the wave in water becomes more oblique to the normal, the angle made with the normal by the refracted ray in air becomes correspondingly greater, and eventually a point is reached when the wave, on emerging into air, just grazes the common surface, *i.e.*, is at right angles to the normal. If the obliquity of the wave in water is increased further, no wave can then emerge into the air, *i.e.*, the wave is totally reflected at the common surface. The angle made by the incident ray with the normal when this occurs is known as the critical angle.

This phenomenon can only occur when the refracted wave makes a greater angle with the normal than the incident wave, *i.e.*, when the refractive index of the first medium is greater than that of the second medium. In other words, total reflection can only occur when the electromagnetic waves attempt to pass into a medium in which they would travel more quickly.

722. These various effects influence the propagation of wireless waves just as they do the propagation of light waves, although the amount of importance attaching to them is different owing to the very different frequencies of these two kinds of electromagnetic vibration. As a simple example in the case of wireless waves, we

may consider the effect of endeavouring to propagate them in a perfect conductor. The dielectric constant K of a perfect conductor is infinite, for no potential difference can be established across it, and so the velocity of electromagnetic waves in it, namely $\frac{3 \times 10^8}{\sqrt{K}}$

metres per second, is zero. In other words, these waves cannot pass through a perfect conductor. It is well known that good conductors, such as copper, will not allow wireless waves to pass through them, and so are extensively used for screening purposes. In such cases the energy in the waves is partly reflected and partly absorbed in the screen. The alternating field produced by the wave in the metal sets its free electrons into oscillation, and the damping out of these oscillations by collisions with molecules converts the energy into energy of molecular vibration, *i.e.*, heat energy.

To illustrate the effect of the different frequencies of electromagnetic waves on their propagation in material media, it may be mentioned that a brick wall is completely opaque to light waves, but allows wireless waves to pass through it, as evidenced by the possibility of reception using indoor aerials, such as are common in portable receivers.

723. An attempt will be made in the next few paragraphs to give some picture of how an oscillatory current flowing in an aerial circuit gives rise to electromagnetic radiation. It must be pointed out, however, that any such picture is crude, and in some ways even misleading. The real proof of the theory depends on mathematical analysis beyond the scope of this book. It was first developed by Clerk Maxwell in 1864 from the general laws of electricity and magnetism, and so the set of relations which he derived, and which are the basis of electromagnetic wave theory, are known as Maxwell's Equations. From these equations it can be shown that changing electric fields and the changing magnetic fields they produce must give rise to electromagnetic radiation, whose velocity of propagation can be determined from the electrical constants of the medium. This calculated velocity was found to be the same as the measured velocity of light waves, and so gave rise to the theory that light waves were of electromagnetic origin, and to experimental attempts to produce similar radiation by means of electrical circuits. The successful production of such radiation by Hertz laid the foundations of wireless telegraphy.

724. Radiation.—Let us take as the simple type of oscillatory circuit with which to illustrate electromagnetic radiation, a condenser whose plates are connected by a vertical wire which has a certain amount of self-inductance (effectively an aerial circuit). An alternator is included in the circuit to represent a source of alternating E.M.F. of high frequency, the frequency being that to which the circuit is resonant. This system is then effectively that of a C.W. oscillator maintaining an oscillation in an aerial circuit.

We shall examine the distribution of the electric field round such an aerial, and its changes during one cycle of the applied E.M.F.

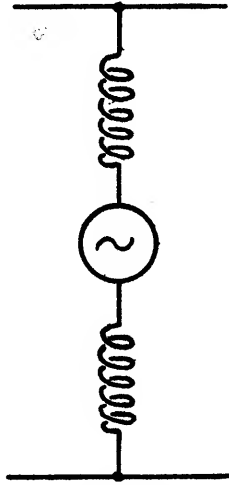


FIG. 429.

At the start of the cycle let the condenser be charged to its maximum P.D., so that the top plate is positive to the bottom plate, and the current is zero. At this instant we may regard the field in the vicinity of the aerial as being entirely electric, and lines

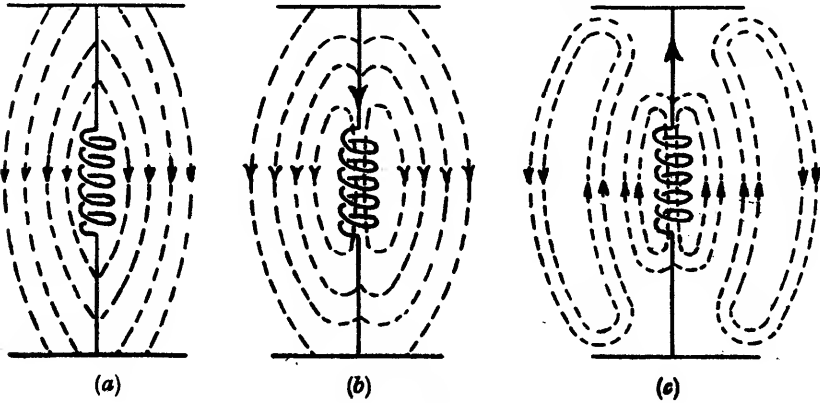


FIG. 430.

of electric strain to be connecting each positive ion on the upper plate to its "opposite number," a negative ion on the lower plate, as in Fig. 430 (a).

When the moment of maximum P.D. has passed, current will start to flow downwards. The electric field starts to collapse, and this effect may be represented as in Fig. 430 (b), showing the

ends of the lines of force coming together along the wire. The current continues to flow after the potential difference across the condenser is reduced to zero, and in so doing starts to charge up the condenser to the opposite polarity, giving rise to new lines of force in the opposite direction to the previous field. We may regard the collapse of the initial field as lagging a little on the changes in potential which cause it to take place, and with this assumption the new electric field starts to build up before the first one has disappeared. The first disturbance is then forced outwards in the form of closed loops by the new electric field, for the direction of the lines in the inner surface of the first and the outer surface of the second are the same.

As the current oscillates in the circuit, a series of closed loops of electric stress is sent off into space radially from the oscillating system, each set repelling its predecessors to make room for the latest-born, and representing lines of force in the opposite direction to the sets next in front and in rear.

Similarly we must regard the circuit as being surrounded by rings of changing magnetic stress, whose intensity varies with the current strength, and whose direction alternates in the same way as the direction of the electric field alternates.

Further, these magnetic lines of force are horizontal, being in a plane at right angles to the current-carrying wire, and consequently at right angles to the electric lines of force. This type of oscillation is produced by an oscillating system excited in the middle, and a practical application may be found in the case of a trailing aeroplane aerial.

725. Earthed Aerial.—The aerial system used in practice, in which the earth is utilised as the bottom plate of the condenser, can be regarded as the upper half of the symmetrical type of oscillating system excited at the middle which has so far been investigated. This assumes that the capacity is concentrated in the "roof" of the aerial. The consequent distribution of the electric and magnetic fields is somewhat as shown in Fig. 431.

The full lines represent the electric field spreading out in the form of annular loops of ever-increasing height but constant width. It may be taken as vertical at points near the earth's surface. These loops are accompanied by horizontal loops of magnetic flux, spreading out from the aerial with the electric loops.

It is important to grasp the fundamental conception that **the moving electric and magnetic fields do not exist separately**, but are simply different ways of expressing the fact that energy is transmitted by an electromagnetic wave.

They are therefore **in time phase with each other, although at right angles to each other in space**. In other words, the maximum strength of magnetic field occurs when the electric field is also a maximum.

In the surface of the earth there will be circular bands of current flowing alternately radially outwards and inwards.

The velocity with which the whole system moves outward is (approximately) 3×10^8 metres per second. The frequency with which consecutive bands of maximum electric or magnetic field are generated is the frequency of the current in the oscillatory system, and the radial distance between two consecutive maxima of electric

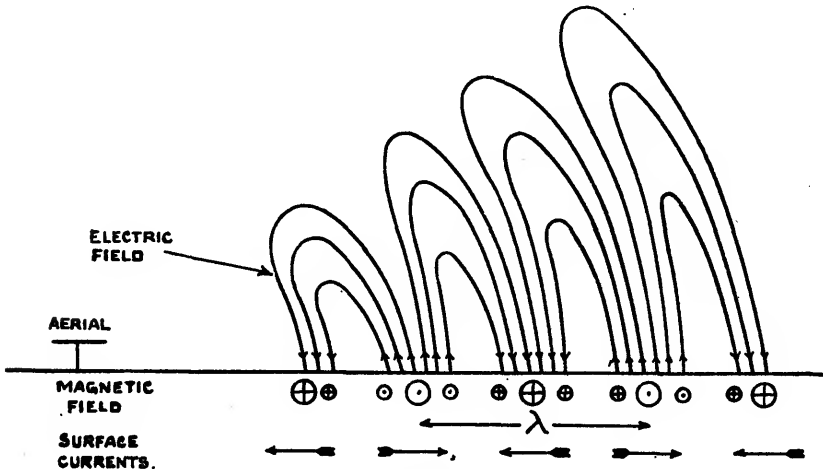


FIG. 431.

or magnetic field in the same direction is the wavelength of the electromagnetic wave. The strength of the electric or magnetic field varies sinusoidally with respect to time at any given point remote from the transmitter, and sinusoidally with respect to distance measured outwards from the transmitter at any given time, (neglecting the attenuation in amplitude discussed in later paragraphs).

726. Radiation and Induction.—It is important to distinguish between “radiation” and “induction.”

The principles of induction have already been explained in earlier chapters.

It may be advisable to state once again the essential characteristics associated with radiation, so as to draw the necessary comparisons with induction.

Radiation is a moving disturbance of the æther, in which varying states of electric and magnetic stress are propagated outwards from the oscillating system, and represent an actual dissipation of energy. The two fields, which are simply two different ways in which the phenomenon may be regarded, are **in time phase** with each other but **in space quadrature** (at right angles), and both are at **right angles** to the direction of propagation. It can be

shown that the intensity of the fields associated with radiation over a perfectly conducting surface falls off **inversely as the first power of the distance** from the oscillating system producing them.

Induction represents a variation of electric and magnetic fields in which there is no dissipation of energy. The electric and magnetic fields are **in both time and space quadrature** (90° out of phase in time, and at right angles to each other in space), and energy simply oscillates from one to the other without any being lost. We have already met this effect in the theory of the oscillations set up in an LC circuit in which resistance losses are assumed to be zero (Chapter VII). It can be shown that the intensity of the electric or magnetic fields of induction at any point varies inversely as the square of the distance of the point from the oscillating system.

Near the aerial the inductive field is stronger than the radiative, and at a great distance from the aerial the reverse is the case. The actual strengths of the two fields can be proved to be proportional to $\frac{2\pi hI}{\lambda d}$ (radiative) and $\frac{hI}{d^2}$ (inductive) respectively, where h is the height of the aerial, I the current, and d the distance of the point at which their intensities are measured.

It follows that at a distance where $\frac{2\pi hI}{\lambda d} = \frac{hI}{d^2}$, *i.e.*, where $d = \frac{\lambda}{2\pi}$, the intensities are equal, while at a distance of even a few wavelengths, say 5λ , the intensity of the inductive field is only about one-thirtieth of that of the radiative field, so that it may be neglected in comparison with the latter.

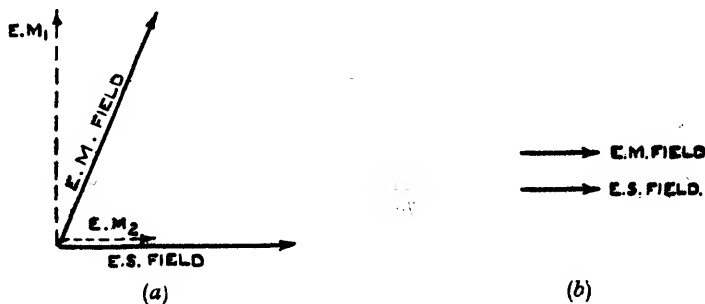


FIG. 432.

At any point, the complete field is, of course, the resultant of the inductive and radiative components. Taking the electric field as a basis of reference, the magnetic field is the resultant of two components, one in phase and the other 90° out of phase (as regards time) with it.

Very near the aerial the inductive component is the larger, so that the resultant magnetic field is as illustrated in Fig. 432 (a);

far from the aerial the inductive component may be neglected altogether, so that the magnetic and electric fields are effectively in time phase.

The intensity of the radiative field will be further referred to quantitatively in the chapter on aerials.

727. An electromagnetic wave of the type shown in Fig. 431 may be represented diagrammatically as in Fig. 433. The vertically oscillating electric field is shown by the vertical vector OZ, and the horizontally oscillating magnetic field by the horizontal vector OX, which is really perpendicular to the plane of the paper. The direction of propagation of the wave, OY, is at right angles to the plane of OX and OZ. The amplitude of the electric field \mathcal{E} , measured in electrostatic units, is equal numerically to the amplitude of the magnetic field \mathcal{H} , measured in electromagnetic units. When both \mathcal{E} and \mathcal{H} are expressed in the same units, the numerical relation connecting them is $\mathcal{E} = \mathcal{H} \times 3 \times 10^{10}$.

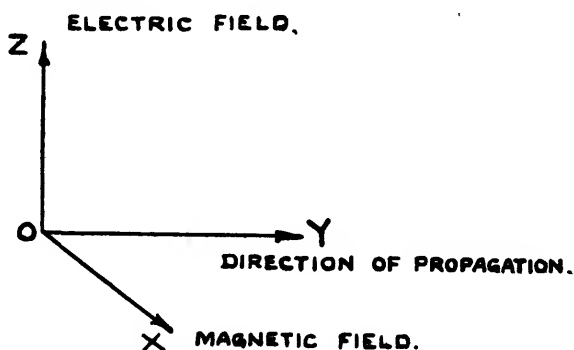


FIG. 433.

It may help to fix ideas of the nature of the wave and its propagation if a numerical example is taken. Consider a wave of frequency 250 kc/s., and therefore of wavelength 1,200 metres in the free æther, and approximately of this wavelength when travelling through the atmosphere close to the surface of the earth. The electric and magnetic fields are in time phase, and at some instant they will have their maximum values simultaneously at the point O in Fig. 433. Suppose that times are reckoned from this instant. The time of a complete cycle of values of the wave fields for a 250 kc/s.

wave is $\frac{1}{250,000}$ second, or 4 microseconds. After a quarter of this time, *i.e.*, one microsecond, the two fields at O will have decreased to zero. They then reverse in direction, and start to increase in magnitude, and after another millionth of a second they attain maximum values in the reverse direction. In this time, 2 microseconds, the wave has travelled 3×10^8 metres per second $\times 2 \times 10^{-6}$

seconds = 600 metres, or half a wavelength, and so the fields at a point P, 600 metres from O in the direction of propagation, have their maximum values 2 microseconds after similar maxima occur at O. As this time corresponds to maximum reversed values at O, it follows that the fields at P and O at the same instant are 180° out of phase.

In 4 microseconds the fields complete a cycle, and their values at O are the same as their initial values. The wave has travelled 1,200 metres in this time to a point Q, say. The fields at O and Q, 1,200 metres apart, are therefore in time phase.

Thus the time lag of the fields at a point R on those at O depends on the time taken by the wave to travel from O to R. 1,200 metres from O, the fields are in phase with those at O; 600 metres from O, they are 180° out of phase; 300 metres from O, they are 90° out of phase; and in the general case the time lag of the fields at R on those at O is $\frac{360d}{\lambda}$ degrees, or $\frac{2\pi d}{\lambda}$ radians, where λ is the wavelength and d is the distance between O and R.

We may also consider the reflection of such a wave when it arrives at the surface of another medium, in the simplest case when the direction of propagation is perpendicular to the surface. The direction of propagation is then completely reversed, and the wave travels back along its original path. From Fig. 433 it will be seen that the electric and magnetic vectors are arranged with respect to the direction of propagation like the forefinger, middle finger and thumb of the right hand extended as in Fleming's rule, and, as in that case, reversing any two of the three quantities together does not alter the direction of the third. Thus, when the direction of propagation is reversed, the direction of either the electric field or the magnetic field will be reversed, but not both. Actually it is the electric field which reverses in direction, or, in other words, changes its phase by 180° at a reflecting surface; the magnetic field is unaltered in direction.

728. Polarisation.—A wave of the type considered above, in which the electric field vector is vertical, is called a vertically polarised wave. When the electric field is horizontal, the wave is said to be horizontally polarised. In these cases the vibration of the æther "particle" undergoing the wave motion may be considered to be either up and down or backwards and forwards horizontally, and so it is a linear vibration. In certain circumstances, however, more complicated motion of the æther is produced. For instance, the æther "particle" may move in a circle or ellipse. The wave is then said to be circularly or elliptically polarised. These various types of polarisation may be illustrated by fixing a long rope at one end. If the free end is then waved up and down, the wave motion in the rope is vertically polarised; if backwards and forwards horizontally, a horizontally polarised wave is produced.

Circular polarisation may be imitated by moving the free end of the rope in a vertical circle.

The wave radiated from a vertical aerial over the surface of a good conductor, such as sea water, may be taken as a good approximation to a vertically polarised wave.

729. Attenuation. Effect of the Earth.—It has been seen that the bases of the electromagnetic waves move outwards from the aerial over the surface of the earth, and also that high-frequency currents are set up in the earth.

On the assumption that the surface of the earth is a perfect conductor, no energy is lost as a result of these currents. This assumption is not justified in practice, because the conductivity of the earth varies widely, depending on its nature. Thus the sea is a very good conductor, and we may pass through the various stages of fresh water, damp soil, etc., to very dry soil, which is a very bad conductor. In some places the surface materials of the earth are, in fact, good insulators.

The attenuation of the electromagnetic wave therefore varies according to the type of surface over which it is passing. As the energy which is being dissipated is supplied by the electromagnetic wave itself, it follows that the wave front must become tilted forward in order to give a horizontal component of the electric field, and so does not remain perpendicular to the earth's surface. It can be shown that the higher the conductivity, the less is the depth to which energy can penetrate, and the less are the losses. For example, a theoretical calculation shows that a station having a range of 1,000 miles over a perfectly conducting expanse would have a range of:—

- 920 miles over sea water ;
- 700 miles over fresh water or very wet soil ;
- 560 miles over wet soil ;
- 270 miles over damp soil ;
- 150 miles over dry soil ;
- 55 miles over very dry soil ;

and these figures accord very well with practical experience.

It can further be shown that the higher the frequency of the waves, the greater are the losses, and so the range of a wave travelling over the earth's surface decreases as the frequency increases.

Since the wave front may be regarded as being tilted forward, so that the bottom of the wave is somewhat retarded, it is obvious that signals may be received by insulated wires buried in the ground or under water. The depth to which effective penetration is possible decreases with conductivity, as was stated above, and the maximum depth possible in salt water may be taken to be about five metres.

The tilting of the wave front also helps to make the waves follow the curvature of the earth.

As a general conclusion, one expects to get the longest ranges over sea, and range falls off if dry ground intervenes.

Great difficulty occurs in communication between two stations which have jungle or dense undergrowth intervening, especially if the jungle grows close up to the station.

A tremendous absorption of energy occurs; moreover, there seems to be a layer of air, level with the tree tops, at the same potential as the earth, and the wave travels along the surface of this, and does not influence a receiving aerial unless the latter be a good deal higher than the trees.

730. Screening.—It is a well-known fact that if a ship is lying close to a very high piece of land, or in a land-locked harbour surrounded by high cliffs, her reception of signals is greatly diminished, while if she steams away clear of the land by a few miles she picks up signals once more. This occurs for two reasons :—

- (a) The electric field of the æther wave is deflected by the land, as illustrated in Fig. 434. This means that the electric lines of force acting on the aerial of the ship (A) are in a horizontal instead of a vertical direction, and thus are made very ineffective.

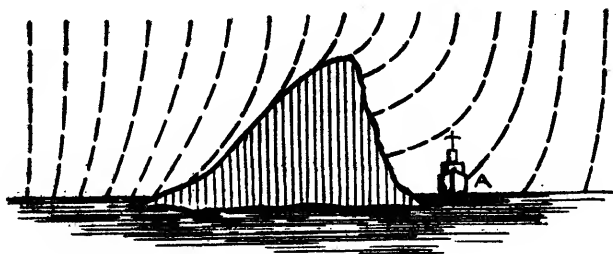


FIG. 434.

- (b) A great deal of the available energy in the portion of the wave front with which the ship is concerned is wasted in the high ground.

This effect is very noticeable if the soil is very permeable, *i.e.*, if it contains a high percentage of iron ores.

A similar screening effect takes place if a ship is lying under a big crane, or a bridge like the Forth Bridge.

731. Atmospheric Effects.—The impossibility of seeing round corners shows that, to a first approximation at least, light waves travel in straight lines, and it might be expected that wireless waves, which are of the same nature, would travel similarly. The question then arises as to how it is possible to transmit wireless signals over the surface of the earth, which is nearly a sphere. Actually, it can be shown that even light waves do not cast absolutely sharp shadows, and can bend round corners to a slight extent

by a process known as diffraction. In the case of wireless waves, which are of much lower frequency, this effect is more pronounced, and helps to explain how they can follow the curvature of the earth's surface. The tilting forward of the wave front due to surface losses was also mentioned in the last paragraph, but even when all such effects are taken into account, they are quite inadequate to account for the strength of received signals at great distances from the transmitter. There are also the observed facts that a signal which is strong at a great distance from the transmitter may not be received at shorter distances, that its strength varies according as light or darkness prevails between transmitter and receiver, and that different effects are obtained in summer and winter.

In order to explain such phenomena, it is necessary to assume that the strength of a received signal is not always that due merely, or in some cases at all, to the wave which is propagated along the earth's surface, and that part of the energy which is radiated upwards from a transmitting aerial returns to the earth's surface by a process of reflection or refraction in the upper atmosphere.

An attempt will be made in the following pages to summarise the main features of this theory, which is still very incomplete in detail. The effects observed are extremely complex, and there is considerable divergence of opinion on the conclusions to be drawn from them.

732. The Atmosphere.—Up to an average height of about 20 miles, the actual height being greatest at the Equator and least at the Poles, the composition of the atmosphere remains approximately the same as at the surface of the earth, since air currents (winds) continue up to this height. The density of the atmosphere naturally falls off as the height increases. Above this height no intermixing takes place due to air currents, and the composition of the atmosphere varies with height, the lighter gases being present in greater proportions as the height increases. There is very little definite knowledge of the composition above a height of 50 miles. It probably contains a large percentage of helium. It has also been suggested that above 70 miles the atmosphere consists mainly of frozen nitrogen particles.

In considering wireless wave propagation, the electrical properties of the atmosphere are of more direct interest than its composition. The air is nowhere a perfect insulator, but its conductivity is small near the surface of the earth. The conductivity in the lower atmosphere is mainly due to ionisation produced by the "cosmic rays" mentioned in Chapter I. Ionisation of this order is insufficient to produce any appreciable effect on the direction of wireless waves, and to provide a basis for the theory that such waves are bent round in the atmosphere and return to the earth's surface, a much greater density of ionisation must be assumed. It was first suggested by Heaviside and Kennelly that such a

densely ionised region must be present in the upper atmosphere, and in consequence this region is generally known as the **Kennelly-Heaviside Layer**. The word "layer," however, is rather misleading, since the region has no sharp boundaries.

The ionisation in this region is mainly due to the presence of free electrons. Its cause is still obscure, but the main factor appears to be the ultra-violet radiation from the sun. Such radiation is most potent at the outer edge of the atmosphere, but the actual number of free electrons it produces also depends, of course, on the number of molecules which are present to be ionised. This number increases as the height above the earth decreases, but the ultra-violet radiation is also being absorbed as it penetrates the atmosphere. The result is that the maximum density of free electrons is produced in some region intermediate between that in which electrons are first liberated and the surface of the earth. The least height at which the ionisation exercises an appreciable effect on the direction of propagation of wireless waves is about 60 miles. This lower limit is by no means sharply defined. It varies considerably from winter to summer and between night and day.

After sunset the ultra-violet radiation ceases to be operative, and in the lower regions where the density of the atmosphere is greater, and collisions between electrons and molecules are more frequent, there is a certain amount of re-combination between electrons and positive ions. The ionisation in these regions therefore decreases between sunset and sunrise, and the effective lower limit of the Heaviside Layer moves upwards, rising slowly through a distance of about twelve miles during the dark hours, reaching a maximum height before sunrise and falling rapidly after sunrise to its daylight value.

The ionisation gradient, *i.e.*, the rate at which the density of ionisation varies with height, appears to be considerably greater from the region of maximum ionisation to the lower limit of the Heaviside Layer than it is above the region of maximum ionisation.

733. Reflection and Refraction in the Heaviside Layer.—The dielectric constant of the atmosphere is altered by its ionisation, the effect being to make K less than unity. Thus wireless waves travel faster in the layer than they do in the free æther. A distinction must, however, be made between the rate at which a signal (consisting of wave trains) travels, and the speed of the individual waves composing it. This may best be realised by considering the effect of dropping a stone into a pond. A disturbance in the form of a wave motion travels outwards over the surface of the water from the point where the stone entered. If one individual wave is watched, it will be seen to travel through the disturbance, and appear to die out when it reaches the outer edge. There are still, however, as many waves in the disturbance as there were before, new waves appearing at the inner edge as the ones at the outer edge

disappear. In other words, the individual waves are travelling faster than the wave train as a whole. The speed of the individual waves is called the **phase** velocity, and that of the train is called the **group** velocity.

In the free æther (and sensibly in the lower atmosphere) these two velocities are the same, but in the Heaviside Layer the **phase** velocity is greater than in the lower atmosphere, and this is what is meant by the statement that wireless waves travel faster in the Heaviside Layer. The velocity of the signal, the group velocity, is no greater and may be less in the layer than in the lower atmosphere.

It has been explained above that when electromagnetic waves pass from one medium into another in which they travel faster, as in the case of light waves travelling from water to air, they are refracted so that their direction of travel is more inclined to the normal to the common surface. When this common surface is approximately horizontal, like that of the Heaviside Layer, the result is that the wave travels in a more horizontal direction in the layer than in the atmosphere below it. The ionisation in the layer is not homogeneous, but increasing with height, and so the phase velocity of the waves also increases in a gradual manner as the wave ascends. Instead of an abrupt change of direction, therefore, the effect of the ionised region is to cause a gradual bending of the direction of the wave. This may be sufficient to bend the wave direction round until it is travelling parallel to the earth's surface, and eventually to direct it downwards so that it again returns to the earth at a distance from the transmitter. The final bending downwards may arise by gradual bending due to refraction, or by a process resembling more closely a reflection when the incident wave reaches the critical angle.

The possibility of the wave returning to earth depends mainly on two factors :—

- (1) The frequency of the wave.
- (2) The angle at which it originally enters the layer.

For a given angle of incidence the frequency determines the actual amount of bending that takes place, since the change in phase velocity in different parts of the ionised region depends on the frequency.

The amount of bending necessary before the wave can be altered sufficiently in direction to travel downwards obviously increases the more nearly perpendicular the original wave is to the surface of the layer when it enters.

It is therefore evident that the height to which a wireless wave penetrates the layer, and the possibility of its return, depend both on the frequency of the wave and the vertical angle at which it is radiated from the transmitter. Further, waves of the same frequency, but differing in angle of incidence, if they return to earth at all, will be refracted differently and pursue different paths in the

layer, and therefore will return to earth at different distances from the transmitter. The radiation from an ordinary vertical aerial is not at one particular angle, but waves are radiated to some extent at nearly all angles to the horizontal. They thus arrive at different points of the Heaviside Layer with different angles of incidence, and the waves returning to earth are spread over a large zone of the earth's surface.

For L.F. waves, the bending takes place in a depth of the Heaviside Layer comparable with the wavelength, and so is more in the nature of reflection, as it is understood for light waves, than refraction. Even in the case of H.F. waves, when the depth of penetration may be many wavelengths, it is customary for simplicity to assume the process of refraction replaced by one of reflection, in which case the height of the equivalent reflecting surface is considerably greater than the actual height to which the waves penetrate. The actual and equivalent processes are illustrated in Fig. 435.

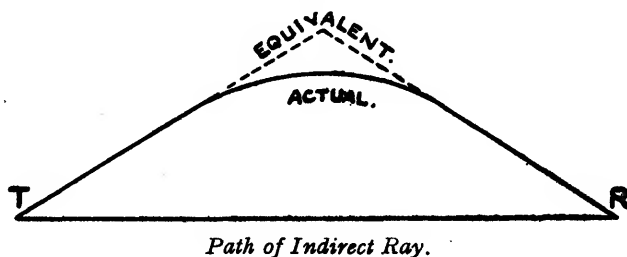


FIG. 435.

734. Skip Distance.—For any given frequency there is one direction which the ascending ray makes with the earth's surface at which it is most quickly bent round in the Heaviside Layer. The point at which this wave reaches the earth is therefore the nearest point to the transmitter at which signals can be received by means of waves returning from the upper atmosphere. The distance from the transmitter to this point (which is, of course, not a point, but a line round the transmitter on the earth's surface, and would be a circle if the radiation and the effect of the Heaviside Layer were the same on all bearings) is known as the **skip distance**. Within this distance the only signals that can be received are those due to the direct wave travelling along the earth's surface.

In the case of H.F. waves the range of the earth-bound ray may be considerably less than the skip distance, and so there appears a zone of silence in which no signals are received. This is shown diagrammatically in Fig. 436. The shaded parts represent the paths of the waves. The diagrams also show that the downcoming wave may be reflected at the earth's surface, travel up again to the Heaviside Layer, be bent round and return to earth still further away, giving rise to other zones of silence. This question of multiple

reflection will be further treated below. The regions where no signals are received are generally called **dead spaces**, being known in order as the first dead space, second dead space, and so on. The parts of the earth's surface where signals due to the refracted radiation can be received are called zones of reception. It will be seen as a matter of simple geometry that the width of the second dead space is less than that of the first dead space, and so on, and that the width of the zones of reception increases correspondingly.

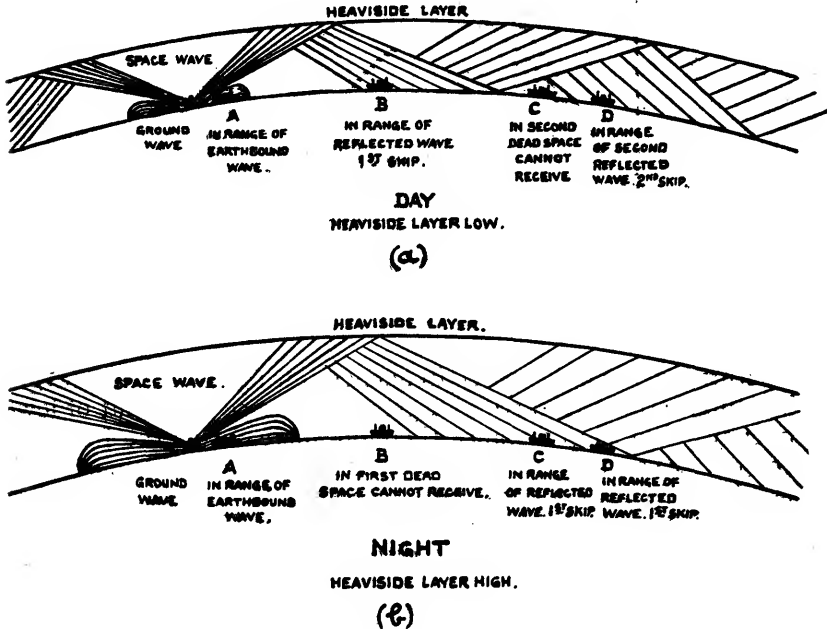


FIG. 436.

It may appear from the diagrams that the refracted ray corresponding to the ray of steepest incident angle that returns at all, returns to earth nearest the transmitter, and so determines the skip distance, but this is not necessarily the case. Actually, rays of different angle penetrate to different heights, and therefore meet different refracting conditions. Of two rays of the same frequency, at different angles of incidence, that at the lesser angle may return to earth nearer the transmitter than the more steeply incident one.

The two diagrams of Fig. 436 also indicate the difference between day and night conditions. The effective equivalent height of the layer is higher at night, and so, for rays of the same frequency and angle of incidence, the skip distance is greater. The skip distance at night is generally three or four times as great as the day skip distance.

The higher the frequency of the transmitted wave, the less is the degree of bending it experiences in the Heaviside Layer, and so the

smaller is the limiting angle of incidence at which rays will be bent round sufficiently to return to earth. This is illustrated in Fig. 437 for three waves of different frequencies radiated at the same angle to the earth. The wave of highest frequency is able **at this angle** to penetrate the layer completely, and does not return to earth.

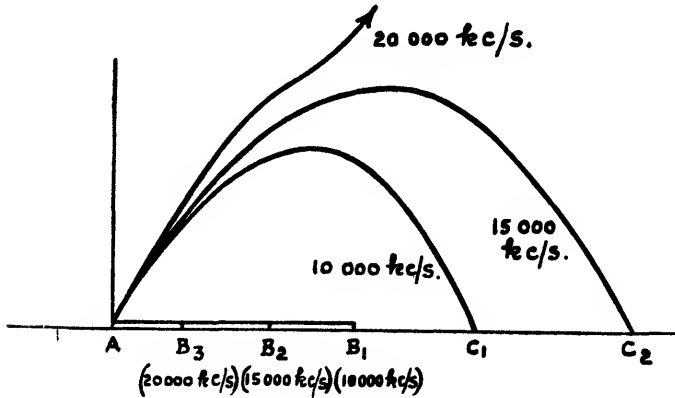


FIG. 437.

The returning wave of higher frequency, owing to its lesser degree of bending, returns to earth at a greater distance than that of lower frequency. The skip distance therefore increases as the frequency increases. Eventually a frequency is reached at which even the most obliquely incident ray fails to return to the earth's surface. This limiting ray probably comes out of the Heaviside Layer, as shown in Fig. 438, but in such a direction that it misses the earth's

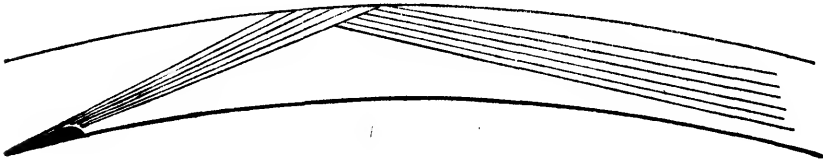


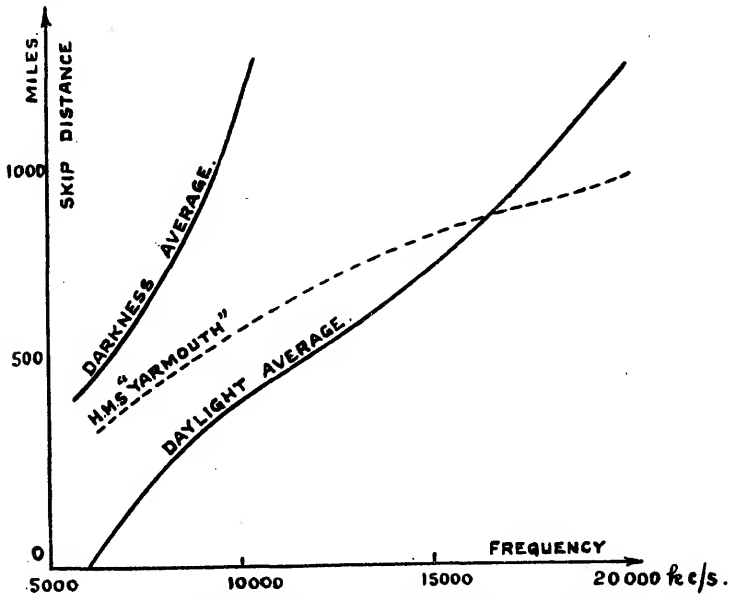
FIG. 438.

surface and returns into the layer, going on in this way until it is completely attenuated. The highest frequency at which **regular** long-distance transmission is possible is about 35,000 kc/s. (8.5 metres) in daylight, and the corresponding frequency at night is about 16,000 kc/s. (18.5 metres).

To illustrate the above remarks, the variation of skip distance with frequency at high frequencies is shown in Fig. 439, the two full lines corresponding to paths between transmitter and receiver lying altogether in daylight and darkness respectively.

The increase of skip distance with frequency and the much larger skip distances at night should be noted. The shapes and

relative positions of the curves are probably fairly accurate, but the actual figures should be treated with reserve, since the experimental evidence is rather conflicting. The values given are average values over a year. In summer the skip distances are less than in winter.



Variation of Skip Distance with Frequency.

FIG. 439.

Some figures obtained in H.M.S. *Yarmouth* on a cruise to Hong Kong and back in 1925 are given in the following table, and are also shown in Fig. 439 by the dotted line.

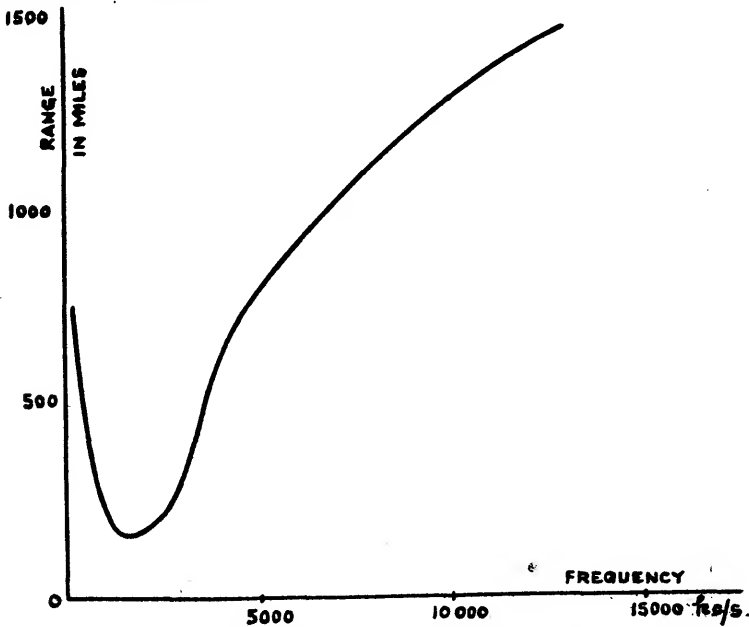
<i>f.</i> (kc/s.).	Skip distance (miles).	Range of earth- bound component (miles).	Width of first dead space (miles).	Width of first zone of reception (miles).	First overlapping zones.
25,000	1,100	30	1,070	800	2nd and 3rd.
12,000	720	80	640	440	2nd and 3rd.
8,500	520	170	350	240	3rd and 4th.
6,250	350	350	Nil	60	No over-lapping up to 6,000 miles.

It is not strictly correct to say that no signals can be received in dead spaces, but the signal intensity is very weak there compared with that in the zones of reception (less than one ten-thousandth). Such signals as are obtained are attributed to "scattering" of the indirect waves in the Heaviside Layer. Instead of being gradually bent round, part of the stream of energy is scattered in various directions, and so may descend to earth at any angle. A similar phenomenon in the case of light waves is responsible for the blue colour of the sky during the day. If the earth possessed no atmosphere, the only sunlight waves reaching it would be from the direction of the sun, and the sun would appear as a bright disc in a black sky (apart from the stars). But the atmosphere scatters the sunlight, and it appears to reach the earth from all directions, giving rise to the familiar blue colour of the sky during the day.

735. Attenuation of Indirect Rays.—Attenuation will necessarily occur in the indirect rays since the atmosphere is not a perfect dielectric. The great ranges attained with small power on H.F. waves shows, however, that their attenuation in the atmosphere is much less than that in the surface of the earth. For H.F. waves the losses in the lower atmosphere are negligible, though they are a considerable cause of attenuation in the case of very L.F. indirect waves. The important attenuation of indirect H.F. waves takes place while they are travelling through the Heaviside Layer. This attenuation is roughly proportional to the density of free electrons. Thus it is greater by day than by night, and so night signals are stronger than day signals. It also varies inversely as the square of the frequency. Hence, other things being equal, the shorter the wave the less the attenuation. This helps to fix a lower limit to the frequencies which can be successfully used for long-distance transmission. The limits appear to be about 8,000 kc/s. for regular day transmission and 4,000 kc/s. for regular night transmission. Waves of low frequency suffer so great losses in transmission that they are useless for long distances unless exceptionally high power is developed in the transmitting aerial. These losses occur partly in the Heaviside Layer, and partly at the earth's surface when multiple reflection takes place. The great attenuation of H.F. waves at the earth's surface has previously been mentioned when considering the short range of the direct ray. In addition, unless the surface is perfectly plane, the wave will not be regularly reflected, and the energy will be scattered in various directions, so that only a small part is available for any particular transmission under consideration. This in itself almost limits long-distance transmission to rays which have only undergone one or two reflections at most at the earth's surface. The energy of these lower frequency waves cannot, of course, penetrate the Heaviside Layer, and so must be dissipated in some such manner as that described above before the wave reaches great distances from the transmitter. In the case of transmission over

shorter distances at these frequencies, or of long-distance transmission at higher frequencies, the strength of a received signal at a particular point may thus depend very greatly on the conditions at a point half-way between it and the transmitter, *e.g.*, a jungle at a distance of 2,000 miles may render reception impossible at 4,000 miles on the same bearing.

736. Since in typical low-frequency transmission the attenuation increases with the frequency, and the reverse is the case for long-range high-frequency transmission, it would be expected that at some intermediate frequency minimum ranges would be obtained. This



Variation of Daylight Range with Frequency.

FIG. 440.

is actually the case, as shown in Fig. 440 by the curve of daylight range against frequency for constant energy radiation from the aerial. It will be seen that there is a decided falling off in range at frequencies between 1,000 and 2,000 kc/s., much larger, indeed, than would be expected from the considerations advanced above.

An attempt has been made to explain this large attenuation by showing that at such frequencies the combined result of the fields of the wave and the earth's magnetic field is to produce a resonance effect in the motion of the free electrons in the Heaviside Layer. This motion then attains a large amplitude, the actual vibratory paths of the electrons being helices like the thread on a screw, and is damped by collisions with molecules. The energy necessary to

maintain the motion is derived from the incident wave, which therefore suffers correspondingly large attenuation. This theory, however, seems to require that the effect should be greater by night than by day, whereas in practice the reverse is the case, the attenuation being much less marked at night.

737. Since the ionisation which constitutes the Heaviside Layer is mainly attributed to solar radiation, it is to be expected that considerable fluctuations in reception will occur when the signal passes from a transmitter in daylight to a receiver in darkness, and *vice versa*. In that part of the path where sunrise or sunset is occurring, the density of ionisation in the lower part of the layer is changing rapidly, and evidence of this is found in the large changes of signal intensity which occur at any particular receiving station under these conditions. The drop in signal intensity seems to be much greater when sunset occurs along the path than when it is night at the transmitter and day at the receiver.

738. Polarisation of Indirect Rays.—Changes in the nature of the polarisation of the indirect ray in its path through the Heaviside Layer are mainly attributed to the effect of the earth's magnetic field. Rotation of the plane of polarisation may take place, *i.e.*, the magnetic field vector ceases to be horizontal as it was in the incident wave, and circular polarisation may also be produced. At high frequencies these effects give rise to difficulties in direction-finding (Chapter XIX). Energy losses due to reflection at the earth's surface are greater when the electric field vector of the down-coming wave is parallel to the surface than when it is perpendicular.

739. Fading.—This is the name given to the occurrence of rapid fluctuations in signal intensity at a receiver. These may take place either at audio frequency or have a considerably longer period of one to four seconds. This latter kind is common on all frequencies, and is of smaller amount as the distance from the transmitter increases. Audio frequency fading is practically confined to frequencies above 400 kc/s. Between 500 and 1,000 kc/s. it is specially noticeable at night at distances of the order of 100 to 1,000 miles from the transmitter. It is violent at night for frequencies between 1,500 and 5,000 kc/s. at distances from the transmitter of 5 to 300 miles. At higher frequencies it occurs mainly at the edges of zones of reception.

Long period fading is presumably due to comparatively slow movements of the clouds of electrons forming the Heaviside Layer.

Audio frequency fading may be due to various causes. It is reasonable to suppose that the edges of zones of reception are not rigidly delimited but subject to small rapid fluctuation, corresponding to similar fluctuations in the effective height of the Heaviside Layer. In the case of overlapping zones, fading due to this cause would also occur. The field at a receiving aerial is the resultant of

that due to the down-coming wave and its reflection from the earth's surface, and small fluctuations in phase of either of these will alter the relative amounts by which they assist or oppose each other. Two or more down-coming waves which entered the layer at different angles may be refracted so that they eventually arrive at the same point, and produce a similar effect on the total signal intensity owing to their varying phases. When the frequency and distance from the transmitter are such that both the direct and indirect rays are received with comparable intensities, changes in the phase of the indirect ray at the receiver will likewise cause it to assist or oppose the direct ray and produce fading.

740. More recent determinations of the equivalent height of the Heaviside Layer seem to provide evidence that there is not only one region in which bending of wireless waves takes place, but two distinct regions. The lower of these, at a mean height of about 60 miles, corresponds to the Heaviside Layer, its equivalent height increasing during darkness and falling rapidly at sunrise to its lower daytime value. Observations made on 750 kc/s. over short distances between transmitter and receiver, and therefore corresponding to steeply incident waves, indicated the presence of this region. Similar observations made on 3,000 kc/s., however, showed that the waves on occasion penetrated this region completely, and were turned back in another region of rich ionisation at rather more than twice the height (140 miles). This occurred always at night, and generally during the day also, but on some days the bending of the indirect ray took place in the lower (Heaviside) layer. Occasionally reflection from both layers occurred simultaneously, or successive observations at short (ten-minute) intervals showed reflection first at one equivalent height and then at the other. The large difference between the two equivalent heights rules out the idea that the change in the conditions is due to variations in ionisation, and seems to establish the existence of two separate densely-ionised regions. The lower layer is presumably the one whose density depends mainly on the sun's ultra-violet radiation, since its height varies with daylight and darkness. The theory has been advanced that the ionisation in the upper layer, which is more stable and permanent, is due to the emission from the sun of α -particles such as are shot out of radio-active substances.

It should be pointed out, however, that it has been found possible to attribute other experimental results of the same nature to multiple reflection between the Heaviside Layer and earth *i.e.*, without recourse to an explanation involving more than one densely ionised region in the upper atmosphere.

741. Atmospherics or Strays.—Since the beginning of the world, æther waves of the order of length used in wireless telegraphy have been continuously traversing its surface, but have only been noticed since the inception of wireless telegraphy.

They are generally known as "atmospherics," "strays," or "X's"; disturbances known as "statics" are a special variety.

Atmospherics have a very variable but generally long wavelength.

Their damping is heavy and their shock effect is severe.

When they encounter an aerial they set it oscillating at its own natural frequency and are consequently very difficult to cut out; they may be reduced in strength by the use of some limiting or balancing device, and as they are non-musical they can be over-read by a skilful operator provided they are not too severe.

The following is a brief summary of the known facts:—

Classification.—For convenience, atmospherics are classified as follows:—

- (a) **Hissing.** An almost continuous noise, especially marked during snow or when heavy clouds are passing over the aerial. (Hissing is not always due to atmospherics. It may occur by heterodyning of the carrier wave of an R/T signal.)
- (b) **Clicks,** or very short noises.
- (c) **Grinders,** a type exactly described by the word given.

The distinction between the various classes, however, does not correspond with any precise idea of their origin and is not always exact.

Annual Periodicity.—In general, atmospherics appear to be more frequent and severe in summer than in winter.

Actual measurements in England indicate a maximum in June and a minimum in March.

In the East Indies, the maxima occur during the changes of the monsoon. A minimum occurs during the N.E. monsoon and a less marked minimum during the S.W. monsoon.

Daily Periodicity.—As a rule, atmospherics are worst just before sunset till about 1 a.m., after which they gradually decrease. This effect varies, however, in different parts of the world.

Direction.—Atmospherics which have a distant origin, as opposed to those generated locally, appear to come from the direction of the equator.

More locally, atmospheric disturbances appear to have their origin in mountainous regions.

Nature.—Atmospherics have either the effect of a shock, *i.e.*, they produce in aerials an E.M.F. of short duration, reaching its maximum in a very short time, or else of a band of frequencies which it is impossible to eliminate by selective tuning. The former is of a similar type to that which would be produced by the sudden discharge of a condenser.

In general, the severity of atmospheric interference increases with decreasing frequency of the wave to which the receiving aerial

is tuned, but observations made at Honolulu showed that stations working on a high frequency experienced numerous and violent disturbances, while neighbouring ones on lower frequencies were less troubled.

Experiments carried out to measure the shape and duration of the discharges have shown that they fall roughly under two headings—aperiodic and semi-periodic. The former are unidirectional and the latter have a second weaker impulse in the opposite sense to the first half-cycle.

Origin.—One class of atmospheric disturbance is due to lightning flashes of local origin.

As regards those coming from a distance, however, which occur even when the sky is perfectly clear, it has already been pointed out that the atmosphere is by no means at a uniform zero of potential, but that large patches of it accumulate excess positive and negative charges.

In addition to ionisation by the sun's rays, local action by wind or rain may also produce similar results.

For example, when a wind blows through cloud or rain, both are electrified, the water drops positively and the air negatively; when this occurs on a large scale, several thousand feet above the earth's surface, in conditions favourable to the formation of large clouds, we get "thunder weather."

Again, evaporation is always accompanied by electrification of the liquid and vapour, so that in the tropics there is always a disturbed electrical state.

When, again, a wind blowing over a sandy or dusty plain raises clouds of dust, there is intense electrification. The potential of the dust, as a system of particles, is raised by their separation. The necessary energy is supplied by the wind.

Both positive and negative ions are formed which collect on the clouds and earth respectively, leading, when the potential difference is high enough, to lightning discharges between them.

742.—Statics.—Statics are a special variety of atmospherics, and are due to electrical charges accumulated by the aerial in the same way as those collected by a lightning conductor. A charged cloud approaching an aerial produces by static induction a charge of opposite sign in the aerial.

For this reason it is always inadvisable to leave an aerial insulated from earth by a receiving condenser, as a charge is liable to accumulate and puncture the condenser (para. 516).

CHAPTER XVIII.

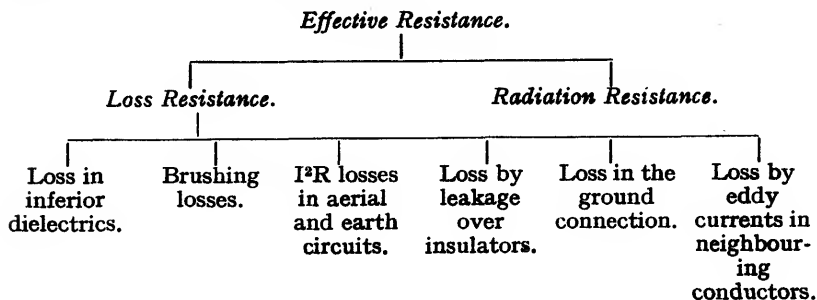
AERIALS AND EARTHS.

743. In this chapter the principles and design of the aerial wire system known as the "Aerial" or "Antenna" will be considered.

In general, aerials differ greatly according as they are intended for transmission or reception.

TRANSMITTING AERIALS.

744. Losses.—The following table indicates graphically the various losses in which is expended the energy supplied to an aerial by the wireless power plant :—

**Table of Losses in Aerial Circuit.**

These various losses, and the methods of minimising them, may now be discussed in detail.

745. Radiation Resistance.—The radiation resistance is that component of the effective resistance which, when multiplied by the square of the aerial current, measures the power radiated.

Let us assume that the aerial circuit is similar to the oscillatory circuit of the last chapter, with the capacity concentrated at both ends and the connecting wire containing only inductance. An actual aerial, whose roof is large, and whose bottom plate is the earth, approximates very well to this theoretical assumption.

In this case **the amplitude of oscillatory current is the same at all points of the connecting wire.**

It can be shown that, under these conditions, the radiation resistance (R_r) is given by the formula $R_r = \frac{160\pi^2 h^2}{\lambda^2}$ ohms, where h is the distance between the concentrated capacities at the ends of the aerial (the height of the aerial), and λ is the wave-length radiated, both being measured in the same units.

The power radiated is then $\frac{160\pi^2 h^2}{\lambda^2} I^2$ watts, I being the R.M.S. value of the oscillatory current. $160\pi^2$ is usually quoted in the above formulæ as 1580, so that $R_r = 1580 \frac{h^2}{\lambda^2}$ ohms.

It is more in accordance with modern practice to express this formula in terms of the frequency in kc/s.

f (in kc/s.) = $\frac{3 \times 10^8}{\lambda}$, where λ is in metres, and so $R_r = \frac{160\pi^2}{9 \times 10^{10}} h^2 f^2$
 $= 1.76 \times 10^{-6} h^2 f^2$ where h is measured in metres.

Height is therefore a factor of great importance in the design of an aerial.

It will also be noted that the radiation resistance varies directly as the square of the transmitted frequency, so that the higher the frequency, the more efficiently is energy radiated from an aerial.

The actual range obtained is dependent on other factors already described in Chapter XVII—absorption, refraction at the Heaviside Layer, etc.—so that the amount of energy radiated is no criterion as to the distance over which communication is possible. It has already been seen that the higher the frequency, the shorter is the range attained by the direct or ground ray, owing to its much greater attenuation.

746. Effective Height.—If the capacity is not concentrated at the upper end of the aerial, the current distribution is not uniform along the aerial. This is the condition occurring in practice, where there is capacity to earth not only from the "roof" of the aerial, but from the vertical connecting wire. Hence the current amplitude falls off the higher the point at which it is measured, and is greatest at a point just above earth. Curves of current distribution will be given later (*see para. 756*). The "average" R.M.S. value of current at different points of the aerial is less than that at the foot, where it is actually measured in practice, and the formula already quoted would give too high a value for the power radiated if this value of current were used.

To compensate for this, the term h is modified in the formula for radiation resistance. A quantity, called the "**Effective Height**," or "**Radiation Height**," is substituted for the actual height, so that the radiation resistance so calculated, multiplied by the square of the current flowing at the foot of the aerial, will give the power radiated. The effective height is obviously less than the actual height.

It may be defined as "the height of an ideal aerial system consisting of an elevated capacity connected to a perfectly conducting earth by a feeder having no capacity which for the same aerial current (measured at the foot of the aerial) and the same frequency, would produce the same electric field at any given distance as the aerial system in question."

The effective height is not easy to measure, and can, in fact, only be determined satisfactorily by quantitative measurements of the actual radiation taken on a receiving aerial in the neighbourhood.

It is decreased in ships by the proximity of stays, funnels and other metallic substances.

747. Aerial Form Factor.—The relationship between the effective height and the actual height is established by the **aerial form factor**.

Effective height = Aerial form factor \times actual height.

The value of the aerial form factor for two common types of aerial is given below.

Let L = length of horizontal part for Γ aerials.

= half total length for flat-top T aerials.

Let h = height of aerial.

Work out the ratio $\frac{L}{h}$.

Then the A.F.F. may be determined from the following table:—

$\frac{L}{h}$	A.F.F.	$\frac{L}{h}$	A.F.F.
0.0	0.639	1.5	0.940
0.1	0.696	2.0	0.958
0.2	0.741	3.0	0.979
0.3	0.777	4.0	0.987
0.4	0.806	5.0	0.993
0.5	0.830	6.0	0.996
0.6	0.850	7.0	0.998
0.7	0.867	8.0	0.999
0.8	0.881	9.0	0.999
0.9	0.893	10.0	1.0
1.0	0.904	—	—

This factor gives a fair approximation to the radiation or effective height for shore station aerials, and for merchant ships with wooden topmasts of equal height, small metal superstructures, and the feeder well away from the funnel.

In ships where these conditions do not obtain it is of no value, and the effective height can only be determined by measurement of the radiation. The power radiated = $1.76 \times 10^{-3} h^2 I^2$ watts, where h is the effective height in metres.

A practice has arisen of indicating the power of a station in "metre-amperes," i.e., the radiation height multiplied by the R.M.S. current at the foot of the aerial (hI).

This expression is termed the "**Radiation Constant**." It is clearly a more accurate method of description than that by which a station is referred to as being of so many kilowatts. The latter

expression leaves doubt as to whether the power input to the generator, to the wireless plant, or to the aerial, is meant. The radiation resistance necessitates an expenditure of energy by the transmitter, but it is a loss we wish to encourage, and it should be as large as possible in proportion to the other losses.

748. Loss in Inferior Dielectrics.—This loss is due to dielectric absorption and is more particularly evident in inferior dielectrics such as wood, concrete, masonry, trees, etc., which may happen to be near the aerial and hence are acted on by the electrostatic field around the aerial.

The air itself may be a bad dielectric when smoke or funnel gases are present, as they partially ionise the air and decrease its dielectric efficiency.

This loss may be taken as being inversely proportional to the frequency.

749. Brushing.—"Brushing"—as distinct from "sparking"—is a phenomenon with which every telegraphist is familiar; it is manifested by a silent bluish discharge from any high potential point in the aerial circuit in the direction of the nearest earthed object; or from the earthed point in the direction of the high potential point. It indicates that the air in the neighbourhood of the conductor is being ionised, and so its insulation is breaking down.

The potential gradient around the aerial wire depends on its diameter. The larger the diameter, the less is the danger of brushing.

Once an aerial has started to brush, it is useless to increase the aerial current; the transmitting range will not be increased, and at night the position of the ship will be disclosed.

It is found in practice that the limit is reached when the peak value of the oscillatory potential in the aerial reaches about 100,000 volts.

The greater the capacity of the aerial, the less inductance will be required for tuning to any particular wave, and consequently the less will be the oscillatory potential of the aerial above that of the

deck insulator $\left(V = I \sqrt{\frac{L}{C}} \right)$ (see para. 304).

An aerial brushes most vigorously from its ends, and from sharp points and angles. If the feeder is immediately abaft the funnels, the smoke and funnel gases will considerably increase its tendency to brush, as they partially ionise the air and decrease its dielectric efficiency. They will also tend to make the aerial wire brittle, and will have a bad effect on spreader discs. For this reason it is best, when possible, to feed the aerial before the funnels, especially in destroyers and other small craft with low aerials.

This loss may also be taken as inversely proportional to the frequency for voltages above the critical value.

750. Conductor Losses in Aerial and Earth Circuits.

The Aerial.—The method of construction of the aerial must be a compromise between the requirements for good conductivity and those for mechanical strength.

In order to keep down "skin effect," it is better to use stranded wire than one solid conductor. Bare iron wire should not be used, as, owing to its permeability, "skin effect" is very marked. Galvanised wire is better, as the current will flow over the galvanised skin; but it is not so good as copper wire. The best material for an aerial wire, is phosphor-bronze but it is expensive. Insulated wire should not be used, as it becomes covered with a semi-conducting layer of soot which, forming a skin to the wire, would carry a certain proportion of the aerial current, and therefore cause great resistance damping. In H.M. ships the wire used is copper, covered with "anti-sulphuric" enamel, to counteract the corrosive effect of smoke and funnel gases.

The importance of making all joints in the wire with the greatest possible care cannot be too much emphasised. Although a badly-made aerial may send nearly as efficiently as a well-made one, on account of the transmission energy being sufficient to "jump" any small break in the continuity of the conductor, yet, when it is used as a receiving aerial, the minute currents will be unable to flow through any high resistance junction, and great loss of efficiency will ensue. The best arrangement is to make aerial and feeders continuous throughout, the aerial wires being taken round a thimble and used to form the feeders. This arrangement is the strongest and the most efficient electrically.

The construction of an aerial is a troublesome task. Accordingly, whenever a new aerial has to be made, it should be made as carefully and strongly as possible, special attention being paid to the measurements of the wire.

The wire should never be soldered at any place which is going to be in a state of tension, for the temper is spoiled by the application of heat, and the wire thereby rendered brittle.

All sharp points, roughnesses, burrs, and sharp bends or kinks should be smoothed off or otherwise avoided, because they assist in the leakage of energy in the form of brushing.

If these precautions are taken, the aerial, when once up, will remain up for a very long time without giving any trouble.

The Earth.—The method of constructing the earth in ship and shore stations so as to afford good conductivity is described later.

Earth losses depend on the high-frequency resistance of conductors, and the latter increases with the square root of the frequency (due to skin effect—see Chapter VI, para. 380).

The losses can therefore be assumed to vary directly as the square root of the frequency.

751. Loss by Leakage over Insulators.—Defective insulation will account for a considerable loss in efficiency, especially on long waves, not only when transmitting, but also when receiving.

Leakage to earth in this way is equivalent to having a resistance in parallel across the aerial capacity, and this can be replaced by an equivalent series resistance, whose value is given by $\frac{1}{\omega^2 C^2 R}$ (see para. 310), where R is the resistance of the leakage path. Hence losses due to leakage are inversely proportional to the square of the frequency. As an approximation, they are generally taken as varying inversely as the frequency.

The points where losses are liable to occur are at the insulators in the trunk, at the deck insulator, and at outhaul or strain insulators.

It is, therefore, the duty of the wireless staff to keep all these scrupulously clean, especially after high speed steaming, heavy rain or bad weather, since under these conditions the insulators will probably be covered with a semi-conducting layer of "stokers," dirt, or dried salt.

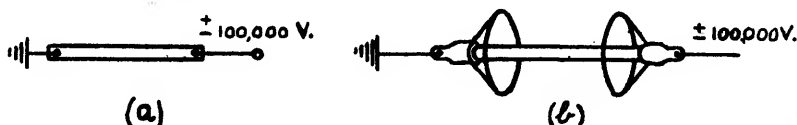


FIG. 441.

Strain insulators are fitted with an "anti-brushing" fitting, as in Fig. 441 (b). The reason for this is as follows:—

Suppose we have a plain porcelain rod, as in Fig. 441 (a), one end of which is connected to earth and the other end to an aerial wire, which may have an oscillatory potential of 100,000 volts, there will then be a great tendency for sparking to occur over the surface of the rod. If, however, "anti-brushing" rings are fitted, the electric strain is imposed on the air between the two rings, i.e., over a larger area of dielectric, and the tendency to brush is reduced.

The number of strain insulators used in staying out the aerial and feeders should be kept as low as possible, as each additional insulator provides another possible leakage path in parallel.

752. Loss due to Eddy Currents in Neighbouring Conductors.—Any metallic bodies which, being near the aerial or feeder, are in the direct inductive field, will have heavy currents induced in them.

The magnetic field due to these currents reacts on the aerial by ordinary "transformer" action, and increases the loss resistance.

For this reason the aerial and feeders should be kept as far away from metal as possible.

Similarly, the feeder should be at the maximum possible distance from, and never parallel to, stays, guys, etc.

In spite of these precautions, currents will be induced in all the stays supporting the masts of a ship or shore station.

These stays may be in one of three conditions:—

- (a) Well insulated from earth, and split up into a number of well-insulated sections.
- (b) Connected to earth by a low resistance path.
- (c) Connected to earth by a high resistance path.

For both transmission and reception, condition (a) is best, then condition (b); condition (c) should never be tolerated.

If, therefore, the rigging insulators become so coated with dirt, paint, or tar, that they cease to insulate satisfactorily and cannot be cleaned, they should be short-circuited.

Stays and shrouds should be carefully earthed at the point where they are connected to the hull.

When transmission is taking place in a ship, all the stays, etc., will be set in oscillation at a frequency depending on their length. Stays should, therefore, be divided up into sections of such a length that minimum interference is caused to reception of short waves.

At a shore station the best condition is that the masts themselves should be insulated from earth. This is not easy, however, for mechanical reasons. If the masts are of metal, they should be so placed that the feeder does not have to run near any of them.

This loss increases with the square root of the frequency, for the same reason as conductor losses (*see* para. 750).

753. Effective Resistance.—The effective resistance of an aerial has already been considered (*see* para. 744). It is that resistance which is equivalent in its effect to all the various losses which affect the aerial circuit, *i.e.*, it is that resistance, which, when multiplied by the square of the aerial current, equals the power put into the aerial system. $\text{Power input} = I^2 R_{\text{eff}}$.

With a given aerial and a given current, we have seen that:—

- (a) Radiation resistance varies directly as f^2 .
- (b) Resistance corresponding to conductor losses in the aerial and earth systems and to eddy current losses, increases as f increases, and may be taken as proportional to \sqrt{f} .
- (c) Resistance corresponding to dielectric absorption losses, leakage and brushing, decreases as f increases, and may be considered as inversely proportional to f .

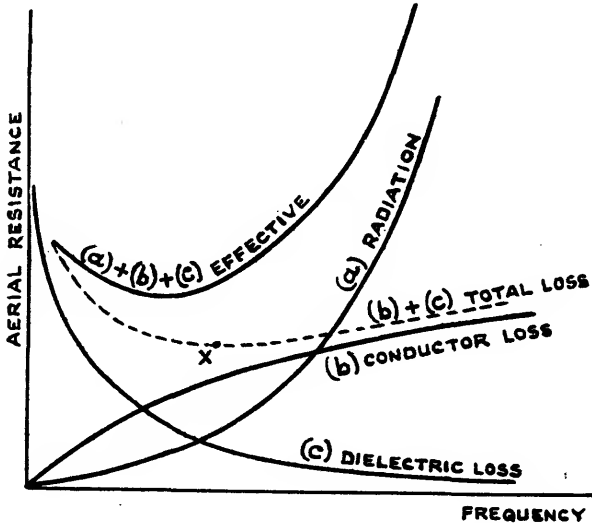
These facts are illustrated in Fig. 442.

Three curves—(a), (b) and (c)—are drawn, showing how the above losses vary with f .

The dotted curve, illustrating the loss resistance, is obtained by adding curves (b) and (c), and shows that at some such point as X the total loss resistance is a minimum.

The top curve, showing the total effective resistance, is obtained by adding curves (a), (b) and (c).

This shows that for any aerial there is always a best frequency at which a given current can be obtained for a minimum power expenditure. To obtain increased radiation, however, this frequency should be increased.



Variation of Aerial Resistance with Frequency.

FIG. 442.

The curve given above is derived from theoretical calculations. It will be found, however, that the "effective resistance" curve of any aerial will have the same general shape as the one given.

754. Aerial Efficiency.—The reduction of loss resistance of aerial systems is especially important in high-power long-range shore stations.

Low frequencies are necessary for transmission over long distances when silent zones must be avoided, but the useful radiation, which is proportional to the square of the aerial height multiplied by the square of the frequency, becomes insignificant compared with the other losses when the frequency is sufficiently low.

The reason for this is obvious when we reflect that there is a practical limit to the height of the aerial, say about 250 metres, so that for waves of frequency less than 30 kc/s. (over 10,000 metres), the radiation resistance cannot be greater than $1,580 \times \frac{250^2}{10,000^2}$ or 1 ohm, and with lower frequencies is reduced in direct proportion to the square of the frequency.

At 15 kc/s. (20,000 metres) the radiation resistance will be less than 0.25 ohm, and the total resistance of an aerial and earth of ordinary type will probably be of the order of 2 ohms, so that only about $\frac{0.25}{2} \times 100$ or $12\frac{1}{2}$ per cent. of the energy delivered to the aerial is radiated, *i.e.*, is employed in useful work.

The only ways of increasing this efficiency are :—

- (a) To increase the height, by erecting very costly masts, which are in fact unpractical above about 250 metres ; and
- (b) To reduce the losses in the aerial and earth circuits, so that they are small compared with the radiated energy (*see paras. 759 and 765*).

755. Aerial Capacity.—The general problem of aerial design is concerned mainly with two factors—efficient radiation and avoidance of brushing.

We have seen that the radiation resistance is proportional to the square of the effective height ; the latter is made large, firstly by making the actual height large, and, secondly, by attaching to the top of the aerial a horizontal roof which represents a large concentrated capacity, and which, therefore, makes the effective height more nearly equal to the actual height (*see paras. 746 and 747*).

Another reason for increasing the capacity by attaching a roof to the aerial is that the peak voltage above earth for a given change in the aerial or a given oscillatory current is thereby diminished (*see para. 749*).

The voltage at which brushing starts is the limiting factor to the power that can usefully be imparted to any aerial. Hence the higher the power the greater the aerial capacity required.

It is necessary to distinguish, however, between “**radiating capacity**” and “**non-radiating capacity**.”

Fig. 443 represents a “**7**” aerial, fed by a feeder going by a long trunk to an office several decks down. Let us assume that there is sufficient clearance in the trunk to prevent sparking over, but that there is considerable capacity between the feeder and the earthed side of the trunk.

This is denoted in Fig. 443 as C_2 . We may term C_2 the “**screened capacity**.”

If C_2 is at all large it will carry a good proportion of the aerial current, which will accordingly not be available to charge up the “**radiating capacity**” shown dotted at C_1 .

It will be remembered that if several condensers are joined in parallel, current divides between them according to their relative sizes, most flowing through the biggest condenser ; thus, if C_2 is equal to C_1 , only half of the available current will do useful work in the aerial, while the other half will flow straight to earth through C_2 .

In order to give a big "radiating capacity," the overhead, or roof, system of wires should have the greatest possible area.

As regards the number of wires used, two wires hoisted up parallel to and at a considerable distance from one another will have a capacity nearly twice that of a single wire, but as the wires approach one another the joint capacity becomes less, so that when they are within (say) a foot of each other, very little extra capacity is gained by the use of the second wire. A few wires spaced far apart are better than many wires near together, so far as total capacity is concerned.

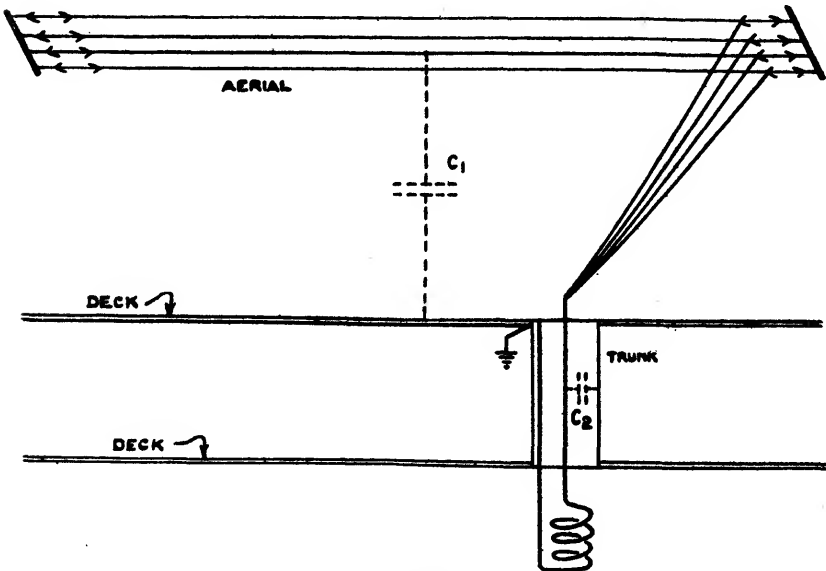


FIG. 443.

It is bad practice to bring a portion of the aerial low down with the idea of increasing the capacity, as this decreases the radiation height (see para. 748).

For an aerial similar in shape to those used with ship installations, the capacity is roughly proportional to the length and independent of the height, after a height equal to the length has been gained.

For low aerials, the capacity decreases as the height increases. Increasing the width of the upper part also increases the capacity, but by no means proportionately.

An aerial should not be made so big that its fundamental wave is longer than the shortest wave which the aerial is required to transmit, as this would necessitate the use of a series condenser for transmission on that wave, which decreases considerably the transmitting range.

The capacity of an aerial will not remain constant unless the aerial is hauled out uniformly at all times. Further, it is probable that it will vary when gun turrets near the aerial or feeder are moved, and according as the awnings are spread or furled, and the decks are wet or dry.

Another possible cause of alteration of aerial capacity is movement of the feeder by the rolling of the ship, or the action of the wind. It is, therefore, important to keep the feeders as rigid as possible, and to check the tuning at intervals.

756. Voltage and Current Distribution.—In an aerial system every small element of the wire has a certain amount of natural inductance, of capacity to earth and of resistance, which is partly loss and partly radiation resistance as described above. Such a system is said to contain distributed inductance, capacity and resistance.

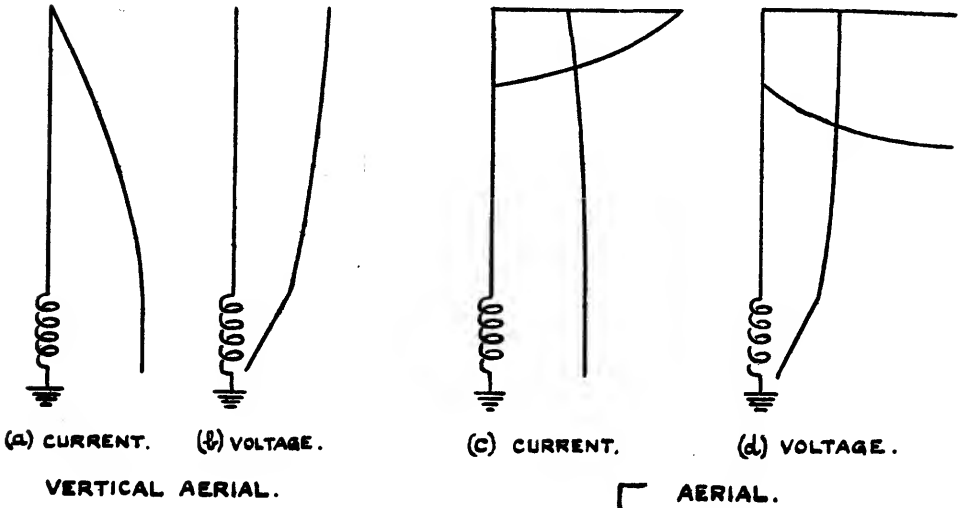


FIG. 444.

At low frequencies, when the length of the transmitted wave is very large in comparison with the length of the aerial, the nett reactance of the aerial is capacitive, and it is tuned by an inductance (Aerial Tuning Inductance). Under such conditions the operation of energising the aerial is most simply considered as the operation of charging the distributed capacity of the aerial to earth. As the oscillatory current travels up the aerial, each capacitive element is charged, and the value of the current falls off accordingly until, at the top of the aerial, the current has fallen to zero. The current at the topmost point of the aerial thus always remains zero, and such a point is called a "node" of current. The current at any other point is oscillatory at the frequency of the circuit, its amplitude,

and therefore its R.M.S. value, decreasing from the bottom to the top of the aerial. This is shown graphically in Fig. 444 (a). From the bottom of the aerial along the feeder to the aerial tuning coil, the R.M.S. current increases, since current flows to earth through the capacity of the feeder system. The capacity to earth of the aerial tuning coil may be considered negligible compared with its inductance, and so the R.M.S. current remains appreciably constant across it.

The charging of each aerial capacity element increases its potential with respect to the element below it, and so the R.M.S. value of potential to earth increases as the aerial is ascended, becoming a maximum at the top. The top of the aerial is therefore an "antinode" of potential. The distribution of R.M.S. potential along the feeder and across the tuning coil is shown in Fig. 444 (b). Owing to the concentrated inductance of the latter and its negligible capacity to earth, the potential rises sharply and uniformly across it. For this reason an aerial coil always requires a greater clearance from earth at its upper than at its lower end.

The corresponding curves of R.M.S. current and voltage for an aerial with a flat roof are shown in Fig. 444 (c) and (d). The distributed capacity of the aerial is mainly from roof to earth in this case, and the decrease of R.M.S. current in the vertical portion is correspondingly diminished.

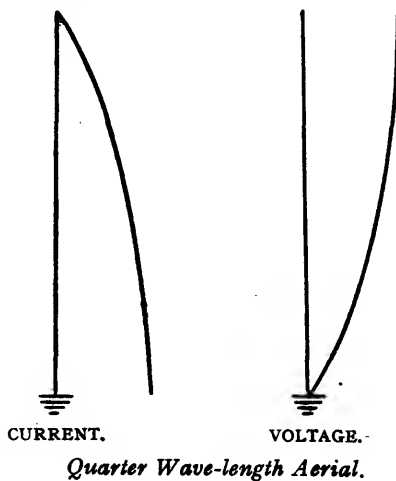


FIG. 445.

As the frequency is increased, the capacitive reactance of the aerial is decreased, and its inductive reactance increased until eventually a frequency is reached at which the aerial tunes by itself. This resonant frequency in the case of an aerial of perfectly distributed inductance and capacity is such that the corresponding wave-length is about four times (4.2 to 4.5 times) the

actual length of the aerial. The aerial at its resonant frequency behaves as a pure resistance and is known as a "**quarter wave-length**" aerial. The curves of R.M.S. current and voltage distribution are shown in Fig. 445. The R.M.S. voltage is a minimum at the bottom of the aerial, but is not actually zero, otherwise no power could be fed across this point into the aerial.

The inductive reactance increases with the frequency until the latter becomes twice the fundamental resonant frequency, *i.e.*, the aerial length is approximately half the length of the transmitted wave. Just beyond this frequency the reactance of the aerial is capacitive and very large. When the aerial length has become three-quarters of the transmitted wave-length, its capacitive reactance has decreased to zero, and it again behaves as a pure resistance. The distribution of R.M.S. current and voltage in half and three-quarter wave-length aerials is shown in Fig. 446. It will be seen that in every

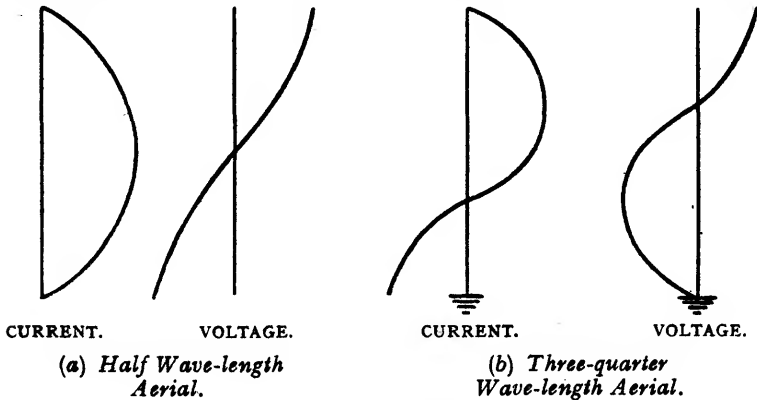


FIG. 446.

case the top of the aerial remains a node of current and an antinode of potential to earth. The other apparent nodes of current and voltage cannot be considered stationary in practice for otherwise no power could be transferred across them.

757. The variation of aerial reactance with frequency is shown in Fig. 447.

At points S and T the frequency is such that the aerial length is one-quarter and three-quarters of a wave-length respectively. The nett reactance is zero, and so the aerial current and voltage are in phase. The impedance, as measured by the ratio of voltage to current at the bottom of the aerial, is very small, since the current has its largest value and the voltage is very nearly zero. An aerial whose length is an odd number of quarter wave-lengths is therefore equivalent to a series resonant or acceptor circuit. The operation of tuning the aerial at lower frequencies by adding inductance in series may thus be considered as making the aerial circuit electrically equivalent to a quarter wave-length aerial.

Similarly, when the frequency is such that the aerial length is intermediate between a quarter and a half wave-length, and the aerial reactance is inductive, the aerial is tuned or made equivalent

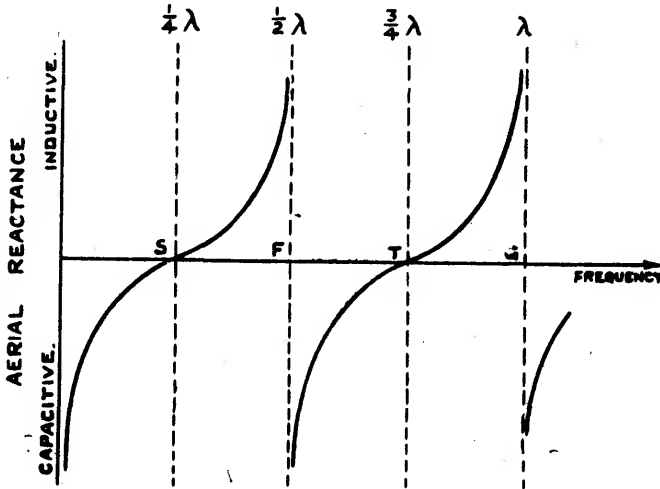


FIG. 447.

to a quarter wave-length aerial by adding capacity in series. The voltage and current distribution curves in this case for the condenser at the bottom of the aerial are shown in Fig. 448.

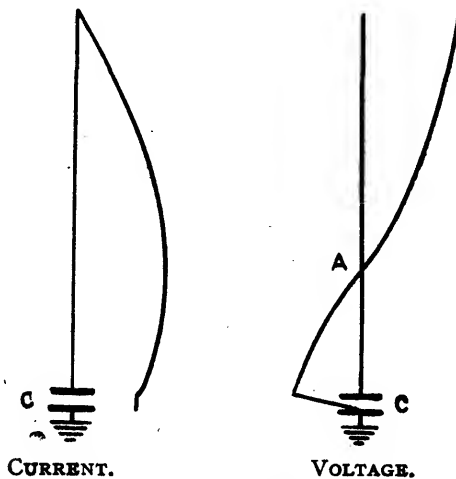


FIG. 448.

The cases of the half and full wave-length aerial correspond to F and G in Fig. 447. Here, again, the net reactance is zero, and the aerial current and voltage are in phase. The impedance, however,

as measured by the ratio of voltage to current at the bottom of the aerial, is very large, being infinite if the effective (loss and radiation) resistance of the aerial is zero. An aerial whose length is an even number of quarter wave-lengths may thus be considered as a parallel resonant or rejector circuit.

It is also to be noted that in a system of such distributed inductance and capacity that it corresponds to an even number of quarter wave-lengths at the frequency in use, the potentials of its two ends have the same numerical value, though they may be different in sign. Thus the bottom of a half wave-length aerial is a high potential point and cannot be directly earthed.

758. Low-frequency Shore Station Aerials.—In designing aerials for shore stations the two chief factors are :—

- (a) Permissible cost.
- (b) Whether directional, or all-round, radiation is required.

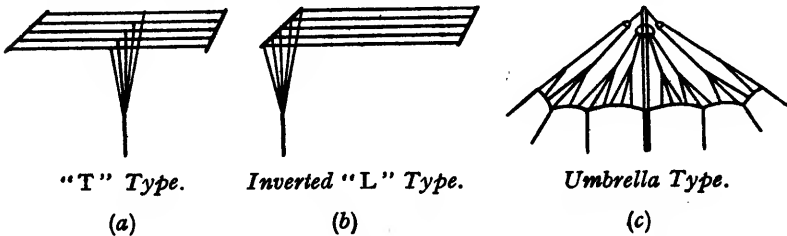


FIG. 449.

For low- and medium-powered stations the most usual types are the "T" (Fig. 449 (a)) and the "inverted L" (Fig. 449 (b)). The "T" type gives the best all-round radiation.

The "inverted L" type is markedly directional, especially if the roof portion is very long in proportion to the height of the aerial; it transmits signals most strongly in the direction opposite to that of its greatest length, and so the horizontal portion of the aerial should be turned backwards from the direction in which the strongest effects are required, either in sending or receiving.

A "T" aerial is generally supported on four or six masts, and consists of a rectangular roof of wires; the office is placed below the centre of the aerial.

The "umbrella" type of aerial, Fig. 449 (c), is used with portable outfits, military stations, and also at certain shore stations.

The main advantage of this type is that only one large mast is required, and that the aerial wires also serve as supporting stays. It is a poor radiator, however, as its radiation height is low for the actual height of its mast.

For high-power stations a very convenient design is as shown in plan in Fig. 450.

This gives good all-round radiation, and is readily increased at a later date by the addition of more masts, as indicated by the dotted lines, if the station is to be increased in power.

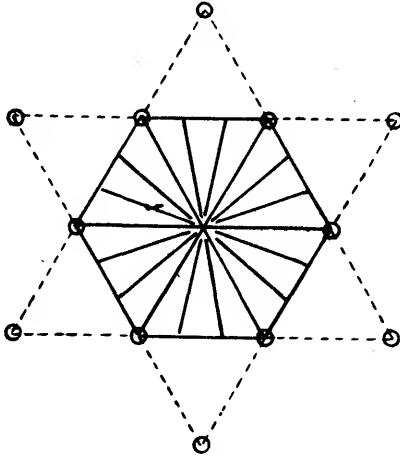


FIG. 450.

759. The Multiple-tuned Aerial.—This is a special type of aerial fitted at certain high-power stations.

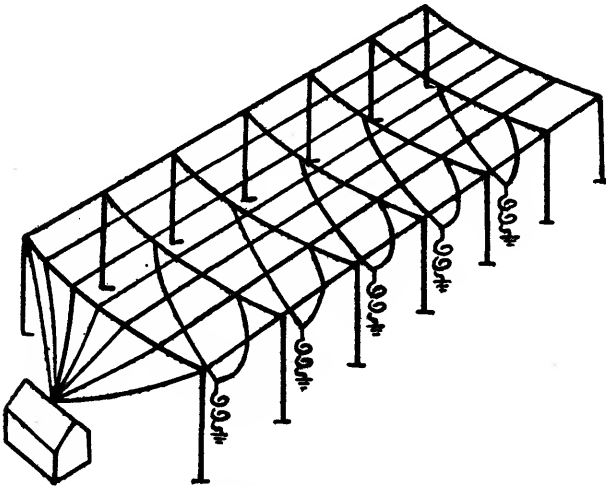
*Multiple-tuned Aerial.*

FIG. 451.

Its objects are to reduce loss resistance and to give directional transmission. It is of particular value when the position of the station renders it difficult to obtain a good earth, *e.g.*, when it is built on stony or sandy soil.

It constitutes in effect a number of separate aerials in parallel ; each has its own aerial tuning coil, and is thereby independently tuned to the desired wave.

The result is that the loss resistance of the whole system is greatly reduced.

For example, the effective resistances of each of the six sections of the aerial fitted at a certain high-power station, and of the whole system, are as follows :—

	One Section. Ohms.	Whole Aerial. Ohms.
Radiation resistance	0.07	0.07
Tuning coil and insulation	0.60	0.10
Earth resistance	2.00	0.33
	<hr/>	<hr/>
Effective resistance	2.67	0.50
	<hr/>	<hr/>

Thus the efficiency of one section = $\frac{I^2 R_r}{I^2 R_{eff}} = \frac{0.07}{2.67} = 2.8$ per cent., whereas the efficiency of the whole aerial = $\frac{0.07}{0.50} = 14$ per cent. With an aerial of equal capacity and mean height arranged in the usual manner, the figures for loss resistance would probably exceed those given above for one section, while the radiation resistance would remain the same, so that a great gain in efficiency is afforded by this method.

The disadvantage of the system is that it is only suitable for working at one particular frequency, since changing wave is obviously a slow process.

760. The Coil or Loop Aerial.—Loop aerials are used mainly for reception, where they have valuable directional properties, but they may be used also for transmitting. Their effective height is proportional to the number of turns and to the area, and the energy radiated is markedly directional, being a maximum in the directions in which the loop points, and zero at right angles to it.

761. Low-frequency Ship Aerials.—In ships we find ourselves very restricted in our choice of design, owing to limitations imposed by the position of funnels, stays, guys, &c., and the necessity of arranging for several aerials.

The principal object in fitting the main aerial should be to make its radiation height as great as possible.

Small aerials should be fitted so as to be as non-inductive as possible to the main aerial, and screened from it, if possible, by a mast.

It does not matter greatly where the feeder is led into the roof portion of the aerial. It is best to feed it centrally, but, if this is not practicable, the aerial may be fed from either end.

The aerial wires are generally arranged symmetrically in the form of a cylinder or cage, held apart by circular, octagonal, hexagonal, or square spreaders.

It is customary to describe an aerial by stating first how many parallel systems of cylinders there are, and then how many wires go to each cylinder, *e.g.*, a "triple tenfold," a "double fourfold," etc.

The main advantage of using a number of wires in parallel, which is the usual Naval practice, is the decrease in resistance gained thereby.

762. The Feeders.—Every wire used in the feeder must make a good contact with the aerial wire to which it is connected. The best practice is, when possible, to make aerial and feeder wires continuous throughout.

If two feeders are used they should be arranged symmetrically with respect to large earthed objects, such as masts and funnels; otherwise their capacities will be unequal and they will take unequal currents.

763. Earth Systems.—In the original experiments of Hertz the capacity of the oscillatory circuit was that between two plates insulated from earth, and connected by an inductance, and this principle was utilised in the Lodge-Muirhead system.

The use of the earth itself as the bottom plate of the aerial capacity in an open oscillator was introduced by Marconi, the aerial forming the upper plate.

Both these methods are now used in practice. If the earth itself is utilised, the system is known as the **Conductive Earth**; if an insulated capacity is used as the lower plate, the system is known as the **Earth Screen**, or **Balanced Aerial**, or **Counterpoise** arrangement.

764. The Conductive Earth in Shore Stations.—When the earth is used as one plate of the open oscillator, it is very important that the connection to it from the aerial should have as low a resistance as possible so as to keep resistance losses low. This low-resistance connection to the surface of the earth is generally made by means of buried metallic wires and plates. The area covered by this system should be at least equal to the area under a flat-roof aerial, and should extend to a distance equal to the height of the aerial in the case of a vertical aerial.

A large number of wires are run radially over the necessary area and are connected to buried copper or zinc plates at the ends distant from the aerial connection. These wires may be buried in the earth, or they may be carried on insulated supports outwards to the point where the buried system of earth plates is situated. It is bad practice to lay wires loosely along the ground from the station. The buried system of conductors should be put deep enough for the

earth round them to be permanently moist, and therefore a good conductor.

The reduction of earth losses is specially important with waves of low frequency, because of the necessity of keeping loss resistance low in comparison with the radiation resistance, whose value is limited by the possible height of the aerial.

The system of earth wires should be symmetrical round the station as regards length and disposition, for any lack of symmetry will result in those wires which are of least inductance carrying more than their share of current, an ohmic loss of energy being thus introduced.

765. The Multiple Earth.—A good example of the importance of this principle is furnished by the "multiple earth" fitted at the Ste. Assise Station in France.

The system consists of a network of copper wires spaced 10 metres apart, divided into sections, each of which carries approximately the same number of lines of force.

Current between these sections and the transmitting plant is conveyed by overhead wires, and in order to ensure that the impedances of these paths are equal, artificial inductances and condensers are inserted in them. No attempt at a perfect balance is made, however, as this would make the adjustment too critical for variations of frequency.

As a result, the total effective resistance of the aerial system is brought down to the very low figure of 0.54 ohms, as compared with 1.9 ohms, which was the resistance before the multiple earth was adopted. This figure is made up as follows:—

	Ohms.
Radiation resistance	0.19
Resistance of inductances	0.1
Resistance of aerial wires	0.05
Resistance of earthing system	0.2
	<hr/>
	0.54
	<hr/>

This corresponds to an aerial efficiency of $\frac{0.19}{0.54} \times 100 = 35$ per cent.

766. The Earth Screen.—The earth screen, as illustrated in Fig. 452, is simply the system previously described in which the wires are run outwards radially on insulated supports, without connection being made to earth plates at the outer end of the system.

The function of the screen is to intercept the lines of force from the aerial to earth, and to carry the return current on the screen wires rather than by the earth. Since no current flows in the earth

itself, the losses due to the poor conductivity of the earth are avoided. In order to get sufficient aerial capacity the surface area of the screen should be large. It should extend on all sides beyond the area covered by a plan view of the aerial system by a distance equal to the height of the aerial.

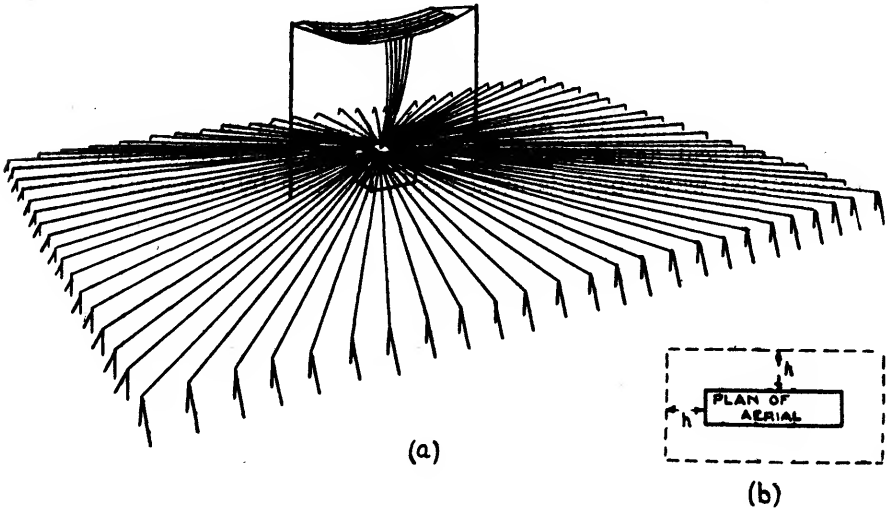


FIG. 452.

787. The Conductive Earth in Ships.—A ship's earth is usually very good compared with that of the average shore station.

In view, however, of the property of high frequency currents of flowing only on the surface of conductors, the precaution must be taken of providing a direct path of good conductivity for the earth currents.

It might be assumed that it would be good enough to tuck the end of the earth connection under a bolt in the side of the wireless office, and that the currents would then flow straight through the hull to earth.

They would not do anything so simple, however, but would take a path up inside the nearest bulkheads to the upper deck, across the deck, and so down the outside of the hull, *i.e.*, always "on the surface."

This path would be one of poor conductivity, because the skin effect in iron is much more marked than in copper, and riveted joints, which have poor conductivity even for direct current, would be encountered.

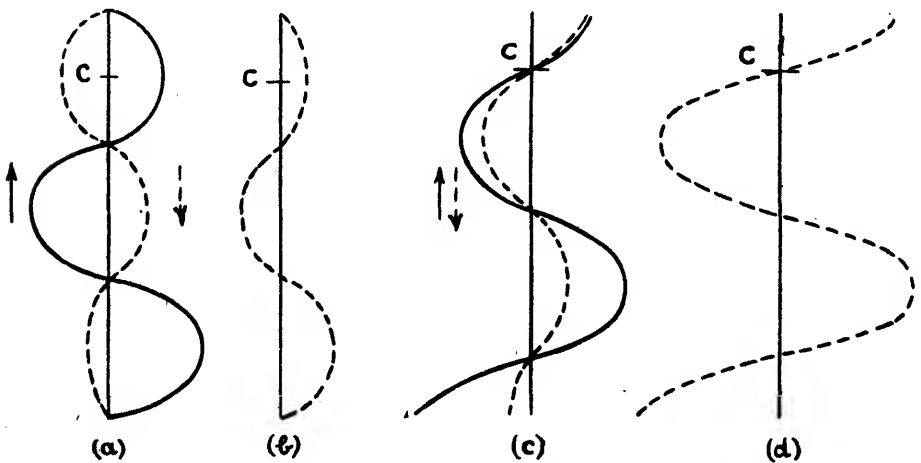
Hence the actual ends of the circuits (both sending and receiving) which it is desired to earth are connected by flat insulated copper strips to the iron ring which is secured round the deck insulator. From this ring, wires are taken up through the aerial trunk to the top deck insulator, where they are connected to another ring.

Alternatively, the trunk itself may be made of copper and used as the earth lead.

From the top insulator the earth currents flow along the iron deck underneath the teak planking before they reach the ship's side.

It must be remembered that the earth connection for receiving sets must be made quite as carefully as that for transmitting sets, as one cannot afford to lose any of the small amount of power available in received signals.

768. High-frequency Aerials.—The operation of energising the simple vertical aerial when its length becomes several times the transmitted wavelength is most easily grasped by considering that



Voltage Distribution.

FIG. 453.

a wave of energy starts from the coupling to the tuned oscillatory circuit, and travels along the feeder system and up the aerial, just as when a rope is held in the hand and the end is shaken up and down, a wave motion travels along the rope. If the other end of the rope is fixed, the wave is reflected from it, and travels back to the hand, and a system of "stationary" waves on the rope is set up if the agitation is continued. The initial and reflected waves reinforce each other at certain points, and nearly balance each other at other points. The points where practically no up and down motion of the rope takes place are called nodes, and the points of maximum displacement are called antinodes. Similarly, when the travelling wave reaches the top of the aerial, it is reflected and travels back down the aerial. Thus there are always two travelling waves in opposite directions in the aerial, and standing waves are produced. As the energy wave travels up the aerial it loses energy,

due to the aerial loss and radiation resistance, and so its amplitude falls off; this applies similarly to the reflected wave in its downward travel.

The voltage (electric) components of the two waves for some distance down the aerial are shown in Fig. 453 (a) for one instant, and in Fig. 453 (c) for an instant a quarter of a period later. At the top of the aerial the reflected voltage component is reversed in phase, *i.e.*, its phase changes by 180° (para. 727), compared with the phase of the upward travelling voltage component. In Fig. 453 (c) the upward wave has travelled another quarter of a wavelength upwards, and the reflected wave the same distance downwards. The instantaneous voltage at any point is the algebraic sum of the upward and downward wave voltages at that point, as shown in Fig. 453 (b) and (d), for the instants of Fig. 453 (a) and (c) respectively.

At the instant of Fig. 453 (b), the voltage at the top of the aerial is zero. A quarter of a period later (Fig. 453 (d)) it is a maximum. The same applies at all points on the aerial an even number of quarter wavelengths from the top. These points are therefore voltage antinodes, and would show a large R.M.S. voltage.

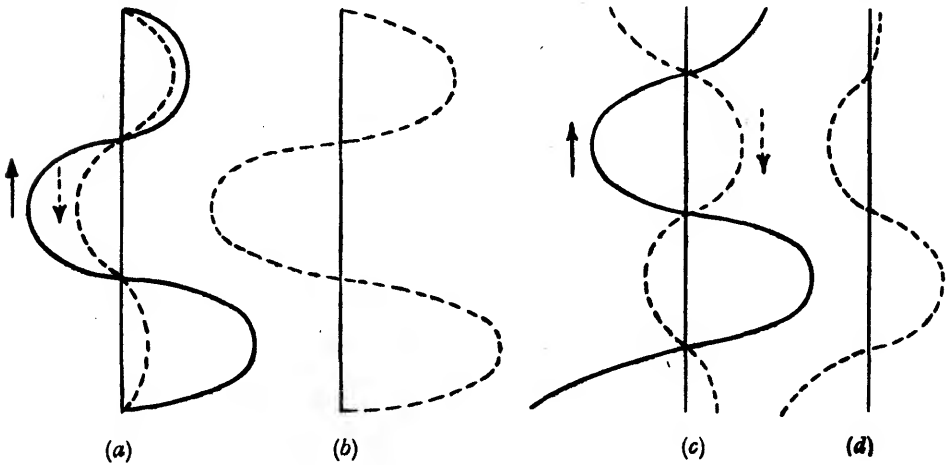
Now consider the point C, a quarter wavelength from the top. In Fig. 453 (a) the upwards and reflected waves are nearly equal and 180° out of phase, and so the net voltage is nearly zero. In Fig. 453 (c) it is zero. Thus the voltage at C is always small, and gives a minimum R.M.S. voltage. Similar conditions apply at other points an odd number of quarter wavelengths from the top, but the actual R.M.S. voltage at such points increases as the aerial is descended. They correspond to some extent to voltage nodes, but the R.M.S. voltage is never exactly zero, for no energy could then travel past them.

Similar conditions apply to the current (magnetic) component of the waves, which are shown in Fig. 454; except that when they are reflected at the top of the aerial, they merely reverse their direction without reversing their phase. Fig. 454 (a) and (c) correspond approximately to the same instants as Fig. 453 (a) and (c). Actually the upward current wave lags by a small angle on the upward voltage wave; owing to the different nature of their reflection, the reflected current wave therefore lags by nearly 180° on the reflected voltage wave. It will be seen that the top of the aerial is a node of current, and points an even number of quarter wavelengths down correspond also to current nodes (with the same limitation as in the case of the voltage wave). Points an odd number of quarter wavelengths down the aerial are current antinodes.

If the aerial is a thin wire, the speed of the travelling waves is very nearly the same as that of the electromagnetic waves radiated into the æther, and the antinodes on the aerial are practically a half wavelength apart. When a tube is used as aerial the velocity

of the travelling wave decreases, and so the distance between antinodes becomes less than a half wavelength.

When the aerial becomes very long compared with the wavelength, as may occur when a ship's main aerial is employed for high-frequency transmission, the decrease in amplitude of the reflected current and voltage waves becomes so large that their effect on the voltage and current at the bottom of the aerial may be neglected.



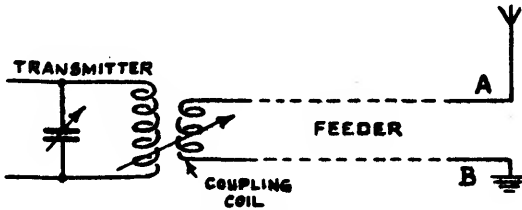
Current Distribution.

FIG. 454.

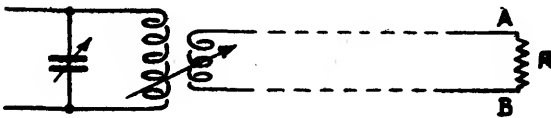
The current and voltage values then correspond to those in the upward travelling wave only, and are therefore nearly in phase. In other words, the aerial behaves as a resistance, no matter what the frequency is, provided it is high enough for the above condition to be fulfilled. Such an aerial is therefore independent of tuning, and can be earthed and used to transmit any frequency.

769. Feeder Systems.—Reference has already been made to one common type of feeder system, consisting of a solid or tubular conductor running along the axis of an earthed outer tube. Another system commonly used consists of two parallel wires mounted in a plane parallel to the ground, and running from the ends of the coupling coil to the aerial, as shown in Fig. 455 (a). The wire earthed at the bottom of the aerial is necessary to concentrate the capacity of the feeder system. It prevents radiation of energy from the other wire, which in its absence would behave like an open aerial, and diminish considerably the energy reaching the aerial proper. This arrangement is called a Lecher wire system. It is generally necessary to insert a variable condenser in each wire at the end nearest the coupling coil, in order to allow the energy distribution to be equalised.

Standing waves are set up on the feeder system corresponding to those on the aerial, owing to the outward and reflected travelling waves. Antinodes of potential then occur at various points on the feeder system, and lead to large dielectric losses. Similarly, at the antinodes of current, large ohmic losses occur. It is thus desirable to suppress the feeder standing waves, if efficient radiation is to be obtained from the aerial.



(a) PARALLEL WIRE FEEDER SYSTEM



(b) EQUIVALENT CIRCUIT WITH TUNED AERIAL.

FIG. 455.

The essential principle of such suppression may be seen from the following considerations. The maximum values of current and voltage due to the travelling wave each occur at points on the feeder system a half wavelength apart. Each half wavelength of the feeder can then be taken to correspond to a tuned circuit (para. 755), of a certain inductance l and capacity c , which would give the same ratio of voltage to current. The relation between peak R.M.S. current and voltage is therefore given by $V = \omega I$ where

$$\omega = 2\pi \times \text{frequency. Since } \omega^2 = \frac{1}{lc}, V = \frac{l}{\sqrt{lc}} I = I \sqrt{\frac{l}{c}}. \text{ If there}$$

is to be no reflected wave along the feeder, the energy carried to the aerial must all be radiated or otherwise used up in the aerial system by the time the reflected wave down the aerial reaches the feeder. If the aerial is tuned its effect is equivalent to a resistance of a certain amount R , across the ends, A and B, of the feeder system, as shown in Fig. 455 (b). To suppress the standing waves on the feeder, this resistance must therefore be of such a value as to absorb all the energy carried from the transmitter to the points A and B of the feeder system. Since I and V are in phase, the power is given by

$IV = I^2 \sqrt{\frac{l}{c}}$. The power absorbed by the resistance R is RI^2 , and so, for complete suppression, these amounts of power must be equal,

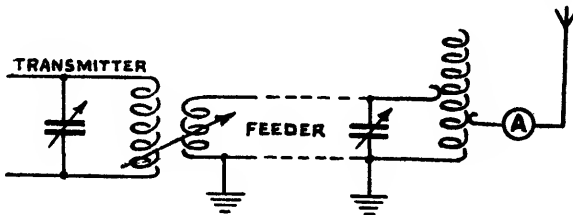
$$\text{i.e., } RI^2 = I^2 \sqrt{\frac{l}{c}}$$

$$\text{or } R = \sqrt{\frac{l}{c}}$$

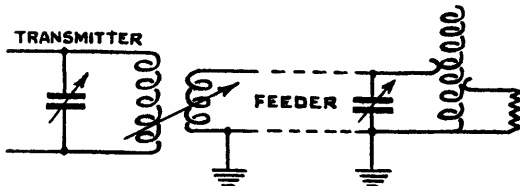
$\sqrt{\frac{l}{c}}$ is called the "Surge Impedance" of the feeder system, and so the condition for the suppression of the standing waves on the feeder system is that its surge impedance should be equal to the effective resistance of the aerial.

If the aerial length is an odd or even number of quarter wavelengths, and is only to be used to transmit the one frequency at which this occurs, its resistance will be constant at this frequency. If the feeder is then designed so that its surge impedance is equal to the aerial resistance, standing waves on the feeder will be suppressed. This is less difficult to arrange with a concentric tube feeder than with a Lecher wire system, but the former is rather expensive.

770. Suppression Unit.—Since the aerial resistance varies with the frequency transmitted, it is not possible to suppress feeder



(a) SUPPRESSION UNIT



(b) EQUIVALENT CIRCUIT

FIG. 456.

standing waves by feeder design alone if the feeder and aerial system is used to transmit more than one frequency, as is commonly

the case on board ship. Suppression may then be accomplished by means of the arrangement shown in Fig. 456 (a). The feeder is connected to a parallel circuit of variable inductance and capacity, and the aerial to a point on the inductance. The parallel circuit is known as a suppression unit. By suitably adjusting the variable condenser and the taps of the inductance, it is possible to make the equivalent resistance across the feeder equal to the surge impedance of the latter. The equivalent circuit is shown in Fig. 456 (b). The inductance with aerial tap may be looked upon as an auto-transformer, so that the equivalent resistance of the secondary (aerial) circuit varies with the position of the aerial tapping point. The variable condenser and its tapping to the inductance provide a means of varying the primary impedance and the transformation ratio of the auto transformer. The correct settings for largest current in the aerial ammeter at various frequencies are found by trial.

771. Distribution of Radiated Energy.—Before the possibility of long-distance communication by short waves was realised, attention was mainly confined to the amount of energy radiated from an aerial on different horizontal bearings. The radiation from a vertical aerial, as would be expected, is symmetrical in this respect, and takes place equally in all directions, but many aerial systems have been devised to give a maximum amount of radiation on one particular bearing and little or none on other bearings. Such "directional" transmission will be further mentioned in the next chapter.

The increasing use of the indirect ray (the radiation which travels up into the Heaviside Layer in its passage between transmitter and receiver) leads to the necessity of discovering how the radiation from an aerial varies at different vertical angles. For instance, in point to point long-distance transmission at high frequencies, suppose that we know the great circle bearing of one point from the other, and also the best frequency and angle at which radiation should enter the Heaviside Layer in order to give the required signal with minimum attenuation; a basis is then provided for designing the most efficient aerial system to satisfy these requirements.

772. Vertical Polar Diagrams.—The relative intensity of the radiation from an aerial at various angles to the horizontal is most readily appreciated from a graphical representation. Such a "vertical polar diagram" is shown in Fig. 457 for the radiation from a vertical aerial. The length of a line drawn from O at any angle to the horizontal to meet the curve is a measure of the amount of radiation (actually of the electric field strength) at that angle.

The shape of the curve depends on the conductivity at the earth's surface. The full-line curve assumes a perfectly conducting earth. The effect at high frequencies of the earth's imperfect

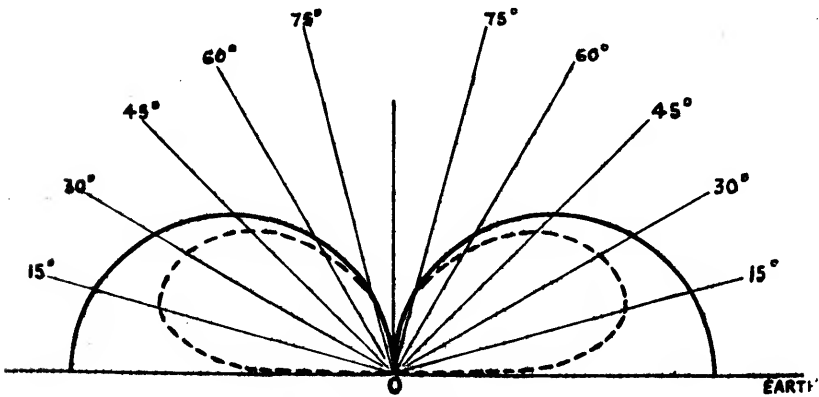
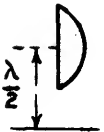
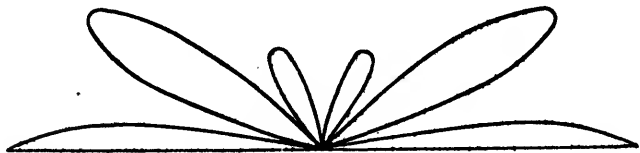
*Vertical Aerial.*

FIG. 457.

(a) CENTRE $\frac{1}{4}$ WAVELENGTH ABOVE EARTH.(b) CENTRE $\frac{1}{2}$ WAVELENGTH ABOVE EARTH

(c) CENTRE ONE WAVELENGTH ABOVE EARTH.

Vertical Half Wavelength Aerial.

FIG. 458.

conductivity is shown by the dotted line polar diagram. This explains, incidentally, the well-known effect that short wave reception improves considerably as the receiver is taken up a hill with a downward slope towards the transmitter.

The vertical polar diagrams for a vertical half wavelength aerial, whose centre is at various heights above the earth's surface, are shown in Fig. 458.

It will be seen that the effect of raising the aerial is to concentrate the upward radiated energy into "beams" at definite angles, and that these beams increase in number and become narrower as the height of the aerial above the ground is increased.

773. Phasing Coils.—The resultant radiation from an aerial at any particular vertical angle and horizontal bearing is obtained by summing the effects of all the individual elements of the aerial system at that angle and bearing. The currents and voltages in these elements differ in amplitude and phase, and so, therefore, do

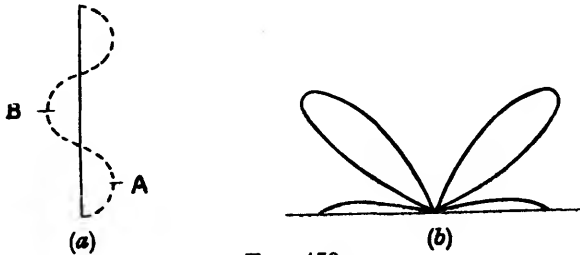


FIG. 459.

the electric fields they produce in various directions. If we consider, for example, the vertical aerial of length three half wavelengths shown in Fig. 459 (a), the currents at the points A and B are in antiphase, *i.e.*, when maximum current is flowing up the aerial past A, maximum current is flowing down the aerial past B. The radiation fields produced by the elements at A and B are similarly in antiphase, and at any point equidistant from A and B would be opposed to each other. At other points where the distances from A and B differed by half a wavelength, the fields would reinforce each other. Summing up the effects of all such aerial elements as A and B gives a resultant field at various angles as shown in Fig. 459 (b).

It follows, therefore, that if any elements of the aerial can be prevented from radiating energy, the resultant fields in various directions will be different from those produced when no attempt is made to alter the distribution of radiation energy. This is accomplished by means of **phasing coils**, which are arranged to be of the same distributed capacity and inductance as, say, a half wavelength of the aerial, but which have practically no capacity to earth and do not produce a radiation field.

Suppose that such a coil is connected between two aerial lengths, each corresponding to half a wavelength. Then, by comparison with Fig. 459 (a), it will be seen that the effect is to suppress the radiation from the central half wavelength, and to give radiation only from the lowest and highest half wavelength of the aerial. To compare the vertical distribution of electric field with that from the aerial of Fig. 459 (a), the vertical polar diagram for an aerial consisting of three half wavelengths radiating in phase is shown in Fig. 460 (b). Fig. 460 (a) shows the two phasing coils necessary to suppress the two intermediate negative half wavelengths. As before, the current standing wave on the aerial is shown.

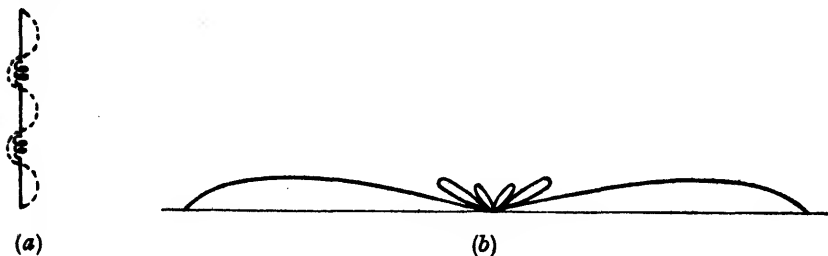


FIG. 460.

The main effect is to produce the chief concentration of energy in a beam at a lower angle than in the three half wavelength aerial in which no suppression is employed. The burden of experimental evidence seems to be in favour of such low angle radiation for long distance transmission at high frequencies.

A further development of phasing in aerials is to arrange the aerial wire itself according to such a pattern that the desired directions of maximum radiation are obtained without the use of phasing coils. This is well illustrated in the Marconi Uniform Aerial.

774. The radiation from any of the vertical aerials mentioned above is approximately symmetrical in any horizontal plane, *i.e.*, the vertical polar diagrams hold for any horizontal bearing. This ceases to be the case when horizontal aerials are used, the radiation then becoming directional in a horizontal as well as in a vertical plane. For directional horizontal radiation, aerial arrays are generally used (para. 817).

RECEIVING AERIALS.

775. Field far from a Transmitting Aerial.—Before considering the effect of an electromagnetic wave in inducing E.M.F.s in receiving aerials of different types, it is convenient to recall the nature of such a wave. The discussion will be confined to a

vertically polarised wave, such as is most nearly represented in practice by a direct (earth-bound) wave passing over a good conducting surface like the sea.

It was pointed out in para. 726 that an electromagnetic wave, representing a transfer of energy by a wave motion, can be looked on as consisting of an electric and a magnetic component, these fields being in time phase, but at right angles to each other in space.

The amplitude of either the electric or the magnetic component at any point depends on the effective height of the transmitting aerial, the current flowing in it, the wave frequency, and the distance of the point from the transmitter. The instantaneous value of either component at a point distant from the transmitter varies sinoidally at the frequency of the wave or the frequency of the oscillatory current in the transmitting aerial.

These conditions may be illustrated as follows:—

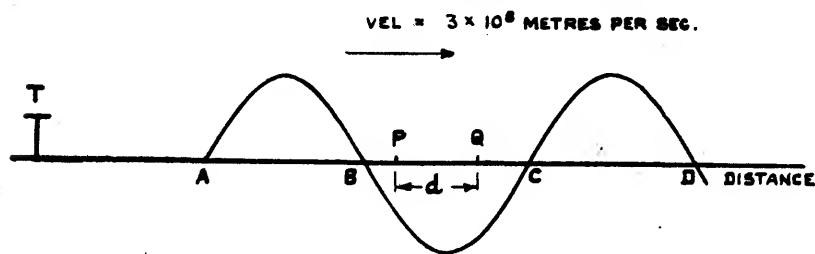


FIG. 461.

At any given instant of time the strength of either the electric or magnetic component in the region of a receiving aerial at a considerable distance from the transmitter is represented by the sine curve ABCD, where $AC = \lambda$; since the wave is itself moving outward with a speed of 3×10^8 metres per second, in one second f complete cycles of change of intensity occur at any given point P.

We may therefore represent the electric field at a given point in the form $\mathcal{E} \sin \omega t$, \mathcal{E} being the amplitude or maximum value, and the magnetic field at the same point in the form $\mathcal{H} \sin \omega t$, \mathcal{H} being likewise the maximum value ($\omega = 2\pi f$).

\mathcal{E} in electrostatic units is numerically equal to \mathcal{H} in electromagnetic units. In the same units, $\mathcal{E} = c\mathcal{H}$, where $c = 3 \times 10^{10}$, and is the numerical value of the velocity of electromagnetic waves in free æther, measured in centimetres per second (para. 717).

At any instant of time the fields at a point Q, distant d metres from P in the direction of the wave, differ from the fields at P by a phase angle $\frac{2\pi d}{\lambda}$ where λ is the wavelength in metres (para. 727).

Thus, if the fields at P are represented by $\mathcal{E} \sin \omega t$ and $\mathcal{H} \sin \omega t$, the fields at Q can be represented by $\mathcal{E} \sin \left(\omega t - \frac{2\pi d}{\lambda} \right)$ and

$\mathcal{H} \sin \left(\omega t - \frac{2\pi d}{\lambda} \right)$, when Q is at a greater distance from the transmitter than P. This can easily be seen by considering that at $t = 0$, the fields at P are both passing through their zero values. The fields at Q then pass through their zero values at a time $\frac{d}{3 \times 10^8 \text{ metres/sec.}}$ seconds later. The formulæ show that the fields at Q are zero when

$$\omega t - \frac{2\pi d}{\lambda} = 0,$$

$$\text{i.e., } t = \frac{2\pi d}{\omega \lambda} = \frac{2\pi d}{2\pi f \lambda} = \frac{d}{f \lambda} = \frac{d}{3 \times 10^8 \text{ metres/sec.}}$$

since frequency \times wavelength = velocity.

If Q is the same distance d nearer the transmitter than P in the line of propagation of the wave, then the fields at Q have their zero values at a time $\frac{d}{3 \times 10^8 \text{ metres/sec.}}$ seconds earlier than those at P, and so may be represented by the expressions

$$\mathcal{H} \sin \left(\omega t + \frac{2\pi d}{\lambda} \right) \text{ and } \mathcal{H} \sin \left(\omega t + \frac{2\pi d}{\lambda} \right)$$

In the following paragraphs, the E.M.F's induced by a wave passing a vertical aerial and a loop aerial will be obtained.

776. Vertical Aerial.—It was shown in para. 103 that potential difference = field strength \times distance. Thus if a vertical conductor is placed in an electrostatic field whose direction is also vertical, a potential difference is produced between the ends of the conductor of value equal to the length of the conductor multiplied by the strength of the field.

The electrostatic field of an electromagnetic wave of the type considered above is vertical, and its strength is represented by the expression $\mathcal{H} \sin \omega t$. Hence it produces a potential difference between the ends of a vertical aerial of height h , whose amount is given by $\mathcal{H} \sin \omega t \times h = \mathcal{H} h \sin \omega t$.

Corresponding units must be chosen for \mathcal{H} and h . Thus, if \mathcal{H} , in practical units, is given in volts per centimetre, then h must be expressed in centimetres. The P.D. is then obtained in volts.

If \mathcal{H} is in electrostatic units of field strength and h in centimetres, the P.D. is obtained in E.S.U. of P.D.

Since the aerial is a conductor, this alternating P.D. between its ends causes an alternating current to flow in it. The frequency of the P.D. and the current is the same as that of the wave. It is also important to notice that the oscillatory voltage in the aerial is in phase with the electric field or magnetic field of the wave. The phase of the current depends, of course, on the impedance of the aerial circuit at that frequency.

It has been pointed out that the electric and magnetic fields in a wave are merely different ways of looking at the same thing, and it is therefore to be expected that the same result as above would be obtained by considering the effect of the magnetic field. This will now be shown.

The magnetic field is of value $\mathcal{H} \sin \omega t$, its direction is horizontal, and it is travelling at right angles to the direction of a vertical aerial. Apart from the fact that the magnetic field is alternating, the case is therefore exactly similar to that of para. 146, and an E.M.F. is induced in the aerial by the lines of flux cutting it. In this case the aerial is stationary and the flux in motion, but this does not affect the argument, since the important thing is **relative** motion of the conductor and the flux lines.

The formula derived in para. 146 for the induced E.M.F. was Blv . In this case $B = \mathcal{H} \sin \omega t$, $l = h$ centimetres, and $v = c$, the velocity of the wave in centimetres/sec. Therefore induced E.M.F. $= c\mathcal{H}h \sin \omega t$, in electromagnetic units when \mathcal{H} is in electromagnetic units.

Expressed in electrostatic units, $c\mathcal{H} = \mathcal{E}$, and so the induced E.M.F. in E.S.U. becomes $\mathcal{E}h \sin \omega t$, which is the same result as was obtained by consideration of the electric field of the wave.

It is again emphasised that the wave does not give rise to two E.M.F.s of this value from the electric and magnetic fields respectively. One E.M.F. is produced, but, as seen above, it is immaterial from which aspect we choose to calculate its amount.

Thus the E.M.F. induced in a vertical aerial is:—

- (1) Proportional to the amplitude of the wave at the aerial.
- (2) Proportional to the vertical height of the aerial
- (3) In phase with the wave fields.
- (4) Independent of the direction of the transmitter (except in so far as the radiation from the transmitter is itself directional).

777. Open Receiving Aerial.—The open aerial is simply a development of the vertical wire aerial, for which theoretical results are worked out above. It is provided generally with a "roof" of concentrated capacity, and so is of similar appearance to a T- or L-shaped transmitting aerial.

From the formulæ above, it is evident that the higher the aerial the greater are the voltages induced in it. It is unnecessary, however, to aim at great height in a receiving aerial, for the following reasons:—

- (a) Reception is generally rendered difficult, not from weakness in incoming signals, but by relatively strong interference. Selectivity is far more difficult to achieve than audibility.

- (b) The use of efficient amplification and reaction makes it possible to achieve audibility with minute induced voltages in the receiving aerial. The important points to watch are that the insulation is good throughout, that the ohmic resistance is a minimum, and that the lead is as direct as possible.

778. Loop Aerial.—An aerial in the form of a vertical rectangular loop of wire is shown in Fig. 462. The top and bottom of the loop are indicated by dotted lines merely to differentiate them from the sides. In deriving the E.M.F. induced in such a loop, two initial assumptions are made, which are obviously justified.

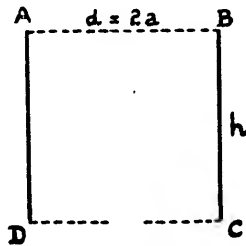


FIG. 462.

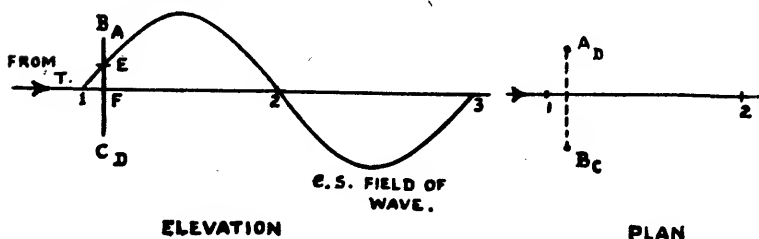
- (1) At distances from the transmitter such as are now being considered, the wave front may be considered plane over the area of the loop, just as a small portion of the earth's surface may be taken as a plane, although it is really part of the surface of a sphere.
- (2) The attenuation of the wave fields in the direction of propagation over a distance equal to that between the vertical sides of the loop may be completely neglected.

These assumptions amount to taking the electric field of the wave as vertical and of the same **maximum** value everywhere in the cylinder obtained by rotating the loop about a vertical axis. The instantaneous electric fields are, of course, different at different points of this cylinder owing to their varying phases, but are the same at all points which are the same distance from the transmitter.

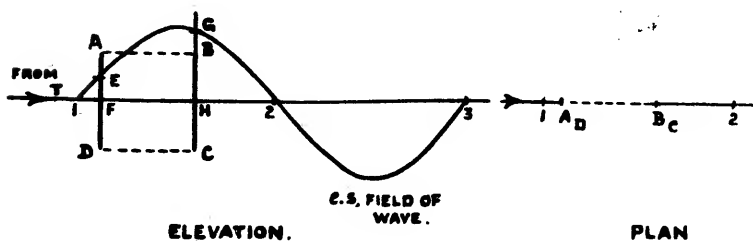
The resultant E.M.F. in the loop will be first obtained by considering the P.D.s produced by the electric fields in the four sides of the loop, each treated individually as a vertical or horizontal aerial. It is at once evident that the horizontal sides can be neglected for a wave whose electric field is vertical, since the P.D.s in themselves are negligible, and in any case are produced at right angles to the direction of these sides and so do not contribute to the E.M.F. **round the loop.**

The E.M.F.s produced in the vertical sides may first be considered in two special cases:—

- (a) Suppose that the plane of the loop is at right angles to the direction of the wave. The vertical sides are then at the same distance from the transmitter and are of the same length. The E.M.F.s induced in them are therefore equal at every instant. This is indicated in Fig. 463 (a). $A_D B_C$ is a plan of the loop and the E.M.F.s induced in the sides AD and BC are equal to EF (the instantaneous field), multiplied by their lengths, *i.e.*, they are equal. Since the E.M.F.s are in the same direction in A_D and B_C they are in **opposite** directions round the loop. The total loop E.M.F. is therefore zero in this case.



(a)



(b)

FIG. 463.

- (b) Suppose that the direction of the wave lies in the plane of the loop. The distances of the two vertical sides from the transmitter then differ by the width of the loop. The electric fields of the wave are therefore different in phase at the two sides, and produce E.M.F.s differing in value, so giving rise to a resultant E.M.F. round the loop equal to the difference between the E.M.F.s in its vertical sides. This is shown diagrammatically in Fig. 463 (b). ABCD is the loop, and the E.M.F.s induced in AD and BC are proportional to EF and GH respectively.

In any position of the plane of the loop other than that of case (a), the vertical sides are at different distances from the transmitter, and so a resultant E.M.F. will be produced round the loop corresponding to the difference in phase of the field of the wave at the two sides. This resultant E.M.F. is obviously a maximum in case (b).

779. The value of the E.M.F. in any orientation of the loop will now be calculated. Suppose that the length of the vertical sides is h and the width of the loop is $2a$, and that the plane of the loop makes an angle θ with the direction of the transmitter, *i.e.*, of the incident wave, as shown in Fig. 464. AZ and BY are drawn

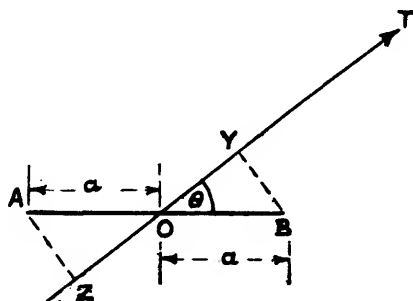


FIG. 464.

perpendicular to the direction of the wave, and therefore lie in the wave fronts passing through A and B respectively. In other words, the field at A is equal to the field at Z, and the field at B is equal to the field at Y. If O is half-way between A and B, *i.e.*, $AO = OB = a$, then $OZ = OY = a \cos \theta$.

Let the electric field of the wave at O be $\mathcal{E} \sin \omega t$. Then the field at Z, at a distance $a \cos \theta$ further from the transmitter is $\mathcal{E} \sin \left(\omega t - \frac{2\pi a \cos \theta}{\lambda} \right)$; and similarly the field at Y is $\mathcal{E} \sin \left(\omega t + \frac{2\pi a \cos \theta}{\lambda} \right)$.

The E.M.F.'s induced in the vertical sides AD and BC are therefore $\mathcal{E}h \sin \left(\omega t - \frac{2\pi a \cos \theta}{\lambda} \right)$ and $\mathcal{E}h \sin \left(\omega t + \frac{2\pi a \cos \theta}{\lambda} \right)$, and the resultant E.M.F. round the loop is the difference of these two expressions, *viz.* :—

$$\begin{aligned} & \mathcal{E}h \left[\sin \left(\omega t + \frac{2\pi a \cos \theta}{\lambda} \right) - \sin \left(\omega t - \frac{2\pi a \cos \theta}{\lambda} \right) \right] \\ &= 2\mathcal{E}h \sin \frac{2\pi a \cos \theta}{\lambda} \cos \omega t. \end{aligned}$$

This is the general expression for the E.M.F. round the loop. It is seen to be an alternating E.M.F., $(\cos \omega t)$, of the same frequency, $\left(\frac{\omega}{2\pi}\right)$, as the wave.

Since $\cos \omega t = \sin \left(\omega t + \frac{\pi}{2} \right)$, the loop E.M.F. differs in phase by 90° from the fields of the wave.

The amplitude of the loop E.M.F. is $2\mathcal{H} \sin \frac{2\pi a \cos \theta}{\lambda}$, or, if $d = 2a$ is written for the width of the loop,

$$\text{loop E.M.F.} = 2\mathcal{H} \sin \frac{\pi d \cos \theta}{\lambda}.$$

If the width of the loop d is small compared with the wavelength λ of the wave being received, $\frac{\pi d \cos \theta}{\lambda}$ is a small angle, and the angle itself (in radian measure) can be substituted for its sine. The amplitude of the loop E.M.F. then becomes

$$\frac{2\mathcal{H} \pi d \cos \theta}{\lambda} = \frac{2\pi \mathcal{H} A \cos \theta}{\lambda},$$

where $A = hd$ is the area of the loop.

It can be expressed in terms of the frequency $f \left(= \frac{\omega}{2\pi} \right)$ of the wave, by using the formula $f\lambda = c$ (velocity of wave).

$$\frac{2\pi \mathcal{H} A \cos \theta}{\lambda} = \frac{2\pi f \mathcal{H} A \cos \theta}{c} = \frac{\omega \mathcal{H} A \cos \theta}{c}$$

The alternating E.M.F. round the loop is therefore proportional to :—

- (1) The frequency of the incoming wave.
- (2) The electric field of the incoming wave.
- (3) The area of the loop.
- (4) The cosine of the angle made by the plane of the loop with the direction of the transmitter.

To complete this summary, it may be mentioned again that it differs in phase by 90° from the wave fields, the full expression for the instantaneous loop E.M.F. being $\frac{\omega \mathcal{H} A \cos \theta}{c} \cos \omega t$.

This value could also be deduced from the magnetic field, as in the case of the vertical aerial, by considering the flux cut by the two vertical sides. It is, however, perhaps more interesting to derive it from the magnetic field in terms of the flux-linkage with the loop.

The direction of the magnetic field is horizontal, and at right angles to the line TO (Fig. 465). Its value at O is $\mathcal{H} \sin \omega t$. This may be taken as its average instantaneous value everywhere inside the loop, if the same assumption, that d is small compared with λ , be made.

The projected area of the loop in the direction of the transmitter, *i.e.*, at right angles to the magnetic field, is

$$h \times d \cos \theta = A \cos \theta.$$

Hence the flux-linkage with the loop is $\mathcal{H}A \cos \theta \sin \omega t$.

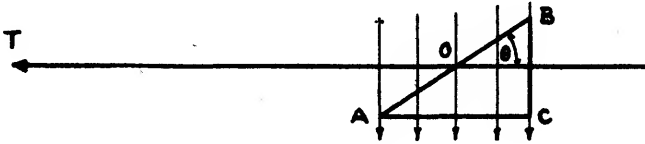


FIG. 465.

The induced E.M.F. is the rate of change of the flux-linkage, or $\frac{d}{dt} (\mathcal{H}A \cos \theta \sin \omega t) = \omega \mathcal{H}A \cos \theta \cos \omega t$, in E.M.U. if \mathcal{H} is in E.M.U. Expressed in E.S.U. [$\mathcal{H} = c\mathcal{H}$], it therefore becomes $\frac{\omega \mathcal{H}A \cos \theta \cos \omega t}{c}$ as before.

780. Polar Diagram of Reception.—The amplitude of the loop E.M.F. is $\frac{\omega \mathcal{H}A \cos \theta}{c}$, and its R.M.S. value is $\frac{\omega \mathcal{H}A \cos \theta}{c\sqrt{2}}$.

Writing E_θ for the R.M.S. value, and E_L for $\frac{\omega \mathcal{H}A}{c\sqrt{2}}$, we have that

$$E_\theta = E_L \cos \theta.$$

E_L is the R.M.S. loop E.M.F. when the plane of the loop is in the direction of the transmitter.

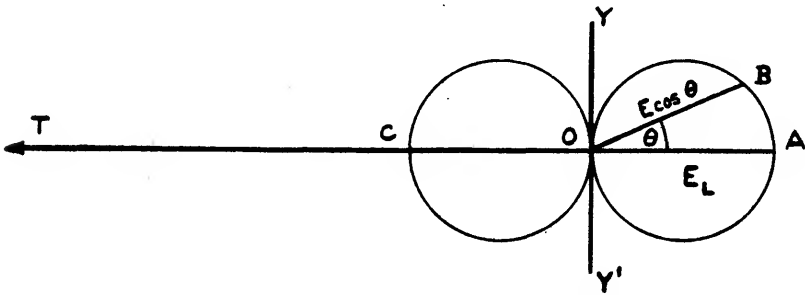


FIG. 466.

A curve may now be plotted showing the value of E_θ for all values of θ from 0° to 360° . Such a curve is called a polar diagram of reception; and in this particular case of the loop aerial it takes the form of two circles, as shown in Fig. 466 (usually called the "figure of eight" diagram).

The value of E_θ for any particular value of θ is obtained by drawing through O to the curve a line making a counter clockwise angle θ with OA, *e.g.*, OB is E_θ for the value of θ shown in Fig. 466; $OA = OC = E_L$.

It can be seen that the curves are circles by considering that they have to satisfy the relation $E_\theta = E_L \cos\theta$. In the particular case shown, this means that $OB = OA \cos\theta$. Hence the angle OBA must be a right angle, and B is therefore a point on a circle whose diameter is OA.

That there must be two circles, arranged as shown, is evident from the fact that in any position the loop may be turned through 180° without affecting the numerical value of the E.M.F. induced round it; but that the phase of the E.M.F. is altered by 180° , *i.e.*, the E.M.F. is in the opposite direction round the loop at any corresponding instant compared with the first position.

The polar curve can be taken as indicating the proportional strength of E.M.F. induced in a loop aerial which is rotated through 360° , the transmitting station being fixed; or, alternatively, the E.M.F. induced in a fixed loop by (say) a ship steaming round it in a circle, and transmitting continuously.

With the loop at right angles to TO, *i.e.*, along YOY', zero signals are received.

In the same way, the polar diagram of a vertical open aerial would be represented by a circle drawn with O as centre, indicating equal receptivity from all directions.

The variable E.M.F. induced in a loop aerial according to its orientation is of great assistance in direction-finding (*see* Chapter XIX), and, in combination with the E.M.F. of a vertical aerial, for sense finding. It is essential to bear in mind, however, that the vertical aerial E.M.F. is in phase with the electromagnetic wave, but that the loop aerial E.M.F. is 90° out of phase with the electromagnetic wave. The elimination of this phase difference, so that the effects may be capable of arithmetical addition or subtraction, is the main feature in the design of sense-finding receivers.

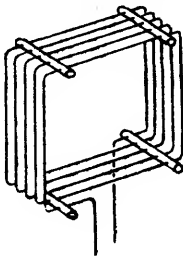
781. The Frame Aerial.—The frame aerial is a development of the loop aerial, consisting of a number of loops instead of one.

If we could imagine n loops of the same size and coplanar, the E.M.F. induced in the frame would be n times that induced in a single loop. This follows, for example, from the proof given above that the E.M.F. is proportional to the rate of change of flux linkage, the latter being n times as much for a loop of n turns as for a single turn.

In practice this theoretical assumption cannot be achieved, and if more than one loop is desired, so as to increase signal strength, the loops must be wound in either (a) box form or (b) pancake form, as illustrated below in Fig. 467.

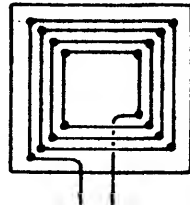
In the box form, the loops are of the same dimensions, but not coplanar, and are equivalent to a single loop of n times the area

in a plane parallel to themselves, plus a loop at right angles of area equal to half the area of the vertical side of the box frame. Hence zero signals will not be obtained when the frame is exactly at right angles to the line joining it to the transmitter.



Box Form.

(a)



Pancake Form.

(b)

Frame Coil Aerials.

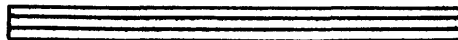
FIG. 467.

In the pancake form the total E.M.F. is the sum of the separate E.M.F.s in the loops, these being proportional to the dimensions in each case. It is equivalent to one loop whose area is the sum of the individual areas, and gives zero signals when its plane is at right angles to the transmitter.

782. Application.—Loop and frame coil aerials may be used in ordinary reception work for three purposes:—

- (a) To facilitate duplex working.
- (b) To cut out interference from other stations.
- (c) To decrease atmospheric interference.

(a) Duplex working means that a station operating a point-to-point service with another station is able to transmit and receive simultaneously, using different frequencies for transmission and reception.



TRANSMITTING AERIAL.



RECEIVING FRAME

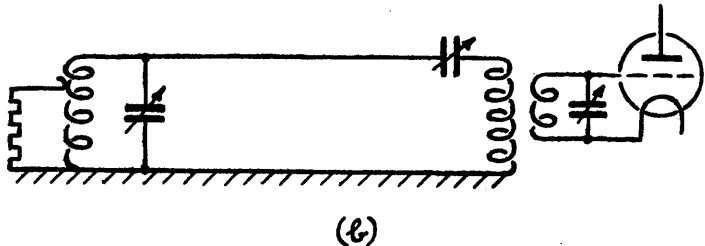
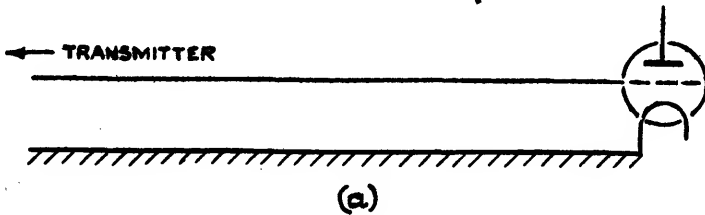
FIG. 468.

To effect this, reception is carried out on a loop aerial placed with its plane parallel to the plane of the transmitting aerial, as in Fig. 468.

It should then be unaffected by the latter, and if the two aerials are pointing at the distant station, as they should be, it will be in the best position for receiving the desired signals.

(b) If interference is experienced from one or more stations which do not lie in the same direction as that of the station from which it is required to receive signals, reception will be considerably improved by the use of a loop aerial with its plane in the correct direction.

783. Beverage Aerial.—This aerial is also directional and is designed for the reception of the indirect wave, which comes down at an angle to the horizontal and so has a horizontally polarised component of electric field. It consists of a wire supported horizontally a few feet above the ground and running in the direction



Beverage Aerial.

FIG. 469.

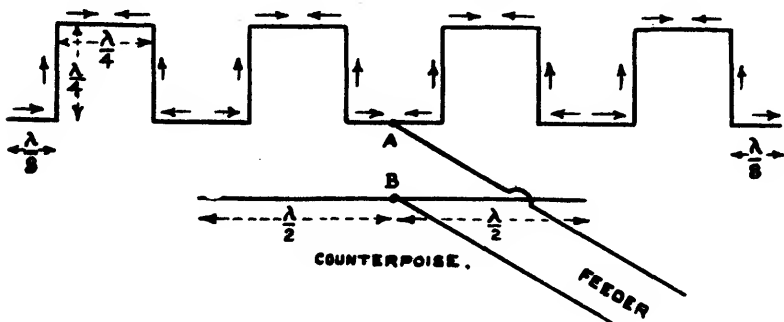
of the great circle bearing of the transmitter, as shown in Fig. 469 (a). The wire should be longer than the wavelength corresponding to the frequency being received. The production of an oscillatory E.M.F. in this aerial by the horizontal electric field of the wave should be compared with the excitation of a vertical aerial which is long compared with the wavelength (para. 768). The wire acts as a system of constant distributed inductance and capacity to earth at any particular frequency and the wave field passing along it builds up travelling waves of voltage and current. Thus, if the far end of the aerial from the transmitter is connected to the grid of a valve whose filament is earthed, an oscillatory P.D. is developed between grid and filament.

Since the distributed impedance alters abruptly at the valve terminals, the travelling wave is there reflected and travels backwards to the open end of the aerial. This is also a point of reflection.

Hence secondary travelling waves towards the receiver would be set up, and would interfere with the original travelling waves produced by the incoming wave field, giving large aerial losses and a diminished P.D. at the valve. To prevent this, the reflected wave must be suppressed at the open end of the aerial. The principle of such suppression has already been explained (para. 789). The open end of the aerial must be closed through a unit whose equivalent resistance is equal to the surge impedance of the aerial (the square root of the ratio of distributed inductance to capacity). No secondary reflection then takes place at this end of the aerial.

One arrangement is shown in Fig. 469 (b). The rejector circuit, with a variable tapping to the resistance, behaves as an auto-transformer of variable transformation ratio, thus enabling the effective resistance to be adjusted to the correct value. The receiver end is closed through a tuned acceptor circuit mutually coupled to a similar circuit between grid and filament. These behave like the primary and secondary of an ordinary tuner. Careful adjustment of the coupling is necessary for maximum signal strength.

784. Extended Aerial.—This is shown in Fig. 470 and is mounted in a plane at right angles to the direction of propagation of the wave to be received. If the electric field vector of the wave lies in a vertical plane, E.M.F.s are induced only in the vertical parts of the aerial. These E.M.F.s are all in the same phase and so assist each other. A horizontal counterpoise may be used with this aerial (as shown in the figure) or a second aerial exactly similar may be arranged below the first aerial as a mirror image. It may also be necessary to include a suppression unit in the feeder system.



Extended Aerial.

FIG. 470.

The arrows in Fig. 470 show the directions of the currents in various parts of the aerial at the same instant. The bending of the aerial into this shape has a somewhat similar effect to that of phasing coils in suppressing the out-of-phase E.M.F.s that would

be produced in a vertical aerial which is long compared with the wavelength.

785. The use of these special aerials and other aerial arrays for directional reception is only practicable for a wave of fixed frequency on a known bearing, as in shore stations for long-distance point-to-point transmission ; but in such cases much greater signal strength is obtainable than with aerials which are required for all-round reception at varying frequencies. The fixed frequency also allows of a more efficient design of the other components of the receiver.

It is further possible in shore stations to choose sites for the aerial clear of all metallic structures and other obstructions. In a ship the aerials are inevitably in close proximity to parts of the rigging and superstructure, and this is particularly harmful on H.F. waves, for many metallic parts of the ship are of such a size that they tune at high frequencies, and so produce considerable re-radiation, which differs in phase from the directly-received signal, and so has a detrimental effect on the signal strength. Directional arrays cannot, of course, be used on board ship, since neither the position nor the course of the ship remains fixed. Thus conditions for H.F. reception are much better on shore. Ship reception may be considerably improved by the use of a short aerial, whose position it is much simpler to adjust so that the reception of re-radiation is minimised. Even on shore the use of a long aerial will not necessarily improve signal strength, since E.M.F.s 180° out of phase with each other are induced in adjacent half wavelengths.

CHAPTER XIX.

DIRECTIONAL RECEPTION AND TRANSMISSION.

786. This chapter is mainly concerned with the principles of circuits which can be used to determine the direction of a transmitter by means of the signals received from it, a process known as direction finding (D/F).

There are obviously two general ways of accomplishing this :—

- (1) As a result of previous information, it may be known that at a certain time a transmitter is radiating energy on one definite bearing only. The reception of the signal by any means then gives the bearing of the transmitter. This is known as Directional Transmission.
- (2) Only the position of the transmitting station may be known, and its bearing must be determined by the use of special apparatus at the receiving station. This is known as Directional Reception, and is the normal method used by ships in fixing their position. Alternatively, the ship may act as transmitter, and its bearing be determined by a shore receiving station, which then communicates the result to the ship; more frequently the position of the ship is fixed by a number of cross bearings at shore receiving stations, and the result communicated. Similarly, a number of ships which know their own positions and are fitted for Directional Reception may determine the position of any transmitting ship.

DIRECTIONAL RECEPTION.

787. The basic principle of directional reception is the use of a loop aerial, or its electrical equivalent. It has been seen in para. **778** that the amplitude of the E.M.F. induced in a loop aerial depends on the direction of its plane relative to the direction of the transmitter, being zero when the plane of the loop is at right angles to the direction of the transmitter, and a maximum when the loop points to the transmitter. In any intermediate direction the amplitude of the E.M.F. round the loop is equal to its maximum amplitude multiplied by the cosine of the angle between the plane of the loop and the direction of the transmitter, provided the width of the loop is small compared with the wavelength. The polar diagram of reception, the figure of eight diagram, was shown in Fig. 466.

788. Rotating Coil Direction Finder.—To determine the direction of a transmitter, use may therefore be made of a vertical coil or loop aerial capable of rotation about a vertical axis. As the coil is rotated, maximum signal strength is obtained when the coil points to the transmitter, and no signal is heard when its plane is at right angles to the direction of the transmitter. The determination of either of these positions is sufficient to determine the direction of the transmitter. It will be observed, however, from the polar diagram, that the change in signal strength with angle of rotation is much more pronounced in the neighbourhood of zero signals than in that of maximum signal strength. Thus a more accurate determination of the bearing can be obtained from the zero signal position of the loop, and this position is always used in practice.

The direction of the transmitter, as thus determined, is ambiguous. For example, if the plane of the loop is East and West when zero signal is obtained, the transmitter may be either due North or due South. All that is known is that it lies on a North-South line through the receiver. It is convenient in this connection to make a distinction between the **direction** and **sense** of a bearing, the direction giving the line of the bearing, and the sense indicating which of the two possible bearings is the correct one, *e.g.*, North or South in the example above. The rotating coil thus determines direction, but not sense.

In practice it is usual, instead of a loop with a single turn, to employ a frame coil carrying several turns. For each frequency there is a best number of turns to give maximum efficiency. The lower the frequency the greater should be the number of turns; but a coil 3 ft. 6 in. in diameter, with six turns, can be used with satisfactory results for any wireless frequency below 500 kc/s. It can also be used for much higher frequencies, but is then appreciably less efficient than a coil with fewer turns. The coil may be rotated directly by mechanical training, or electrically from an office at a distance. All that is necessary to determine the direction of the wave is that some means should be available for training the coil through 360°, and that a suitable receiver should be connected to the terminals of the coil. The shaft which rotates the coil usually carries a pointer moving over a scale graduated from 0 to 360°, and the pointer is set so that the reading when zero signal is obtained is the true geographical bearing of the transmitter. If, for example, the wave is arriving from due North, the zero is obtained when the coil is pointing East and West, and the pointer is therefore set so that the reading is then 0°, *i.e.*, true North.

789. Antenna or Vertical Effect of a Loop.—The use of a loop for direction finding depends entirely on the fact that when the loop is at right angles to the direction of propagation no signal is heard. Anything which may cause signals to be heard when the

loop is in this position is detrimental to accurate direction finding. One such effect is due to the action of the two sides of the loop as vertical aerials and is described as "vertical" or "antenna" action.

It was seen in para. 778 that the E.M.F. round the loop was the difference between the E.M.F.s induced in the two vertical sides. Now consider the simple D/F receiver shown in Fig. 471.

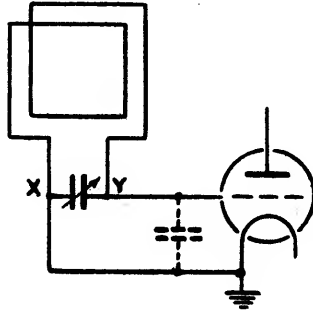


FIG. 471.

The loop (shown as consisting of two turns) is tuned by a condenser, and the voltage developed across the condenser is applied between the grid and filament of the first valve of a receiver. When the plane of the loop is at right angles to the direction of the transmitter, there is no resultant E.M.F. round the loop, and therefore no voltage across the condenser due to this cause. There are, however, large equal E.M.F.s in the vertical sides of the loop. These vertical sides each have a certain amount of capacity to earth, one side from Y via the capacity to earth of the grid, and the other side from X via the filament. The sides of the loop therefore behave as earthed vertical aerials, and, further, their paths to earth are unsymmetrical, and therefore of different impedance. It is obvious that the impedance from Y to earth via the grid will be different from the impedance from X to earth via the filament circuits. Thus, even when equal E.M.F.s are induced in the two vertical sides (*i.e.*, in the position of the loop where zero signals would be expected), unequal currents will flow in them due to their unequal impedances, and the points X and Y will not be at the same potential. Hence a P.D. is developed across the condenser, and a signal will be heard. Due to this vertical effect, either no zero signal position will be obtained, or, if it is, it will not occur when the plane of the loop is at right angles to the direction of the transmitter.

790. Elimination of Vertical Effect.—The obvious method of preventing vertical effect due to unequal capacity of the vertical sides of the coil to earth, is either to equalise these capacities or to short-circuit them by providing symmetrical paths to earth of

much smaller impedance for the vertical currents. A simple solution employing this latter method, which is found to be sufficient at low frequencies, is to earth directly the mid-point of the coil, as shown in Fig. 472 (a). The vertical currents in the two sides of

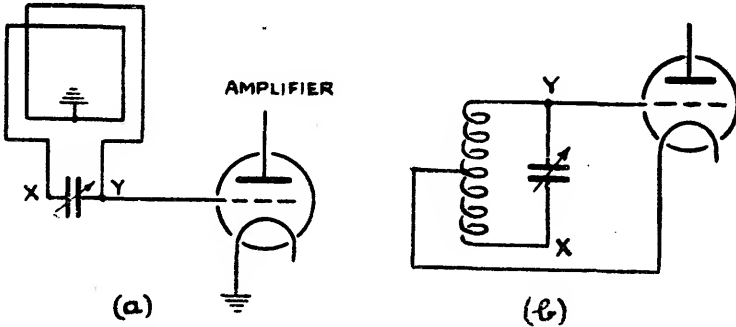


FIG. 472.

the loop then flow straight to earth, being provided with paths of much smaller (and equal) impedance than those through the capacities to earth of the valve electrodes. Since the paths are symmetrical, X and Y will be at the same potential, as far as vertical

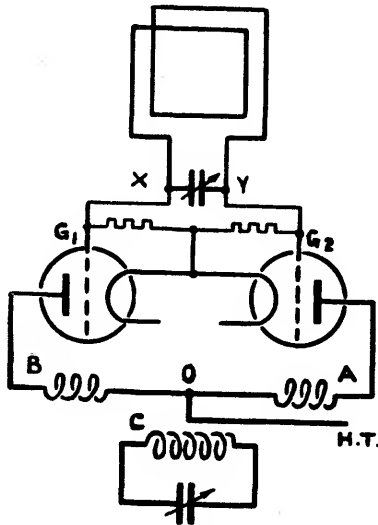


FIG. 473.

currents are concerned. It will be noted that in this circuit only half the voltage developed across the tuning condenser by the loop E.M.F. is applied between grid and filament of the valve, Fig. 472 (b) showing the equivalent circuit.

At higher frequencies the reactance of the paths to earth via the valve electrodes becomes comparable with that of the vertical

sides themselves, and the only solution is to equalise these capacities. A push-pull arrangement of valves readily lends itself to this purpose, and a circuit suitable for medium frequencies is shown in Fig. 473. The vertical earths are then via the capacities to earth of the grids of the two matched valves, and so are of very nearly equal impedance. As far as vertical effect is concerned, the instantaneous potentials of the grids of the two valves are therefore always equal. The corresponding anode currents are also equal, and, flowing in opposite directions through the two halves of the output choke, give rise to zero resultant flux-linkage with the secondary. The loop E.M.F., of course, produces a P.D. across the tuning condenser, and therefore causes anti-phase potential variations on the two grids, as in the ordinary push-pull amplifier (para. 595), giving a resultant E.M.F. in the secondary.

On high frequencies, *e.g.*, 15,000 kc/s., it is desirable to keep the receiver symmetrical as far as the telephones.

791. Screened Coils.—Another method of reducing antenna action is to surround the coil by a number of earthed conductors, which are not allowed, however, to form closed conducting paths. They may, for example, consist of a large number of vertical aerials immediately surrounding the coil; but a simpler solution in practice is to enclose the windings of the coil in a metal tube, as shown in Fig. 474. Currents are prevented from circulating round the tube by the insulating segment shown at S, which is commonly made of porcelain or ebonite. The tube itself is earthed at the bottom at its centre point.

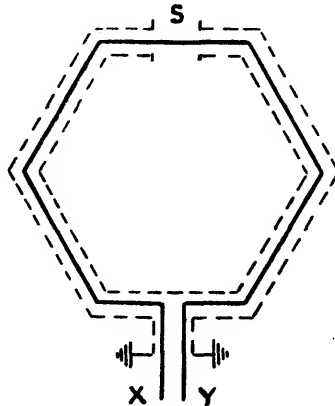


FIG. 474.

It is clear that comparatively small currents will flow up and down the screen; for although the two vertical sides of the screen will certainly act as open aerials, they will be far from resonance. At all but very high frequencies, the impedance of each screen acting as an aerial will be very high, and the magnetic fields of the

vertical currents in the screen very small. Thus the screen currents will have little effect on the total magnetic flux threading the circuit of the frame. This effect is entirely dependent on the segment S being a good insulator. If S is short-circuited, quite large currents will circulate round the screen and cancel the magnetic flux of the wave itself.

By Faraday's Law (para. 141) the total E.M.F. acting round a closed conducting circuit is equal to the rate of change of flux-linkage through the circuit. As the amount of flux is scarcely influenced by the earthed screen, the E.M.F. induced in the loop is practically the same as if there were no screen.

The capacity between the sides of the loop and the earthed screen, however, provides much the greater proportion of the total capacity of the sides to earth, by virtue of which they behave as vertical aërials, and the valve electrode capacities to earth carry very little or none of the vertical current. Since the screen is symmetrical about the sides, the points X and Y are at the same potential as far as their potentials are a result of vertical effect.

It is common practice to use earthed screens for rotating coil direction finders for all frequencies up to 1,500 kc/s.

792. Bellini-Tosi or Fixed Loop Direction Finders.—Instead of rotating the aerial itself it is possible to use a system of fixed aërials with an instrument called a radio-goniometer in the office. The principle will first be illustrated by a special case.

Referring to Fig. 475, suppose P to be a loop in a vertical plane pointing East and West, and Q a loop pointing North and South. P is joined by leads as shown to a coil P' inside the office. In Fig. 475, P and P' are shown in the same plane for simplicity, but this is not necessary in practice. In a similar manner Q is joined to a coil Q', which is mounted at right angles to the coil P'. The coil S, which is joined to the receiver, can be rotated and is coupled magnetically to P' and Q'. Consider a wave coming from a direction due North (or due South). No E.M.F. is induced in the loop P, and therefore no current flows in P'; maximum E.M.F. is induced in Q, and a current flows in Q'. If now the coil S is rotated it has maximum E.M.F. induced in it, and maximum signals are obtained, when it is parallel to Q', *i.e.*, at right angles to the magnetic field of Q', and a zero is obtained when it is parallel to P'. Similarly, for a wave incident from due East (or due West), a zero is obtained when S is parallel to Q'. Again, suppose the wave to be incident on a bearing of 045°. Equal E.M.F.s are then induced in P and Q, giving rise to equal currents in P' and Q'. Zero signal thus occurs when S is inclined equally to P' and Q'. Finally, we see that for a wave on a bearing of 000°, the position of S for a zero is 0° from the coil P'; for a wave incident at 090° the position of S for a zero is 90° from P'; and similarly, for a wave incident at 045°, the position of S for a zero is 45° from P'. In fact, as the bearing of the wave

changes, the direction of S for a zero changes by the same amount. This will be proved in the next paragraph for any angle of incidence of the wave.

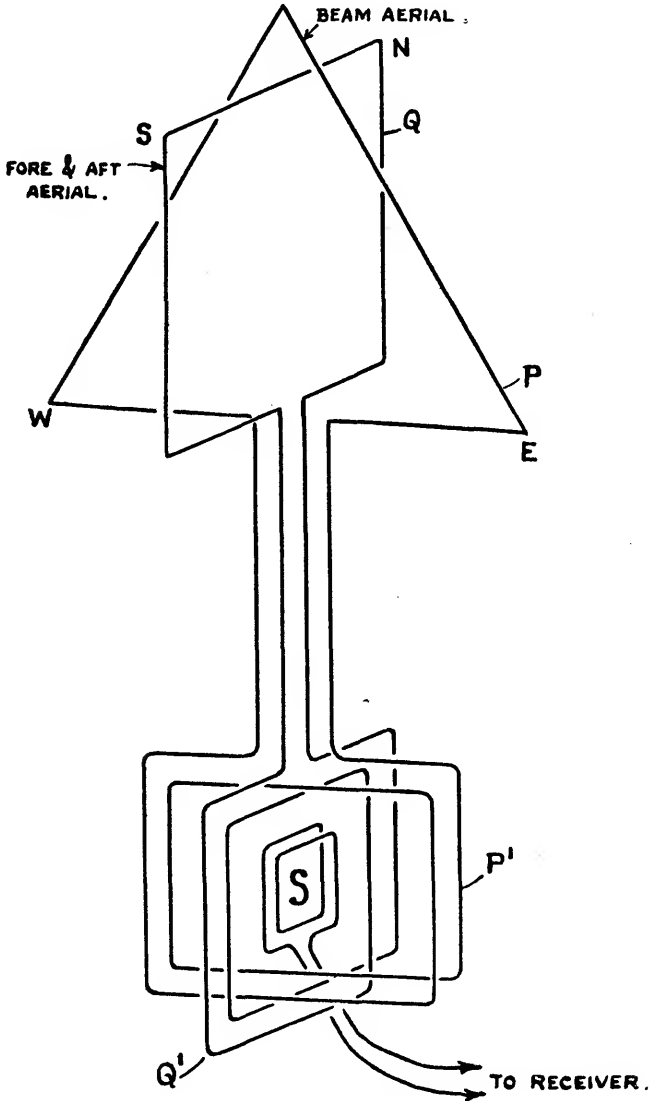


FIG. 475.

*793. The general case will now be considered. The two loops P and Q may be set up in any two vertical planes at right angles to each other. In order that each of them, treated as a separate loop aerial, may have the same E.M.F. induced in it by a wave

passing over it in its own plane, both loops must have the same area, since the E.M.F. is proportional to the area. (It will be seen below that this may not be the case in practice in order to correct certain errors that arise.) Suppose, now, that a wave whose electric field is $\mathcal{E} \sin \omega t$ is incident on the system at an angle θ with the plane of loop Q, and therefore at an angle $90^\circ - \theta$ with the plane of loop P (Fig. 476 (a)).

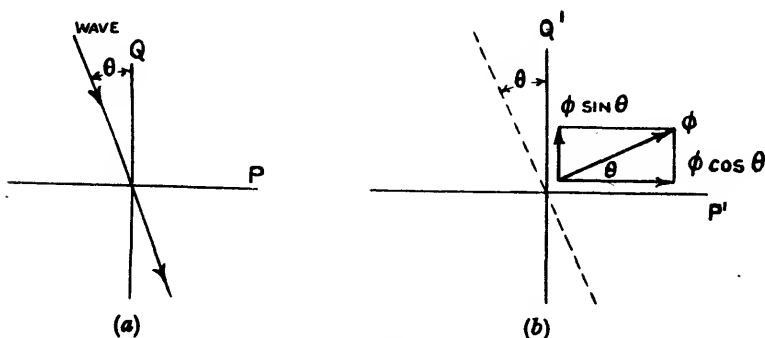


FIG. 476.

From para. 770, the E.M.F. in Q is $\frac{\omega \mathcal{E} A}{c} \cos \theta \cos \omega t$, and the E.M.F. in P is $\frac{\omega \mathcal{E} A}{c} \cos(90^\circ - \theta) \cos \omega t = \frac{\omega \mathcal{E} A}{c} \sin \theta \cos \omega t$, provided the widths of P and Q are small compared with the wavelength. The currents produced by these E.M.F.s depend on the impedances of the two circuits, including their goniometer coils. In practice untuned aërials are used, and their resistance may be neglected, compared with their inductive reactance. If both circuits have the same inductance L, currents proportional to the E.M.F.s and lagging 90° in phase on the E.M.F.s flow in the two goniometer windings P' and Q', the currents being given by

$$\frac{\omega \mathcal{E} A}{c \omega L} \cos \theta \sin \omega t = \frac{\mathcal{E} A}{c L} \cos \theta \sin \omega t \text{ in } Q',$$

$$\text{and similarly by } \frac{\mathcal{E} A}{c L} \sin \theta \sin \omega t \text{ in } P'.$$

It may be noted in passing that these current amplitudes are independent of frequency. If the coils P' and Q' are identical, except that they are at right angles to each other, the fluxes produced by the currents bear the same proportion to the currents, and may therefore be written as $\phi \cos \theta \sin \omega t$ through Q' and $\phi \sin \theta \sin \omega t$ through P'. They are therefore equivalent to a resultant flux $\phi \sin \omega t$ at right angles to a plane making an angle θ with the plane of Q' (Fig. 476 (b)). If, then, the plane of the search coil S is set at right angles to this plane, *i.e.*, in the direction of the resultant flux, S will have no flux-linkage and no E.M.F.

will be produced. Thus zero signals are obtained when the plane of coil S makes the same angle θ with the plane of coil P' , as the direction of the incident wave makes with the plane of the loop Q , and so the Bellini-Tosi system enables the direction of a transmitter to be determined.

An alternative statement of this result is that the plane of the search coil in the position for zero signals makes the same angle with coil P' as the magnetic field of the wave makes with the plane of the loop aerial P to which coil P' is connected.

It is interesting to note that the maximum E.M.F. in the search coil, S , (when coil S is at right angles to the direction of the resultant flux), is independent of the direction of the incident wave, since the maximum flux is always $\phi \sin \omega t$.

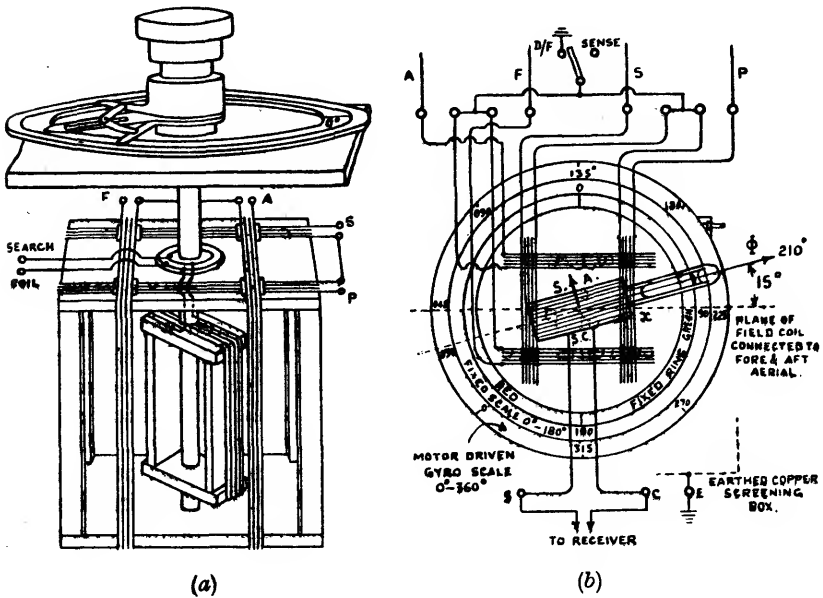


FIG. 477.

In order that the E.M.F. induced in S by mutual inductance with P' should be proportional to the cosine of the angle between P' and S , as was assumed above, the instantaneous density of flux produced by the current in P' must be the same over the whole area of S ; and the same applies to Q' and S . This is attained by winding the coils P' and Q' in two halves, as shown in Fig. 477, their distance apart being equal to their mean radius. The instantaneous field due to either coil then has an approximately constant value over the whole region in which the search coil S moves.

794. On shore, Bellini-Tosi aerials will normally be set up in the North-South and East-West planes, and the goniometer arranged in any convenient position in the office. A pointer is attached to the search coil, and moves over a scale graduated from 0° to 360° while the search coil turns through 360° . The pointer is adjusted so that it reads 0° when zero signals are obtained with a wave known to be incident from due North. The true geographical bearing of any transmitter is then given directly by the pointer reading for the position of zero signals. In a ship, the aerials are normally placed with their planes in the fore and aft line and athwartships. If the goniometer pointer is then adjusted so that it reads 0° in the zero signal position, with a wave known to be incident from right ahead, its readings in direction-finding determinations give bearings relative to the fore and aft line of the ship.

A numerical illustration of this is given in Fig. 477 (b), which represents a plan view of the Service radiogoniometer. The ship is supposed to be steaming on a course of 135° , and so the scale marking, 135° , on the motor-driven gyro scale is opposite 0° on the fixed goniometer scale. Suppose that the transmitter whose direction is to be found bears 210° from the ship; the magnetic component of the wave is then in a direction 120° , making an angle of 15° with the fore and aft loop aerial. Hence the resultant flux in the goniometer makes an angle of 15° with the plane of the field coil connected to the fore and aft aerial (Fig. 476).

Zero E.M.F. will then be produced in the search coil if it is placed as shown, its plane making an angle of 15° with the plane of the coil connected to the terminals FA. The D/F pointer, which is permanently fixed in the plane of the search coil (the angling device not being in use), then indicates starboard or Green 75° on the relative bearing scale and 210° on the gyro compass scale.

The determination of a bearing, after all adjustments have been made, thus consists of the simple operation of rotating the search coil until zero signals are obtained in the telephones, and of reading off the bearing on the gyro compass scale opposite the D/F pointer.

Zero signals would also be obtained if the search coil were rotated through 180° ; the pointer would then indicate 030° , the reciprocal of the correct bearing. The reciprocal should be taken as a check if time permits, and the sense subsequently determined by the sensefinder, which eliminates the 180° ambiguity (para. 798).

In order to reduce antenna action with Bellini-Tosi aerials, it is usual to join the mid-points of the windings P', Q' to earth. When the loops are not unduly large, screened tubes of the type shown for the rotating coil in Fig. 474 are frequently used for the same purpose. Where space is available, it is best to use large loops and a single turn of wire; where space is restricted, loops or frame coils with several turns may be employed, but it must be remembered that increasing the number of turns will not generally compensate for

the reduction in size. For multi-turn coils used in a Bellini-Tosi system, four turns are usually found satisfactory, and little improvement is obtained by using a larger number of turns. For best results, the inductances of P' and Q' should be approximately equal to the inductances of the loops or coils.

795. Comparison between Rotating Coil and Bellini-Tosi Systems.

—A rotating coil and a Bellini-Tosi system are electrically equivalent. On shore they give the same result, and there is no question of one system being more accurate than the other. In a ship a difference in their performance is often found, but this can always be shown to be due to the fact that Bellini-Tosi aerials are made larger than rotating coils in order to obtain the same signal strength. For aerials of the same size in the same position, there is no difference in the bearings which the two systems give; the only difference is one of signal strength.

The rotating coil, provided it can be fitted in a suitable position, has the merit of simplicity from an electrical point of view. Bad contacts or other defects result in a loss of signal strength, and possibly in no bearing being obtainable, but do not easily give rise to false bearings. Moreover, the coil can be tuned directly by a condenser, the full resonant voltage thus obtained across the condenser being available for direct application to the amplifier.

Untuned aerials are always used in the Bellini-Tosi system, and, in order to obtain an accurate scale, it is necessary that the mutual inductance between the search coil and each fixed coil should vary as the cosine of the angle between them. This involves an appreciable loss of signal strength in the goniometer, and, to obtain the same range as a rotating coil, fixed loops must be correspondingly larger. When, however, this can be achieved, the system has the merit that the only element to be rotated is a light coil. For rapid working this is an advantage. The disability of the system is the necessity for perfect electrical balance between the two separate aerial circuits. This balance is at once destroyed by a bad contact in one circuit, and a bearing up to 90° in error can result.

In order to obtain the same range with a rotating coil as with the large Bellini-Tosi aerials fitted in a battleship, it is necessary to use a coil about 4 ft. in diameter. The same range can also be obtained with fixed frames and a goniometer, provided the frames are about 7 ft. side. A frame coil 4 ft. in diameter, with six turns, can be used with good efficiency over the frequency range 60–600 kc/s., and with suitable circuits can be used up to 15,000 kc/s., although at this frequency it is not so efficient as a single turn coil tuned directly by a condenser.

796. Instrumental Design.—In all direction finders it is essential that the whole of the energy received by the amplifier reaches it through the aerials in a known and definite manner. For this reason

every part of the apparatus, including all tuners, amplifiers, batteries, etc., must be so well screened from the direct action of the wave that it does not pick up an E.M.F. capable of giving an appreciable signal, *i.e.*, not more than $\frac{1}{2}$ per cent. of the signal picked up by the aerials.

Apart from imperfect screening, errors may sometimes be produced by undesirable coupling between instruments inside the office, or by unsuitable circuit arrangements. Instrumental defects frequently have the characteristic that the E.M.F. due to them does not change appreciably as the frame or search coil is rotated. The result of this is that opposite minima are either not exactly 180° apart, or are of different quality, *i.e.*, one is more blurred than the other. Whenever symptoms of this kind are present, suspicion should be directed to:—

- (a) Pick-up by the receiver directly from the wave or from other aerials coming into the office ;
- (b) Antenna action of the aerials ; or
- (c) Defects in the circuits.

From this point of view it may be said that the rotating coil is less liable to instrumental errors than the Bellini-Tosi system. The only precaution necessary with the rotating coil is to ensure that the circuit to which the coil is connected is symmetrical about the two ends of the coil. The higher the frequency the more essential is perfect symmetry.

With Bellini-Tosi systems the two aerial systems should have identical electric constants (except in so far as they are deliberately altered to allow for the effect of local structures). The goniometer must be completely screened from all other instruments. To make certain that it is itself free from error, it is subjected to a number of tests before being issued for service. These tests ensure that when the winding which is joined to the fore and aft aerial is alone in use, two zeros are obtained at 0° and 180° , and that when the other winding is alone in use the zeros are exactly at 90° and 270° . The

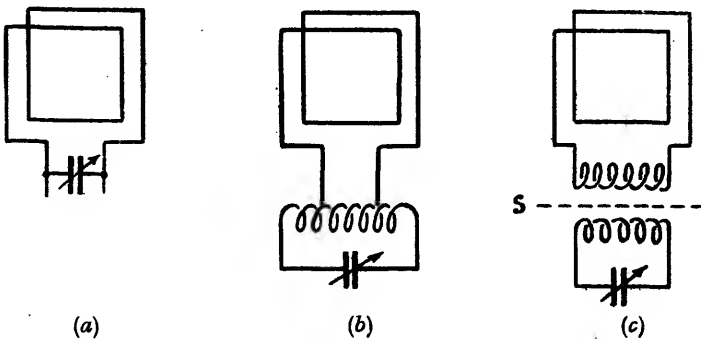
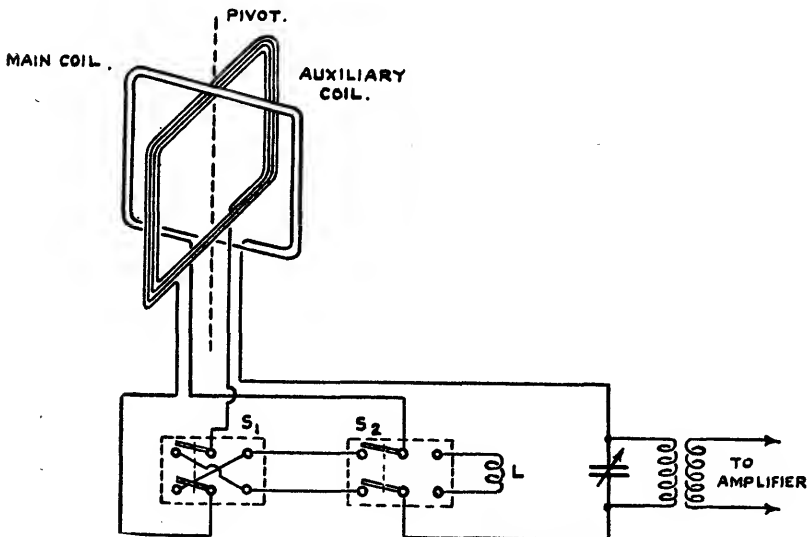


FIG. 478.

accuracy of the instrument at intermediate points is ensured by a proper choice of dimensions for the search coil.

Between 300 and 1,000 kc/s. it is the practice to tune the frame coil directly by a condenser, as shown in Fig. 478 (a). For lower frequencies an auto-transformer is used, as shown in Fig. 478 (b), and for very high frequencies the circuit is as shown in Fig. 478 (c), in which the dotted line S indicates that there is an electrostatic earth screen between the windings. This prevents capacity action between the two coils, which might introduce a lack of symmetry, but does not prevent the magnetic flux, due to current in the aerial coil, from linking with the coupling coil. Similar circuits may, of course, be applied to the search coil of the goniometer in the Bellini-Tosi system.

797. The Robinson System.—The Robinson system enables signals to be heard over the whole time that bearings are being taken, and obviates the necessity of swinging the frame rapidly in order to take a bearing.



Robinson D/F Circuit (Diagrammatic).

FIG. 479.

The method consists essentially in the use of two frame aerals (Fig. 479), fixed at right angles to each other, the system being free to rotate in a vertical plane.

The two coils are known respectively as the main coil and auxiliary coil; the auxiliary coil has approximately two and a half times as many turns as the main coil.

The two coils are connected in series with one another, but the direction in which the auxiliary coil is connected may be reversed

by the switch S_1 . The switch S_2 is used to cut the auxiliary coil out of circuit, and to replace it by a coil L of equal inductance, so that the tuning is unchanged.

If, now, the main coil is directed exactly towards the transmitting station, no E.M.F. will be induced in the auxiliary coil. If, however, as is probable, the main coil is a little off the bearing, a small E.M.F. will be induced in the auxiliary coil.

Let us assume that with switch S_1 to the left, the E.M.F.s due to the main and auxiliary coils are acting in the same direction round the circuit.

Now throw the switch S_1 over to the right; the E.M.F. due to the auxiliary coil will be in opposition to that due to the main coil, and signals will be reduced in strength.

If, therefore, the switch is rapidly reversed, signals will vary in strength until the main coil has been swung to the true bearing of the transmitting station, when reversals of the switch will have no effect on the strength of signals.

When the main coil is near the true bearing, the E.M.F. round the auxiliary coil is in the neighbourhood of its zero value. It is therefore necessary to have a larger number of turns in the auxiliary than in the main coil if the reversal of the auxiliary E.M.F. is to produce an appreciable change in the signal strength.

This system has the great advantage that signals can be read continuously while the bearing is being taken. It is used extensively in the Royal Air Force, since the coils can be fixed, and the course of the aeroplane altered so as to direct the main coil towards the transmitter.

798. Determination of Sense.—The ambiguity of the bearing obtained by the direction finders described above has already been remarked. For any direction of the incident wave, zero signals are found in two positions, 180° apart, of either a rotating coil or the search coil of a fixed loop system, corresponding respectively to the bearing of the transmitter and its reciprocal bearing. It is therefore necessary to devise some means of determining which of the two pointer readings is the correct one. Instruments for this purpose are called sensefinders. The usual principle on which they operate is as follows: according as the transmitter lies on a certain bearing or its reciprocal, the loop E.M.F., when the plane of the loop is in the direction of propagation, will have the same amplitude, but will be altered in phase by 180° . If at any instant it was directed clockwise round the loop for the one bearing, then for a transmitter on the reciprocal bearing it would be counter-clockwise round the loop at the same instant. This, however, does not apply to the E.M.F. in a vertical aerial, the amplitude and phase of which are unaffected by the direction of the transmitter.

Suppose, therefore, that in addition to the rotating coil we have an open aerial which can be coupled to the receiver. The wave

induces an E.M.F. in this aerial at the same moment as an E.M.F. is induced in the loop. The E.M.F. which is induced in the open aerial does not vary with the direction of the wave. The E.M.F. induced in the loop is a maximum when the loop is in the direction of propagation of the wave, and changes sign when the coil is rotated through 180° , or when the wave is reversed in direction. Therefore if these two E.M.F.s, one from the loop and one from the aerial, can be made to act on the receiver at the same time, it will be seen that for one position of the coil, viz., when it is pointing towards the transmitting station, the two E.M.F.s will reinforce each other, but that the two E.M.F.s will be opposed when the coil is rotated through 180° from this position. If, now, we make the two E.M.F.s equal in magnitude, they will cancel each other when

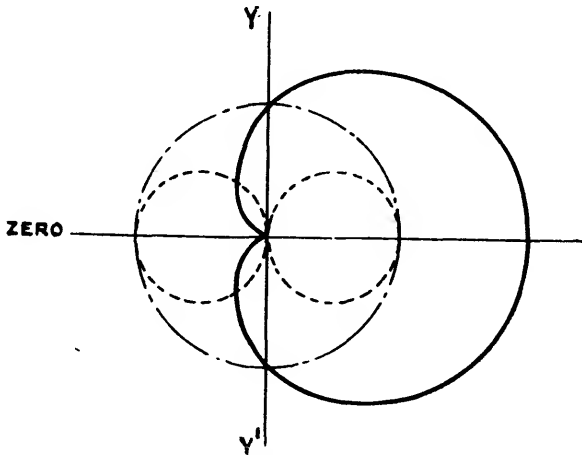


FIG. 480.

opposed, and a zero will be obtained. In the position of the coil 180° from this there will be a maximum due to the combined effect of the two E.M.F.s assisting each other. When the coil is at right angles to the direction of the wave the E.M.F. from the open aerial will still be operative, so that, instead of a zero, the signal will continue to be heard. In fact, there is only one position for a zero during a rotation of the coil through 360° .

In order that the amplitudes of the vertical and loop E.M.F.s may be added or subtracted in this manner, these E.M.F.s must either be in phase or 180° out of phase. But it has been pointed out that the loop E.M.F. is 90° out of phase with the vertical E.M.F. The actual effects required to be additive or subtractive are not the E.M.F.s themselves, but the P.D.s they produce across the first stage of the amplifier. Hence, in order that sensefinding may be possible, an alteration in phase by 90° of one effect relative to the other must be brought about. Methods of accomplishing this are

explained below. If we assume for the moment that it has been successfully achieved, the variation of resultant input P.D. to the amplifier, due to both E.M.F.s as the coil is rotated, may be obtained graphically, as in Fig. 480.

The P.D. due to the loop E.M.F. is represented by the figure of eight diagram (para. 780), and that due to the vertical aerial by a circle. The amplitude of the vertical P.D. is adjusted to equality with the maximum P.D. produced by the loop E.M.F. In other words, the radius of the vertical aerial circle is equal to the diameter of either circle of the loop figure of eight diagram. In the right-hand circle of the loop polar diagram, the P.D. is supposed to be in phase with the vertical P.D., and to be 180° out of phase in the left-hand circle. Radii vectors to the right of YY' are therefore added, and the difference is taken of those to the left. Thus the variation of the resultant P.D. with direction of the loop is represented by the heart-shaped curve, which is called a "cardioid." It will be seen that this curve has only one zero value, and so there is only one orientation of the rotating coil in which zero signals are obtained. It is therefore possible to distinguish between the correct bearing and its reciprocal. The zero will be either in the direction of the transmitter or its reciprocal direction, according to the phases of the two circles of the figure-of-eight diagram, compared with the phase of the vertical aerial circle. This can be determined for a known bearing, and the pointer set accordingly. It will then give the correct reading for any bearing of the transmitter.

It will be seen that the zero of the cardioid is at 90° from either of the zeros obtained with the frame coil alone. Provided that the cardioid is perfect in shape, there is no reason why it should not be used directly for taking bearings without ambiguity. But in practice, and particularly in a ship, it is found to be very difficult to obtain a cardioid sufficiently perfect to give accurate bearings in all directions. Another disadvantage is that the change in signal strength as the coil is rotated in the neighbourhood of the cardioid zero is much less than the corresponding change on the figure-of-eight diagram; although a polar diagram of reception with only one zero over 360° and considerably improved sensitivity is obtainable by mounting a subsidiary loop at right angles to the main loop. It is therefore usual to take the accurate bearing with the rotating coil alone, that is on the figure-of-eight characteristic, and to switch in a separate aerial to give a cardioid for the purpose of determining "sense." As the zero of the cardioid is 90° from either zero of the figure of eight, a separate pointer (*e.g.*, the sense arrow S.A. in Fig. 477 (b)) is required for sense determination.

If the E.M.F. at the amplifier due to the vertical aerial is not exactly equal to that due to the rotating coil, the cardioid is distorted.

The resultant polar diagrams of reception for the cases when the vertical aerial E.M.F. is less and greater than the loop E.M.F., are shown in Fig. 481 (a) and (b) respectively. In Fig. 481 (a) two zeros are obtained at equal angles from the correct direction of the transmitter, the size of the angles depending on the ratio of the E.M.F.s. In Fig. 481 (b) a minimum of sound is obtained instead

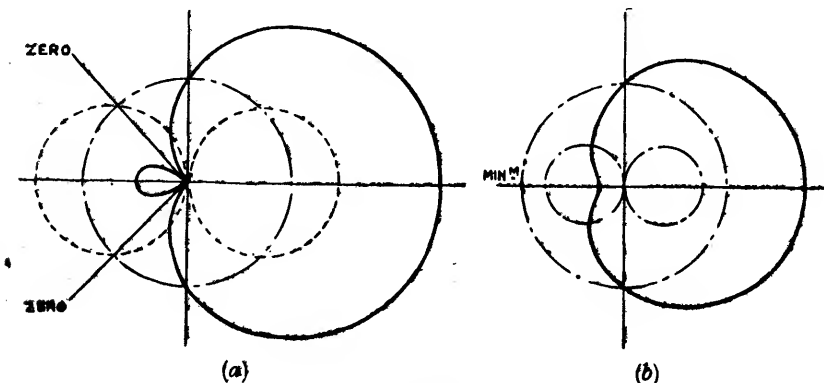


FIG. 481.

of a zero in the correct position. When the direction of the transmitter has previously been obtained and only its sense is required, these diagrams show that, provided the inequality of loop and vertical E.M.F.s is small, there is sufficient difference between the maximum signal and the signal on the reciprocal bearing to allow of sense determination.

799. The problem of sensefinder design therefore reduces to:—

- (a) Producing a change in phase of 90° in the effect of the vertical aerial E.M.F. relative to that of the loop E.M.F.
- (b) Adjusting the effects to equality in magnitude.

One form of sensefinder is shown in Fig. 482. The vertical aerial is tuned by the inductance L_1 . The current in L_1 is therefore in phase with the E.M.F. produced by the wave in the vertical aerial, and so in phase with the wave itself. L_1 is mutually coupled to the inductances L_2 in the loop circuit. The E.M.F. induced into L_2 from L_1 lags or leads by 90° on the current in L_1 , and is therefore 90° out of phase with the incident wave. Hence it is either in phase or in antiphase with the loop E.M.F. produced by the wave, and the two effects can be added. The value of the mutually induced E.M.F. is made equal to that of the loop E.M.F. by adjusting the mutual coupling.

In Fig. 482 an aerial is shown combined with a rotating frame coil, but the same result may equally well be obtained when using Bellini-Tosi aeriaks by connecting the search coil of a goniometer in place of the rotating coil.

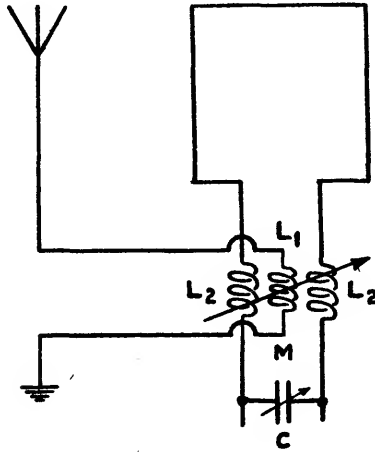


FIG. 482.

*800. The operation of the sensefinder circuit of Fig. 482 may be more directly analysed as follows.

Suppose $\mathcal{E} \sin \omega t$ to be the electric field of the wave incident at an angle θ from true North. If the plane of the loop is set at an angle α from true North, the angle between the loop and the direction of propagation is $\theta - \alpha$, and the loop E.M.F. is

$$\frac{\omega \mathcal{E} A}{c} \cos (\theta - \alpha) \cos \omega t$$

(para. 779).

The E.M.F. in the open aerial is $\mathcal{E} h \sin \omega t$, where h is its effective height. Since the aerial is tuned, the current flowing in it is $\frac{\mathcal{E} h \sin \omega t}{R}$ where R is its total resistance. If the mutual inductance between L_1 and L_2 is M , the E.M.F. induced into L_2 is therefore $\frac{\omega M \mathcal{E} h \cos \omega t}{R}$. The total E.M.F. round the loop is therefore

$$\begin{aligned} & \frac{\omega \mathcal{E} A}{c} \cos (\theta - \alpha) \cos \omega t + \frac{\omega M \mathcal{E} h}{R} \cos \omega t \\ &= \left[\frac{\omega \mathcal{E} A}{c} \cos (\theta - \alpha) + \frac{\omega M \mathcal{E} h}{R} \right] \cos \omega t \end{aligned}$$

If M is now adjusted so that $\frac{\omega \mathcal{E} A}{c} = \frac{\omega M \mathcal{E} h}{R}$, the total loop E.M.F. becomes $\frac{\omega \mathcal{E} A}{c} [\cos (\theta - \alpha) + 1] \cos \omega t$.

This expression shows how the signal strength varies for different values of α . Between 0° and 360° it has only one zero value, viz., when $\cos (\theta - \alpha) = -1$, or $\alpha = \theta \pm 180^\circ$. For the value of α 180°

different from this, *i.e.*, $\alpha = 0$, the signal strength is a maximum, the amplitude of the E.M.F. being $2\omega\mathcal{A}/c$ (since $\cos(\theta - \alpha) = 1$). The horizontal polar diagram obtained by plotting total loop E.M.F. against α is, of course, the cardioid, as shown in Fig. 483.

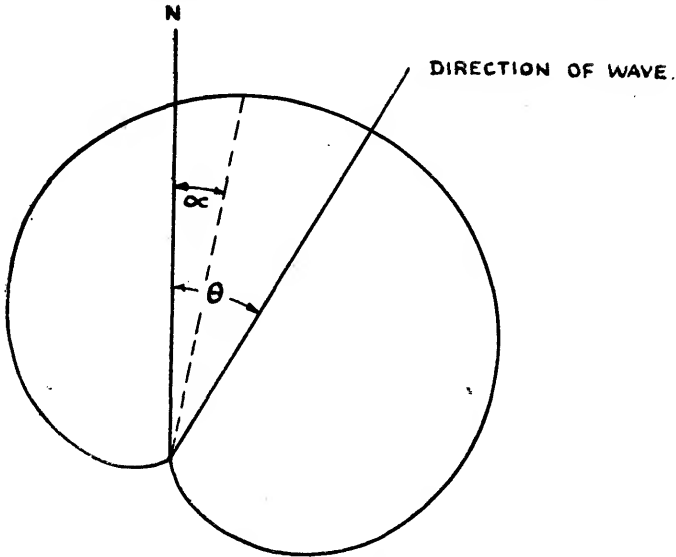


FIG. 483.

801. A sensefinder which avoids the necessity of tuning a circuit to resonance is shown in Fig. 484. In this arrangement, use is made of the properties of a valve to give a current in the anode circuit

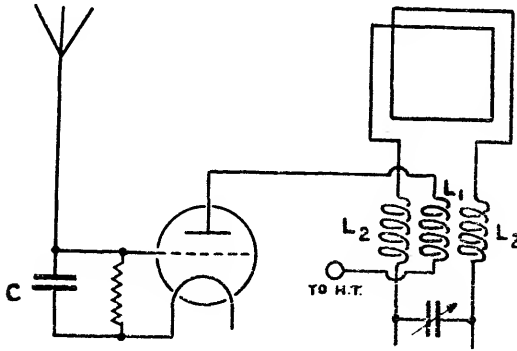


FIG. 484.

which is in phase with the voltage applied between grid and filament. The aerial circuit is kept far from resonance by the small condenser C, while the impedance of the coil L_1 is kept small relative to the A.C. resistance of the valve.

The current in the vertical aerial is 90° out of phase with the E.M.F., and therefore with the electric field of the wave. The voltage developed across the condenser, which is the grid filament P.D., is 90° out of phase with the current, and so is either in phase or antiphase with the field of the wave. The anode current is practically in phase with the grid voltage owing to the small impedance of L_1 , and so the E.M.F. induced into L_2 , which is 90° out of phase with the current flowing in L_2 , is 90° out of phase with the field of the wave. It is therefore in phase or antiphase with the loop E.M.F., and a cardioid diagram of reception can be obtained by adjusting the value of the mutual coupling between L_1 and L_2 . This circuit has the advantage that the correct phase relation is obtained over a wide range of frequencies without adjustment of tuning.

Sometimes the separate aerial may be dispensed with by making use of the antenna action of the loops themselves. This is most readily arranged with the large loops used in Bellini-Tosi systems. In the "sense" position the electrical mid-points of the fixed windings of the goniometer are joined to earth through a circuit coupled to the receiver so as to give a voltage in the correct phase. The loops thus act at the same time in two distinct ways, first as loops with circulating currents flowing round them, and secondly as vertical aerials giving antenna currents down the lead to earth. Where the antenna action of the aerials has been deliberately eliminated, as, for example, by the use of an electrostatic screen, this method is not applicable, and a separate aerial is necessary.

802. A Service direction and sense-finding circuit in which the Bellini-Tosi loops are made to act as vertical aerials as well as loops is shown in Fig. 485 (a). In the D/F position of the two mechanically-coupled switches, the electrical mid-points of the goniometer windings are directly earthed. In the sense position they are earthed through a variable inductance L_1 and resistance R , and the effect of the vertical current flowing through R is applied to the grid tuned circuit via the coupling condenser C_1 . It should be noted that the tuning of the grid circuit is unaffected by switching over from D/F to sense, since in both cases the condenser C_1 is in the same relative position.

The phase of the grid-filament voltage due to the loop E.M.F.s will be first considered. The loop E.M.F.s are 90° out of phase with the wave field, and as the loops are untuned the loop currents are 90° out of phase with the E.M.F.s, or 180° out of phase with the wave field. Thus the search coil E.M.F. is 90° out of phase with the wave field. This E.M.F. is applied in the tuned grid circuit, and produces a current in phase with itself; and this current, flowing through the grid tuning condenser, gives a P.D. across the condenser lagging on the current by 90° . Hence, finally, the grid-filament voltage is in phase, or 180° out of phase, with the wave field.

The vertical E.M.F. is in phase with the wave field, and the loops used as vertical aerials are de-tuned capacitively by first tuning by means of L_1 , and then decreasing the amount of L_1 in circuit. The vertical current thus leads 90° on the vertical E.M.F., and produces a P.D. across R in phase with itself. The P.D. across R therefore leads the wave field by 90° . The equivalent circuit by which this P.D. is applied to the grid is shown in Fig. 485 (b). This circuit is tuned to the incoming frequency, and since C_1 has capacitive reactance, the combination of C in parallel with L and the search coil in series must possess a nett inductive reactance. It is the P.D.

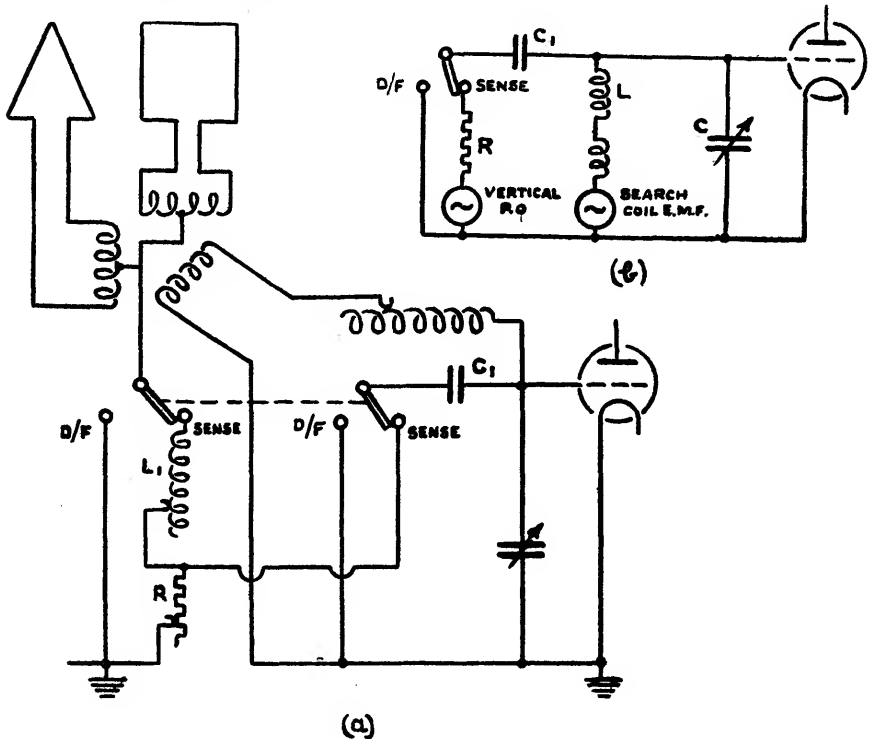


FIG. 485.

across this combination which constitutes the grid-filament voltage due to the vertical effect. The P.D. across the combination is 90° out of phase with the make-up current to it, because of its equivalent inductive reactance; but the make-up current is the current round the total acceptor circuit consisting of R , C_1 , and the parallel combination all in series. This current is therefore in phase with the P.D. across R , and so is 90° out of phase with the wave field. The grid-filament voltage due to the vertical effect is thus in phase, or 180° out of phase, with the wave field. This is the same phase relationship as was found for the loop E.M.F., and therefore the

correct condition for sense finding. The magnitude of the vertical effect is made equal to that of the loop effect by adjusting the value of the resistance R , and hence of the P.D. developed across it.

It should be noted that if the vertical aerial is de-tuned inductively by adding more inductance in L_1 than is found necessary for resonance, the grid-filament voltage due to the vertical effect will be altered in phase by 180° . Hence, if the sense arrow has been adjusted so that it gives the correct bearing with capacitive de-tuning, it will give the reciprocal bearing with inductive de-tuning.

The D/F circuits of the Service receiver employing this sense-finding circuit are described later (para. 810 and Fig. 498). It will then be seen that the D/F tuning circuit shown above is made more selective by adding an intermediate circuit. According as this circuit is tuned or untuned, it alters the phase of the grid-filament voltage due to the loop effect by 90° or 180° , and so must not be used when sense is being determined.

803. Night Effect.—The theory of determining the direction of a transmitter by the circuits described above has been based on the assumption that the wave is propagated in a direction parallel to the earth's surface, with its electric field vertical and its magnetic field horizontal. This is approximately the case for the direct or

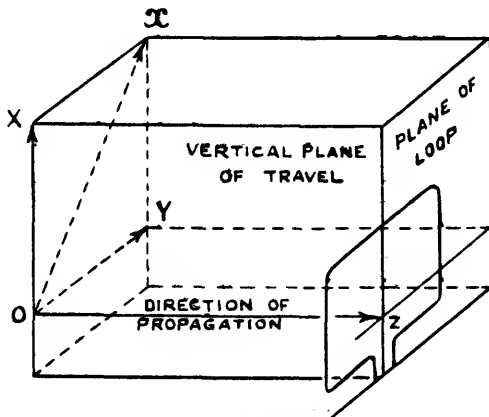


FIG. 486.

ground radiation from a transmitter, but it has been explained in Chapter XVII that in many cases reception is either partly or wholly due to the indirect radiation which has travelled up to the ionised parts of the atmosphere and has been bent round so that it returns to earth at some distance from the transmitter. The effect of such radiation on direction-finding apparatus will now be considered.

Suppose that the plane of the loop is set at right angles to the vertical plane in which the wave is supposed to be travelling. If the wave were propagated horizontally, no resultant E.M.F. would act round the loop in this position, and zero signals would be obtained. If, however, the direction of the wave is inclined to the horizontal, cases may arise in which a signal will be heard. One such case is due to the fact that the plane of polarisation of the wave has been rotated in its passage through the Heaviside Layer (para. 732). The electric field is then no longer in the vertical plane of travel of the wave (the vertical plane through transmitter and receiver). This is shown in Fig. 486, where XOZ is the plane of

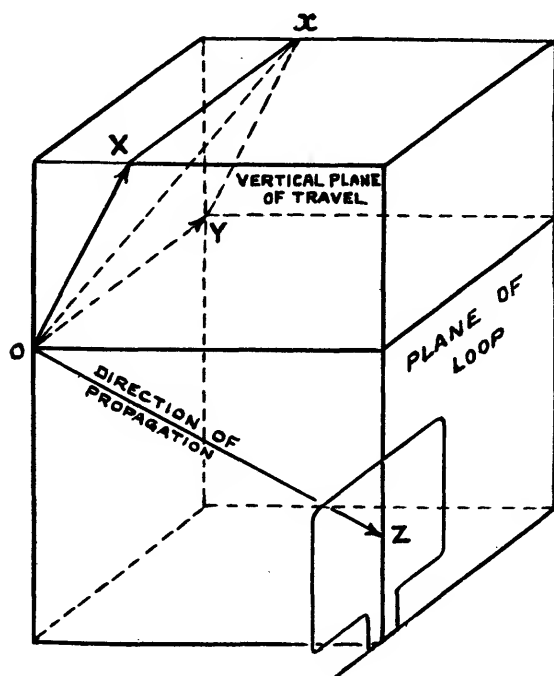


FIG. 487.

travel, and \mathcal{X} is shown making an angle with that plane. It is obvious that \mathcal{X} can be resolved into two components OX in the plane of travel, and at right angles to the direction of propagation, and OY horizontal and at right angles to OX in the plane perpendicular to the direction of propagation. Suppose, now, that the wave is incident at an angle to the horizon, as in Fig. 487. The direction of OY is unaltered, and OX, although no longer vertical, is still in the vertical plane of travel. OX therefore produces equal E.M.F.s in the vertical sides of the loop and no E.M.F.s along the horizontal sides, and so gives no resultant loop E.M.F. OY produces no E.M.F.s along the vertical sides, since it is horizontal. It

will, however, produce E.M.F.s along the horizontal sides, and, since it reaches the top of the loop before the bottom, these E.M.F.s, though equal in amplitude if the loop is symmetrical, differ in phase, and so give rise to a resultant E.M.F. round the loop. The loop, in fact, behaves towards the field OY as a loop pointing to the transmitter would behave to OX when the wave is travelling horizontally. Thus a signal is heard in the position of the loop where a zero should be obtained, and an error is introduced into the determination of direction.

It should be noted that the inclined direction of travel of the wave does not in itself give rise to this particular error, provided the electric field remains in the plane of travel. The error arises when this field has a horizontal component not in the plane of travel. More complex changes in the nature of the wave such as circular polarisation (para. 738) will also produce errors in direction determinations.

For simplicity, the ray reflected from the earth's surface at the loop has been neglected in the above discussion, but, obviously, its contribution to the loop E.M.F. should also be taken into account.

Near a transmitting station the indirect ray is weak or non-existent on all frequencies, and good bearings can then be obtained. On low and medium frequencies the indirect ray is relatively weak during daylight hours for transmissions over sea up to 100 miles. Good bearings are then possible. At night the indirect ray becomes important, though relatively less important over sea than over land, because the direct ray is less attenuated over sea, and therefore forms a larger percentage of the whole signal received.

It is the ratio of the intensity of the indirect to the direct ray in the total received signal which determines the liability to error of loop direction finders. Since this is greater at night for the usual D/F frequencies and distances, such errors are then most common and of largest amount, whence the name of "night effect" given to this phenomenon.

804. Adcock Direction Finder.—Night effect errors have been shown above to be due to the voltages induced in the horizontal parts of the loop. The Adcock system aims at reducing such errors by removing the top horizontal side of the loop altogether and screening the lower horizontal leads to the receiver by an underground earthed metal tube so that no E.M.F.s can possibly be induced in them by an incident wave, no matter what may be its state of polarisation.

The direction-finding circuit is thus modified as shown in Fig. 488. AD and BC are two vertical aerials corresponding to the vertical sides of a rotating loop, and are joined by an underground screened cable to an inductance L in the office, this inductance being coupled to the first stage of the receiver. If the circuit is arranged symmetrically, the two aerials may be considered as

earthed in common from the electrical mid-point of L . The currents flowing in opposite directions through the two halves of L from the vertical aerials to earth are then in the same proportion as the E.M.F.s induced in the aerials by the incident wave. The resultant flux through L is thus proportional to the difference of these two

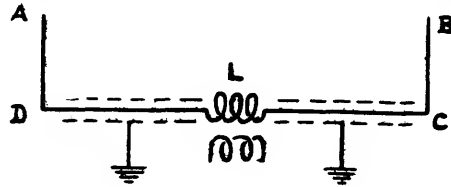


FIG. 488.

E.M.F.s, and the system is electrically equivalent to a loop aerial, except that it is only affected by the vertical component of the electric field of the wave. For example, when the plane of the aerials is at right angles to the direction of propagation of the wave, equal E.M.F.s are induced in the aerials, equal currents flow in opposite directions through the two halves of L , and there is no resultant flux through L . Zero signal is therefore obtained in this position, as in the case of a loop aerial receiving the direct ray only.

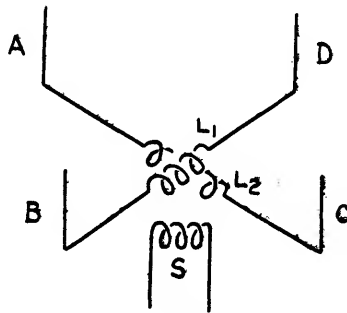


FIG. 489.

The Adcock equivalent of the Bellini-Tosi system is shown in Fig. 489. Four equal vertical aerials—A, B, C, and D—are situated at the corner of a square. Opposite aerials AC, BD, are joined by means of buried cables through two equal inductances L_1 , L_2 , mounted at right angles as in an ordinary goniometer. The search coil S rotates, and is joined to a receiver suitable for the frequencies in use. The practical operation of such a type of Adcock system is exactly the same as for a Bellini-Tosi system.

The result obtained in para. 779, that the loop E.M.F. is proportional to the cosine of the angle between the plane of the loop and the direction of the transmitter, was seen only to be justified provided that the width of the loop (or the distance between the

two vertical aerials in the Adcock system) was small compared with the wavelength. At the higher frequencies at which these Adcock systems are most necessary, this condition may not be realised; particularly since their high impedance makes it necessary for them to be of large dimensions to give sufficient signal strength. Provided the bearing is obtained by a zero on the rotating coil or its Adcock equivalent, this limitation does not affect the bearing, since the resultant E.M.F. is zero whatever the distance between the aerials in any practical arrangement. (The sine of the angle in the proof given in para. 779 is obviously always zero when the angle is zero.)

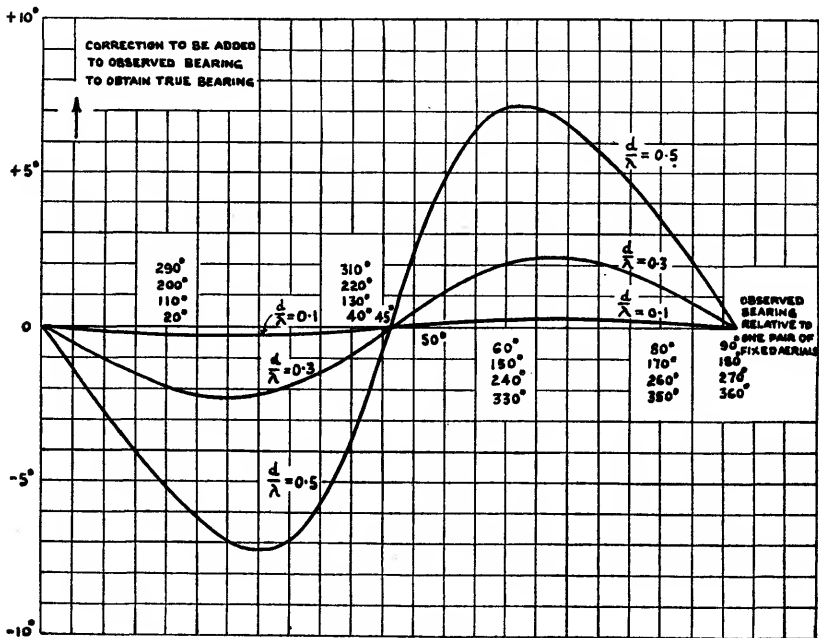


FIG. 490.

In the Bellini-Tosi system or its Adcock equivalent, however, the E.M.F.s round the two loops, or the resultant fluxes in the two coils L_1 and L_2 , are not zero when the bearing is being obtained. In this case, therefore, unless the distance d between the vertical aerials, or sides of the loops, is small compared with the wavelength λ (in practice less than about a quarter of the wavelength), a correction must be applied to the pointer readings according to the ratio $\frac{d}{\lambda}$ and the direction of the wave. This correction is shown graphically in Fig. 490 for three values of $\frac{d}{\lambda}$.

It is found in practice that better results are obtained if Hertzian aerials are used instead of earthed aerials. Fig. 491 shows a circuit arrangement of this type corresponding to the circuit of Fig. 488 with earthed aerials, *i.e.*, the Adcock equivalent of the rotating loop.

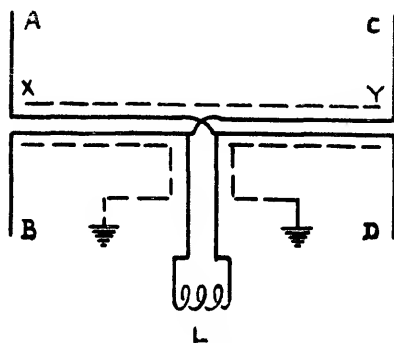


FIG. 491.

The Hertzian aerials AB, CD, are mounted vertically at the opposite ends of a horizontal tube XY, which can be rotated about a vertical axis through its centre point. The leads from the aerials are brought along inside the metal tube shown by the dotted lines. The current which flows through the coil L is then due to the difference of the E.M.F.s induced in the two doublets AB, CD. If the whole system is rotated until XY is perpendicular to the direction of the wave, no current flows through L, and a zero is obtained in the receiver. As pointed out above, this system has the advantage that no correction is required for the distance between the aerials even when this is a large fraction of a wavelength, and so is suitable for high frequencies.

In order to obtain satisfactory signal strengths it is desirable that the separation between opposite aerials should be about one-tenth of a wavelength. At 300 kc/s. this means about 100 yards. The space required for Adcock aerials working on the medium and lower frequencies is therefore considerable. It should also be remembered that exact equality in the receiving power of opposite aerials is necessary if accurate bearings are to be obtained. This is sometimes difficult to secure in a ship, even with aerials of exactly the same height, owing to the influence of structures nearer to one aerial than to the other.

In the range of the direct ray, and where the indirect ray is negligible, there is nothing to choose in point of accuracy between the closed loop and the Adcock direction finder. Where the indirect ray is important, careful tests have shown that the Adcock system has the advantage. But it is clear that even this system will fail when a number of rays from the transmitting station reach the receiver at the same instant from different directions. This appears

to happen at high frequencies with a receiver just outside the range of the direct ray, and is supposed to be due to scattering of the rays in the upper atmosphere, like the scattering of the sun's rays by fog. At very great distances it appears that the main indirect stream of energy arrives in a fairly uniform direction in the great circle plane between transmitter and receiver, and a definite bearing is again obtained.

805. Coastal Refraction.—The refraction or change in direction of propagation of electromagnetic waves when they pass from one medium to another has been described in Chapter XVII. It provides the reason for the return of the indirect ray to the earth's surface, and, even in the case of the direct ray, similar effects are produced by changes in the conductivity and dielectric constant of the surface over which the wave is being propagated. The velocity of a wireless wave over sea water may be up to 5 per cent. greater than its velocity over land. Thus the direction of a wave is altered when it passes over a coastline. A direction-finder determines the great circle bearing on which the wave is travelling when it passes the receiver, and if the wave has been refracted this will not be the bearing of the transmitter. Errors due to this cause are not appreciable unless the direction of the wave makes an angle of less than about 20° with the coastline, and all bearings of shore stations should be examined on the chart to ensure that this cause of error is not likely to have arisen.

By analogy with the dispersion of light, the effect would be expected to vary with the frequency of the wave. This is found to be the case for frequencies above 150 kc/s., but below this frequency the refraction appears to be almost independent of the frequency.

DIRECTION FINDERS IN SHIPS.

806. When a wireless wave strikes a ship, every conducting part of the ship behaves like an aerial and has an alternating current produced in it. The magnetic fields due to these currents flowing in hull, superstructure and rigging are naturally in different directions, and they also differ from each other in phase since the wave reaches the various parts of the ship at different times. These fields produce flux-linkage with the direction-finding aerial, and so give rise to errors in its determinations.

For any particular bearing of the wave the effect of all these component fields may be represented by two resultant fields in different directions, one in time-phase with the magnetic field of the wave, and the other 90° out of time-phase. This is indicated in Fig. 492. $H \sin \omega t$ is the magnetic field of the wave at right angles to the direction of propagation, $P \sin \omega t$ is the resultant field of the

ship in phase with $\mathcal{H} \sin \omega t$, and $Q \cos \omega t$ is the ship field 90° out of phase with $\mathcal{H} \sin \omega t$. The types of error to which these two fields give rise will now be considered.

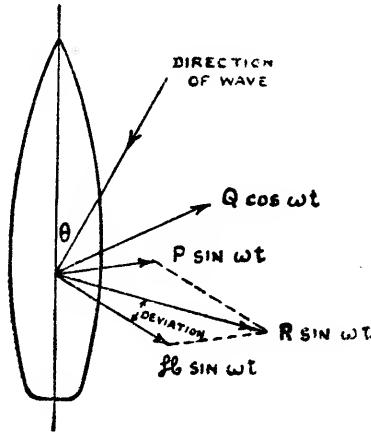


FIG. 492.

(a) **Ship field in phase with wave field.**—Since $\mathcal{H} \sin \omega t$ and $P \sin \omega t$ are in phase, they may be combined to give a resultant field $R \sin \omega t$. A zero would then be obtained when the plane of the loop was set at right angles to the resultant field. The effect of this ship field is therefore to displace the zeros.

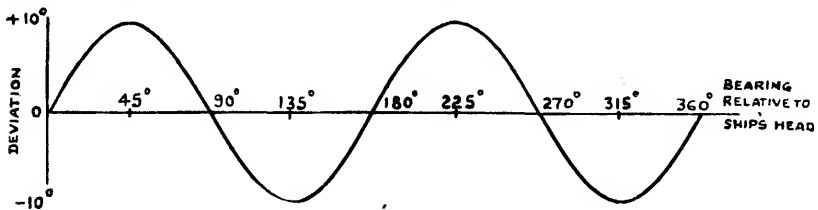
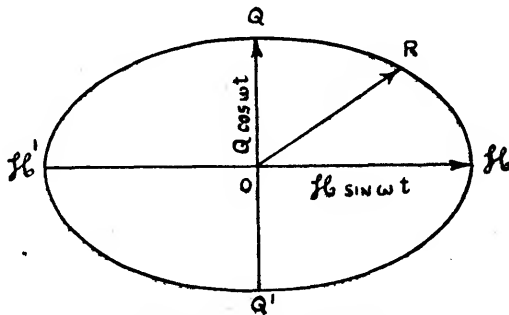


FIG. 493.

For a direction-finder situated on the centre line of the ship it is found that the effect is to make the wave appear to be arriving from a direction nearer to the fore and aft direction than it actually is. The difference between the true bearing, and the apparent bearing as given by the direction-finder, is termed "deviation" (by analogy with the deviation produced in magnetic compass bearings by the ordinary magnetic field of the ship). It is known from experience that the deviation is often zero for the four directions 0° , 90° , 180° , 270° , measured from ship's head, but reaches a maximum for bearings 45° , 135° , 225° , and 315° . Such deviation is called **quadrantal**, and is shown graphically in Fig. 493. The amount of the deviation depends upon the position of the direction-finder and upon the size of the ship. It is not uncommon for a

deviation to amount to 10° for a direction-finder situated on the upper deck, while for a direction-finder high up the mast it may be 3° or 4° , depending upon the size of superstructure in its immediate vicinity. Where the direction-finder is far from the centre line, the deviation will not be of this simple type, and the shape of the curve of deviation can only be found by trial.

(b) **Ship field 90° out of phase with wave field.**—This is the field $Q \cos \omega t$. Since it differs both in direction and phase from the field of the wave, the two cannot be combined to give a resultant alternating field in any particular direction. The resultant field turns through every direction from 0° to 360° in one cycle of the components, and so is known as a rotating field.



Elliptical Rotating Field.

FIG. 494.

This effect is most simply understood when the fields are supposed to be at right angles to each other in space, as shown in Fig. 494. Suppose that time is reckoned from the instant when $H \sin \omega t$ has its maximum value H . The value of $Q \cos \omega t$ is then zero, and so the resultant field is OH . A quarter of a period later, $H \sin \omega t$ is zero and $Q \cos \omega t$ has its maximum value Q . The resultant field is therefore of value OQ in the direction of $Q \cos \omega t$. After another-quarter of a period, $Q \cos \omega t$ is again zero, and $H \sin \omega t$ has its maximum value in the opposite direction, giving the resultant OH' . Three-quarters of a period from the initial moment, $H \sin \omega t$ is zero and $Q \cos \omega t$ has a maximum negative value. Hence the resultant is OQ' . At the end of one period the resultant returns to its initial value. At intermediate times the resultant field is the sum of the instantaneous values of the two components, neither of which is zero, and so is in a direction intermediate between the directions of the components. It is shown for an instant in the first quarter period by OR . The ends R of all the instantaneous resultant fields during one cycle form the curve shown in the figure, which is called an ellipse. The resultant field represented by the vector OR sweeps out the area of this ellipse during one cycle of

values of the components, and is called an elliptical rotating field. When $\mathcal{H} \sin \omega t$ and $Q \cos \omega t$ are not at right angles, their resultant is still an elliptical rotating field, and this is also the case if the effect of $P \sin \omega t$ is taken into account.

It is obvious that when such a field is present there is no position of a rotating loop in which zero signal can be obtained. The effect of this ship field is therefore to blur the zeros.

It is frequently found in practice that the blurring of the zeros is most pronounced when the wave arrives on the beam, *i.e.*, on relative bearings of 90° and 270° , and is absent when the transmitter is exactly ahead or astern, *i.e.*, on relative bearings of 0° and 180° . The effect is therefore said to be **semicircular**, as opposed to the quadrantal error which has a maximum value in every quadrant and four zeros in 360° .

807. Quadrantal Correctors.—In order to obtain true bearings without the necessity for applying corrections for quadrantal deviation, a number of quadrantal correctors have been devised.

(1) **Bellini-Tosi Systems.**

- (a) One loop is set up fore and aft and the other loop athwartships. To counteract the effect of the ship, which tends to make the wave appear to be coming from a direction more nearly right ahead than it is, the fore and aft aerial is reduced in size until, with a transmitting station bearing 45° , the currents flowing in the two primary windings of the goniometer are equal. The goniometer then gives the true bearing 45° .
- (b) Equality of currents for a wave incident at 45° may also be obtained by adding impedance to the circuit of the fore and aft loop. This takes the form of two equal inductances, one in each leg of the aerial, but inside the office. They are adjusted by trial until the zero is at 45° .
- (c) A similar result may be obtained by shunting the winding of the goniometer connected to the fore and aft aerial by inductance. The larger the inductance the less the shunting effect, and it is possible to calculate the amount of inductance required to correct any given amount of deviation. The principal advantage of this method of correcting quadrantal error is that a defective contact in the shunt inductance can at most give an error equal to the deviation, whereas a defective contact in a series inductance of method (b) can lead to an error up to 90° .

(2) **Rotating Coils.**—The most usual practice is some form of cam corrector by which the pointer is made to lag or lead on the coil so that the readings given by the pointer are true bearings,

while the coil itself merely determines the apparent bearing. The advantage of a cam is that it can be so shaped as to correct any type of deviation due to the ship, whether it is of the simple quadrantal or a more complicated type.

808. The quadrantal deviation due to a ship is usually found to be constant for frequencies corresponding to wave-lengths longer than about five times the length of the ship, but increases appreciably on higher frequencies. For this reason it is necessary to adjust quadrantal correctors for the frequency in use at the time, and to find out by trial how the correct adjustment varies with the frequency.

In Bellini-Tosi systems having a long length of cable between the aerals and the office, the behaviour of quadrantal correctors is appreciably modified by the capacity of the cable. They will only behave in a normal manner if the circuit consisting of the aerial inductance and the cable capacity is very far from resonance.

809. Semicircular Correctors.—To compensate for semicircular effect it is necessary to introduce into the loop an E.M.F. equal and opposite to the ship field which blurs the zeros. From the previous analysis of this error it will be seen that this correcting E.M.F. must be 90° out of phase with the E.M.F. induced directly in the loop by the incoming wave.

The semicircular corrector may consist either of a vertical aerial or a loop aerial circuit.

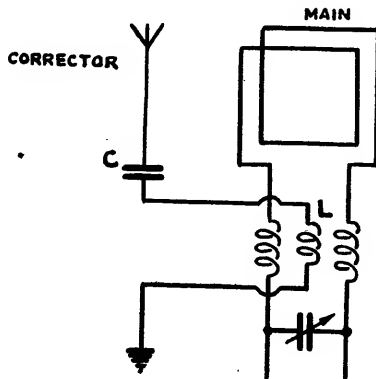


FIG. 495.

A vertical aerial corrector is shown in Fig. 495. The aerial is kept far from resonance by the series condenser C , and so the current flowing in the inductance L is 90° out of phase with the vertical aerial E.M.F., *i.e.*, 90° out of phase with the field of the wave. The mutually induced E.M.F. into the loop circuit is therefore in phase or antiphase with the field of the wave. The loop E.M.F. directly induced by the wave is 90° out of phase with the wave field.

Hence the correcting E.M.F. is 90° out of phase with the directly induced loop E.M.F.

The amplitude of the correcting E.M.F. is varied by adjusting the mutual coupling. The amount of coupling required varies with the direction of the wave, and usually, if it is positive for starboard bearings, it is negative for port bearings, being zero for bearings exactly ahead and astern.

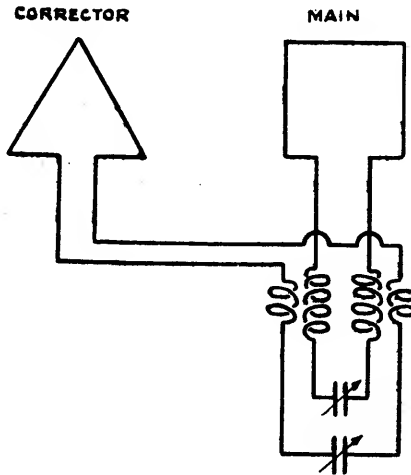


FIG. 496.

With a loop corrector, the E.M.F. induced by the wave is 90° out of phase with the wave, and so is in phase with the main loop E.M.F. Hence, the final correcting E.M.F. introduced into the main loop circuit must differ in phase by 90° from the E.M.F. induced in the corrector loop. Corrector loop aerial circuits may therefore be used which are similar to those already described for a vertical aerial sense-finding circuit (paras. 799 and 801), where the same change of phase of 90° in the original and final E.M.F.s is necessary, *i.e.*, the corrector loop circuit may be tuned by a condenser, as in Fig. 496, or a valve circuit may be used in which the external anode impedance is small enough to enable the anode current to remain approximately in phase with the grid voltage, as in Fig. 497 (*cf.* Figs. 482 and 484).

The symmetrical arrangement of the corrector loop circuits to prevent vertical effect should be noted. Such an effect in the corrector circuit would induce an E.M.F. in the main loop circuit which would be in phase (or antiphase) with the E.M.F. directly induced from the wave, and would therefore displace the zeros.

With a loop corrector placed athwartships, the correct mutual coupling is approximately constant for all directions of the incident wave.

For both aerial and loop correctors, the exact amount of coupling to obtain a perfect zero is critical; but if time does not permit of perfect adjustment, no error in bearing is produced by the slight residual blurring.

Semicircular effect blurs equally both the zeros (180° apart) obtained for any particular direction of the wave, *i.e.*, minima are obtained in both settings. Either of the above types of semicircular corrector converts one minimum into a true zero, while the

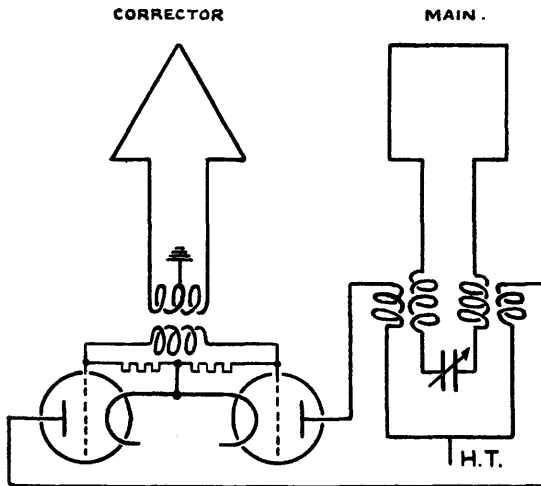


FIG. 497.

opposite minimum becomes more blurred than before; but it should be noticed that the perfect zero does not necessarily give the correct sense. With the beam loop type of corrector the sign of the coupling can be fixed so that the true zero does always give the correct bearing, but with the vertical aerial corrector this is not possible, and the sense has to be determined by other means.

810. Typical Receivers.—An early Service D/F receiver employing fixed loops is shown in Fig. 498. The variable inductances in the loop circuits for the final correction of quadrantal error should be noted. For any particular receiver, of course, inductance need only be included in one of the two loop circuits. In the D/F position shown, the centre points of the goniometer windings are earthed to minimise "vertical" effect. The sense position for this circuit is shown in Fig. 485. The search coil is connected to a tuner, the switches shown giving three possible tuner circuits. With the switch to the right, the search coil is directly in series with the variable coil *L* and the variable condenser *C*. This corresponds to the stand-by position in an ordinary tuner, and is unselective, but simplifies searching for signals over a range of frequencies. To increase the selectivity, the switch is made to the

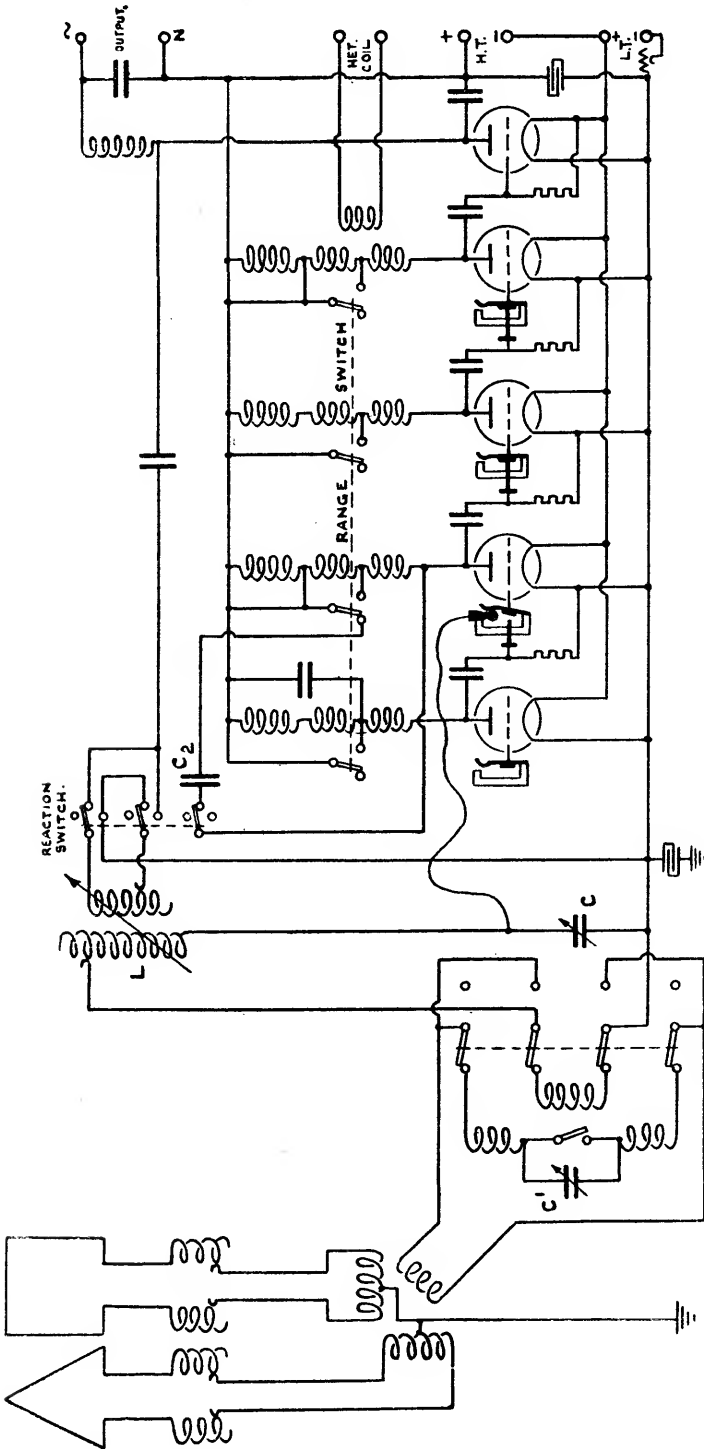


FIG. 498.

left. This introduces an intermediate linking circuit, mutually coupled to the LC circuit. The tuning of this latter circuit is unaltered, since the coupling coil takes the place of the search coil. The intermediate circuit may be tuned to increase strength of signals, or left untuned, according as the condenser C' is put in circuit or not.

Some points may be noted about the amplifier. The plug and jack arrangement enables any desired number of stages to be used. Reaction is supplied to the tuner from the output circuit of the last valve, and its phase changes according to the number of valves in use (para. 601). The reaction switch, by reversing the direction of the current through the reaction coil, enables the phase to be altered so as to give regeneration independently of the number of stages employed.

The amplifier output impedances are made to tune at different frequencies by the various short-circuiting arrangements shown, thus preventing peaky amplification and reducing the risk of self-oscillation. The whole amplifier, however, tunes at a certain frequency, and so the phase of the reaction reverses when this frequency is passed through. Searching for signals in the neighbourhood of this frequency would entail continual alteration of the reaction switch. To avoid this the reaction switch is made to its central position. This brings the condenser C_3 into the output circuit of the second valve, and alters the resonant frequency of the complete amplifier sufficiently to allow searching to be carried out without reversal of the reaction being necessary. For C.W. signals, the reaction may be increased to allow the amplifier to function as an autodyne receiver, or a separate heterodyne may be used.

A super-heterodyne D/F receiver is shown in Fig. 499. The reasons for earthing the frame coil at the centre, for the electrostatic screen between the coupling coils, and for the use of a push-pull valve arrangement, have previously been explained. Cumulative grid detection takes place in this first stage, capacity-controlled magnetic reaction being used to decrease the damping of the grid circuit. The output circuit of the push-pull stage is tuned to the supersonic frequency oscillation (about 150 kc/s.), which passes on to the I.F. amplifying stages, symmetry being preserved until the grid circuit of the first I.F. amplifying stage. This amplifier is actually the same as that shown in more detail in Fig. 498, and described earlier in this paragraph. The separate heterodyne necessary to produce the intermediate frequency beat by interaction with the incoming signal is of the familiar type described as tuned circuit between anode and grid, direct grid excitation, and series feed. It is carefully screened to prevent stray coupling to the detector grid circuit, and the heterodyne oscillation is fed to this circuit through the small condenser C_2 , a lead being taken from C_2 to the mid-point of the grid tuned circuit inductance. This symmetrical arrangement prevents any alteration in the grid tuning condenser C_1 from affecting the heterodyne circuit.

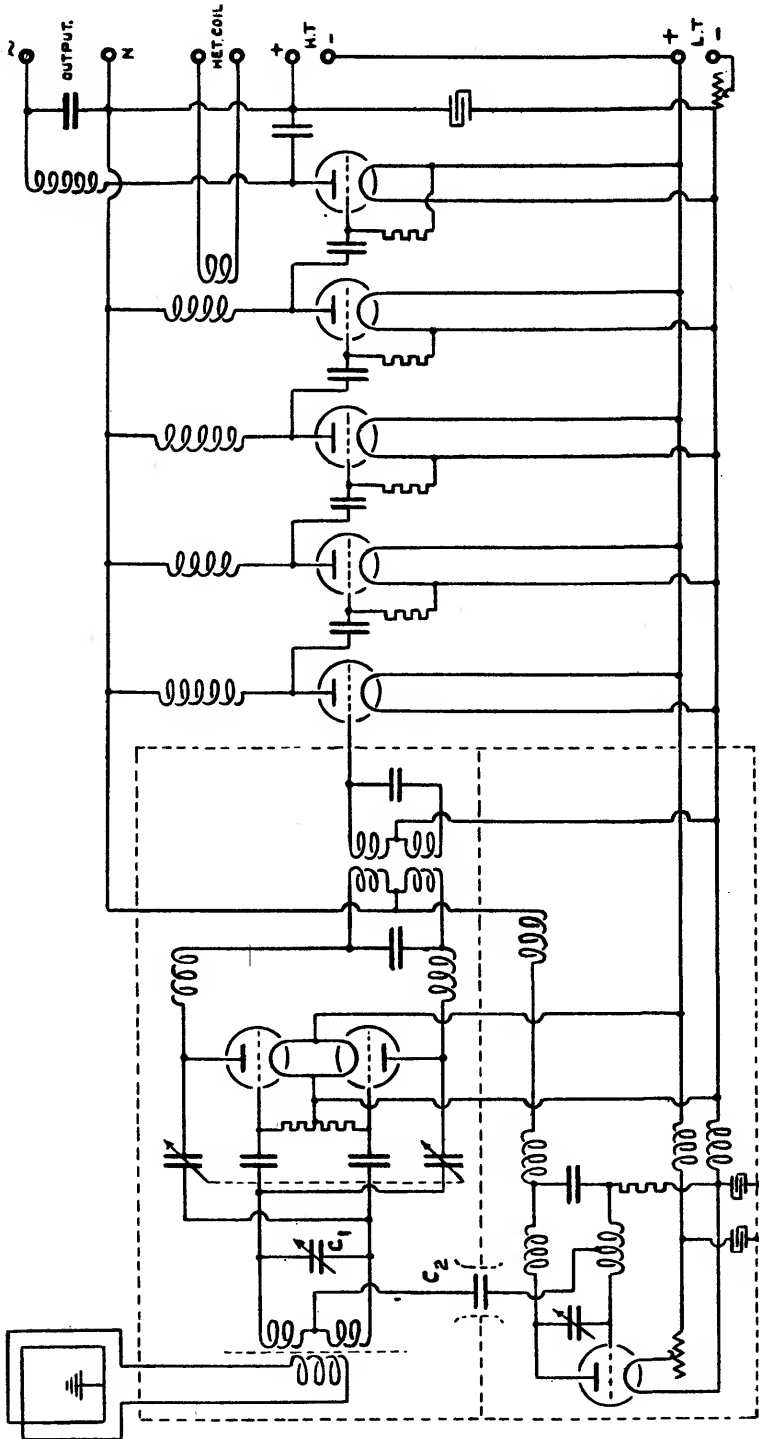


FIG. 499.

811. Calibration of a Ship Direction-Finder.—Prior to a calibration it is essential that tests should be carried out to ensure that the aerials and instruments are in perfect order. For this reason each type of direction-finder has a scheduled list of tests to be carried out before calibration.

For the calibration itself, the ship takes up a position three to five miles from a transmitting station, but without intervening land. The transmitting station is brought to bear right ahead, and the pointer is set so that the zero is obtained at 0° . The transmitting station is now brought to bear Green (starboard) or Red (port) 45° , and adjustments are made to the quadrantal correctors until a zero is obtained at Green or Red 45° . In this operation the true bearing of the station is obtained by visual bearings taken from the bridge at exactly the same moment as the wireless bearings are taken in the office. All bearings should be relative to the centre line of the ship, and should not involve the use of the ship's compass. A frequency commonly used for this work is 170 kc/s.

The station is then directed to transmit successively on a number of frequencies in order to obtain the appropriate quadrantal correction for each. The ship must be maintained throughout so that the bearing of the transmitting station is within 3° of Green (or Red) 45° . The next operation consists of a slow swing through 360° with the station transmitting on 170 kc/s., or on any other frequency of special interest to the ship. It is important that the speed of turning of the ship should not exceed 6° a minute, and, if time permits, it is preferable for the ship to be steadied every $22\frac{1}{2}^{\circ}$.

Direct comparisons of visual and wireless bearings are made for as many directions as possible, and if it is suspected that the curve of error changes appreciably with the frequency, it may be necessary to repeat the operation on several frequencies. If zeros are found to be blurred, use is made of a semi-circular corrector, and a record should be kept of the necessary coupling to obtain a perfect zero for each bearing. This amount of coupling is always the correct amount to use, and if it is found that a value very widely different is required on any subsequent occasion, then a source of error either in the instrument or due to night effect is to be suspected. Although the greater portion of the deviation is quadrantal in type, it is frequently found that small errors remain in certain directions, which have to be allowed for as a result of the calibration. A curve is issued after a calibration showing the corrections to be applied to the observed D/F readings in order to obtain true bearings. These are commonly given on the relative Red or Green scales, but in order to obtain true geographical bearings, it is necessary to allow for the ship's head. The formula is

$$\text{Relative Bearing} + \text{Ship's Head} = \text{True Bearing.}$$

In practice this calculation is avoided by fitting a scale to the direction-finder, which is controlled by the master gyro of the ship.

In effect, this scale remains fixed in direction as the ship turns, and it is set initially so that the reading opposite the zero of the relative bearing scale corresponds to the ship's head. Readings taken on the gyro scale then give true geographical bearings. It should be remembered that this scale is graduated from 0° to 360° , and that therefore the sign of the corrections on the port bearings is changed. It is very important that corrections of the right sign in accordance with the results of calibration should always be applied to observed bearings.

812. Sources of Error in Ship's Direction-Finders.—It has been said that all conductors in a ship have currents induced in them which are liable to affect the direction-finder. It follows, therefore, that the position and state of tuning of other aerials in the ship may have an important effect on any bearings which are obtained. This is particularly true of the main aerial when it is above the direction-finder, or if the direction-finder is in the vicinity of the feeders. Best readings are obtained on the lower frequencies when aerials are insulated. With the main aerial earthed, but far from resonance with the wave on which the direction-finder is working, there is usually a slight blurring of the zero. This becomes worse as the aerial comes near to resonance, and at resonance it is possible for a bearing to be very greatly in error.

It is sometimes possible to have in a ship a single wire aerial working on the same frequency as that to which the direction-finder is tuned, but it must be at a considerable distance in order to be safe. It is best to have it below the level of the direction-finder if possible.

It is important that calibrations should take place under the conditions which obtain in practice, and that, whenever bearings are taken, these conditions should be strictly fulfilled as regards the position and tuning of all other aerials.

It sometimes happens that the gyro scale, which is fitted either to the goniometer or to the training unit of the frame coil, becomes out of step with the master gyro of the ship. It should therefore be checked frequently—say once each watch—to ensure that no error has arisen from this cause. Sometimes signals are so weak that it is not possible to read the position of the zero directly on the scale. Recourse may then be had to an angle-dividing device which is fitted to all goniometers and training units. Some practice is needed to attain proficiency in judging the two positions in which the signal is of equal intensity, especially if interference is present; for the interference will probably not be of the same strength for two positions of the coil equally inclined to the direction of the signal whose bearing is required. The angle-divider is intended to reduce computation, and it should be remembered that the true bearing is always given by the pointer, without calculation, even when a cam corrector is fitted.

With Bellini-Tosi aerials consisting of single wire loops, it is important that the positions of these loops should be fixed very definitely immediately after a calibration, and that whenever it becomes necessary to rig a fresh aerial, steps are taken to ensure that the new aerial is in exactly the same position as the one used during the calibration. Whenever unusual results or large errors are found at a subsequent date, the appropriate tests for the model in use should be carried out.

813. Best Position for a Direction-Finder in a Ship.—From an electrical point of view, a direction-finder should be situated as high above the hull of the ship as possible. This tends to reduce the amount of quadrantal deviation, but regard has to be paid to the position of aerials and other conductors such as main aerial halyards, lightning conductors, lead-cased cables for masthead lights, etc., all of which are capable of acting as vertical aerials. The currents in these conductors have the effect of blurring the zeros. Thus the best position for a direction-finder is at the top of a mast, above the level of the roof of the main aerial and above all other conductors. When this is not possible, every endeavour should be made to obtain a position which is as symmetrical as possible both about the centre line and athwartships, *e.g.*, Bellini-Tosi aerials between two equal vertical funnels are superior to aerials abaft a single funnel. The same consideration should be borne in mind in the rig of the aerials themselves, so that they may be as symmetrical as possible with respect to surrounding structures, and about the vertical line.

In general, of course, the position of the D/F aerials cannot be settled entirely by electrical considerations, but regard must be paid to questions of vulnerability, and the positions of the main and auxiliary wireless offices. This sometimes leads to the aerials being at a considerable distance from the office. Technically this is bad, and every effort should be made to reduce the length of cable which has to be run from the aerials to the office. Even with the lowest capacity paper-insulated cables in use at the present time, every 100 ft. represents an effective capacity across the loop or frame of 0.5 jar, and a capacity to earth of 2 jars, which sets a serious limit to the highest frequency on which bearings can be taken.

Whether it is better to fit a rotating coil or a Bellini-Tosi system depends upon the circumstances which obtain in the particular ship under consideration. Where space is available, a Bellini-Tosi system, with large aerials on the centre line and approximately amidships, is recommended; otherwise, it is better to use a rotating coil at the top of the mast. Where the office can be placed immediately under the aerials, it may be possible to take bearings on the higher frequencies (above 1,000 kc/s.). In general, it is possible to work more quickly with a Bellini-Tosi system than with a rotating coil, and this gives it an advantage for tactical purposes.

As the frequency increases, the errors due to the ship likewise increase rapidly in amount and complexity. Their correction becomes correspondingly more difficult, and in every ship installation there is a limiting frequency (varying according to the class of ship and the nature and position of the direction-finder), above which reliable D/F bearings cannot be obtained.

814. Applications of D/F.

(1) **As an Aid to Navigation.**—For this purpose bearings should be taken of stations on the coast so that the transmission is, as far as possible, entirely over sea. The nearer the station the better, for an error of bearing then gives the least error in position, and, in addition, the indirect ray will be weak compared with the direct ray. In general, an attempt should be made to obtain bearings of stations not more than 100 miles distant. At greater distances it may be necessary to apply a correction to allow for the fact that a wireless wave follows the great circle path of shortest distance, and not the straight line path on a Mercator chart, which is a rhumb line.

This correction is equal to half the "convergency" of the two meridians of longitude passing through the transmitter and the receiver. The convergency is the difference between the angles which the great circle passing through the transmitter and the receiver makes with their respective meridians.

The "half-convergency" correction is given in magnitude by the expression

$$\frac{1}{2} \text{ d. Long.} \times \sin \text{ Mid. Lat.}$$

Its sign is determined by the following rule.

To obtain the true bearing from the D/F bearing :—

- (a) In the Northern Hemisphere, add the correction to the D/F bearing for bearings between 0° and 180° , and subtract it for bearings between 180° and 360° .
- (b) In the Southern Hemisphere, subtract the correction from the D/F bearing for bearings between 0° and 180° , and add it for bearings between 180° and 360° .

In many countries beacon stations are maintained on 300 kc/s. for the express purpose of assisting navigation. These stations give such a type of transmission, and are so situated, as to be specially suitable for taking D/F bearings. They should therefore be used for navigational purposes whenever possible.

It is probable that a shore D/F station, suitably placed, will be able to achieve a greater accuracy than is possible with a direction-finder in a ship, owing to the fact that conditions at a shore station are more favourable for setting up the best possible aerial system ; but as there are comparatively few shore D/F stations, it is not always possible for satisfactory three-point fixes to be obtained from them. A ship direction-finder, however, can make use of any transmitting station which happens to be working. As there are

usually a large number of such stations, including special beacon stations for D/F, it is possible to obtain bearings in many different directions to act as checks on each other.

(2) **Safety of Aircraft.**—In this case the important thing is to obtain an approximate bearing with absolute certainty of "sense." Errors have sometimes been found to be due to the direction of the trailing aerial of the aircraft making the transmissions. This error is a maximum when the aircraft is travelling at right angles to the line joining the direction-finder to the aircraft. It is also frequently found that zeros are blurred when the aeroplane is in the immediate vicinity of the direction-finder, so that the waves come down at an angle. Such errors can, of course, be reduced if there is sufficient room to fit an Adcock direction-finder.

(3) **Checking the Reported Position of Reconnaissance Aircraft.** This is sometimes necessary under conditions where the aircraft have considerable difficulty in knowing their own position exactly when sighting the enemy.

(4) **Bearings of Enemy Aircraft and Ships.**—This is an obvious application, but perhaps the most difficult of all, for it is clear that the enemy will always use the minimum power necessary for his own purpose and therefore that the signals will probably be weak. A further difficulty is that it is essential to obtain not only the bearing but also sufficient information to identify the nature of the enemy.

DIRECTIONAL TRANSMISSION.

815. Principle of Spaced Aerials.—Consider two transmitting aerials A and B (Fig. 500) at a distance apart equal to half a wavelength. Suppose that both aerials are transmitting at the same time on the same frequency; the effect they produce at any point C will be the sum of the effects due to the separate aerials. If the alternating currents in the aerials are in the same phase, it is clear that at a point such as D on the line through AB, the wave from A will, at every instant, be cancelled by the wave from B, because the distance DA differs by half a wavelength from the distance DB. At any point, such as P on the line at right angles to AB, and through a point O midway between them, the separate waves from A and B will reinforce each other. Thus a maximum of radiation takes place along OP, and no radiation takes place along OD. At any other point C the radiation will be the resultant of a component from A and a component from B, the phases of the components depending on the difference of time taken by the wave to travel from A to C and from B to C.

This is the simplest example of the use of two transmitting aerials to produce directional transmission. In order to concentrate a large percentage of the total energy in one direction, it is necessary

to employ a number of radiating aerials spaced over a distance of several wavelengths. The currents in these aerials must be maintained in a definite phase relation to each other, though not necessarily all in the same phase. Many different arrangements of aerials have been used to give directional transmission. Theoretically, they all depend upon summing up the separate effects due to each element of current in the "array," with due regard to its phase and distance from the point under consideration. It may also be necessary to take into consideration the effect of the conductivity of the sea or earth immediately under the aerials. This modifies the radiation appreciably.

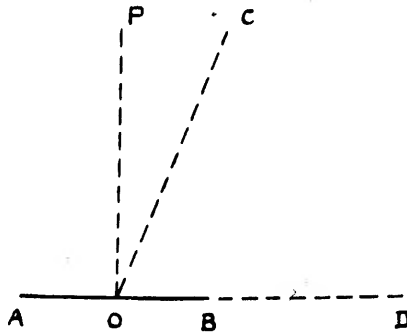


FIG. 500.

The currents flowing in the several aerials may be derived directly from a generator, or some of them may be the result of induction from other aerials carrying currents. In the latter case the wires carrying the induced currents are called reflector wires, and it is obvious that they must be located very carefully, relative to the main current-carrying wires, in order to get the desired relation of phase between the main and induced currents. At present this is done rather by trial than by calculation.

In the Marconi Beam Stations a large number of vertical aerials are placed in a line at about half a wavelength apart, with a reflecting system consisting of a parallel line of vertical aerials about a quarter of a wavelength behind. The currents in the main aerials, which are several half-wavelengths high, are in the same phase. This array gives a very considerable concentration of energy in a direction at right angles to the line of aerials, and on the opposite side from the reflectors; but since communication at very great distances depends entirely on the indirect ray, directive radiation of the ground ray is of no importance. The essential requisite is concentration of energy at the correct angle upwards as well as on the right bearing, and this problem cannot be considered as completely solved at the present time. It appears from experience that directional transmission can assist in reducing fading

(para. 739), and also in reducing interference to other lines of communication, but a considerable amount of scattering of the beam in the upper layers of the atmosphere does occur.

To produce a beam so sharp that it can be used for navigational purposes, it is necessary to use frequencies of 30,000 kc/s. or more. One of the first of such beacons was installed at Inchkeith by the Marconi Company, and had a single wire vertical aerial transmitter, with a parabolic reflector which could be rotated about this aerial. The reflector consisted of a number of vertical wires in the form of a parabola, with the transmitting aerial at the focus. The beam could be made to revolve by rotating the reflector system about the transmitting aerial as axis. Although the width or aperture of the parabola was only about two wavelengths, the beam was sufficiently sharp to give an angular accuracy of 2° – 3° at a range of ten miles. Its working range was, however, rather restricted, and it had the disadvantage that a ship making use of the beam needed a special receiver suited to the very high frequency in use.

Beams working on frequencies of the order of 80,000 kc/s. have been used in America for assisting aircraft to make good landings in fog.

816. Radio Beacons.—If an alternating current is made to flow round a loop, a portion of the energy will be radiated in the form of electric waves. The maximum radiation is in the plane of the loop, while in the plane at right angles to the loop there is zero radiation. Thus, if such a coil is made to rotate clockwise through 360° , say once a minute, the direction of zero radiation also turns through 360° . Now suppose that the plane of the coil is due East and West when the revolution commences, and a signal is sent out, such as the letter "N," to indicate the exact instant at which the coil starts to rotate. Imagine an observer at a point bearing 135° (true) from the rotating coil receiving signals on an ordinary open aerial. At the instant at which he hears the letter "N" he starts a stop watch. The signal gradually increases in strength until, after $7\frac{1}{2}$ seconds, it reaches a maximum. It then commences to decrease, and at the end of $22\frac{1}{2}$ seconds the rotating coil is at right angles to the direction of the receiver, and a zero is obtained. The signal again commences to increase, reaches a maximum, and once more returns to zero after a total time of $52\frac{1}{2}$ seconds from the commencement of the revolution. It is clear that by taking the time at which the zero is obtained in the receiver, the bearing of the transmitter can be obtained. If the zero occurs t seconds after the commencement of the swing, then the bearing is $6t^{\circ}$ or $(6t + 180)^{\circ}$,

since the rate of rotation is 6° per second.

It is only possible to say which of the bearings is the correct one, provided sufficient is known of the probable direction of the transmitter, for at present no arrangements are made to remove

the 180° ambiguity. In case the letter "N" should not be heard because the receiver is due North or South of the beacon, a second signal is sent out when the plane of the coil is North and South.

This system of directional transmission, in which there is some radiation in every direction but one, has the great advantage that it can be used at medium frequencies, and demands no special apparatus at the receiving end. An ordinary open aerial and receiver is all that is necessary, together with a stop watch. As the time of the revolution of the beacon is maintained very accurately at 60 seconds, it is possible to check the rate of the stop watch. This should always be done and allowed for in the calculation of the bearing. For example, if the zero is obtained 20 seconds after the commencement of a revolution, but the time of revolution appears to be 61 seconds by the stop watch, the true bearing is not 120°, but

$$\frac{20}{61} \times 360 = 118^\circ.$$

In order to get sufficient radiation from a coil of moderate dimensions it is necessary that it should carry very large currents. For example, one beacon at present in use has four turns on a frame 10 ft. \times 11 ft., and carries a current of 80 amperes, at a frequency of 300 kc/s.

Instead of a rotating coil, two large loops at right angles can be coupled to a transmitting goniometer. Fixed loop transmitters of this type are used in America and in France to assist aircraft to fly on a prescribed course in bad weather.

In one system a letter A is heard when the aircraft is to the right of the course, and a letter N when it is to the left of the course. These signals are so timed that when the aircraft is on the course they coalesce, and produce a continuous dash.

In another system a directional transmitter is used, having two different frequencies of modulation, and visual indication is obtained in the aircraft by means of two reeds tuned to the modulation frequencies after the manner of a frequency meter. The pilot observes the amplitudes of vibration of the two reeds. On the course the amplitudes are equal. Off the course they are unequal, the reed vibrating with the greater amplitude being on the side to which the aircraft has deviated.

***817.** To understand how this may be achieved, consider two loops Z and Y (Fig. 501) which are radiating in phase on a 300 kc/s. wave. The radiation from Z is modulated at 60 cycles per second, and the modulation from Y at 80 cycles per second. At a point distant r from the point O, and bearing θ° from North, the electric field is of the form

$$x = \mathcal{E} \sin \omega t \{ \cos \theta (1 + k \cos n_2 t) + \sin \theta (1 + k \cos n_1 t) \}$$

$$\text{where } \omega = 2\pi \times 300,000$$

$$n_1 = 2\pi \times 60$$

$$n_2 = 2\pi \times 80$$

The E.M.F. applied to the amplifier is also of this form, and, after rectification, the low frequency output is approximately proportional to x^3 . The components of the output current which are capable of affecting the tuned reeds are the terms in $n_1 t$ and $n_2 t$.

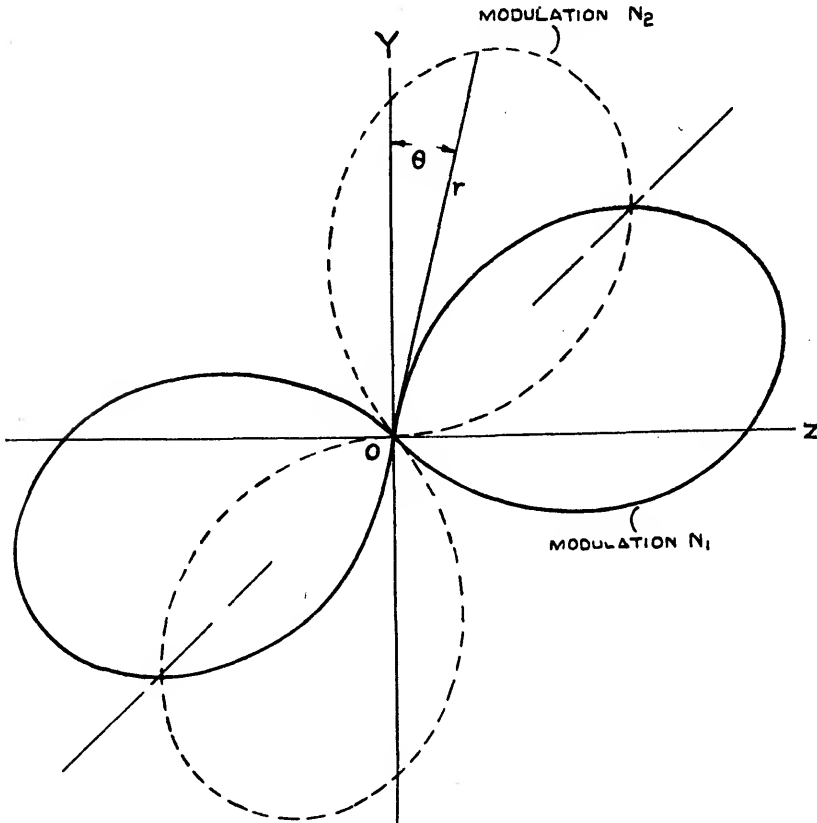


FIG. 501.

The amplitude of the $n_2 t$ term is proportional to $\cos\theta (\sin\theta + \cos\theta)$,
and that of the $n_1 t$ term is proportional to $\sin\theta (\sin\theta + \cos\theta)$.

The relative amplitudes of these terms are shown in Fig. 501.

In the plane bearing $022\frac{1}{2}^\circ$ from North, the 60-cycle modulation is a maximum. In the plane bearing $067\frac{1}{2}^\circ$, the 80 cycles modulation is a maximum. In the plane 045° , the two terms are equal, and on this course the reeds will vibrate with equal amplitude. At right angles to this there is no radiation.

Secondary maxima of amplitude about one-sixth of the main radiation occur at $112\frac{1}{2}^\circ$ and $157\frac{1}{2}^\circ$, but these will not generally cause any confusion, owing to their relative weakness.

In the above example the course is at 045° . *i.e.*, midway between the aerial loops, but by modifying the relative amplitudes and phases of the radiations from the several aerials, it is possible to put the course in any direction required. The system has the advantage that the curves for n_1 and n_2 cross at a point where the amplitudes of the two components are changing comparatively rapidly. Thus the arc over which both reeds vibrate with the same amplitude is a very narrow one, and immediate warning is given to the aircraft that it has deviated from the correct course.

Directional transmitters are not at present used in H.M. Ships, but aerial arrays with directional properties are used at shore stations for transmission and reception over great distances at very high frequencies.

The radio beacon is just as liable to night error as the rotating coil direction-finder, and, theoretically, an improvement should be obtained by using the transmitting equivalent of the Adcock receiver; but this is at present only in an experimental stage.

CHAPTER XX.

THE WAVEMETER.

818. The measurement of frequencies of the order used in the A.C. supply to valve filament heating circuits, rectifiers, spark transmitter charging circuits, &c., has already been described in Chapter VI, paras. 375 and 376. In this chapter the methods employed to measure accurately the radio-frequencies produced in wireless transmitters and receivers will be considered. Instruments for this purpose are called "wavemeters."

819. The measurement of wireless frequencies may be effected in several ways:—

- (a) **Absorption Wavemeters.**—These consist essentially of a closed oscillatory circuit in which either the inductance or the capacity can be varied to cover a range of frequencies, together with some device for indicating when the current in the circuit is a maximum.

Such wavemeters absorb a certain amount of power from the source; hence their name. In the Service the title "absorption" is usually dropped and these instruments are simply called "wavemeters." They are suitable for measuring the frequency of medium-power and high-power transmitters, but are unsuitable for use with low-power transmitters or with receivers of the self-oscillatory type.

- (b) **Heterodyne and Self-quenching Oscillators.**—These are instruments comprising a valve and an oscillatory circuit and capable of producing C.W. or I.C.W. oscillations over a range of frequencies determined by the constants of the oscillatory circuit. They can be used for measuring the frequency of either low or high-power transmitters and also for oscillating receivers. For this purpose the self-quenching oscillator is used in conjunction with an absorption wavemeter. When used for tuning receivers, these instruments are called "wave-testers" in the Service.

- (c) **Multivibrators.**—These are instruments comprising valves with associated circuits arranged to give a type of oscillation very rich in harmonics. The series of harmonics provides a number of known frequencies covering a wide range. The fundamental of the series may be determined by the frequency of a tuning fork or by that of a quartz crystal, and hence the method is

applicable at both low and high frequencies. For obvious reasons such apparatus is unsuitable for general use, though it is capable of giving the extremely high accuracy required in wavemeter calibration work. It will not be further considered in this book.

ABSORPTION WAVEMETERS ("WAVEMETERS").

820. If an alternating E.M.F. of constant amplitude is applied to a circuit consisting of inductance, capacity and resistance in series, then the current has its maximum value when the circuit is tuned to resonance with the applied E.M.F. The resonant frequency " f " in kc/s. is connected with the inductance L in microhenries, and the capacity C in jars by the formula

$$f = \frac{3 \times 10^4}{2\pi\sqrt{LC}} \text{ (para. 299).}$$

A wavemeter thus consists essentially of a closed oscillatory circuit in which either the inductance or the capacity can be varied to give a range of accurately-known frequencies and a device for indicating when the current in the circuit is a maximum. According to the manner of its operation, this indicating device may be either in series or parallel with the oscillatory circuit. The E.M.F. is usually obtained by **loose** magnetic coupling between the wavemeter circuit and the oscillator whose frequency it is desired to measure. It will be remembered (para. 334) that if two oscillatory circuits are tightly coupled together, a double-frequency effect is produced; hence the importance of **loose** coupling for wavemeter measurements. With tight coupling, the large currents produced may also damage the indicating device.

The frequency at resonance may be read off directly on a scale attached to the instrument, or from the calibration curves issued with the wavemeter.

In one early type of Service wavemeter the calibration curves give the LC value, from which the frequency can be calculated or read off on an "abac" at the end of the calibration book.

821. Condensers and Inductances.—In a condenser with semi-circular plates the capacity is proportional to the angular overlap of the moving plates and the fixed plates, so that equal divisions on the scale correspond to equal changes in LC value of the wavemeter circuit. The resonant frequency is, however, inversely proportional to the square root of the LC value, and so on the lower condenser settings, *i.e.*, on the lower scale readings or parts of the calibration curves, frequencies are crowded together compared with the higher settings, and interpolation is rendered difficult. Consequently, the plates for a modern wavemeter condenser are shaped so that equal changes in overlap give equal changes in frequency. This

gives equal divisions on a scale calibrated in frequencies, or a straight line calibration curve of frequency against condenser setting, as the case may be, and more accurate interpolation is possible.

In earlier instruments, condensers may be found either with semi-circular plates or plates shaped so as to give a linear calibration curve of condenser setting against wavelength.

The condenser is generally the variable component because of the difficulty of constructing variable inductances which will preserve their values unchanged. This is mainly due to "dead-end" effects (para. 382).

When a wavemeter is required to cover a large range of frequencies, this is usually effected by supplying a number of interchangeable coils, whose inductances are selected so that the LC values obtained with any one coil and a high value of the variable condenser may also be obtained with the next coil in the series and a low value of the variable condenser. In this way a continuous range of frequencies is covered.

The self-capacity of the coils is taken account of in the calibration of the wavemeter, and care is taken that the resonant frequency of the coil itself is considerably above the range of frequencies over which it is to be used.

822. Indicating Devices.—The device for indicating when the current in the wavemeter circuit is a maximum may take several forms. The commonest are :—

- (a) A lamp.
- (b) A neon tube.
- (c) A thermo-junction and a galvanometer.
- (d) A two-electrode valve and a galvanometer.
- (e) A crystal and a pair of telephones.

(a) A low-voltage lamp may be inserted in series in the wavemeter circuit, or in a separate circuit mutually coupled to the main wavemeter circuit. The coupling of the wavemeter to the oscillating

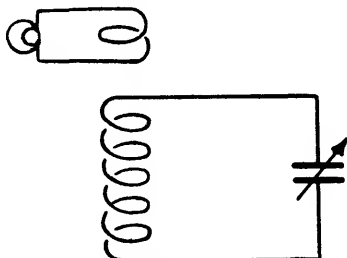


FIG. 502.

circuit under test should be so adjusted that the lamp just glows when the wavemeter current is a maximum. Sometimes a battery is inserted in the circuit to bring the lamp up to a point very near

glowing, so that only a small current and loose coupling are necessary to bring it to indicating point. Fig. 502 shows a simple wavemeter circuit, with a mutually coupled lamp as indicating device.

(b) The **neon tube** is connected in parallel in the wavemeter circuit, *i.e.*, across the condenser. It depends for its action on the fact that, when sufficient potential difference is impressed across a tube containing neon gas at low pressure, ionisation occurs, and the gas glows. When maximum current flows in the wavemeter circuit, the potential difference across the condenser is a maximum, and hence the neon tube serves as an indicator of maximum current.

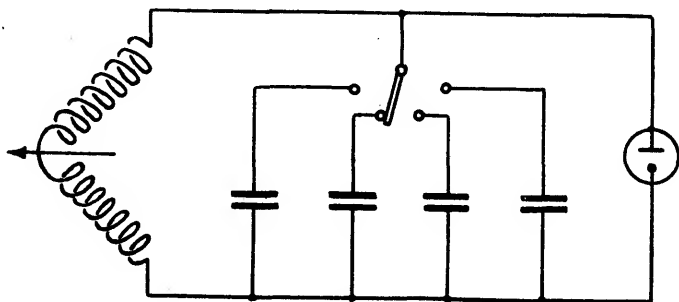


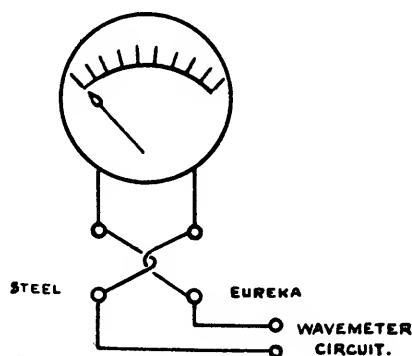
FIG. 503.

Fig. 503 shows a wavemeter circuit using a neon tube as indicating device. The variable component in this circuit is the inductance, which is a variometer, and so avoids the difficulty of variable tapping points. The range is altered by switching in different fixed condensers.

(c) **Thermo-junction and Galvanometer.**—If two wires of certain dissimilar metals are joined together, it is found that an E.M.F. is developed across the junction. This is probably due to the different concentration of the free electrons in the two metals, which tries to equalise itself across the junction. If the other ends of the metal wires are also joined so that a complete electrical circuit is formed, an equal E.M.F. will be developed at the second junction; but as the two E.M.F.s are in opposite directions round the circuit, there will be no resultant E.M.F., and no current will flow round the circuit. The E.M.F. produced in this way, however, depends on the temperature of the junction, and, within certain limits of temperature, the size of the E.M.F. is found to be roughly proportional to the temperature. If one of the two junctions be now heated so that it is kept at a higher temperature than the other, the E.M.F. developed at the hotter junction is greater than that at the colder junction, there will be a resultant E.M.F., and a current will flow round the circuit. This is called the "Seebeck Effect," after its discoverer.

It is found that the resultant E.M.F. is unaffected by including a wire of some third metal as part of the circuit, and so a galvanometer, for instance, may be inserted in the circuit without altering the value of the resultant E.M.F. The extra resistance will, of course, alter the value of the current produced, and, as the thermoelectric E.M.F. is itself small, the galvanometer which indicates the size of the current should have a low resistance.

The utilisation of this effect in an indicating device is shown in Fig. 504. Two fine wires of dissimilar metals (usually steel and eureka) are interlinked, and lightly soldered at the junction. One pair of ends of the two wires is joined through a sensitive galvanometer, and the thermo-junction is connected in series with the wavemeter circuit by means of the other pair.



Thermo-Junction and Galvanometer.

FIG. 504.

The necessary rise in temperature of the thermo-junction is provided by the heating effect of the radio-frequency current flowing round the wavemeter circuit. The heating effect is proportional to the square of the current, and so the rise in temperature of the junction is greatest when the current in the wavemeter circuit has its maximum value, *i.e.*, when the wavemeter circuit is tuned to resonance with the oscillatory circuit whose frequency it is desired to measure. In consequence, the reading of the galvanometer is greatest at this setting of the wavemeter condenser.

The current through the galvanometer is a direct current, due to the unidirectional thermo-electric E.M.F. produced at the hot junction. It should be clearly understood that no radio-frequency current flows in the galvanometer which, as regards such current, is short-circuited by the junction.

As the accuracy of this method depends on the difference in temperature between the hot junction and the rest of the circuit, the hot junction should be protected from draughts. To increase the sensitivity the thermo-junction may be enclosed in an evacuated

glass bulb. The minimum loss of heat by conduction and convection will then be experienced, and the greatest rise in temperature for a given heating effect will be obtained.

(d) A wavemeter circuit employing a **two-electrode valve and galvanometer** as indicating device is shown in Fig. 505.

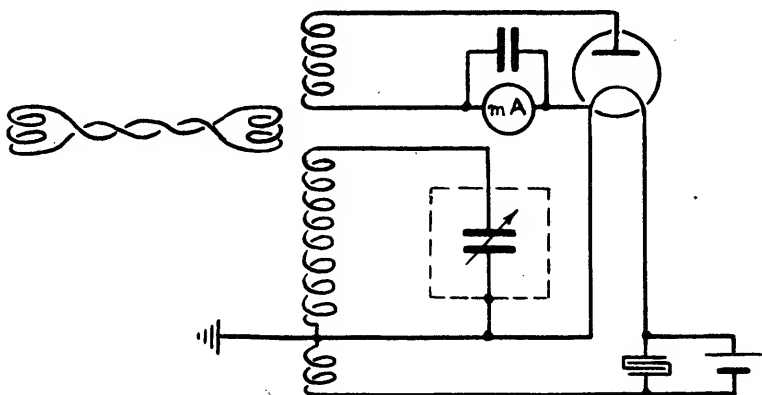


FIG. 505.

The diode acts in its usual manner as a rectifier, *i.e.*, current only flows when the anode is positive to the filament. The D.C. milliammeter in the external circuit from anode to filament reads the mean value of the anode current pulses. This will have its maximum value when the P.D. between anode and filament is a maximum, provided that saturation current is not reached.

The E.M.F. is introduced into the wavemeter circuit by an untuned intermediate coupling circuit. Maximum current flows in the wavemeter circuit when it is tuned to this E.M.F., and the maximum flux then produced induces maximum E.M.F.s in the coils which give the anode voltage and filament current. When the filament current produced by this means is insufficient, a cell is provided which may be switched into the filament heating circuit.

(e) When the frequency of an I.C.W. transmitter is being measured, a **crystal and pair of telephones** in parallel with the wavemeter condenser may be used as indicating device. The loudness of the sound in the telephones gives a criterion of resonance. This method is not used in Service wavemeters, as it cannot give accurate results. A special use of a pair of telephones as indicating device, depending on a change in the pitch of the note heard in them, is described below (para. 846).

823. Comparison of Indicating Devices :—

- (a) Lamp. Capable of moderate accuracy ; cheap and robust.
- (b) Neon tube. Capable of moderate accuracy ; fairly cheap and robust.

(c) Vacuum thermo-junction and galvanometer. Capable of high accuracy; expensive and fragile.

(d) Diode. Capable of high accuracy; fairly cheap and robust.

(e) Crystal. Not capable of even moderate accuracy; simple and cheap.

824. Accuracy of Wavemeters.—The overall accuracy of the frequency determined by a wavemeter circuit may be considered under four heads:—

- (1) Constancy of components.
- (2) Constancy of amplitude of E.M.F. induced in wavemeter circuit.
- (3) Sharpness of tuning of wavemeter circuit.
- (4) Accuracy in estimating maximum current and in reading scale.

825. Constancy of Components.—It is not in general necessary that the inductance and capacity of a wavemeter circuit should be accurately known, since the ordinary wavemeter is calibrated as a circuit by comparison with a standard. The question of how the standard is obtained will not be considered here. The important point is that the values of the components should not vary from those they possessed in the circumstances under which the calibration took place. This is rendered difficult by the necessarily small values of the coils and condensers which must be used in wavemeters to reach small enough LC values. The inductance and capacity of the indicating device, between leads, and between the circuit and its surroundings, thus become important in spite of their small values.

With fixed inductances most of these effects can be kept constant as far as the total inductance of the circuit is concerned.

When the mutual coil is connected to the wavemeter circuit by flexible leads, the length of these leads should not be altered, nor should they be gathered in a coil for convenience. By keeping them extended at full length their inductance should be the same under all conditions. In modern wavemeters, to prevent such alterations, the leads are made stiff (as indicated in Fig. 506), variations in the coupling being obtained by rotating the coupling coil on a fixed frame.

For an accurate instrument the use of a coupling coil with long flexible leads is most undesirable; it is only permissible in a second-grade instrument employed in measuring low and intermediate frequencies.

The capacity presents a more difficult problem. Changes in the value of the variable condenser itself, owing to mechanical wear, may be obviated by good design and construction, but the effect of the stray capacities to earth of the various parts of the circuit is more difficult to deal with, and generally constitutes the final

limitation upon the accuracy of a wavemeter. Every point not at earth potential in the wavemeter circuit has a capacity to earth, and this may be large enough to be cumulatively appreciable.

Such stray capacity varies with the position of the wavemeter circuit with regard to its operator (hand capacity), other apparatus in the room, and the walls of the room itself.

As such stray capacity cannot be avoided, the principal object aimed at is to keep its value constant and independent of the position of the wavemeter with respect to neighbouring objects. This may be achieved to a large extent by suitable screening. The stray capacity is then almost entirely capacity to the earthed metal screen. Such screens are shown by dotted lines in the diagrams of wavemeter circuits in this chapter.

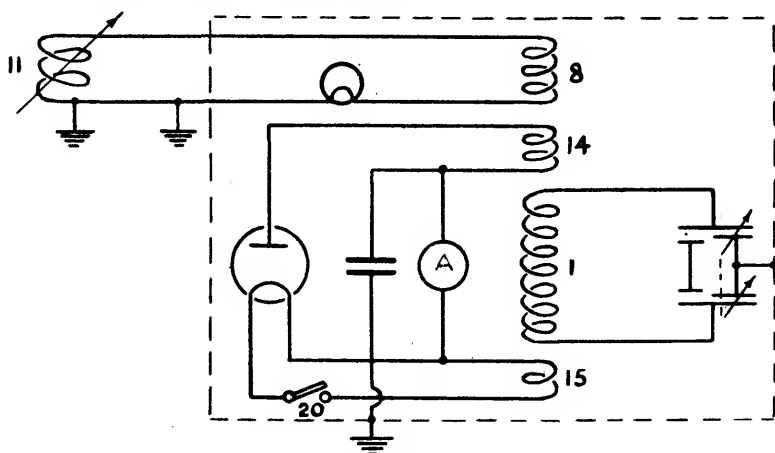


FIG. 506.

Hand capacity is avoided in the circuit shown in Fig. 506 by a cylindrical condenser of special construction. The spindle which rotates the moving plates is connected to the earthed screen, and so the handle is always at earth potential.

The capacity of the indicating device must also be taken into account. The neon tube, for example, in Fig. 503 is in parallel with the wavemeter condenser in use. This would be taken account of in calibrating the set, and would not matter if it remained constant. Its capacity, however, depends on the distribution of electric field between the electrodes, and this varies with the brilliancy of glow.

The effect of this on the frequency will be most marked when the minimum wavemeter condenser setting is being used, for the neon tube capacity is then a larger proportion of the total capacity. The largest possible condenser setting should always be used in such a wavemeter. Further, the glow at resonance should be adjusted to the same brilliancy as it had under the conditions of its calibration.

This is generally the minimum glow that can be observed. The adjustment will be effected, of course, by varying the coupling between the wavemeter coupling coil (the variometer in Fig. 503), and the source of the oscillation whose frequency is required.

Similar remarks apply to other indicating devices. The lamp, for instance, is normally at minimum brilliancy when calibration is carried out, and should be adjusted so that it just lights up at the resonant frequency when measurements are being made.

826. Constancy of E.M.F.—It is important that the only E.M.F. acting in the circuit should be that induced in the mutual coil. The screening of the main inductance and connecting leads in the wavemeter circuit should be sufficiently complete for this to be the case.

827. Sharpness of Tuning.—The accuracy with which the resonant frequency may be determined depends on how sharply the current falls on either side of its peak value at resonance, *i.e.*, on the selectivity of the wavemeter circuit.

Selectivity is governed by two considerations (para. 509):—

- (1) The ratio of inductance to capacity.
- (2) The total resistance or damping.

The greater the L/C ratio, the more pronounced is the resonance peak of the current-frequency curve (*cf.* Fig. 137). It is undesirable, however, to increase this ratio by decreasing the capacity too much on account of the effect of stray capacity discussed above, and improvements in selectivity are generally obtained by decrease of damping. The resistances in the circuit are the high frequency ohmic resistances of the inductance, connecting leads and indicating device, and the equivalent resistance of the power losses in the condenser. With inductances and leads of stranded wire, and air condensers, the principal cause of damping is the indicating device. This damping may be reduced by arranging the indicating device in a separate circuit loosely coupled to the main wavemeter circuit, as shown in Fig. 502. The equivalent resistance of the lamp is then less than its actual resistance by a factor depending on the coupling.

A wavemeter in which more elaborate precautions have been taken to decrease damping is shown in Fig. 506. The indicating device is a diode and galvanometer.

The two coils (14) and (15) which give respectively the anode-filament P.D. and the filament heating current are mutually coupled to the main wavemeter inductance (1). Coil (8), wound on the same former, is connected via a lamp to coil (11), in which an E.M.F. is induced by magnetic coupling from the circuit whose frequency it is desired to measure.

Maximum current will be produced in the tuned circuit when it is in resonance with the applied E.M.F., and this in turn will induce maximum E.M.F.s in coils (14) and (15), hence giving a maximum reading on the milliammeter.

The lamp is not, of course, the indicating device. It merely provides an indication of the current flowing in the wavemeter circuit so that the valve filament will not be damaged by too great a heating current. The coupling of coil (11) should be loosened until the lamp ceases to glow. It will be seen that opening the switch (20) protects the filament circuit while this adjustment is being made.

828. Accuracy in Estimation of Resonant Frequency.—Apart from sharpness of tuning, the nature of the indicating device plays a large part in determining within what limits the frequency for maximum current may be estimated. A lamp or neon tube, for example, in which the brilliancy of the light is the criterion, does not lend itself to such accurate determinations as an instrument in which a pointer reading may be observed, *e.g.*, a galvanometer. The same remark applies to a pair of telephones, in which the criterion is intensity of the noise produced. The diode valve also has the advantage over the thermo-couple in this respect. A short time elapses before the heating effect of the high frequency current produces the temperature, and therefore the thermo-electric E.M.F. which should correspond to it, and so the pointer reading lags slightly on the wavemeter condenser variation. Owing to their minute mass, the electrons in the diode accommodate themselves instantaneously to changes in anode voltage, etc., and no pointer lag is experienced.

Finally, the accuracy of a determination obviously depends on the accuracy with which readings may be made on the scale or curve of calibration. This has been mentioned above when considering the design of wavemeter condensers (para. 821).

829. An example of an early type of Service wavemeter, which is deficient in many of the qualities noted above as essential for accuracy, is illustrated in Fig. 507. The variable condenser is of the semicircular plate type, and is unscreened. An attempt is made to minimise hand capacity by fitting a long insulating handle to the condenser scale. When using this wavemeter the operator should not alter his position with respect to it. This will stabilise the stray capacity as far as possible under the circumstances. The calibration curves for this instrument actually give the LC values for a given coil and variable condenser setting. To facilitate accurate interpolation, a condenser setting near the middle of the scale should be aimed at, *i.e.*, an endeavour should be made to work on the most uniform part of the calibration curve, where the greatest percentage accuracy is attained.

The range of frequencies covered by this instrument is extended by inserting two fixed condensers in parallel with the variable condenser. It will be observed that the upper fixed condenser may be switched in by itself, but that the lower fixed condenser can only be inserted if the upper one is already in circuit.

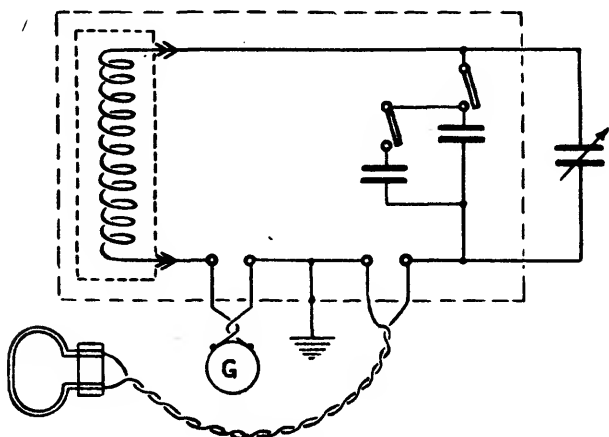


FIG. 507.

Other defects of this circuit are that the mutual coil is connected to the main circuit by long flexible leads, and that the indicating device is inserted directly, and not by means of coupling.

830. The wavemeters described above may be used either

- (1) To find the frequency of the waves emitted from a transmitting circuit ; or
- (2) To tune a transmitting circuit to a given frequency.

In the first case, the setting of the wavemeter condenser is varied until the indicating device shows a maximum value of current. The two circuits are then in resonance. The corresponding frequency is obtained from the wavemeter scale or calibration curve.

Where a galvanometer or milliammeter is used as indicating device, an accurate method of determining the resonant frequency is to note the two condenser settings giving equal currents slightly lower than the maximum, and to take the mean of the corresponding frequencies. This method is particularly valuable when using a vacuum thermo-junction and galvanometer, with which it is difficult to determine the condenser setting for resonance owing to the lag between condenser variation and rise or fall of galvanometer current.

To tune a transmitting circuit to a given frequency, the necessary condenser setting for the inductance in use is found from the wavemeter scale or calibration curve. The inductance of the transmitting circuit is then varied—increased or decreased—until a maximum reading on the indicating device is observed when the transmitter is energised and the wavemeter is coupled to it.

831. An accurately calibrated LC circuit can be used to tune a transmitting installation by bringing it close to the aerial circuit.

If the aerial circuit is in tune with it, the current in the wavemeter circuit reaches a maximum value, and in consequence the additional damping resistance thus introduced into the oscillatory circuit has its maximum effect. Therefore the current in the oscillatory (aerial) circuit has then a minimum value, which is registered on the aerial ammeter. Thus resonance between the two circuits is shown by a drop in the aerial current, *i.e.*, the "indicating device" is actually in the oscillatory and not in the wavemeter circuit, and registers by a diminution, instead of an increase, of current there.

832. Practical Points in Tuning Spark Primaries.—The following special precautions should be observed in tuning the primaries of spark transmitting circuits:—

- (a) Use a coupling between wavemeter mutual and primary which gives a maximum of about half the scale reading on the galvanometer.
- (b) Sparking must be steady and even. The key should be pressed so as to give evenly-spaced "shorts." A continuous "long" should not be made, as the gap will get very hot, and the plugs be burnt away.
- (c) The plugs should be clean and not pitted.
- (d) In sets where a rotary gap is fitted this should be kept running.
- (e) Great care must be taken in adjusting a fixed gap and suiting the A.C. voltage to it. If the gap is too short for the supply voltage, "arcing" will occur, *i.e.*, current from the transformer will flow across the gap without charging the condenser. If too long, a combination of "one spark per cycle" and "one spark per half-cycle" may be produced, with resultant uneven sparking voltage.
- (f) Sparking in a wavemeter condenser should not be allowed. This can generally be obviated by loosening the coupling to the oscillatory circuit.
- (g) The mutual coil of the aerial circuit should be disconnected.

833. Tuning the Aerial. The Two Waves.—In tuning a spark-transmitting installation the primary is tuned first, the aerial being disconnected. It is, however, impossible to tune the aerial circuit unless it is energised by the primary, and if the coupling is tight the interaction of the two circuits sets up a pronounced double-frequency effect, as was explained in Chapter VII.

If the primary and aerial circuits are separately tuned to the same LC value, then, owing to the transfer and re-transfer of energy between them via the mutual coil, the aerial oscillates at two frequencies:—

$$f_1 = \frac{f}{\sqrt{1+K}} \text{ and } f_2 = \frac{f}{\sqrt{1-K}},$$

where f is the frequency of either circuit taken alone, and K (the coupling factor) $= \frac{M}{\sqrt{L_1 L_2}}$.

834. There are two methods used for tuning accurately the aerial circuit of a spark installation :—

(a) By spark.

(b) By buzzer.

The buzzer method is the easier and more accurate, but installations not fitted with a buzzer must be tuned by the spark method.

835. Tuning Aerial by Spark.—(a) The first step is to render the aerial approximately in tune with the primary by altering the turns in circuit on the aerial coil until a maximum reading is given on the aerial ammeter.

(b) Loosen the coupling as much as possible.

Place the wavemeter inductance so that it is influenced by the aerial circuit only. In small offices the wave should be measured at the top deck insulator, or at some suitable section of the trunk.

Under these conditions, if the primary and aerial circuits are exactly in tune, the aerial will transmit a wave of practically single frequency.

If the two circuits are not in tune, both the primary and aerial tuning frequencies, or the latter alone, may be shown on the wavemeter, and the aerial tuning should be corrected until the aerial has the primary tuning frequency only.

836. An alternative procedure to (b) is to make the coupling between the aerial and primary circuits as tight as possible. Owing to the double frequency effect, the galvanometer pointer will give maximum deflections at two condenser settings. In general these two settings do not bear a simple relationship to the setting obtained when tuning the primary circuit. In the general case, the relations between the corresponding frequencies (obtained from the wavemeter scale or calibration curve) are

$$f_1 = \frac{f}{\sqrt{1+K}} \text{ and } f_2 = \frac{f}{\sqrt{1-K}} \text{ (para. 421).}$$

A relation between the frequencies not involving K may be found as follows. From the above equations,

$$1 + K = \frac{f^2}{f_1^2}, \text{ and } 1 - K = \frac{f^2}{f_2^2}.$$

$$\therefore \frac{f^2}{f_1^2} + \frac{f^2}{f_2^2} = 2$$

$$\text{or } \frac{1}{f_1^2} + \frac{1}{f_2^2} = \frac{2}{f^2}$$

The aerial turns would have to be altered until this relationship was satisfied. It will be seen that this is not a very practical method in the general case.

837. In special cases where there is a simple relationship between condenser setting and frequency, this method may be practicable. Three cases occurring in practice will now be considered :—

- (i) Condenser setting proportional to frequency.
- (ii) Condenser setting proportional to wavelength.
- (iii) Condenser setting proportional to capacity (semi-circular plates).

The condenser setting in degrees corresponding to the primary tuning position will be taken as θ , and the two settings giving maximum deflection when tuning the aerial as θ_1 and θ_2 .

- (i) **Condenser setting proportional to frequency.**—In this case

$$\theta \propto f, \theta_1 \propto f_1 \text{ and } \theta_2 \propto f_2$$

Hence the equation $\frac{1}{f_1^2} + \frac{1}{f_2^2} = \frac{2}{f^2}$ gives $\frac{1}{\theta_1^2} + \frac{1}{\theta_2^2} = \frac{2}{\theta^2}$ as the relation which must be obtained between the three condenser settings.

- (ii) **Condenser setting proportional to wavelength.**—Since $\lambda \propto \frac{1}{f}$ and $\theta \propto \lambda$, therefore $\theta \propto \frac{1}{f}$. The general relation therefore becomes $\theta_1^2 + \theta_2^2 = 2\theta^2$.

- (iii) **Condenser setting proportional to capacity.**

$$f = \frac{3 \times 10^4}{2\pi\sqrt{LC}}, \text{ i.e., } f \propto \frac{1}{\sqrt{C}} \text{ or } C \propto \frac{1}{f^2}$$

In this particular case the frequency relationship may therefore be expressed in terms of condenser settings as

$$\theta_1 + \theta_2 = 2\theta,$$

or

$$\theta_1 - \theta = \theta - \theta_2.$$

In other words, when the primary and aerial circuits are in tune, the two maximum current values occur at condenser settings which are equidistant from the setting used to tune the primary.

838. From the above results it will be seen that the method of tuning the aerial by making use of the double-frequency effect is likely to be tedious in cases (i) and (ii).

In case (iii), however, *i.e.*, **if the wavemeter condenser is of the semicircular type**, it provides a simple and practical method.

The procedure is then as follows :—

Make the coupling to the aerial as tight as possible. Place the wavemeter mutual so as to be influenced by the aerial circuit, and note the condenser positions corresponding to the two maxima swings of the wavemeter galvanometer. Alter the turns on the aerial coil until these two positions are an equal amount on either side of the positions used for the primary tuning.

The following example illustrates this method.

Example 63.

It is required to tune to a frequency of 625 kc/s.

$$f \text{ (in kc/s.)} = \frac{3 \times 10^4}{2\pi \sqrt{LC}} = 625$$

$$\text{Hence } LC = \left(\frac{3 \times 10^4}{2\pi \times 625} \right)^2 = \left(\frac{24}{\pi} \right)^2 = 58.34 \text{ mic-jars.}$$

Let the inductance used in the wavemeter be 100 mics. Then the required wavemeter capacity is 0.583 jar.

An inspection of the wavemeter curve shows that this is given at, say, 110° on the condenser scale. The primary should now be tuned to this position as described in paras. 830 and 832.

Join up the aerial circuit and tune it roughly by aerial ammeter as described in para. 835 (a). Measure the two condenser positions at which maximum readings are registered on the galvanometer. Suppose they are 102° and 124° , *i.e.*, 8° below and 12° above the mean. Reduce the turns on the aerial coil until the positions of maximum readings are 100° and 120° , *i.e.*, an equal amount (10°) above and below the mean.

839. Tuning Aerial by Buzzer.—The tuning of a spark set fitted with a motor buzzer is a much simpler matter. This is because "quenching" may be assumed to occur with a buzzer, and therefore no energy is re-transferred from aerial to primary once it has been transferred to the aerial circuit.

In consequence, the aerial oscillates at its own natural frequency only, and neither the tightness of coupling nor the primary tuning makes any difference to the wave radiated by the aerial. The only result consequent on the aerial and primary circuits being out of resonance will be that the amount of energy transferred to the aerial circuit will not be a maximum.

Hence buzzer tuning should be carried out as follows:—

Tune the primary circuit by spark as before. Tune the aerial roughly to the primary by aerial ammeter.

Join up the buzzer in the primary. Place the wavemeter mutual on top of the aerial coil.

Press the transmitting key, and vary the aerial tuning until the maximum galvanometer swing occurs at the same condenser position as that to which the primary circuit was tuned. The primary has now been tuned by spark, and the aerial by buzzer, to the same frequency.

A slight correction has to be made to find the primary tuning position for buzzer, on account of the extra inductance added in the primary circuit by the leads from the buzzer brushes to the spark gap. This can easily be obtained by keeping the aerial circuit unaltered, and varying the primary inductance until a maximum reading is obtained either on the aerial ammeter or on the wavemeter galvanometer.

It may be possible, with this method, to get three positions at which maxima are observed, corresponding to the natural frequency of the aerial, and to the two frequencies at which the system of coupled circuits oscillates during the short period in which interaction is taking place and the energy is being transferred from primary to aerial.

840. Measurement of Coupling.—Sometimes it is required to measure the coupling factor, or the percentage coupling, given by certain positions of the mutual coil when the primary and aerial circuits have both been tuned to the same value.

For this purpose, advantage may be taken in spark transmitters of the double frequency effect. The coupling factor may be expressed in terms of the two frequencies by making use of the relationships

$$f_1 = \frac{f}{\sqrt{1+K}}, \text{ and } f_2 = \frac{f}{\sqrt{1-K}}.$$

This gives as before

$$1+K = \frac{f^2}{f_1^2} \text{ and } 1-K = \frac{f^2}{f_2^2}$$

$$\therefore \frac{1+K}{1-K} = \frac{f^2}{f_1^2} \times \frac{f_2^2}{f^2} = \frac{f_2^2}{f_1^2}$$

$$\therefore f_1^2 + Kf_1^2 = f_2^2 - Kf_2^2$$

$$K(f_2^2 + f_1^2) = f_2^2 - f_1^2$$

$$\therefore K = \frac{f_2^2 - f_1^2}{f_2^2 + f_1^2},$$

$$\text{and percentage coupling} = \frac{100(f_2^2 - f_1^2)}{f_2^2 + f_1^2}.$$

In the special case when the wavemeter condenser is of the semi-circular type, this equation for K may be simply expressed in terms of condenser settings. As before, let θ_1 and θ_2 be the condenser settings corresponding to f_1 and f_2 .

$$f^2 \propto \frac{1}{C} \propto \frac{1}{\theta}$$

$$\therefore K = \frac{\frac{1}{\theta_2} - \frac{1}{\theta_1}}{\frac{1}{\theta_2} + \frac{1}{\theta_1}} = \frac{\theta_1 - \theta_2}{\theta_1 + \theta_2}$$

$$\text{and percentage coupling} = \frac{100(\theta_1 - \theta_2)}{\theta_1 + \theta_2}$$

Suppose that in this special case the correct condenser position for the primary tuning adjustment is 90° , and that the following condenser positions for the two waves are found as the mutual coil

is pulled away from the primary; then the percentage coupling will be as indicated in the right-hand column :—

Distance of Mutual from Primary.	θ_1 .	θ_2 .	Percentage coupling = $\frac{100 (\theta_1 - \theta_2)}{\theta_1 + \theta_2}$
Inches.	Degrees.	Degrees.	Per cent.
0	101.7	78.3	13
2	101.25	78.75	12.5
4	100.8	79.2	12
6	99	81	12
8	96.3	83.7	7
10	94.5	85.5	5
12	93.4	86.6	3.8
14	92.5	87.5	2.8

These results can be plotted as a "coupling" curve thus :—

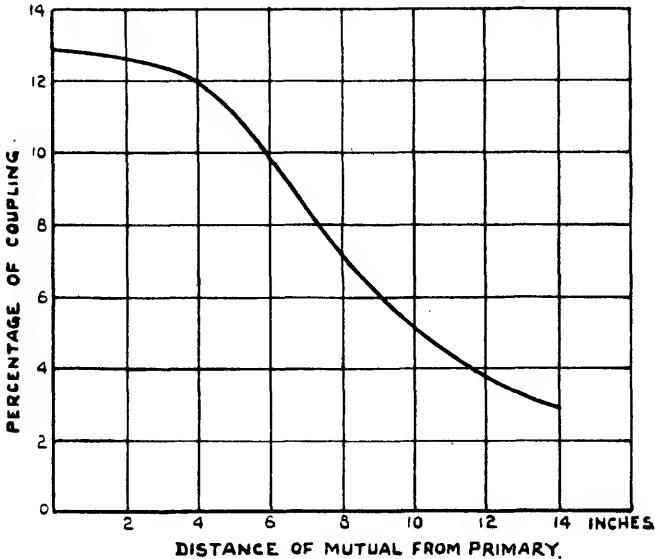


FIG. 508.

In the general case, the coupling curve would be plotted by calculating $100K$ from the formula

$$100K = \frac{100 (f_2^2 - f_1^2)}{f_2^2 + f_1^2}$$

for various distances between the mutual and primary coils, f_1 and f_2 being obtained from the wavemeter scale or calibration curve.

Since the coupling factor $K = \frac{M}{\sqrt{L_1 L_2}}$ is different for each wave tuned to (since L_1 and L_2 alter), a coupling curve showing percentage coupling (or K) against position of mutual (or mutual inductance M) is only correct at one frequency.

In practice it is seldom necessary to go to the trouble of plotting a coupling curve, as Service spark installations are designed so that the coupling will not be excessively tight when the mutual is close up to the primary.

841. Measurement of Aerial Inductance and Capacity.—It is possible to measure the natural inductance and natural capacity of an aerial system by using a wavemeter circuit. In the case of the aerial being energised by a spark set, it is necessary to use the buzzer method of excitation, or very loose coupling with the spark method, as only one wave frequency is wanted in the aerial system, and hence one condenser setting of the wavemeter which will give a maximum reading.

This method is as follows. The frequency of the aerial circuit is determined. A known alteration is then made in the aerial inductance, and a second frequency is determined. From these two results the two unknown quantities, aerial natural inductance and capacity, can be calculated.

The results can be expressed in a formula, as under :—

Let L_{as} and σ represent the natural inductance and capacity, respectively, of the aerial circuit.

Let L_1 be the value of any known inductances, such as vario-meter, spacing coil, portion of the aerial coil, etc., in the circuit in both cases.

Let L_2 be the value of an additional known inductance added to the circuit in the second case (generally by adding a few turns to the aerial coil, the value being determined from the handbook of the set).

Let f_1 and f_2 be the two frequencies in k/cs. as determined from the wavemeter scale or calibration curve.

$$\text{Then } f_1 = \frac{3 \times 10^4}{2\pi\sqrt{(L_{as} + L_1)\sigma}}, \text{ and } f_2 = \frac{3 \times 10^4}{2\pi\sqrt{(L_{as} + L_1 + L_2)\sigma}}.$$

$$\text{Hence} \quad (L_{as} + L_1)\sigma = \frac{9 \times 10^8}{4\pi^2 f_1^2},$$

$$\text{and} \quad (L_{as} + L_1 + L_2)\sigma = \frac{9 \times 10^8}{4\pi^2 f_2^2}.$$

$$\begin{aligned} \text{By subtraction,} \quad L_2\sigma &= \frac{9 \times 10^8}{4\pi^2} \left(\frac{1}{f_2^2} - \frac{1}{f_1^2} \right) \\ \therefore \sigma &= \frac{9 \times 10^8}{4\pi^2 L_2} \left[\frac{1}{f_2^2} - \frac{1}{f_1^2} \right] \text{ jars.} \end{aligned}$$

By substitution for σ ,

$$\begin{aligned} (L_{as} + L_1)\sigma &= \frac{(L_{as} + L_1)}{4\pi^2 L_2} \left[\frac{1}{f_2^2} - \frac{1}{f_1^2} \right] \times 9 \times 10^8 \\ &= \frac{(L_{as} + L_1)}{4\pi^2 f_1^2 L_2} \left[\frac{f_1^2 - f_2^2}{f_2^2} \right] \times 9 \times 10^8. \end{aligned}$$

$$\therefore \frac{(L_{\infty} + L_1)}{4\pi^2 f_1^2 L_2} \left[\frac{f_1^2 - f_2^2}{f_2^2} \right] \times 9 \times 10^8 = \frac{9 \times 10^8}{4\pi^2 f_1^2}$$

$$\therefore L_{\infty} + L_1 = \frac{L_2 f_2^2}{f_1^2 - f_2^2},$$

and

$$L_{\infty} = \frac{L_2 f_2^2}{f_1^2 - f_2^2} - L_1.$$

For use when the LC value corresponding to a given wavemeter condenser setting is given instead of the frequency by the calibration curves, these formulæ may be modified as below.

Let L_m be the wavemeter inductance and C_1 and C_2 the capacity values corresponding to f_1 and f_2 respectively. Then

$$L_m C_1 = \frac{9 \times 10^8}{4\pi^2 f_1^2}, \text{ and } L_m C_2 = \frac{9 \times 10^8}{4\pi^2 f_2^2}.$$

$$\begin{aligned} \sigma &= \frac{1}{L_2} \left[\frac{9 \times 10^8}{4\pi^2 f_2^2} - \frac{9 \times 10^8}{4\pi^2 f_1^2} \right] = \frac{L_m C_2 - L_m C_1}{L_2} \\ &= \frac{L_m (C_2 - C_1)}{L_2}. \end{aligned}$$

$$\text{Also } (L_{\infty} + L_1) \sigma = \frac{9 \times 10^8}{4\pi^2 f_1^2} = L_m C_1$$

$$\begin{aligned} \therefore \frac{L_m (C_2 - C_1) (L_{\infty} + L_1)}{L_2} &= L_m C_1 \\ \therefore L_{\infty} + L_1 &= \frac{L_2 C_1}{C_2 - C_1}. \end{aligned}$$

Two numerical examples illustrating these methods will now be given.

Example 64.

In a valve transmitter with 180 mics. in the aerial circuit, the resonant frequency was 600 kc/s., and with 300 mics. added it fell to 400 kc/s. Find the aerial inductance and capacity.

$$(L_{\infty} + 180) \sigma = \frac{9 \times 10^8}{4\pi^2 \times 36 \times 10^4} = \frac{10^4}{16\pi^2}$$

$$(L_{\infty} + 180 + 300) \sigma = \frac{9 \times 10^8}{4\pi^2 \times 16 \times 10^4} = \frac{9 \times 10^4}{4 \times 16\pi^2}$$

$$\begin{aligned} \therefore 300\sigma &= \frac{9 \times 10^4}{4 \times 16\pi^2} - \frac{10^4}{16\pi^2} = \left(\frac{9}{4} - 1 \right) \times \frac{10^4}{16\pi^2} \\ &= \frac{5 \times 10^4}{64\pi^2} \end{aligned}$$

$$\therefore \sigma = \frac{5 \times 10^8}{3 \times 64\pi^2} = 0.264 \text{ jar.}$$

$$L_{aa} + 180 = \frac{10^4}{16\pi^2\sigma} = \frac{10^4 \times 3 \times 64\pi^2}{5 \times 10^3 \times 16\pi^2} = 240 \text{ mics.}$$

$$\therefore L_{aa} = 60 \text{ mics.}$$

Example 65.

In an arc set, with spacing coil and variometer inductances amounting to 160 mics in circuit, the following results were obtained.

With 600 mics in aerial coil, LC value = 1,000 mic jars.

With 1,000 mics in aerial coil, *i.e.*, 400 mics added, LC value = 1,500 mic jars.

$$\text{Then } (L_{aa} + 160 + 600) \sigma = 1,000$$

$$(L_{aa} + 160 + 1,000) \sigma = 1,500$$

$$\text{By subtraction, } 400 \sigma = 500$$

$$\therefore \sigma = 1.25 \text{ jars.}$$

$$(L_{aa} + 760) \times 1.25 = 1,000$$

$$L_{aa} + 760 = \frac{1,000}{1.25} = 800$$

$$\therefore L_{aa} = 40 \text{ mics.}$$

842. Construction of a Tuning Curve.—When the various tuning positions for primary and aerial have been obtained, it is a good plan to plot them in the form of a curve so that if an intermediate wave is required at any time it can be picked off from the curve.

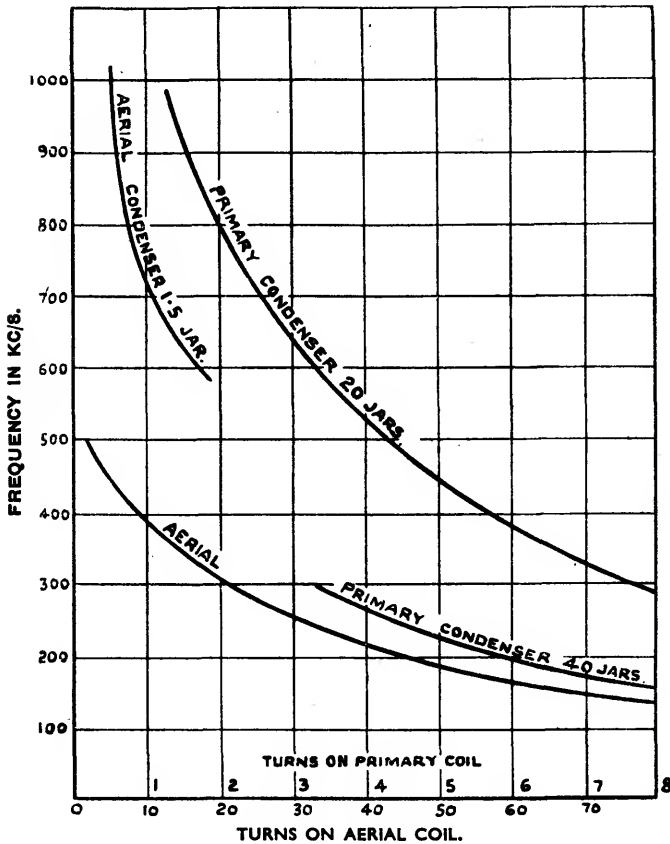
Let us suppose we have tuned a set, the primary condenser of which has two alternative values, 20 and 40 jars.

The following tuning adjustments were obtained :—

Wave Frequency in kc/s.	Primary Turns with		Aerial Turns.
	20 Jars.	40 Jars.	
1,000	1.3	—	6*
750	2.3	—	9*
600	3.3	—	18*
500	4.3	—	2
375	6.2	—	11.5
300	7.7	3.3	21
250	—	4.3	30.5
214	—	5.4	40
187	—	6.2	51
167	—	7.1	60
150	—	7.8	68

* 1.5 jar series condenser used.

These results can then be plotted as in Fig. 509, showing the turns used on the primary and aerial coils.



Tuning Curve.

FIG. 509.

843. Tuning Continuous Wave Installations.—Tuning continuous wave installations—both arc and valve—is an easy matter, as only one wave is emitted. Great accuracy in tuning can be obtained and should be insisted on, for on the higher wave frequencies a very small error in the position of the wavemeter condenser will lead to a very big error in the result.

It has been found in tuning valve transmitters that the wave frequency radiated is liable to be affected by any one of the factors given below, in addition to a variation of the aerial inductance, and these changes may be serious if a very high wave frequency is being used.

When any one of the changes below is made, the **wave frequency increases** :—

- (1) Decrease of steady grid voltage.
- (2) Decrease of anode voltage.
- (3) Decrease of coupling of grid circuit to anode circuit.
- (4) Increase of aerial damping.
- (5) Increase or decrease of filament current from the point which gives the **minimum** wave frequency.

OSCILLATORS.

844. Methods have been developed whereby an accurately calibrated LC circuit may be used for many other purposes than tuning or measuring the frequency of a local transmitter as described above. In Service practice, where the LC circuit is capable of generating self-oscillations, it is commonly used for the following purposes :—

- (1) Tuning or measuring the frequency of a local transmitter, *i.e.*, as a wavemeter in the manner described above.
- (2) Tuning receiving apparatus to any desired frequency. It is then said to be used as a **wave-tester**.
- (3) Tuning or measuring the frequency of a distant transmitter.

Such circuits are merely low power transmitters whose frequencies have been calibrated for various settings of the oscillatory circuit condenser. They are known as oscillators, and the two types used in the Service are :—

- (a) The heterodyne oscillator ; and
- (b) The self-quenching oscillator.

The principles by which these circuits generate oscillations have been described in paras. **567** and **706** respectively.

845. A circuit which can be used either as a C.W. oscillator of the heterodyne type or as a self-quenching oscillator is shown in Fig. 510. Two grid leaks are provided, either of which can be introduced into the circuit, and one of which is of much higher resistance than the other. When the leak of lower resistance is in circuit, C.W. oscillations are produced. With the leak of higher resistance, the grid runs so far negative that I.C.W. oscillations are produced with an audio-frequency modulation.

Each of these oscillators may be used as a receiver. The heterodyne oscillator then acts as an autodyne receiver, *i.e.*, it is mistuned by an audio-frequency amount to the incoming signal. The self-quenching oscillator indicates the presence of an incoming signal by a rise in the pitch of the note heard in the telephones, the rise in pitch being a maximum when the oscillator is tuned to the incoming signal (para. **709**).

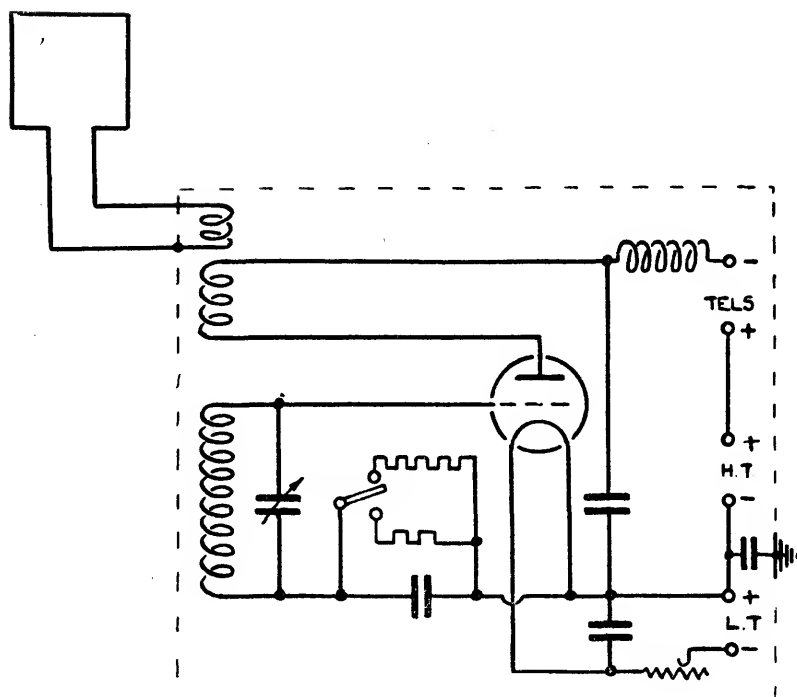


FIG. 510.

846. When used as wavemeters, these circuits are acting as receivers. To measure the frequency of a C.W. transmitter, the heterodyne oscillator is adjusted until it is in the centre of the dead space. In practice the two limits of the dead space are found and the half-way position taken. For an I.C.W. transmitter, the condenser setting is varied until loudest undistorted signals are heard in the telephones. In any other setting at least two notes will be heard, one the I.C.W. note of the transmitter and the other the beat note between the transmitter and the local oscillator. Actually a confused "mush" of sound is heard. This "clears up" to give a true note when the oscillator and the transmitter are in tune. This note is also the loudest, as, when the transmitter and oscillator are in tune, maximum current will be produced in the oscillator tuned circuit by the E.M.F. induced from the transmitter.

The self-quenching oscillator is only used for tuning C.W. transmitters, and is adjusted until the largest rise in pitch is heard in the telephone note.

In each case the frequency of the transmitter is then read off from the scale or calibration curve of the local oscillator. If the oscillator is not calibrated, its frequency must be determined by a wavemeter.

847. As a wave-tester, *i.e.*, to tune receiving apparatus, the heterodyne oscillator is first set to the required frequency by means of its calibration curve. The receiver is then made to oscillate, and its settings adjusted until the receiver telephones indicate that the receiver is in the centre of the dead space. The oscillator telephone terminals are short-circuited.

For a non-oscillating receiver, some method of producing I.C.W. in the heterodyne oscillator must be available. This is usually a buzzer in the grid circuit.

848. The self-quenching oscillator is used to tune an oscillating receiver in exactly the same way as when tuning a transmitter, *i.e.*, the receiver adjustments are altered until the greatest rise in pitch of the self-quenched note is observed.

The frequency of an incoming signal is measured by an extension of the above method. An oscillating receiver is tuned to the incoming signal, and the frequency of the receiver is then determined by means of the local oscillator, and its appropriate wavemeter, if necessary.

A distant transmitter may be tuned by the reverse process. An oscillating receiver is tuned by the local oscillator to the required wave, the distant station then transmits, and is directed by signal to increase or decrease its frequency until it is in the middle of the receiver's dead space.

849. Calibration of Oscillator Circuits.—The use of an oscillator for the above purposes requires an accurate knowledge of the frequency produced by it at different condenser settings, *i.e.*, it must be calibrated just as a wavemeter circuit is. The various factors that affect the frequency of a valve oscillator have been discussed in Chapter XIV. It was there seen that the frequency is partly determined by the constants of the valve used and by the filament heating current and anode voltage applied. Thus, when the latter are altered or a new valve has to be inserted, the oscillator must be re-calibrated. It is to minimise the changes in frequency for a given condenser setting caused by such alterations that a grid leak and condenser are inserted in the heterodyne oscillator. The flow of grid current during oscillations is thereby reduced, with a beneficial effect on the frequency constancy.

In a circuit such as that shown in Fig. 510, for example, the alteration from the C.W. grid leak to the self-quenching grid leak produces a noticeable change in frequency, and two different calibration curves or scales are necessary in an oscillator of this type.

The methods of calibrating the two Service types of local oscillator will now be given.

850. Calibration of Heterodyne Oscillator.—For this purpose certain stations belonging to the three fighting services transmit standard waves according to a definite programme, once or twice a month.

It is usual to set the transmitter as closely as possible to the desired wave frequency, and then to check the actual frequency of the transmitted signal by means of a standard wavemeter. A message is then sent out advising stations concerned what wave frequency has actually been transmitted.

The standard waves actually sent out are recorded in the first instance on the heterodyne by listening out, and the reference points thus obtained may conveniently be recorded in pencil on the scale of the instrument; or, alternatively, the procedure given above for measuring the frequency of an incoming signal may be used.

Further reference points may then be obtained as follows:—

Any convenient C.W. transmitter is tuned accurately (by listening in) to the waves thus obtained (say 100, 150, 250 and 350 kc/s.).

The C.W. transmitter is then set to one of these frequencies (say 250 kc/s.), and the heterodyne is set to approximately double this frequency.

It will be found that a note can be heard which is due to the second harmonic of the original oscillation, and the heterodyne is carefully adjusted to this frequency (500 kc/s.).

The result gives a new reference point which corresponds to a wave frequency of twice the original frequency; others corresponding to three or four times the original frequency can be determined in the same way.

Similarly, a further series of reference points may be obtained by tuning the transmitter to wave frequencies of one-half, one-third, etc., of the known frequencies.

In this manner the whole scale of wave-frequencies used in the Service may be covered, starting from the few standard waves as originally transmitted.

851. Calibration of Self-quenching Oscillator.—The rise in the pitch of the note heard in the telephones which enables a self-quenching receiver to detect incoming signals, was explained in para. 709. It is due to a decrease in audio-frequency period of grid voltage variation, because the energy supplied by the incoming signal allows the circuit to start self-oscillations at a more negative mean grid voltage than when all the energy must be supplied by the oscillator circuit itself.

This is illustrated in Fig. 511 (a).

The same effect would be produced if the oscillations could be quenched at a more positive grid voltage than normal, provided that the mean grid voltage for oscillations to begin remained the same. There would be a smaller negative charge on the grid when oscillations were quenched, and a smaller period would therefore elapse before this charge had leaked away sufficiently to bring the grid to the potential at which oscillations would start to build up again. The audio-frequency period of grid voltage change would be lessened, and a rise in the pitch of the note heard in the telephones would be observed. This case is illustrated in Fig. 511 (b).

This effect may be produced in practice by increasing the damping of the grid oscillatory circuit. The extra resistance does not noticeably affect the grid voltage at which oscillations start. The condition for generation of oscillations is $\frac{Mg_m}{C} > R$ (para. 567). The slope

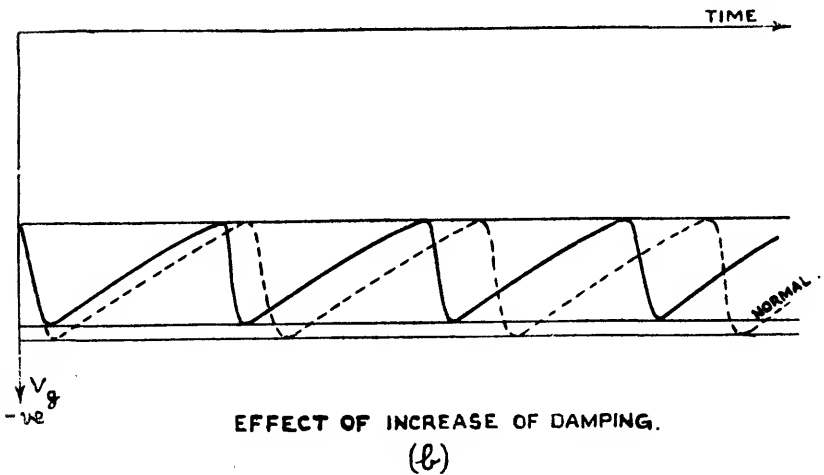
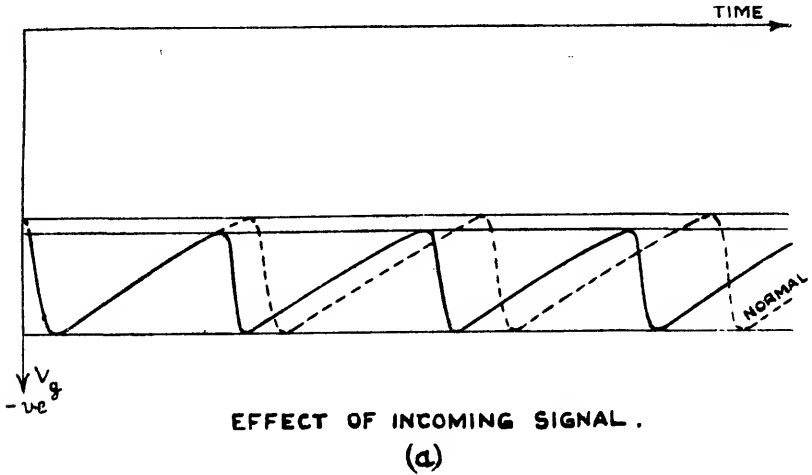


FIG. 511.

of the static mutual characteristic, g_m , is increasing so rapidly at this grid voltage that an undetectably small increase in grid voltage enables the condition to be fulfilled in spite of the greater value of R .

The effect of the increased damping in raising the grid voltage at which self-quenching takes place is much more marked; g_m is then the average slope of the dynamic characteristic (para. 568), and this varies very little with change in mean grid voltage at large negative grid voltages. It thus becomes too small to enable oscillations to be maintained at appreciably more positive mean grid potentials as the resistance of the oscillatory circuit increases. Thus, the greater the damping, the higher is the pitch of the note heard in the telephones.

Advantage is taken of this fact to calibrate a self-quenching oscillator. The oscillator is magnetically coupled to a wavemeter circuit, which then behaves as in para. 831. Maximum current flows in the wavemeter when it is tuned to the oscillator. Thus maximum damping is introduced into the oscillator circuit, and the greatest rise in pitch of the note heard in the telephones is observed, when the two circuits are in tune. This frequency is then determined from the wavemeter calibration.

852. A wavemeter circuit designed for use both as an ordinary wavemeter and for calibrating a self-quenching oscillator is shown in Fig. 512.

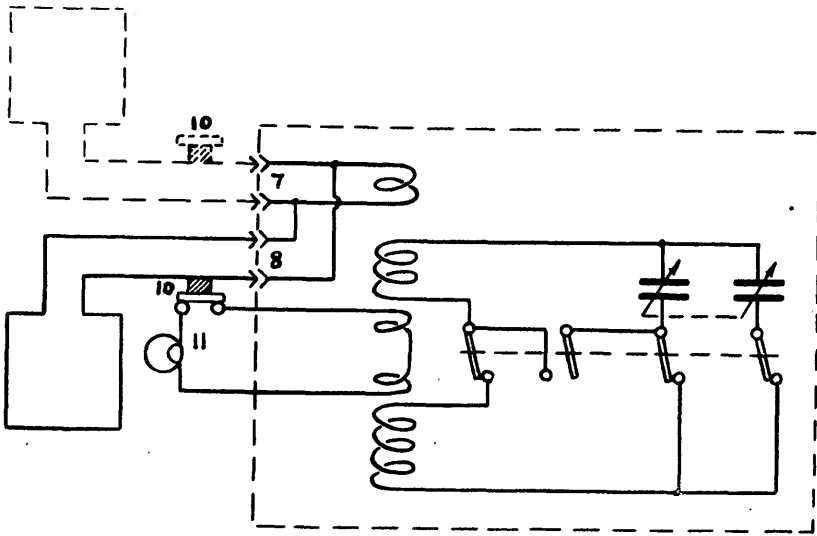


FIG. 512.

The coupling coil, via its stiff connecting leads, may be connected in either of two positions as shown. The full line drawing shows the coupling coil in the wavemeter position. A metal contact (10) which is carried on one lead, but is insulated from it by an ebonite block, completes in this position the circuit containing the indicating device, a lamp. The switching arrangements in the main wavemeter circuit for altering the range should also be noted.

To calibrate the self-quenching oscillator, the dotted line position of the coupling coil is used. The indicating device is then the telephones in the oscillator circuit, and the lamp is not required. The lamp circuit is therefore broken to decrease the damping, and so sharpen the tuning of the main wavemeter circuit.

The two positions of the coupling coil necessitate two sets of calibration curves for this instrument.

APPENDIX A.

ELEMENTARY APPENDIX FOR JUNIOR OFFICERS AND RATINGS.

Wave Motion.—Suppose that you wish to attract the attention of a person some little distance away who does not happen to be looking in your direction. Various methods might be employed. You might write out your message, wrap it round a stone and throw the result so as to hit the desired person. Besides depending on your skill in marksmanship and the distance you can throw, this method has the added disadvantage that you are unlikely to have engendered in your target the most favourable state of mind for a consideration of your request.

Another method would be to shout. It is not denied that this may suffer from some of the disadvantages of the previous method, according to the strength of your vocal chords and the nerves of the person you are addressing, but the point we are interested in is that you are making use of an entirely different principle of communication. You make a noise in one place and it is perceived by somebody in another place, yet nothing tangible seems to have passed between you.

The bed of the North Sea subsides a few feet and a nervous householder in Hampstead grasps the poker more firmly as he sets out to look for the burglars; in the morning he discovers that an earthquake has occurred. A large liner passes rather fast up Spithead and a few minutes later the trippers on Southsea beach grab their chattels and families and run for their lives; a tidal wave is arriving.

In all these instances a disturbance at one point has been transferred so as to create a disturbance at another point, and the obvious thing is to enquire how this has come about. The cause of the consternation on the beach is easily realised. The passage of the liner through a shallow channel has set up large waves which eventually arrive at the shore. An important point to grasp is that the water which soaks the feet of the laggard on the beach is not the same water as that lapping the liner's sides when the disturbance was caused. If it were, any boats between the liner and the shore would be thrown up on the shore. But if you were in a boat at the moment, all that would happen would be that the boat would rock up and down violently as the wave passed it.

It thus becomes evident that in the transference of disturbances by this method there is no actual transfer from transmitter to

receiver of the medium in which the disturbance is created. The disturbance travels through the medium, and wherever it passes it sets up a to-and-fro motion of the medium which we call a wave. In the case of water waves this motion is up and down, and so is at right angles to the direction in which the wave is travelling.

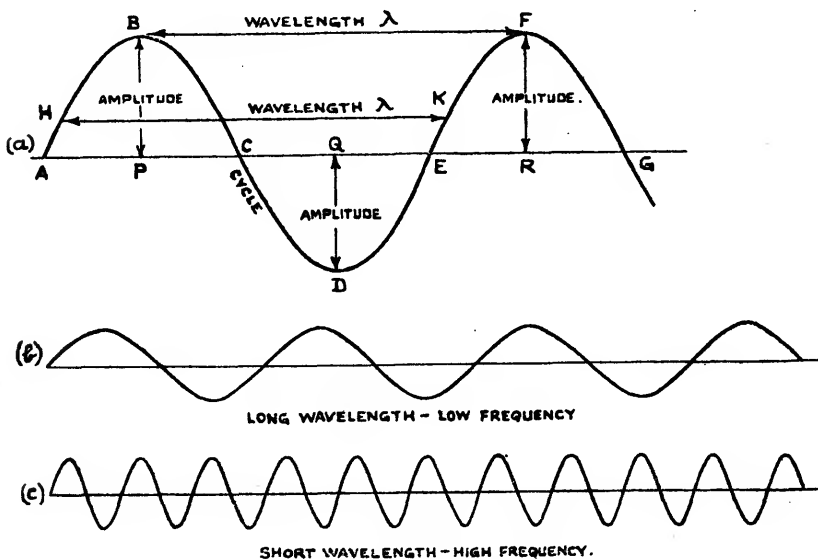
Similarly, in the case of sound waves, the air particles near the mouth of the speaker are set in vibration by the movements of the vocal chords and communicate this vibration to the air further away. Thus the vibration is passed on until it reaches the air in the neighbourhood of the listener's ear, and, impinging on his ear drum, sets up a vibration there which is recognised as a sound. In this case the vibratory motion of the air is in the same direction as the direction of travel of the disturbance, or "direction of propagation," but no air is transferred bodily from transmitter to receiver.

Returning to the case of water waves, where the process is readily apprehended, it is well known that when a disturbance is caused in water, not one solitary wave, but a number of waves following each other at regular intervals is emitted from the centre of the disturbance. We recognise this by fixing our eyes on any convenient point on the water's surface and watching it move up and down, or by observing the series of crests and troughs that follow each other over the surface. If we count the number of crests that pass a given point in a minute, say, this will be found to be the same for any minute during the time the train of waves is passing. The number of waves (or crests, since each wave has one crest and one trough) passing a given point in a given period of time is known as the **frequency** of the waves.

While one wave is passing over a point in the water surface, that point first moves upwards until it reaches a maximum height (the crest) and then starts to move down again. It does not stop when it reaches its original level, but continues to move downwards to a greatest depth (the trough), after which it again rises to its original level and the process is repeated during the passage of succeeding waves. The series of positions which the water goes through while one wave is passing is called a **cycle**. At the end of a cycle the water has returned to the same position and is about to move in the same direction again as it was at the beginning of the cycle. Fig. 1 (a) is a conventional representation of a cycle. AG can be looked on as a time axis. At the moment represented by A the water is at its normal level. At a time AP later, *i.e.*, at the instant represented by P, the water is at its highest point and its displacement from the mean position is shown by PB. By the instant represented by C it has fallen to its mean level again and is still descending, and so on. A cycle of displacement of the water surface is represented by the curve ABCDE. Another similar cycle then commences, its first half being shown in the figure by the curve EFG.

The maximum distance that the medium (in this case the water) is displaced from its normal or mean position is called the **amplitude** of the wave. In Fig. 1 (a), PB or QD represents the amplitude.

Fig. 1 (a) may also be looked upon as a snapshot of a wave motion at some definite instant of time, the axis AG then representing distance measured in the direction in which the waves are travelling. B and F then correspond to two adjacent crests and D to the trough between them. The distance from B to F, *i.e.*, from the crest of any individual wave to the crest of the wave preceding it, is called the **wavelength**. It is obvious that the wavelength need not



NOTE.—Wavelength of (b) is three times that of (c),
Frequency of (b) is one third that of (c).

Wave Definitions.

FIG. 1.

be measured from crest to crest; any two corresponding points in successive waves (such as H and K in Fig. 1 (a)) will be the same distance apart as the crests.

The series of waves, or "wave-train," as it is called, travels through the water at a certain rate or velocity. Though it is not strictly true for water waves, this velocity does not usually depend at all on the frequency in the case of the wave motions we are concerned with here. In sound waves, for example, waves of different frequency are recognised by their difference in pitch. A low note corresponds to an air vibration of a lesser frequency than a high note. But if low notes travelled, say, more slowly than high notes, it is obvious that the tune we should hear when listening

to a band some distance away would depend on how far we were from the band. The more quickly travelling high notes would crowd on top of the slower low notes and the relative time in which they were played by the band would not be preserved, so that the music would be distorted. But the only effect of distance is to decrease the loudness of the music, not to distort it. Thus the speed of sound waves is independent of their frequency. This enables us to arrive at a simple relationship between the frequency and the wavelength in a wave motion. Any one crest travels a certain distance in a second, this distance depending on its velocity. The longer the wavelength (the distance between successive crests) the less is therefore the number of crests that pass any particular point in a second. But the number of crests that pass per second is what we have called the frequency; hence **the longer the wavelength, the lower the frequency**. This is illustrated in Fig. 1 (b) and (c).

Another distinction between waves is shown in Fig. 2, and may be illustrated by the difference in the sound waves produced by a ship's bell and a siren. When a ship's bell is struck, the sound dies away rapidly. The note from a siren continues with the same intensity. Another example is provided by the notes produced by a piano and organ respectively. The sound wave from a bell or piano is called a damped wave, and that from a siren or organ an undamped or continuous wave.

So far we have rather loosely spoken of a "disturbance" being created at a certain point and being transferred to distant points by a wave motion in the substance or "medium" in which the disturbance is created. We may now proceed to the discussion of what is transferred from one point to the other. It has been seen that there is no actual transference of the medium itself, but that something is passed on from point to point of the medium. The clue to this something is gleaned from the results produced. In the case of water waves, for example, a distant boat rocks up and down. This shows that a considerable amount of work is being done on the boat. Anybody who was called upon to give the boat the same motion by his own muscular efforts would no doubt consider that he was working harder than he cared to. The wave motion thus possesses the capacity for doing work or, as is more commonly said, it possesses energy. It is a fundamental scientific principle that energy cannot be created out of nothing. If the wave motion possesses energy it must have acquired it from somewhere, and the source is fairly obvious in this case. To set up the wave motion at all required the expenditure of a certain amount of energy by the liner, and it is this energy which is carried outwards by the waves and is made available at distant points.

In order to have a wave motion set up in it, the medium must possess two qualities which may be described as **elasticity** and **inertia**. When we say that a substance possesses **elasticity** we mean

that it normally has a certain shape, and if it is deformed from that shape it tends to return to it. A good example is a stiff spring, which may be pulled out, but tends to go back to its normal size as soon as it is released.

The possession of **inertia** means that the substance possessing it has a marked disinclination to do anything else but the particular thing it happens to be doing at the moment. If it is at rest it objects to being set in motion; if it is in motion it objects just as strongly to being brought to rest. It is the golden example of doing with all its might whatever it is doing. The spring quoted above also possesses inertia, and the effect is easily seen. When pulled out and let go, the spring is set in motion to return to its usual undeformed shape. But once it is set in motion it objects to

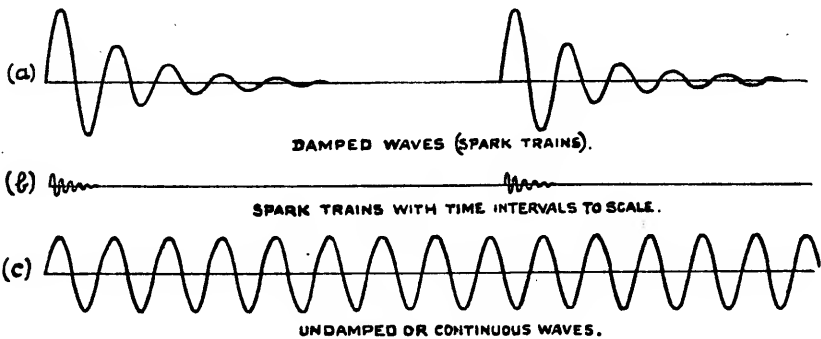


FIG. 2.

being brought to rest again. It is still travelling fast when it returns to the undeformed position and its inertia causes it to keep on. It thus shoots beyond its equilibrium point and begins to be compressed. This brings its elasticity into play to resist compression and the elastic opposition eventually overcomes the inertial effect, but the result is that the spring does not come to rest until it is compressed to some extent. The elastic forces then operate to bring the spring back to its equilibrium point, but in so doing the spring is set in motion and owing to its inertia shoots past this point and becomes extended once more. The whole process is then repeated several times, but with diminishing extension and compression as the energy given to the spring on its initial extension is gradually dissipated in overcoming friction and air resistance while the spring is in motion. The spring thus returns to its equilibrium position after a series of oscillations of diminishing size.

All material substances possess inertia. The property corresponding to elasticity in the case of water is given to some extent by gravitation. Any water particles displaced above or below the mean level obviously have a tendency to return to mean level. Similarly, in the case of sound waves, the movement of the vocal

chords or diaphragm of the instrument producing the sounds compresses or rarefies the air in its neighbourhood, and the air, whose pressure is thus altered, endeavours to assume once more the steady pressure corresponding to atmospheric conditions.

In order to set up **regular** wave motion in a material substance there must be immersed in it some mechanical device capable of executing oscillations. This communicates the oscillatory motion constituting the wave to the neighbouring parts of the medium, and it is then transferred to the more distant parts as described above. The original oscillating device is the cause of the wave motion in the material, but its motion is not itself a wave motion. It is a mechanical oscillation of the parts of the device, which does not shift its mean position in the medium. The wave motion set up by it travels through the medium, the energy sustaining the waves being detached from that originally associated with the oscillations. On the other hand, the wave-making device must also possess the properties of inertia and elasticity or it will not be possible to set it in oscillation. This will be evident from the case of the oscillating spring described above.

The energy which sustains the wave motion in the medium comes from the energy imparted to the oscillating mechanism, and the waves travelling through the medium resemble in form the oscillations of the mechanism. Since the oscillations of the spring are diminishing, the waves set up by it in the medium surrounding it are of proportionately decreasing amplitude. They are damped waves. A tuning fork which has its prongs pulled apart and then let go will produce a damped sound wave. If the fork is kept in continuous oscillation by impulses supplied to it at suitable intervals, it will set up continuous sound waves in the surrounding medium.

We may summarise this by saying that the necessary conditions for a regular wave motion to be set up in a medium are :—

- (1) That the medium must possess the properties of elasticity and inertia ;
- (2) That at some point in the medium there must be a device executing oscillations. For this to be possible the said device must also possess the properties of elasticity and inertia.

Æther Waves.—It has long been known that the stimulus causing the sensation which our eyes recognise as light reaches the eye from the source of light by a process which is of the nature of a wave motion. There is no question of actual particles being emitted by the light source and impinging on the eye. From what we have now learned about wave motion we can therefore deduce two things about the production and transmission of light :—

- (1) There must be some kind of motion going on in the source of light.

- (2) The medium between the source of light and the eye has waves set up in it by this motion and is able to propagate them so that they reach the eye.

The two questions arising from these deductions are therefore :—

- (1) What is the nature of the original motion in the source ?
- (2) What is the medium that propagates the wave motion ?

We shall deal with the second question first. It soon appears that, whatever the medium is, it is certainly not of a material nature. The sun is a familiar source of light, and it is well known that while the distance from the sun to the earth is about 91 million miles, only the last hundred miles or so of this distance is occupied by any material substance, namely the earth's atmosphere. Until recently also the ordinary electric lamp had its bulb completely exhausted of air, and yet the light could pass from the glowing filament to the glass bulb and thence to the eyes. Hence it appears that the wave motion of which light is known to consist can be propagated in space entirely empty of material substances (including gases such as air). Such space is called a vacuum. This is not the case with sound waves, for example. If a bell is rung in a vacuum, no sound will be heard. It is obviously also necessary for water to be present when water waves are set up. In these cases there is a mechanical displacement of the particles of the material while the wave is passing through it. Such a displacement is obviously out of the question when there are no particles to displace, as in a vacuum.

The case of light waves is familiar to everyone, since they are directly detected by the eye, but there are many other wave motions which are of exactly the same nature. The only difference is that the wave motion does not take place at the same frequencies as in the case of light waves, and the eye is so constructed that it only responds to a certain range of frequencies. Examples are the ultra-violet rays, which can be detected by their action on a photographic plate, "radiant heat" and X-rays, which are in common use for medical purposes.

It therefore seems that it is a property of empty space or vacuum to be able to propagate wave motions of the type exemplified by light. When considered from this point of view, empty space is known as the **æther**. Light and similar wave motions are called **æther waves**. As is well known, they can also be propagated in material substances—light, for example, travels through the atmosphere. The fact, however, that the atmosphere is not necessary for the transmission of light shows that such transmission occurs not primarily because the atmosphere is a mixture of certain gases, but because it is situated in space. The light waves continue to travel through the atmosphere because they find there essentially the same space background as if there were no material substance. They are still propagated, as it were, in the æther or underlying

background of space that would persist if the atmosphere were taken away.

As might be expected, however, the nature of any material substance occupying the space through which they are passing influences the propagation of æther waves, the general effect being to alter their velocity and direction. Some material substances can also reverse the direction of æther waves when they encounter them, *e.g.*, light waves are reflected from a mirror; others absorb the energy which the waves are transmitting, with the result that the wave motion dies away altogether inside them, *e.g.*, such substances as are opaque to light waves. Both these latter effects generally occur together. Part of the energy is absorbed and part reflected. Opacity varies with frequency. A substance opaque to æther waves of one frequency may be transparent to those of other frequencies.

We may now go on to deal with our query as to the nature of the disturbances that set up æther waves. The fact that æther waves are propagated in empty space would suggest that the devices which originate them are not of the same mechanical type as those which give rise to sound and water waves, and they have been shown to be of an electrical nature. It is by no means surprising that this should be the case, for it is now supposed that the whole of the material universe, including the apparently solid earth on which we live, is mainly emptiness, held together, as it were, by myriads of minute bits of electricity, most of which are in extremely rapid motion.

If we take some water in a glass, we can pour half of it away and the remaining liquid will still be water. Pouring away half of this remainder would still leave water (in diminished quantity) in the glass. Suppose we tried to repeat this process indefinitely. We should be left with smaller and smaller quantities of water in the glass, and the practical difficulties of pouring out half of the remaining quantity would soon become insuperable. But even if these could be overcome we should eventually reach a point where we should either have to pour away all the water or none. It would be impossible to divide the remaining quantity in two (neglecting practical difficulties), because we had arrived at the smallest possible quantity of water that can exist. This quantity is called a water **molecule**. It can be divided into smaller parts, but they are no longer water. They are, in fact, small quantities of gases called hydrogen and oxygen. One might think that these small quantities of gas might in turn be divided into smaller parts giving, say, some other gases, but this is not the case. For this reason they were called **atoms**, and the basic substances like hydrogen and oxygen, into which more complex substances like water could be split, were called elements. All these molecules and atoms are of almost inconceivably small size. In a drop of water magnified to the size of the earth, the molecules would appear to be about the size of tennis balls.

Even atoms can be split up into smaller parts, but these parts are not material in the ordinary sense, *i.e.*, they do not correspond as do the atoms to any solids, liquids or gases that we are familiar with in larger quantities.

The dissection of the atom reveals it as a system arranged in much the same way as the sun and the planets. There is a central speck corresponding to the sun which contains nearly all the mass of the atom, and around it there revolve a number of other specks. The scale of the atom is, of course, immeasurably smaller than that of the solar system, but its parts preserve much the same relative proportions. The actual bulk of the planets and the sun forms a negligible proportion of the total space which the solar system occupies. It is mainly empty space. Similarly, in the atom its constituent parts are specks in a void. The sun and planets are held together as a system by their mutual gravitational attractions; the parts of the atom are held together to form an atom by **electrical** attraction. The central speck of the atom is called the nucleus and those revolving round the nucleus are called **electrons**.

Investigation of these electrons shows that they owe their existence purely to the fact that they are electrically charged, the charge being of the type conventionally known as a negative charge. Now, if we take a fountain pen, for example, it can be electrically charged by rubbing in the hair, and discharged (*i.e.*, have its electric charge removed) by touching it to earth. Throughout the process it will not appear noticeably different to the casual observer from any other pen of the same make. But the electron is in a different category. If an electron could be "discharged," we should find that, in losing its charge, the electron itself had vanished. It is not merely electrically charged, it is an electric charge. The electron is, as it were, an atom of disembodied electricity, and all electrons are identical, no matter what nucleus they are associated with.

No such definite statement can yet be made about the nucleus. The atom as a whole, however, is electrically neutral. It is neither positively nor negatively charged. This is obvious since substances in bulk consist of atoms and show no sign of possessing an electric charge. It follows, therefore, that the nucleus of any particular atom must have a positive electric charge exactly equal to the total negative charge of the electrons revolving round it in that atom.

From our point of view, this lack of knowledge of the nucleus is unimportant, since only the electrons are operative in the ordinary phenomena which are studied in electricity. Thus, when two substances are rubbed together, one acquiring a positive charge and the other a negative charge, what has occurred is that some electrons have been transferred by the friction from the atoms of one substance to the atoms of the other. The substance that loses the electrons acquires a positive charge, since it was originally neutral and has lost a negative (electron) charge. Similarly, the

substance that gains the transferred electrons acquires a negative charge. Again, the flow of an electric current along a wire is a flow of electrons, the electrons being passed on from molecule to molecule of the material of the wire. Here it must be noticed that in ordinary electrical language the direction of the current is from "positive" to "negative," since it refers to the direction in which "positive electricity" was supposed to flow. The electrons, which are now known to constitute the current, are negative charges, and so they actually move in the opposite direction to the conventional current, *i.e.*, from "negative" to "positive." It is well known that a current flowing in a wire heats the wire, this fact being used practically in electric fires, irons and other heating apparatus. The heat arises from the motion of the electrons being retarded by their collision with the molecules of the wire. This may be most simply realised by comparing it to the heat generated in a target by the impact of a rapidly-moving shell.

The motion of electrons is the wave-making device for æther waves that corresponds to the vibrating tuning fork for sound waves. In the case of light waves and X-rays, this motion is confined within the limits of the atom. In the solar system, if Mercury were summarily thrown out and the earth moved in to take its place, we might expect that there would be some commotion in the system before it settled down to a tranquil acceptance of the new state of affairs. This is precisely the kind of thing that may happen in atoms. Due to various causes, an electron that is peacefully proceeding in its normal orbit round the nucleus is ejected altogether from the atom. Another electron moves in to take its place, and the commotion is made evident to the outside world by an æther wave sent out from the atom, its frequency depending on the violence of the atomic disturbance.

We cannot do much to influence things happening inside atoms, our efforts in that direction having been aptly compared to an attempt to obtain a musical result from a grand piano by letting it fall downstairs. But the realisation that æther waves are produced by some sort of electron motion encouraged attempts to obtain similar waves by producing various types of motion of electrons on the larger scale with which we are familiar in ordinary electrical effects, such as the flow of a current along a wire.

The motion of material bodies is regulated by adjusting the conditions under which it is possible for them to move. For instance, a ship can move in any direction in the open sea, but when going up harbour her motion is limited by the extent of the navigable channel. In the same way, the motion of electrons in electrical circuits is determined by the nature of the circuit. Special circuits have been devised in which the motion of the electrons is of such a nature that æther waves are produced by it. It is found that these waves are of much lower frequency than those produced by disturbances inside the atom such as give rise to light waves.

The æther waves which travel out from these special circuits are known as **wireless waves**. It has been uncharitably suggested that the name is paradoxically due to the maze of wires which characterises the circuits. A more reasonable derivation, however, lies in the fact that, although of electrical origin, these waves travel out into the surrounding space like light waves, and their use for communication does not involve a visible (wire) connection between the transmitter and the receiver as is the case, for example, in cable telegraphy.

It may again be emphasised that wireless waves are of the same nature as light waves and differ only in frequency. Visual signalling and wireless signalling both depend on the fact that the æther, or space background in which all our activities are set, is capable of propagating the special type of wave motion originally communicated to it by the motion of electrons. Our ability to exist on the earth at all depends on the same thing, since it is directly due to the light and heat waves that reach the earth from the sun.

The advantage of visual signalling is that it is immediately obvious to the eye of the person receiving the signal (or should be!). The fact, however, that light waves cannot follow the curvature of the earth's surface to any great extent limits its use to distances up to that of the horizon.

Wireless waves do not make a direct impression on any of the human senses and so require special arrangements before their presence and meaning can be detected; but their greatly differing frequency from that of light waves has the advantage that they are enabled to follow the curvature of the earth over much longer distances and are therefore well adapted for communication where visual signalling would be out of the question.

All æther waves travel at the same speed in free space and this speed is not sensibly affected by the presence of the atmosphere. That this speed is extremely large is evident from the case of light waves. Over terrestrial distances no lag can possibly be perceived between the instant when a light signal is made and the instant when it is visible to the receiver. Light waves travel much faster than sound waves. The lightning flash and the thunder clap occur together, but we estimate the distance of a thunderstorm by the interval between the instants when we are aware of them. The actual speed of æther waves is 186,000 miles per second in empty space (and is practically the same in the atmosphere). Thus wireless signalling is practically instantaneous, a signal made in England, for example, being heard in New Zealand about one-fourteenth of a second later.

Production of Wireless Waves.—It has been seen that the essential qualities of a wave-making device for sound and water waves are elasticity and inertia. When energy is imparted to a device with these properties, a mechanical oscillation is set up and part of the energy is detached and propagated through the medium

as a wavemotion. It is found that the wave-making device for æther waves must be such as to impress on the motion of the electrons exactly the same two properties. It must be arranged so that when the electrons are displaced from their equilibrium arrangement they tend to return to it (corresponding to elasticity); and so that when they are in process of returning their tendency to remain in motion carries them beyond the equilibrium point (corresponding to inertia). Circuit arrangements that are capable of introducing these qualities into the motion of electrons were known for some time before the realisation that such a motion was likely to give rise to æther waves.

Capacity.—The piece of electrical apparatus which confers the property corresponding to elasticity on the motion of electrons is called a **condenser**. It is well known that all materials can be divided broadly into two classes from the electrical point of view—those which allow an electric current (electrons) to flow through them and those which refuse to allow the passage of electrons. The former class are called conductors and are for the most part metals. The substances which act as a bar to electrons are known as insulators. All gases are normally insulators, and examples of solid insulators are glass, porcelain and mica. A condenser consists of two pieces of conducting material (usually two parallel metal plates), separated by a sheet of insulating material. A condenser (C) is illustrated diagrammatically in Fig. 3 (b). The two plates are shown as connected to two conducting wires, which can be joined by closing a switch, or kept apart by leaving the switch open (the position shown), so that there is no complete conducting circuit from one plate of the condenser to the other. If the ends of the wires, with the switch open, are connected to the terminals of a battery or dynamo, one condenser plate acquires a positive charge and the other plate acquires an equal negative charge. The battery or dynamo is able to set the electrons in motion until a certain number have moved round the circuit from one condenser plate to the other, thus leaving one plate with a deficit of electrons or positive charge, and piling up a surplus of electrons on the other plate, which therefore acquires an equal negative charge.

The number of electrons transferred in this manner by any given dynamo or battery depends on the area of the condenser plates and the nature and thickness of the insulating substance between them, and this dependence is expressed by a quantity called the **capacity** of the condenser. The greater the capacity, the greater is the charge acquired by the plates when connected to the terminals of a particular battery or dynamo. As long as the switch is kept open, the plates will retain their charges even when the battery or dynamo connections are broken, for the electrons cannot find a conducting path along which they can move so as to return to their original distribution. Like the stretched spring, however, they have a

strong tendency to do so, and the interposition of the opened switch may be compared to fastening the spring when extended to a nail in the wall. The spring will tend to pull the nail out of the wall, and even if it is unsuccessful there will be a considerable strain on the nail. If the nail comes out the strain will be eased.

In the same way an electrical strain is set up in the insulation (air or oil) of the switch as long as it remains insulating. The "voltage" across the switch (or between the condenser plates) is a measure of this strain. If it is too great the insulating properties of the air or oil break down, and a current flows across the switch until the electrons have regained their normal arrangement and the strain is eased. This appears as a spark across the switch. There is, of course, a corresponding strain across the insulating material between the plates of the condenser and it may happen that the voltage is too great for this insulation. The strain is then eased by an electrical breakdown of the insulator, which becomes momentarily conducting and allows the electrons to equalise their distribution across it. Closing the switch has the effect of providing a complete conducting path for the electrons to regain their normal distribution and thus removes the strain from the insulation between the condenser plates without a breakdown of the insulator.

Inductance.—The property corresponding to inertia in electrical circuits is called **inductance**. It has already been mentioned that one sign of the motion of electrons in a wire is the production of heat in the wire. Another sign is that if a compass needle or other magnet is anywhere near the wire it is deflected from the North and South direction in which it normally points. This action shows that the effects of a current or electron motion are not confined only to the wire, but extend into the air in the vicinity. It is called the magnetic effect of a current, and since to deflect a compass needle or magnet requires energy, it indicates that all the energy associated with the electrons in motion is not being dissipated as heat due to their collisions with molecules of the wire, but that part of it is being carried along outside the wire altogether, and manifests its presence by these magnetic effects.

This "magnetic energy" associated with a current depends on the value of the current. If the current is increased, the magnetic energy is increased. The other thing on which the amount of magnetic energy depends is the shape into which the wire is wound. A coil of wire, for example in which a current is flowing, has a much greater amount of magnetic energy associated with it than the same wire pulled out straight and with the same current flowing through it. This dependence of the magnetic energy associated with a current on the arrangement of the circuit in which the current is flowing is expressed by a quantity known as the **self-inductance** of the circuit. The greater the self-inductance, the greater is the magnetic energy for a specified current. A circuit with self-inductance is represented symbolically by a coil as shown by L in Fig. 3 (c).

The effect of self-inductance in producing an effect corresponding to inertia in the motion of electrons may be seen as follows. Inertia is the objection of a body to having its motion altered. To start a railway truck in motion requires a much stronger effort (or force) than that necessary to keep it moving along at a steady rate once it is started, and similarly to stop it or slow it down requires a strong force in the opposite direction. The greater the inertia of the truck, the greater is the effort required to alter its motion.

A steady current flowing in a wire is equivalent to electrons passing along the wire at a uniform rate, losing a certain amount of energy as heat in the wire and providing a certain amount of magnetic energy outside the wire. This magnetic energy is constant in amount all the time that the current remains steady and so the battery or dynamo which keeps the current flowing has only to provide for the energy loss as heat. When the current is increased to a new steady value, *i.e.*, when the electrons are travelling at a faster steady rate through the wire, the battery correspondingly supplies a greater quantity of energy to meet the increased heat losses. But the greater current also has a greater quantity of magnetic energy associated with it. This increase of magnetic energy must also have been supplied from the battery, and to do this takes time. The result is that the current does not jump instantaneously to its higher value, but gradually increases at a rate depending on the rate at which the extra magnetic energy can be supplied. In the same way, the railway truck has to gather speed from rest and does not instantaneously start off at a steady rate as soon as a push is applied to it.

The greater the inertia of the truck the longer does it take to increase speed. Similarly, the greater the self-inductance of an electrical circuit, the greater is the amount of magnetic energy required by a given increase in current, and the longer it takes for this increase to be brought about. The circuit, in fact, opposes any tendency to increase the current through it much more than it opposes the passage of a steady current, and extra energy is required to overcome the opposition.

The same tendency is evinced when an attempt is made to decrease the current in a circuit. The smaller current corresponds to a smaller amount of magnetic energy in the surrounding space, and while the current is decreasing some magnetic energy has to be reabsorbed into the circuit. Until it has been used up, the current cannot fall to a lower steady value. The energy is actually used up in the same way as that taken from the battery when a steady current is flowing. It goes to supply the heat losses incidental to the flow of electrons along the wire. It thus operates in such a way as to tend to keep the current up to its higher value. As it becomes dissipated in heat, so the current decreases. Hence if a steady current is flowing in a circuit possessing self-inductance and the circuit is suddenly broken, the current cannot immediately fall to zero. It tends to

go on flowing across the break in the circuit, and if there is a large amount of magnetic energy to be dissipated, a spark or arc may take place across the break. In the same way, if the push keeping the railway truck in motion is removed, the truck does not come to a stop at once, but travels along the line until it has lost its "way," or, in other words, until the energy associated with its steady motion has all been dissipated against the friction and air resistance which are trying to bring it to a standstill.

The disinclination of an inductive circuit to allow alterations of the current flowing in it, or what comes to the same thing, the amount of magnetic energy in the space around it, is exhibited still more remarkably if there are other conducting circuits in the vicinity. There may be no battery or other contrivance in these circuits capable of giving rise to a current, but when the current in the original circuit is altering in value, it is found that currents flow in them. Such currents do not flow as long as the original current remains at a steady value—only when the original current is actually changing—and the direction of these "induced" currents, as they are called, is such as to try to keep unchanged the original amount of magnetic energy in the space around the circuits. When a current is induced in this way by an alteration of the current in another circuit, there is said to be "mutual inductance" between the two circuits.

The Closed Oscillatory Circuit.—Suppose now that we connect up a condenser and a coil of wire possessing self-inductance in a circuit like that shown in Fig. 3 (c) and that by some means or other we have given the condenser a charge. The electrons are straining to return to their normal distribution and therefore to discharge the condenser, but this is not possible as long as the switch is open and the insulation of the condenser and switch is sufficient. If now the switch is closed, the surplus electrons on the negative plate of the condenser start moving round the circuit to supply the deficit of electrons on the positive plate. In other words, a current flows in the circuit. If the inductance were not present, this current would have a value depending on the strain between the condenser plates, and as this was eased the current would decrease, falling to nothing when the condenser was fully discharged. Due to the inductance, however, the current does not reach such high values at the beginning of the discharge. Further, when the condenser is completely discharged, the current for the same reason cannot stop immediately. The presence of the inductance keeps the current flowing (in diminishing quantity) after this point, and the extra electrons thus reaching the plate that was originally positive give it a negative charge, the originally negative plate similarly losing electrons and acquiring a positive charge.

When the current finally falls to zero the condenser is therefore charged in the opposite direction. Because of this charge it will again start to discharge, the current flowing the other way round the circuit,

and as the same process is repeated, it will end up by being charged in the same direction as it was to start with. During this time, therefore, a current has flowed first in one direction and then in the other direction round the circuit. The connecting wires and the inductance, however, have a certain amount of energy dissipated as heat in them while the current is flowing, and so the inertial effect of the inductance does not succeed in piling up as large a supply of electrons on whichever condenser plate is negative as there was on the negative plate at the beginning. Each surge of current results in a loss of energy as heat, less energy is therefore available in the succeeding

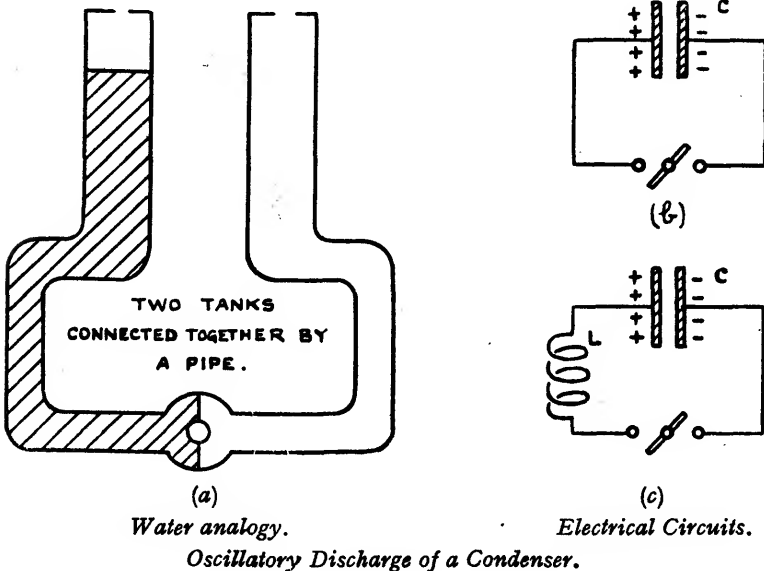


FIG. 3.

surge, the current does not reach such a high value and the condenser acquires a smaller charge. Eventually all the original energy in the charged condenser is used up and the process ceases.

The number of to-and-fro surges of current will obviously be less the greater the amount of energy lost as heat during each surge. Representing currents in one direction by heights above a horizontal time axis and currents in the other direction by depths below it, the variation of the current in the circuit from the time the switch is closed until the condenser is finally discharged may be shown by the graph already used to illustrate a damped wave (Fig. 2 (a)). It is a damped oscillatory current.

This oscillatory discharge of a condenser through an inductance may be compared to the oscillations executed by a stretched string when the tension is released. Another useful comparison is furnished by the arrangement shown in Fig. 3 (a), which consists of

two tanks united by a connecting pipe in which a valve is fitted. While the valve is closed the left-hand tank is filled with water. When the valve is opened, water flows through the pipe so as to make the water level in both tanks the same. Once the water is in motion, however, its inertia causes it to overshoot this point and it rises to a higher level in the right-hand tank. To equalise the levels, water must then flow in the opposite direction, and so an oscillatory flow of water from one tank to the other is set up. The motion of the water, however, is being continually retarded, by friction against the sides of the tanks and connecting pipe and so each oscillatory flow is of smaller amount and the water eventually settles down at the same level in each tank. (In this analogy the "damping" properties of the friction between the water and the sides of the tanks should not be confused with those of the water itself!).

During the condenser discharge the time that each surge of current lasts and therefore the frequency of the oscillatory current depends on two things:—

- (1) The actual number of electrons which have to be transferred from one plate of the condenser to the other. It will be remembered that for a given strain this increases with the capacity of the condenser. The greater the capacity the longer does each surge last.
- (2) The opposition to the growth and decay of the current which is provided by the inductance. If the current could rise to a large value immediately, the greater would be the rate at which electrons were transferred from one plate to the other; and, similarly, the faster the current could die away again, the more quickly would the surge be over. The greater the inductance of the circuit, the greater are the limitations imposed on these processes.

Hence increasing either the inductance or the capacity or both lengthens the period of the current surges and so decreases their frequency. The frequency can therefore be adjusted to any desired value by choosing suitable values of inductance and capacity. The operation of altering either the inductance or the capacity so as to adjust the frequency is called **tuning** the oscillatory circuit.

Open Oscillatory Circuit.—The damping of the oscillatory current has so far been ascribed solely to the heat generated by the electrons in their passage round the circuit. But it must be remembered that our primary reason for choosing this particular type of circuit was that the oscillatory motion imposed on the electrons by it furnished an effective æther wave-making device. In other words, part of the energy originally given to the circuit when the condenser is charged leaves the circuit altogether when the oscillatory current is flowing, and travels out into space as an æther wave train, the

frequency and damping of the wave train being the same as those of the current oscillation. This loss of energy also contributes to the damping of the current; and even if no energy were lost as heat, the oscillation would still be a damped one. We wish, of course, to increase as far as is practicable the energy lost in producing æther waves, and it is found that in the oscillatory circuit so far considered, in which the condenser consists of two metal plates separated by a thin strip of insulating substance, very little energy is detached in this way.

The energy which goes off as æther waves can be very greatly increased by separating the condenser plates until there is a large amount of insulating material between them. The practical form then assumed by the condenser is that of a long wire which is fixed in the ground at one end and has its other end free in the air. The air acts as the insulating substance, the wire as one plate of the condenser,

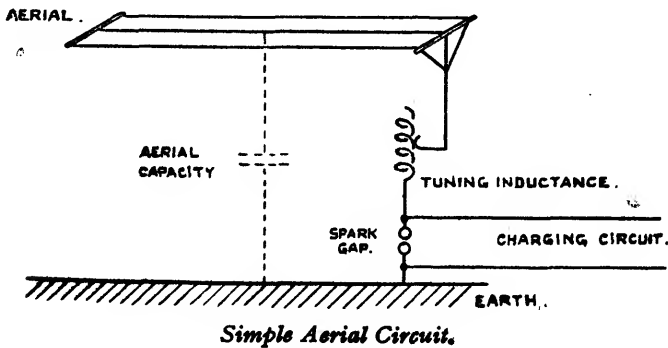


FIG. 4.

and the other plate is provided by the ground in the vicinity, since the earth is a fairly good conductor. The higher the free end of the wire is above the ground, the more efficient is this condenser from the point of view of losing energy as æther waves when it is charged up and allowed to discharge in an oscillatory manner. The wire is therefore usually run up vertically as high as is practicable and there may also be horizontal lengths of wire attached at the top to increase its size as a condenser plate and therefore its capacity to earth. It is called an "aerial" or "antenna." In a ship the "earth" is the hull of the ship. The inductance necessary to obtain an oscillatory discharge is partly provided by the aerial wire, and a coil is also inserted at some convenient point between the top of the aerial and earth. The desired frequency of oscillatory current is obtained by adjusting this coil to have the requisite amount of inductance. An aerial circuit is shown in Fig. 4.

Every time the aerial capacity is charged up, a damped oscillatory discharge takes place and a damped æther wave-train is emitted into space. From the time the discharge stops until the condenser

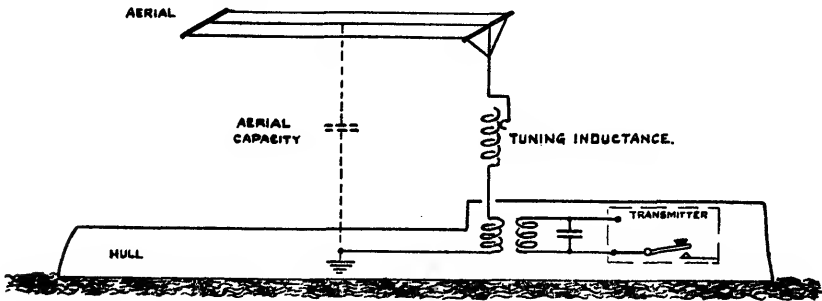
is again discharged nothing happens (Fig. 2 (a)). Thus the number of æther wave-trains sent out in a given time depends on the number of times the condenser is charged during that time. When we come to consider the reception of wireless signals we shall see that it is desirable for this number to be about 500 to 1,000 times every second. This means that the switch shown in the oscillatory circuit of Fig. 3 (b) would have to be closed this number of times a second to allow the condenser to discharge and opened the same number of times to allow the condenser to be charged.

It is more or less out of the question to devise an efficient switch closing and opening mechanically as fast as this. Use is therefore made of a property of insulators already described, namely, that as the electrical strain across them is increased, a point is reached where the insulation breaks down and becomes conducting. The "switch" used consists of two metal plugs with an air gap between them. As the condenser is charged the insulation across this air gap becomes more and more strained. When the condenser has received a certain charge depending on its capacity, the insulation of the air gap breaks down and a current passes across it in the form of a spark. The gap remains conducting as long as the oscillatory condenser discharge continues, but, once this ceases, it is able to return to its former insulating condition. The condenser may then be again charged in the same way, and discharges when the same strain is produced across the gap. The "spark gap," as it is called, thus behaves as a switch which closes automatically whenever the condenser acquires a certain charge and opens again when the condenser is fully discharged.

A complete arrangement for producing æther wave-trains is therefore as shown in Fig. 4. It consists of an aerial (the capacity), a tuning inductance and a spark gap. The machine for charging the condenser is connected to the two sides of the spark gap, which is the same as connecting it to the two plates of the condenser.

The oscillatory current produced in a circuit of this nature is very highly damped, since the energy losses in the spark gap as heat, light and noise are considerable. This is objectionable for several reasons, and in practice the oscillatory current is usually set up in a closed oscillatory circuit with a spark gap and transferred as quickly as possible to an aerial circuit tuned to the same frequency as the generating circuit, as shown in Fig. 5. The transfer is accomplished by making use of the property of an inductive circuit already described, namely, that when the current is changing in it, currents are set up in neighbouring circuits. A coil is inserted in the aerial circuit and brought near the primary (generating) circuit inductance. The current induced in this coil by the oscillatory current in the primary inductance is also oscillatory and of the same frequency. By putting out the spark quickly the oscillation is then confined to the aerial circuit and the spark gap losses in the primary circuit are avoided.

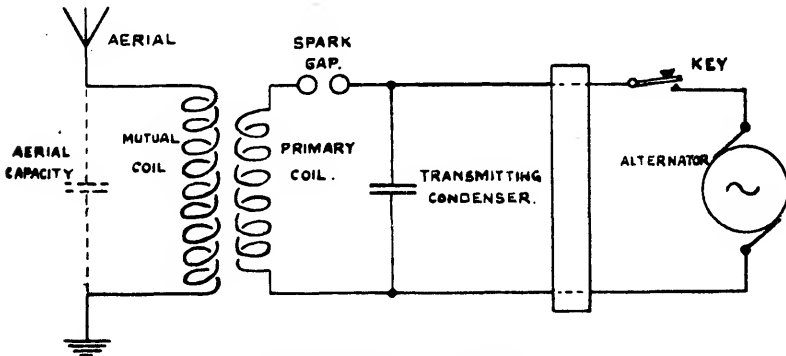
In order to send intelligible signals, a means must be provided whereby the process of charging and discharging the condenser, and therefore the radiation of æther waves, can be controlled at will.



Simple Aerial Circuit (Mutually Coupled).

FIG. 5.

This is accomplished by inserting a Morse key in one of the leads to the charging machine (Fig. 6). Whenever the key is pressed, the condenser may be charged and discharged; when the key is released the machine cannot charge the condenser. The charging arrangement itself usually consists of an alternator and transformer. The effect of this will be understood by those with a



Simple Spark Transmitter.

FIG. 6.

knowledge of electrical machinery. All that can be said about it here is that it provides a convenient means of charging the condenser at a frequency of about 500 times a second.

There are two frequencies which must be carefully distinguished in such a spark transmitting circuit :—

- 1) The frequency with which the condenser is charged by the alternator and transformer. This is about 500 times a second and is the frequency of the æther wave-trains,

i.e., the number of such wave-trains per second. This is called the "spark-train" frequency.

- (2) The frequency of the oscillatory current produced by the discharge of the condenser. This depends, as stated above, on the values of the inductance and capacity which constitute the oscillatory circuit. It is normally of the order of 500,000 cycles per second. In dealing with frequencies of this order the unit of reference is usually taken as the kilocycle per second (kc./s.), which is 1,000 cycles per second. Thus 500,000 cycles per second is 500 kc./s.

This is the frequency with which the individual cycles occur in any one wave-train. It is called a Radio Frequency. It may be mentioned in passing that the higher the radio frequency, the greater is the proportion of energy in the oscillatory circuit which is emitted as æther waves. This is one principal reason why it is necessary to have such high-frequency oscillatory currents.

Taking the above figures for illustrative purposes, it will be seen that the condenser receives a charge from the alternator and transformer once in every $\frac{1}{500}$ second. If we assume that there are 100 cycles in the oscillatory discharge, the time taken before the condenser is fully discharged is $\frac{100}{500,000}$ second, since each cycle takes $\frac{1}{500,000}$ second. $\frac{100}{500,000}$ second = $\frac{1}{5,000}$ second and is therefore only a tenth of the time ($\frac{1}{500}$ second) between condenser charges.

Thus for nine-tenths of the time that the key is pressed no energy is being sent out into space as æther waves, which is one reason for the comparative inefficiency of spark transmission. (See Fig. 2 (b).)

Reception.—Æther waves travel out in all directions from the ordinary vertical aerial, but just as a light produces a smaller effect on the eye, the further it is away, so does the effect of æther waves of wireless frequency fall off as the distance from the transmitter increases. The range of a transmitter depends on the power developed in the aerial circuit and also on the frequency of the waves. Even within this range, however, as has already been explained, special arrangements must be made if the waves are to produce any effect which the human senses can appreciate. The arrangement for this purpose is called a wireless receiving circuit, and the sense which is usually stimulated is the sense of hearing. In other words, the wave is made to produce a sound, which reflects in its character the nature of the signal made by pressing the key at the transmitter.

When discussing wave motion it was seen that the waves set up in water produce an up-and-down oscillatory motion of objects floating on the surface. Boats, for example, rock up and down in step with the waves passing under them. Water waves are produced in the first place by a similar oscillatory movement of the material wave-making device. Since æther waves are set up by an oscillatory motion of electrons, we may reasonably expect that whenever an æther wave encounters electrons it will set them in oscillation in step with itself, and this proves to be the case. Hence the first objective in a receiving circuit is to provide a supply of electrons in a convenient place to be set in oscillation by passing æther waves. This is attained by erecting a long vertical wire at the receiving station. The electrons in the wire then oscillate up and down whenever an æther wave passes the wire. The higher the wire, the greater is the effect, but since at long distances from the transmitter the amplitude of the æther wave is extremely minute, the effect produced is correspondingly small even with long wires. In order to obtain an electron oscillation of reasonable amount, a further adjustment is necessary. The clue to this we may seek as before from a comparison with a mechanical case.

Suppose that a child sitting in a swing succeeds in inducing a playmate to push it for him. The other child will only have the strength to administer weak pushes. The first push only carries the swing a little way, but the effect of further pushes depends very much on the instants that are chosen for their delivery. If they are delivered at any odd times when the moderately unselfish benefactor is not engrossed in his toffee, the unfortunate in the swing is not likely to derive much pleasure from his pastime. His heart will never come remotely near his mouth. But if the swinger chooses his moments well, even his puny efforts will soon work up quite a large swing. It is all a question of timing the impulses appropriately, the proper moments being those at which the swing has just executed one to-and-fro movement and is about to start another. The actual time this takes depends on the construction of the swing, but if the swinger "waits for it" he will obtain more value from his efforts than if he expended much more energy in a series of stronger but ill-timed pushes. When the swing is going well and he stands back to survey his efforts, he will see that it continues swinging without his help at the same frequency. There is, in fact, a certain frequency at which the swing tends to oscillate naturally, and he has timed his swings so as to fit in with this frequency.

Now we have already seen that when the electrons in an oscillatory circuit, *i.e.*, a circuit containing inductance and capacity, are set in motion, their oscillations take place at a certain frequency depending on the amounts of capacity and inductance. In order that the electron oscillation in our vertical wire may reach an appreciable size, we see by analogy with the case of the swing that

the wire must be made part of an oscillatory circuit whose total inductance and capacity are so chosen that the electrons in it tend to swing naturally with the same frequency as the passing æther wave. This is a comparatively simple matter to arrange. It has been explained that an earthed vertical wire with an inductance inserted in it constitutes an oscillatory circuit whose frequency can be adjusted to any desired value by altering the amount of inductance. The vertical wire or aerial has a certain capacity to earth, which cannot be altered except by altering the aerial, but we may if we choose insert another condenser somewhere in the aerial circuit. Varying this condenser alters the total capacity and therefore the "resonant" frequency of the aerial circuit, as it is called, *i.e.*, the frequency at which an æther wave of given amplitude produces the greatest amount of oscillatory motion of the electrons in the circuit. In practice it is often more convenient in receiving sets to have a variable capacity than a variable inductance in the aerial circuit, and turning the dial labelled "Aerial Tuning" in the normal broadcast receiver usually has the effect of varying the total capacity of the aerial circuit by altering the effective area of the plates, and hence the capacity, of a condenser fixed inside the panel opposite the dial.

The nett effect of the receiving aerial and its associated components is therefore to set up an oscillatory current in the aerial at the frequency of the æther waves. The electrons do not respond to anything like the same extent to æther waves of other frequencies, for, although the impulses they are given may be just as strong or stronger, the timing of the impulses is wrong—either too fast or too slow. The aerial circuit thus "selects" the wireless wave to which it is tuned and practically ignores any other which may be passing it at the same moment. This **selectivity** is an important point in the design of receivers, and various other arrangements are usually incorporated in a receiver so as to increase still further the degree of selectivity imparted to it by tuning the aerial circuit.

It will be observed that the first stage in receiving a wireless signal is the reverse of the last stage in transmitting it. At the transmitter an oscillatory current in a tuned circuit produces æther waves of its own frequency. At the receiver these æther waves produce an oscillatory current in a circuit tuned to the same frequency.

The next stage is to enable this current to make its presence known—generally by causing it to produce an audible sound. Now it is quite a simple matter to cause an oscillating current to give rise to a sound. It is the principle used in ordinary telephones. When you speak into a telephone mouthpiece you cause a diaphragm in the mouthpiece to vibrate. The oscillatory movement of this diaphragm sets up an oscillating current at the same frequency in the line and this current flows along the line till it reaches the ear-piece of the receiver, where its magnetic effect operates another

diaphragm. As the current changes in value, the pull on the diaphragm alters correspondingly. The diaphragm thus vibrates at the frequency of the current, and, agitating the air in its vicinity, produces a sound of the same frequency in the ear of the listener. The range of frequencies of the sound waves that affect the human ear is, however, limited. The highest note that can be heard varies with the individual. Most people of normal hearing can hear a bat squeak, but some elderly people cannot. The average upper limit is about 16,000 cycles per second, which gives a very high-pitched squeak. For comparison it may be said that middle C on a piano has a pitch of about 250 cycles per second.

The frequencies of the æther waves used in wireless telegraphy are generally far above 16,000 cycles per second, and so even if we could pass the oscillatory current in the receiving aerial circuit through the windings of a telephone earpiece, we should not hear the corresponding sound. In point of fact, however, there are two other reasons why we should hear no sound in the telephones even if radio-frequency sound waves were intrinsically audible. One is that the pulls and pushes on the telephone diaphragm would succeed each other so rapidly that it could not respond to them. The other is an electrical reason. The telephone windings have a high inductance and so object strongly to any change in the current through them. As a radio-frequency current is a current which is changing with great rapidity, the inductance of the windings presents a most formidable obstacle to its passage. It would choose almost any other path in preference. For all these reasons, therefore, it is quite impracticable to attempt to pass the radio-frequency current straight through the telephones and obtain a perceptible response. By some means or other we have to produce from our radio-frequency current a current oscillating at an audible frequency. This audio-frequency current flowing through the telephone windings will then give an audible note.

The operation whereby an audio-frequency variation of current is produced from a radio-frequency current is known as **rectification** and the electrical apparatus that is most frequently used to bring it about is called a thermionic valve.

The Thermionic Valve.—The simplest form of the thermionic valve is shown diagrammatically in Fig. 7 (a). It resembles an electric lamp in that it consists of an evacuated glass bulb in which is inserted a fine wire **filament** capable of being heated by the passage of an electric current through it. In addition, a metal plate is also inserted in the bulb and a wire passes from this plate through the glass to a terminal outside.

In an electric lamp the wire filament is heated so that it may become white hot and emit light waves. The reason for heating it in a thermionic valve is quite different. The filament, like all material substances, contains electrons, and the effect of heating it

is, as it were, to cause the electrons to boil vigorously inside it. Now, if we boil some water in a dish, the water evaporates and passes as steam into the surrounding atmosphere. Similarly, when the electrons boil, some of them jump right out of the filament and escape into the bulb. The number of electrons escaping from the filament in this way increases very rapidly as the filament temperature rises, which provides the reason for heating it. It is heated by an electric battery merely for convenience; any other method of heating it would be just as efficient in causing it to emit electrons.

A variable resistance or "rheostat" is included in the filament heating circuit to enable the heating current, and therefore the filament temperature and the number of electrons emitted, to be adjusted to any desired value.

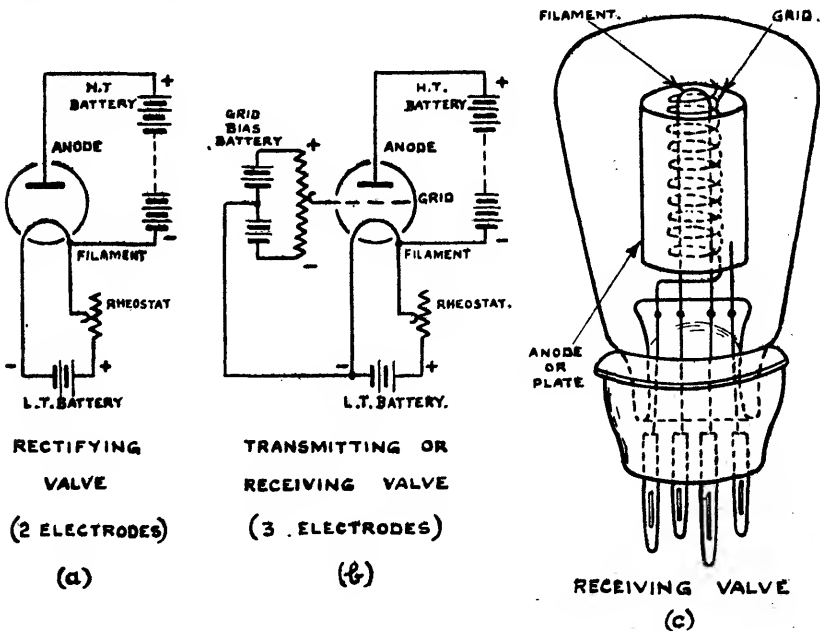


FIG. 7.

If no other provision is made for their disposal these electrons hang about in the space around the filament and by their accumulated negative charge add to the difficulty of later electrons in escaping, since they tend to repel them back into the filament. Suppose, however, that we connect the metal plate, which is usually called the **anode**, to the positive terminal of a battery (called the H.T. battery), and the filament to the negative terminal. This has the effect, as already explained, of creating a deficit of electrons in the anode, and the electrons in the space between anode and filament are attracted to the anode to supply the deficit. As long as the battery is connected in this way there is a permanent deficit of

electrons at the anode, and so a steady stream of the electrons emitted by the filament flows across the intervening space. In other words, a current flows through the valve. If now the battery is reversed, *i.e.*, if the anode is connected to the negative terminal and the filament to the positive terminal, the anode acquires a surplus of electrons and the electrons in the space have no tendency to stream over to it. They are, in fact, repelled from it, since its surplus of electrons gives it a negative charge. Under these conditions, therefore, no current will flow through the valve.

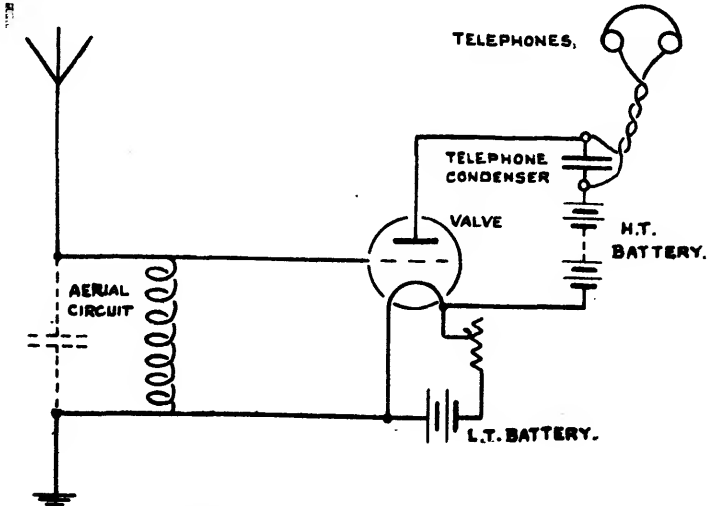
The name "valve" for this appliance will now be understood, since it only allows current to flow through it in one direction. It is called a "thermionic" valve because the electrons are emitted by heating (therm) the filament and because they travel to the anode when it is positive, "ion" being the Greek word for a traveller.

In this form the thermionic valve is quite capable of rectification, but its action may be improved by inserting yet another piece of metal inside the glass bulb (Fig. 7 (b)). This usually takes the form of a wire spiral or mesh, and it is placed much nearer the filament than is the anode. It is called the **grid**, and may be made either positive or negative to the filament by a battery in the same way as the anode. Since it is so much nearer the filament, however, than is the anode, it is correspondingly more efficient than the anode in setting the electrons emitted by the filament in motion across the valve when it is positive and in holding them back when it is negative. It thus gives a more efficient control of the electron flow than does the anode. Its chief virtue, however, lies in the fact that when it is positive it does not collect many of the electrons which it sets in motion across the valve. Due to its "open-work" construction, the vast majority of these electrons pass it and travel to the anode. It is like the traffic policeman, whose physical proportions need not be sufficient to block the roadway completely. He can either hold up the traffic or allow it to pass at will by a small effort on his part, and when he does allow it to go on, it passes him quite easily, although he is still present. The anode might similarly be compared to a drawbridge which, when hoisted (negative), blocks the traffic completely by the gap it causes in the roadway. When it is let down (made positive) the traffic must all pass over it before it can proceed.

The grid therefore provides a very efficient control of the electron traffic through the valve while exacting a negligible toll in electrons actually impounded for itself.

Rectification.—It is now possible to consider how the thermionic valve produces rectification of the radio-frequency current. We left the electrons swinging vigorously in the aerial circuit at the frequency of the passing æther wave. Suppose now that two points on the aerial (generally the two ends of the tuning inductance or the two plates of the tuning condenser) are connected to the grid

and filament of a valve, as shown in Fig. 8. Electrons swing backwards and forwards between these two points. One moment the point connected to the grid has a surplus of electrons and that connected to the filament has a deficit. The next moment the



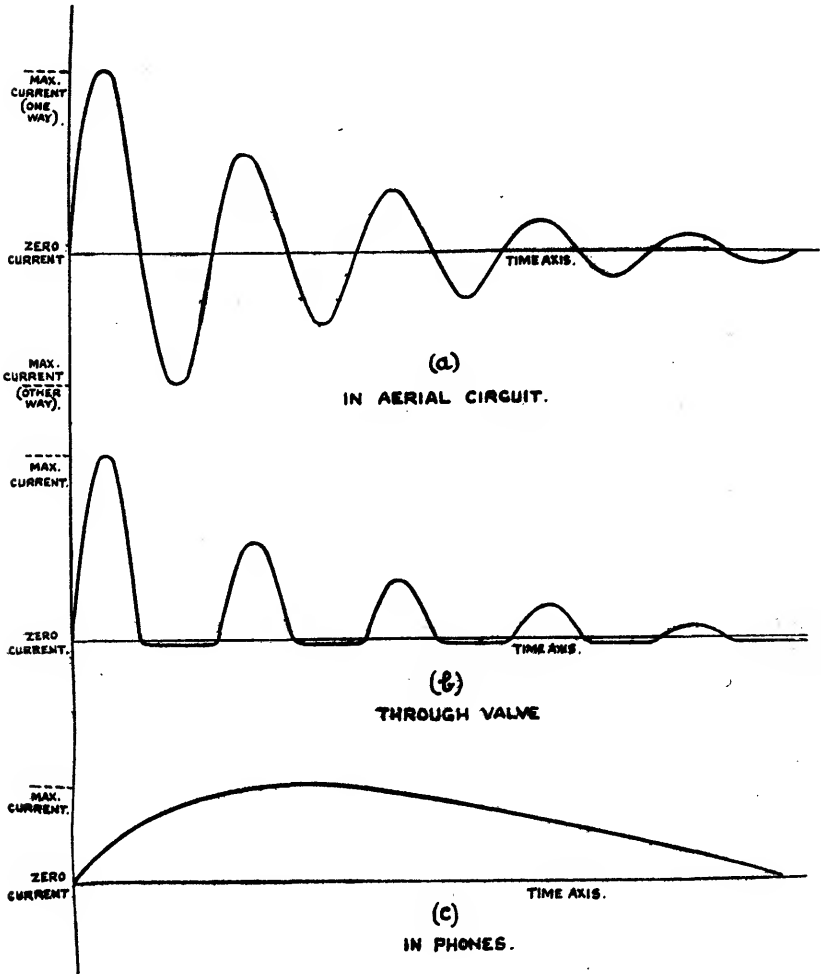
Simple Valve Receiver for Spark Signals.

FIG. 8.

point connected to the grid has a deficit of electrons and that connected to the filament has a surplus. The effect is thus much the same as if a battery were connected between grid and filament and the battery connections were being reversed with the frequency of the electron swing. The grid becomes alternately positive and negative to the filament. When it is positive electrons flow to the anode, when it is negative they are held back. Thus, during every cycle of the æther wave, a pulse of current flows through the valve. This is illustrated in Fig. 9 (a) and (b).

The telephones in which we eventually hope to hear a sound are connected in the anode circuit between the anode and the battery. Thus when the radio-frequency pulses of current through the valve endeavour to continue their journey round the circuit through the battery and back to filament, they arrive at the telephones. As they are at radio-frequency they will not, as previously explained, evoke any response from the telephones. Suppose, however, that a condenser is connected across the telephones, as in Fig. 8; the high inductance of the telephone windings holds up the rapid current pulses and they find it easier to flow into the condenser and charge it up. Since all the current pulses are in the same direction, each as it arrives at the condenser helps to add to the total charge. It should be noticed that if the telephones and the condenser had

been inserted directly in the aerial circuit this would not be the case; for the electrons in their swing would first surge into the condenser plate and then surge out of it and there would be no nett accumulation of charge on the condenser. It is the interposition



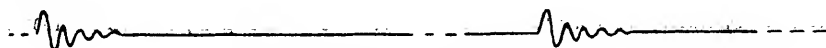
Flow of Current in Receiver due to one Incoming Spark Train.

FIG. 9.

of the valve which ensures that all the electrons reaching the telephones and condenser are travelling in the same direction.

As soon as the condenser begins to charge up, it will, of course, start to discharge through the inductance of the telephones. At first it accumulates charge more quickly than the inductance will allow it to flow away, but as the effect of the passing wave-train

dies away and no more current pulses flow through the valve, the condenser charge eventually leaks away completely through the telephones. While this discharge current (shown in Fig. 9 (c)) is flowing through the telephones, the diaphragm is pulled over, and it returns to its normal position when the condenser is fully discharged. The effect of the complete wave-train is therefore to displace the diaphragm once only. When the next wave-train arrives it produces exactly the same effect. The result is that the telephone diaphragm receives as many impulses in a second as there are wave-trains in a second. Now it was seen when considering spark transmission that the number of wave-trains per second was of the order of 500. The diaphragm is therefore vibrating 500 times a second and so gives rise to a sound wave whose frequency is 500 cycles per second, and which is therefore well within the audible range. The note heard in this case would be about the octave note of middle C.



(a) IN AERIAL CURRENT.



(b) THROUGH VALVE.



(c) IN PHONES

Flow of Current in Receiver due to a Series of Spark Trains.

FIG. 10.

Amplification.—The above is an elementary description of the essential features of a wireless receiver for spark transmission. It may often happen, however that even when the aerial circuit is tuned to resonance with the incoming signal, the latter is so weak that it produces only a very slight noise in the telephones. In such a case the electron swings may be increased in size or **amplified**, before they are applied to the grid and filament of the rectifying valve.

The thermionic valve is also available for this purpose. When used to **rectify**, the grid is made so negative to the filament normally that practically no electrons reach the anode except when the grid swings positive during the oscillations of the electrons in the aerial

circuit. To use the valve as an **amplifier**, however, the anode and grid batteries are so adjusted that there is a steady stream of electrons from filament to anode. When a signal arrives the electron oscillation in the aerial circuit causes the grid to swing alternately positive and negative to the filament compared with its steady value, and the current to the anode increases and decreases correspondingly. Since a small effort between grid and filament produces a much greater effect on the anode current, the original electron swing on the grid gives rise to a much greater electron swing on the anode, and this swing may then be applied to the grid of a valve adjusted for rectification; or if it is still too small, to the grid of another valve also arranged for amplification, before it is finally rectified. Since the electron swings are amplified while they are still at radio-frequency this kind of amplifier is called a **radio-frequency amplifier**.

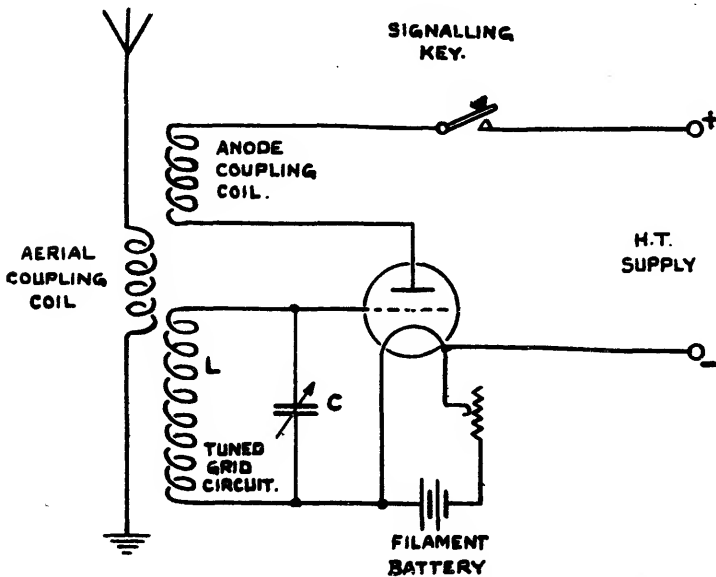
In the same way, the audio-frequency current which is produced by detection may be amplified before it is finally passed through the telephone windings. Valves used for this purpose are known as audio-frequency amplifiers or **note magnifiers**.

Production of Continuous (Undamped) Waves.—The waves set up in the æther by an oscillatory current flowing in the aerial circuit of a spark transmitter have been seen to be damped waves. The reason for this is that energy is continuously being used up during the condenser discharge, either as heat losses or in the æther waves emitted, and there is no correspondingly continuous supply of energy to make up for these losses. As a result the aerial is only energised for a fraction of the time that the key is pressed and the transmission is inefficient. In addition, a damped æther wave tends to have a shock effect on receiving aërials and to set their electrons oscillating quite irrespective of the frequency to which they are tuned. This is much the same effect as is produced if you throw yourself heavily on a spring mattress. Before it settles, it will bounce up and down two or three times at a frequency depending only on the elasticity and inertia of the springs. A damped æther wave gives a receiving aerial a similar jerk as it passes and so it is difficult to obtain selectivity when such waves are used for transmission.

Both these disadvantages are obviated by the use of continuous or undamped wave transmission. The transmitter aerial is energised to the same extent all the time the key is pressed, and the shock effect on receiving aërials is avoided. Thus continuous waves (C.W.) are practically always used in modern wireless communication. C.W. transmitters, however, are more fragile than spark transmitters and so the latter are generally fitted as emergency attachments in case of breakdown of the main set. Spark transmission is also useful for distress calls, since its agitation of all receiving aërials, irrespective of their tuning, is then obviously an advantage.

The early C.W. transmitters made use of an arc, but this type is now nearly obsolete in the Service, and only the later type of C.W. transmitter, which utilises the properties of the thermionic valve, will be considered here.

Valve Transmitter.—It should now be recognised that the æther waves set up by the flow of an oscillatory current in an aerial circuit are of the same nature as the current. If the current is damped, the æther waves are similarly damped. If an undamped oscillatory current can be produced, then undamped æther waves will be sent out from the aerial.



Simple Valve Transmitter.

FIG. 11.

A simple valve circuit capable of producing an undamped oscillatory current is shown in Fig. 11. The filament heating circuit and the H.T. battery between anode and filament are already familiar. It is to be noted that the positive terminal of the H.T. battery is not fastened directly to the anode, but to one end of an inductance, the other end of which is connected to the anode. This does not affect the permanent deficit of electrons on the anode under such conditions, but it does mean that the electrons flowing through the valve to the anode to supply the deficiency (when the grid allows them), then flow through the inductance in the anode lead on their way back to filament through the key and the battery. Between the grid and filament of the valve there is connected an oscillatory circuit.

Until the key is pressed no current will flow through the valve, but there will be an emission of electrons from the filament, their number depending on the heating current. When the key is pressed, the anode is made positive to the filament, and a current starts to flow through the valve. Since there is an inductance in the anode lead, this current will not rise instantaneously to the value it would have if the positive terminal of the battery were directly connected to the anode. Hence a current whose value is changing (increasing) flows through the inductance. This has the effect of causing an induced current to flow (by mutual inductance) in the inductance L of the oscillatory circuit between grid and filament, and this induced current therefore charges up the oscillatory circuit condenser C . As we know, this condenser then starts to discharge, and if the oscillatory circuit were now disconnected from the grid and filament, this discharge would produce a damped oscillatory current.

The oscillatory circuit condenser C , however, is connected between grid and filament, and the direction of the induced current is such that the electrons constituting it flow upwards through L . The top plate of C therefore becomes negative, acquiring a surplus of electrons, and the bottom plate positive, *i.e.*, the grid becomes negative to the filament. This cuts down the flow of electrons from filament to anode in the valve, and the current through the anode coil starts to decrease. Now an increasing current through the anode coil made electrons flow upwards in L , and consequently a decreasing current through the anode coil makes electrons flow downwards through L . Electrons are already flowing downwards in L as the condenser discharges, but it has been seen that the impetus of these electrons would not, by itself, be sufficient to give the condenser the same charge in the opposite direction (top plate positive, bottom plate negative) as it originally had. It will be remembered that this was because of the energy lost as heat and æther waves during the flow of electrons. In the present case, however, the downward flowing electrons are given an extra "kick" by the mutual inductance effect of the decreasing current in the coil in the anode lead. The nearer the anode coil is to the grid inductance, the greater is this "kick," and it can be made sufficient to compensate for the drag on the electron motion caused by heat and æther wave losses. The result is that the condenser acquires as large a surplus of electrons on its bottom plate as it originally had on its top plate.

During this surge of electrons from top to bottom plate of the condenser the grid is becoming less negative to the filament, and the current through the valve, which was previously decreasing, is starting to increase again. Thus when the condenser again starts to discharge and electrons flow upwards through the coil L to the top plate, there is an increasing current through the valve and the anode coil, and a tendency for electrons in coil L to be given an upward kick by mutual inductance, with the result that heat and

æther wave losses are again compensated and the condenser acquires as large a surplus of electrons on its top plate as it had originally. The cycle of events is then repeated.

It will be seen that as long as the key is pressed, the alternately increasing and decreasing currents in the anode coil assist the to-and-fro swings of electrons in the oscillatory circuit, and the condenser receives the same charges during each oscillation. The oscillatory current therefore remains of constant amplitude and so gives rise to an undamped æther wave.

At first sight, the process may look rather like the attempt of a man to lift himself by pulling at his own bootstraps, since the changing currents in the anode coil are caused by the grid becoming alternately positive and negative to filament, and the maintenance of this in constant amount depends in turn on the same changing currents in the anode coil. It must be remembered, however, that it is the deficit of electrons on the anode due to the battery between anode and filament which causes any current to flow through the anode coil at all. The grid is not responsible for this current any more than the policeman on point duty is responsible for the presence of traffic in his street. He merely controls its movements along the street, just as the grid controls the flow of current to the anode and therefore through the anode coil. The power which carries the vehicles along is their own engine power. The power which carries the electrons through the valve and the anode coil and which ultimately supplies the heat and æther wave power losses in the oscillatory circuit is derived from the H.T. battery. When the key is released and the battery connection broken, the oscillatory current in the grid circuit dies away because of heat and radiation losses, just as it did in the spark transmitter circuit. The condenser is, of course, not charged again until the key is pressed once more. Thus the oscillation is continuous all the time the key is pressed, dies out as a damped oscillation when the key is released, and nothing more happens until the key is again pressed.

In the circuit shown in Fig 11, the undamped oscillatory current is not generated in the aerial circuit, but is transferred to it by mutual inductance from the primary oscillatory circuit between grid and filament of the valve. The primary circuit could, however, be utilised as an open oscillatory circuit and a separate aerial circuit be dispensed with, by substituting an aerial for the primary condenser. Both these types of oscillatory circuit are in common use in valve transmitters.

Reception of Continuous Waves.—The use of continuous waves for signalling raises a new problem in reception. It will be recalled that the radio frequency oscillatory current produced by an æther wave in a receiving aerial must be made to produce an audio frequency oscillatory current before there is any response in the telephones. This is achieved for spark signals by making use of

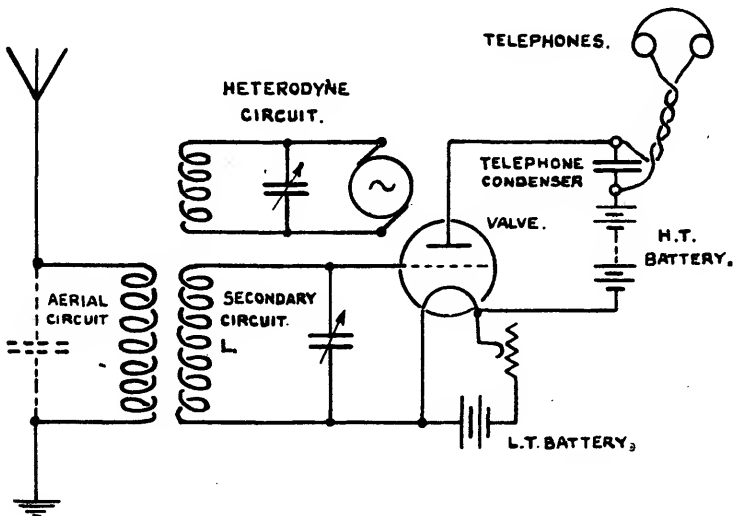
the fact that the damped æther wave-trains follow each other from the transmitter at times corresponding to an audible frequency. There is no such audio frequency stamp on C.W. transmission, and the rectified current flowing through the telephones in the receiver would therefore remain of the same amount all the time the key was pressed. The only intimation in the telephones would be a click as the diaphragm moved over when the signal arrived and the rectified current began to flow through the windings, and a corresponding click as the diaphragm moved back when the signal stopped. It therefore becomes necessary to introduce a variation of the rectified current at audible frequency by means of some device incorporated in the receiver itself. The device normally used is known as a **heterodyne oscillator** and the process is called **heterodyning**.

Visualise two men walking along the street together. One is tall and lanky and takes long strides, the other is short and cannot take such long strides as his partner. Thus, if the two keep abreast, the short gentleman must be moving his legs faster than the tall one and taking more strides each minute. If they start off in step with the left foot they will become more and more out of step, the tall one starting his left foot forward after the short one. This goes on until the tall walker is just advancing his right foot as the short one is advancing his left. The two are completely out of step at this moment. They now start to work more into step again and will eventually come into step for a moment just as they were at the beginning. The whole process is then repeated. Thus, as they walk along, they are alternately exactly in step and completely out of step at certain moments; at intermediate times they are more or less out of step.

If one takes 50 steps a minute and the other 60, then in one-tenth of a minute the first one will have taken exactly five steps and the second one exactly six steps. At this moment, therefore, the first one is stepping off with his right foot just as the second one is stepping off with his left. They are completely out of step. In one-fifth of a minute the first completes ten steps and the second twelve; after one-fifth of a minute, therefore, they are exactly in step again. In another tenth of a minute they will be completely out of step and so on. It will be obvious that the intervals between the moments at which they are in step or completely out of step are considerably longer than the time occupied by one step of either. Thus one step of the short gentleman takes one-sixtieth of a minute, but the interval between his "in step" moments is one-fifth of a minute. He takes 12 steps in the interval. We may say that the frequency of his steps (60 per minute) is 12 times the frequency at which he is in step (5 per minute).

The above is the principle which is applied to the reception of continuous waves in order to obtain an audio frequency variation from a radio frequency one. For steps we merely substitute electron swings between grid and filament. One of the walkers, or

electron-swingers, is provided for us by the passing æther wave. The other we have to provide for ourselves, and this is accomplished by means of a small valve circuit at the receiving station, of the same type as that shown in Fig. 11. This is the only essential difference between Fig. 12 and the receiver for spark signals shown in Fig. 8. It will be observed, however, that in this receiver the electron swings set up in the aerial circuit by the passing wireless wave are made to set up similar swings of the electrons in a tuned circuit between the grid and filament of the detector valve by making use of a mutual inductance effect. This makes the receiver more selective, but otherwise does not alter the situation. The electron swings between grid and filament are still at the same frequency as the signal.



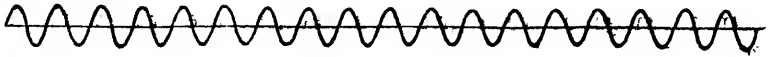
Simple Valve Receiver for C.W. Signals.

FIG. 12.

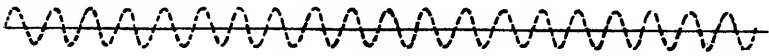
Now suppose that the heterodyne circuit is also set in oscillation, and is brought near the inductance L of the tuned circuit between grid and filament of the detector valve. There will be a mutual inductance effect and the electrons in L will tend to be set swinging at the frequency of the heterodyne oscillations. They are also tending to swing at the frequency of the incoming signal oscillation and their nett motion is the result of these two tendencies. Either tendency by itself may make electrons move from the grid end of L to the filament end at some particular instant. If they coincide in their efforts to do this more electrons will be moved than either would move by itself and the grid becomes correspondingly more positive to the filament. If the incoming signal is driving electrons from grid to filament while the heterodyne oscillation is driving them

from filament to grid, there will only be a small nett movement of electrons and a correspondingly small surplus of electrons at grid or filament. Now our two walkers worked into and out of step, because their step frequencies were different (50 and 60 steps per minute respectively). In the same way the effects of the heterodyne and incoming signal on the electrons in the grid coil L can be made to be in the same or opposite directions at comparatively long intervals by causing the heterodyne circuit to produce oscillations at a frequency different from that of the incoming signal.

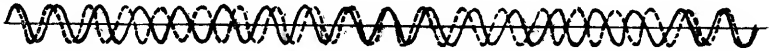
(a) INCOMING SIGNAL.



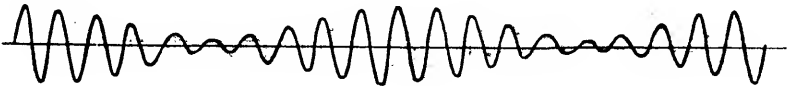
(b) HETERODYNE FREQUENCY.



(c) ABOVE EFFECTS SUPER-IMPOSED.



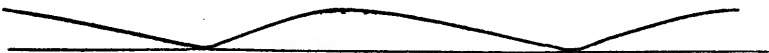
(d) AMOUNT THAT GRID SWINGS POSITIVE OR NEGATIVE TO FILAMENT.



(e) RECTIFIED CURRENT THROUGH VALVE.



(f) CURRENT IN PHONES.



Heterodyne Frequencies.

FIG. 13.

This is illustrated in Fig. 13. Fig. 13 (a) shows the electron swings in coil L that would be produced by the incoming signal alone, and Fig. 13 (b) those produced by the heterodyne oscillator alone. They are shown together in Fig. 13 (c), and it will easily be seen that they start off in step and so help each other, then work out of step and oppose each other, then work into step again and so on. The nett electron swings due to both oscillations are seen in Fig. 13 (d), which is merely the addition of the two simultaneous tendencies of Fig. 13 (c). The grid is swinging considerably negative

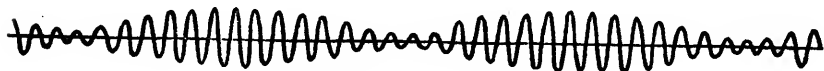
and positive to filament at the beginning when the effects are in step, the swing falls off as the effects work out of step, there is practically no swing when they are completely out of step, and as they work into step again the swing begins to increase, reaching its greatest value when they are once more exactly in step.

As previously explained under spark signal rectification, the electron flow to the anode of the valve is proportional to the grid swing when the grid is positive to filament, and there is no electron flow when the grid is negative to the filament. The current flowing through the valve is therefore as shown in Fig. 13 (e), a series of pulses of current at radio frequency, but with their amount rising and falling at a much lower frequency. In the figure there are actually two "peak" pulses and two nearly negligible pulses of current over about eighteen pulses altogether. The charge flowing into the telephone condenser at each pulse depends, of course, on the size of the pulse. In the neighbourhood of the peak pulses the condenser acquires a large charge, its discharge current through the telephone windings is correspondingly large, and there is a large displacement of the telephone diaphragm. When the rectified current pulses are weak and there is only a small charge on the condenser, the telephone diaphragm returns nearly to its normal position. Thus the diaphragm vibrates at the same frequency as there are peak pulses of rectified current flowing into the condenser; in other words, at the frequency with which the incoming signal oscillation and the heterodyne oscillation work into step with each other. This frequency can be made of any value we choose by adjusting the difference between the heterodyne and incoming signal frequencies—the heterodyne frequency is entirely under the control of the receiving station—and if it is arranged to lie in the audible range, a note will be heard in the telephones during the time that the incoming signal is operative. The signal can therefore be read without difficulty.

The frequency with which the two oscillations come into step, or "beat" frequency as it is called, is actually the difference between their individual frequencies, as may be verified by counting the cycles in Fig. 13. Thus, if the incoming signal has a frequency of 100 kc/s. and the oscillatory circuit of the heterodyne oscillator is adjusted until its frequency is 101 kc/s. or 99 kc/s., the "beat" frequency is 1 kc/s. or 1,000 cycles per second, which gives an easily audible note in the telephones.

Interrupted Continuous Wave Transmission.—It was seen, in discussing the valve C.W. transmitter, that it was possible to produce an undamped oscillatory current because of the power supplied from the H.T. battery. It is further the case that the amplitude of the oscillatory current depends on the amount of power supplied by the battery. The more powerful the battery, the larger is the oscillatory current. Hence, if the battery is varied in amount, the amplitude of the oscillatory current also alters.

This can be accomplished in practice by using an alternator instead of a battery (or other direct-current generator). The alternator can be looked upon as a battery whose ability to supply power varies regularly, the frequency of variation being adjustable by the operator. When such an arrangement is applied between the anode and filament of the valve circuit of Fig. 11, the amplitude of the oscillatory current varies at the alternator frequency, some idea of the amplitude variation being given by Fig. 14. (In practice



Interrupted Continuous Waves.

FIG. 14.

the substitution of a battery by an alternator alone would not give nearly such a regular amplitude variation).

The æther waves emitted from the aerial circuit of such a transmitter have a similar variation in amplitude. They are known as Interrupted Continuous Waves (I.C.W.). When they affect a receiving aerial they will, of course, produce electron swings of varying amplitude corresponding to their own variation. These swings applied between grid and filament of a detector valve will be seen to give almost exactly the same type of grid swing as that shown in Fig. 13 (*d*). Hence, if the alternator frequency is in the audible range, the I.C.W. signals will produce an audible note in the telephones of the receiving circuit, and, further, there has been no necessity to use a separate heterodyne circuit at the receiving station.

Radio Telephony (R/T).—The distinction between Radio or Wireless Telephony and Wireless Telegraphy is the same as that between ordinary line telephony and telegraphy, *i.e.*, speech and music are heard in the receiver instead of Morse dots and dashes.

The principle by which æther (wireless) waves may be used to transmit speech or music is merely an extension of the principle used in I.C.W. transmission. It was there seen that if the H.T. supply to the anode of the valve was made to vary at an audible frequency, then a note of the same frequency would be heard in the telephones of the receiver. Both speech and music consist of a succession of sound waves of frequencies differing according to the pitch of the sounds. It has already been seen that such waves can be converted to oscillating currents (or electron swings) of the same frequency by the use of a microphone. These electron swings may therefore be superimposed on the permanent deficit of electrons created at the anode of a transmitting valve by the H.T. supply. The actual deficit of electrons at the anode then varies according to the frequency of the original speech or music note, and so the amplitude of the oscillatory current and the wave emitted from

the aerial varies at the same frequency. The note in the telephones at the receiver is therefore the same as that which actuates the microphone at the transmitter.

The principle of R/T transmission is thus exactly the same in kind as that of I.C.W. transmission, but it is, of course, as infinitely more complex in detail as is an orchestral symphony compared with the monotone of a tuning fork, and correspondingly greater complexity is required both in the transmitting and receiving circuits to ensure that the sound produced in the telephones or loud-speaker of the receiving set will be a faithful replica of the sound which energises the transmitter microphone.

Direction Finding (D/F).—The wireless waves from the aerial of an ordinary transmitting set travel out into the æther in all directions and so can be picked up by any receiver within the range of the transmitter. Like light waves, however, they travel in straight lines, and so the wave which energises the aerial of any particular receiver has travelled in a straight line from the transmitter to that receiver. If we can determine the direction from which the wave is coming, we have then found the direction or bearing of the transmitter from the receiver. A straight line on the earth's surface is a great circle, and so the bearing determined in this way is the great circle bearing of the transmitter from the receiver.

The ordinary straight vertical wire receiving aerial is energised to the same extent by all passing wireless waves, no matter what may be their direction. It is thus useless for direction finding. If, however, the aerial wire is arranged in the form of a vertical loop and its ends are connected to the grid and filament of a detecting valve, the strength of the note produced in the telephones by a passing wireless wave varies according to the direction in which the loop is pointing. If it points towards the transmitter or directly away from it, the loudest note is heard in the telephones, and as it is turned round from either of these directions the strength of the note falls off. When the loop is "beam on" to the direction of the transmitter no sound at all is heard in the telephones. The direction of a transmitter may thus be determined by mechanically rotating the vertical loop aerial until no signal is heard. The transmitter is then known to be in the direction at right angles to the plane of the loop.

The reason for this behaviour on the part of a vertical loop aerial may be gathered by referring back to the tanks and connecting pipe illustrated in Fig. 3 (c). The vertical tanks correspond to the vertical sides of the loop. If desired, another pipe may be run between the tops of the tanks to represent the top horizontal side of the loop. It is not, however, necessary, and in the loop aerial the top side may also be omitted without affecting its behaviour; it often is omitted in modern D/F aerials. The pipe connecting the tanks at the

bottom corresponds to the horizontal lower ends of the wire connected to grid and filament of the detector valve.

The whole arrangement of tanks and pipe is moored in the sea so that it floats upright. The valve in the pipe is always kept open, and so water stands at the same level in both tanks as long as the surface is smooth. A disturbance of the sea surface is now made at some distance away and waves spread out from it in all directions. When they reach the tanks the behaviour of the water inside the tanks will depend on how their plane is pointing. If they are "beam on" to the centre of the disturbance, the crest of any wave reaches both of them at the same moment. The whole arrangement will rock up and down as the waves pass, but the water in the tanks will always remain at the same level in each tank. No water will flow through the connecting pipe.

Suppose, however, that the arrangement of tanks and connecting pipe is "bows on" to the centre of the disturbance. The wave crest now reaches one tank before the other and tilts it up; water flows through the connecting pipe from the nearer to the farther tank to keep the levels in the two tanks the same. As the wave passes on, the crest reaches the farther tank and the nearer tank sinks in the following trough. Water has now to flow through the pipe from the farther to the nearer tank to equalise the levels. Thus, as the arrangement rocks with the passing waves in the "bows on" position, water runs to and fro through the connecting pipe. It is obvious that in any oblique position of the arrangement there will still be an oscillatory flow of water through the pipe, but not of such great amount as in the "bows on" position.

Now apply this analogy to the vertical loop aerial. The passing æther wave sets up electron swings in the two vertical sides of the aerial. If the latter is "beam on" to the transmitter, electrons are swinging either upwards or downwards at the same instant in the vertical sides. There is thus the same surplus or deficit of electrons at any particular moment on both grid and filament of the detecting valve, and so the grid becomes neither positive nor negative to the filament and no current flows through the valve. But if the loop is "bows on" to the transmitter, one vertical side experiences the crest of the wave before the other, and the electron swings in the two sides are out of step. Electrons are swinging upwards in the nearer side before they start to swing upwards in the farther side, and if the grid is connected to the nearer side there is a greater deficit of electrons at the grid than at the filament; the grid is positive to the filament and a pulse of current flows through the valve. The next moment the nearer side of the loop is in the trough. Electrons swing down it, though they are still swinging up the farther side. The grid accumulates a surplus of electrons and the filament is increasing its deficit; the grid, in other words, swings negative to the filament and no current flows through the valve. The next æther wave brings back the same state of

affairs as the previous crest. The farther side of the loop in the trough of the previous wave is producing a surplus of electrons at the filament ; the nearer side on the crest is pulling electrons from the grid. Thus in the " bows on " position of the loop the grid swings alternately positive and negative to filament at the frequency of the passing æther wave, just as in a vertical receiving aerial, and a sound is produced in the telephones.

In positions of the loop intermediate between " bows on " and " beam on " to the transmitter, there will still be produced out-of-step electron swings in the vertical sides, but the swings will be of smaller amount than in the " bows on " position. The grid swing is not so pronounced and the note in the telephones is weaker. Hence the loudest note is heard in the telephones in the " bows on " position, and the note decreases in intensity as the loop is rotated, until in the " beam on " position no note is heard at all.

It may be noted that the loop cannot tell whether it is pointing towards the transmitter or directly away from it, any more than the tanks in the water wave analogy can tell on which side of them the disturbance lies. A disturbance in the opposite direction from the actual one would produce exactly the same effect. This ambiguity in the direction of a transmitter is resolved in practice by devices known as sense-finders.

APPENDIX B.

MATHEMATICS.

The following signs are in frequent use :—

- ∞ Infinity.
- \propto "Varies as," or "is proportional to."
- \therefore Therefore.
- \because Because.
- \sim Difference between.
- $>$ Is greater than.
- \nlessgtr Is not greater than.
- $<$ Is less than.
- \nlessgtr Is not less than.
- $=$ Equals or is equal to.
- \times Multiply by.
- \div Divide by. Also $/$, $2 \div 3$, $\frac{2}{3}$, $2/3$.

The following sections are written to cover the mathematics required for the purposes of this book, having in view the average attainments of the ratings who will use it. Those who have little or no knowledge of what follows, and who desire fuller knowledge, are referred to a book on mathematics such as Castle's "Practical Mathematics for Beginners."

ARITHMETIC.

It is assumed that readers have a good working knowledge of the ordinary operations of arithmetic, including fractions and decimals. The following notes are intended to be merely supplementary.

Squares and Square Roots.

When a number is multiplied by itself we obtain the **second power** of that number—we have "**squared**" it, *i.e.*, $3 \times 3 =$ the second power of 3, written 3^2 . The figure 2 is called the **index** of the power.

The reverse process is called "finding the square root"—symbol $\sqrt{\quad}$ or $\sqrt[2]{\quad}$

Thus $\sqrt{36} = 6$.

Example.

To find the value of $\sqrt{738 \cdot 238}$ to 2 decimal places.

	2 7 . 1 7 (answer)
	7,38·23,80
	4
	338
47	329
7	923
541	541
1	38280
5427	37989

(1) Mark off the figures in pairs to the left and to the right of the decimal point.

(2) The square number next below 7 is 4 and $\sqrt{4} = 2$. Put 2 in the answer. Square 2, ($= 4$), place it underneath the 7, and subtract.

(3) Double the 2 (4) and place it at the left opposite the 3, and bring down the next two figures (38).

(4) 4 into 32, 8 times; but this will be too great, as will be seen from what follows. Try 7. Place 7 in answer and alongside the 4. Multiply 47 by 7, and subtract.

(We have now finished with the whole number part (738), so the decimal point in the answer will be placed after the 7.)

(5) Repeat the 7 under the 47, and add (or, what amounts to the same, double the 27 in the answer (54) and place it at the left). Bring down the next two figures (23).

And so on.

If more figures are required in the answer, bring down pairs of 0's at each step.

$$\sqrt{.029}$$

$$\begin{array}{r} .02,90,00,00 \quad (.1702 \\ 1 \\ 27 \quad \underline{190} \\ 7 \quad \underline{189} \\ 340 \quad \underline{100*} \\ 3402 \quad \underline{10000} \\ \quad \underline{6804} \\ \quad \quad 3196 \end{array}$$

One figure in answer for each pair of figures in original number.

*Note.—34 into 10, 0 times. Put 0 in answer and at left. Then bring down two more 0's. 3402 into 10000, twice.

$$\sqrt{.0004} = .02$$

$$\begin{aligned} \sqrt{.004} &= \sqrt{.004000} \\ &= .063. \end{aligned}$$

Square Root by Tables.

Using Tables of Square Roots find $\sqrt{897.6}$

There will be two whole numbers, the first being 2. Answer, 29.96.

$$\sqrt{30,00,00} = 547.7. \quad \sqrt{10.8659} = 3.297.$$

Variation.

When two quantities are so related that an **increase** in the value of one results in an **increase** in the value of the other, the second quantity is said to **vary directly** as the first. Symbol \propto = varies as.

If an **increase** in the first quantity results in a **decrease** in the second quantity, the latter is said to **vary inversely** as the former.

E.g., Ohm's Law :— $I \propto E$ means that the current varies **directly** as the E.M.F.

$I \propto \frac{1}{R}$ means that the current varies **inversely** as the Resistance.

Ratio.

Ratio is the comparison of two (or more) quantities of the **same kind**.

A Ratio is expressed as a pure number—it has no units, e.g., A 4 kW. generator has twice the power of a 2 kW. generator. The **ratio** of the powers is as 2 is to 1.

If P_1 = Power of 4 kW. generator and

P_2 = Power of 2 kW. generator

$P_1 : P_2 = 4 : 2 = 2 : 1$, which may also be written—

$$\frac{P_1}{P_2} = \frac{4 \text{ kW.}}{2 \text{ kW.}} = \frac{2}{1} = 2 \text{ (not 2 kW.)}$$

Collection of Signs.

$4 - 2 + 1 - 3 - 6 + 8$ can be written $4 + 1 + 8 - 2 - 3 - 6$, i.e., $+13 - 11 = 2$. If the negative quantity had been the larger, the result would have been negative. Thus, $8 - 12 = -4$.

ALGEBRA.

Definitions.

$2ax - 3bcy + dz$ is an **algebraic expression**. The three distinct parts connected by the $+$ and $-$ signs are the **terms** of the expression. If a term is without a sign, $+$ is understood. In the term $-3bcy$, the 3 is called the **numerical coefficient** of the rest of the term.

N.B.— $3bcy$ means $3 \times b \times c \times y$. **Like terms** are those which differ in the numerical coefficient only. Thus $2ax$ and $-3ax$ are like terms.

Substitution.

Find the value of $2pq^2 - 3pq + q^3$ when $p = 2$, $q = 3$.
 $= 2 \times 2 \times 9 - 3 \times 2 \times 3 + 3 \times 3 \times 3$.
 $= 36 + 27 - 18$.
 $= 45$.

If $A = m \frac{R}{r_a + R}$, find A when $m = 10$, $r_a = 30,000$ and $R = 10,000$,
 20,000 70,000.

$$A = \frac{10 \times 10^4}{30,000 + 10,000} = \frac{10^5}{40,000} = \frac{10^5}{4 \times 10^4} = \frac{10}{4} = 2.5.$$

Other answers:—4, 5, 5.71, 6.25, 6.66, 7.

Brackets.

() simple ;
 { } brace ;
 [] square ;

$\overline{a + x}$ vinculum.

A bracket is used to collect or bind together the terms placed inside it. The contents of a bracket must be completely operated upon by whatever coefficient or sign is placed in front of the bracket before the latter is removed.

Thus, $4(x + 2y) = 4x + 8y$.

A minus sign in front of a bracket changes all the signs inside when the bracket is removed.

Thus, $-2(2a - 3b + 4c) = -4a + 6b - 8c$.

Exs.

(1) $2(a + 3) + 3(a + 2) = 2a + 6 + 3a + 6 = 5a + 12$.

(2) $4(a + 2b + 3c) + 3(2a + b - 4c) = 10a + 11b$.

(3) $5x - 2y + 3 + 2x + 3y - 2 - 7x + y + 1 = 2y + 2$.

(4) $3[x - \{2x - 2x + 6 + x\} - 6x - 12]$
 $= -18x - 54$ or $-18(x + 3)$.

(5) $3[a - 2\{b - 4(c - d)\}] - 4[a - 3\{b + 4(c + d)\}]$
 $= 3[a - 2\{b - 4c + 4d\}] - 4[a - 3\{b + 4c + 4d\}]$
 $= 3[a - 2b + 8c - 8d] - 4[a - 3b - 12c - 12d]$
 $= 3a - 6b + 24c - 24d - 4a + 12b + 48c + 48d$
 $= -a + 6b + 72c + 24d$.

Rule of Signs.

Two LIKE signs multiplied or divided give PLUS.

Two UNLIKE signs multiplied or divided give MINUS.

e.g., -2 multiplied by $-3 = +6$, $\frac{-6}{-2} = +3$.

-2 multiplied by $+3 = -6$, $\frac{-6}{+2} = -3$.

N.B. $-2(-x - a) = 2x + 2a$.

Fractions.

$$\begin{aligned} \text{(i) Simplify } & \frac{3x-1}{4} - \frac{2-x}{5} + \frac{1}{5} \\ &= \frac{5(3x-1) - 4(2-x) + 4}{20} \\ &= \frac{15x-5-8+4x+4}{20} \\ &= \frac{19x-9}{20} \end{aligned}$$

$$\begin{aligned} \text{(ii) Simplify } & \frac{1}{7}(3x+5) - \frac{1}{3}(2x+7) - \frac{3x}{5} \\ &= \frac{3x+5}{7} - \frac{2x+7}{3} - \frac{3x}{5} \\ &= \frac{45x+75-70x-245-63x}{105} \\ &= \frac{-88x-170}{105} \text{ or } = -\frac{88x+170}{105} \end{aligned}$$

Addition.Add together $3x + 2y - 4$, $-2x + 3 - 3y$, $x - 2$, $7y - 4x + 1$.

First method :—

$$\begin{aligned} & (3x + 2y - 4) + (-2x + 3 - 3y) + (x - 2) + (7y - 4x + 1) \\ &= 3x + 2y - 4 - 2x + 3 - 3y + x - 2 + 7y - 4x + 1 \\ &= 3x - 2x + x - 4x + 2y - 3y + 7y - 4 + 3 - 2 + 1 \\ &= -2x + 6y - 2. \end{aligned}$$

Second method :—

$$\begin{array}{r} 3x + 2y - 4 \\ - 2x - 3y + 3 \\ \quad x \quad - 2 \\ - 4x + 7y + 1 \\ \hline - 2x + 6y - 2 \end{array}$$

Subtraction.From $3ax^2 - 2ay + 5$ take $3ay - ax^2 + 4$.

First method :—

$$\begin{aligned} & (3ax^2 - 2ay + 5) - (3ay - ax^2 + 4) \\ &= 3ax^2 - 2ay + 5 - 3ay + ax^2 - 4 \\ &= 3ax^2 + ax^2 - 2ay - 3ay + 5 - 4 \\ &= 4ax^2 - 5ay + 1 \end{aligned}$$

Second method :—

$$\begin{array}{r} 3ax^2 - 2ay + 5 \\ -ax^2 + 3ay + 4 \\ \hline 4ax^2 - 5ay + 1 \end{array}$$

Change (mentally) the sign of the bottom line and proceed as in addition.

Indices.

In " a^2 ," 2 is the "index"; it indicates the power to which " a " is raised.

a^4 means $a \times a \times a \times a$ (not, as one frequently hears, " a " multiplied by itself four times, as there are only three multiplications).

a^4 is "unity (1) multiplied by ' a ' 4 times," i.e., $1 \times a \times a \times a \times a$.

Rule 1.—To multiply together powers of the same quantity add the indices.

Thus $a \times a \times a = a^3$ and $a \times a = a^2$.

$\therefore a^3 \times a^2 = (a \times a \times a) \times (a \times a) = a^5$,

i.e., $a^3 \times a^2 = a^{(3+2)} = a^5$.

Rule 2.—To divide powers of the same quantity subtract the indices.

Thus $\frac{a^7}{a^4} = a^{(7-4)} = a^3$.

Negative Indices.

Consider $\frac{a^4}{a^7} = \frac{\cancel{a} \times \cancel{a} \times \cancel{a} \times \cancel{a}}{\cancel{a} \times \cancel{a} \times \cancel{a} \times \cancel{a} \times \cancel{a} \times \cancel{a} \times a} = \frac{1}{a^3}$

By the rule for division—

$$\frac{a^4}{a^7} = a^{(4-7)} = a^{-3}$$

$$\therefore a^{-3} = \frac{1}{a^3}$$

From this we can find the meaning of a^0 .

$$\frac{a^3}{a^3} = 1, \text{ but } \frac{a^3}{a^3} = a^{3-3} = a^0 \therefore a^0 = 1$$

Rule 3.—To raise a power to any power, multiply the indices.

Thus $(a^3)^4 = a^3 \times a^3 \times a^3 \times a^3 = a^{(3+3+3+3)} = a^{12}$.

i.e., $(a^3)^4 = a^3 \times 4 = a^{12}$.

Fractional Indices.—

$$\sqrt{a} \times \sqrt{a} = a = a^1 \text{ but } a^{\frac{1}{2}} \times a^{\frac{1}{2}} = a^1 \therefore \sqrt{a} = a^{\frac{1}{2}}$$

Similarly $\sqrt[3]{a} \times \sqrt[3]{a} \times \sqrt[3]{a} = a^1 \times a^{\frac{1}{3}} \times a^{\frac{1}{3}}$, i.e., $\sqrt[3]{a} = a^{\frac{1}{3}}$

Also $\sqrt[3]{x} = x^{\frac{1}{3}}$ and $\frac{1}{\sqrt[3]{x}} = \frac{1}{x^{\frac{1}{3}}} = x^{-\frac{1}{3}}$ and $(\sqrt[3]{x})^3 = (x^{\frac{1}{3}})^3 = x^1$

Rule 4.—To find the root of a power divide the index of the power by the root.

Thus $\sqrt[3]{a^6} = (a^6)^{\frac{1}{3}} = a^{\frac{6}{3}} = a^2$

1. Simplify—

$$\frac{x^2 (\sqrt[3]{x^4})^6}{(\sqrt{x})^{12}} = \frac{x^2 (x^{\frac{4}{3}})^6}{(x^{\frac{1}{2}})^{12}} = \frac{x^2 \times x^8}{x^6} = \frac{x^{10}}{x^6} = x^4$$

2. Simplify—

$$\frac{y^{\frac{1}{2}} \times \frac{1}{\sqrt{y^{\frac{1}{2}}}}}{y^{\frac{1}{2}} \times y^{-\frac{1}{2}}} = \frac{y^{\frac{1}{2}} \times \frac{1}{y^{\frac{1}{2}}}}{y^{\frac{1}{2}} \times \frac{1}{y^{\frac{1}{2}}}} = \frac{1}{1} = y$$

Equations.

A statement that two algebraic expressions are equal to each other is called an equation; and the two equal expressions are called the sides of the equation. The two sides of the equation are only equal for certain particular values of the letters involved, and to **solve** the equation is to find what are these particular values. These values are said to **satisfy** the equation.

The letters representing the unknown quantities whose values are to be found, are usually taken from the end of the alphabet, *e.g.*, x, y, z , etc. If it is desired to represent quantities which are supposed to be known, without actually giving their arithmetical values, letters at the beginning of the alphabet are used, *e.g.*, a, b, c , etc.

Equations are classified according to the highest power of the unknown quantity.

A **simple** equation is one in which the unknown quantity occurs only to the first power.

e.g., $3x = 9$.

There is only **one** value of x , *i.e.*, $x = 3$, which satisfies this equation.

In a **quadratic** equation, the unknown quantity is raised to the second power,

e.g., $x^2 = 16$.

There are **two** values of x satisfying this equation, $x = +4$ and $x = -4$.

Solution of Simple Equations.

Solve the equation $9x + 5 = 3x + 17$.

Arrange the terms containing x on one side and the plain figures on the other side.

The change must be made so that the left side will still remain equal to the right side, *i.e.*, any term added to or subtracted from one side must be added to or subtracted from the other side.

We subtract 5 from each side and $3x$ from each side, and obtain $9x - 3x = 17 - 5$.

Thus the method is, move $3x$ to the left side and change its sign; move $+5$ to the right and change its sign.

Then $6x = 12$ and $x = 2$.

Examples.

(a) $x + 7 = 17$. Subtract 7 from both sides.

$$x = 17 - 7$$

$$x = 10.$$

(b) $4x - 2 = 18$. Add 2 to both sides.

$$4x = 18 + 2.$$

$4x = 20$. Divide both sides by 4.

$$x = 5.$$

(c) $\sqrt{x} = 3$. Square both sides.

$$x = 9.$$

(d) $5x - 4 = 2(x + 1)$

$$5x - 4 = 2x + 2$$

$$5x - 2x = 2 + 4.$$

$$3x = 6.$$

$$x = 2.$$

Transpose.

(e) $3(4x - 2) - 5(x + 5) = 2(x - 4) - 3(x + 1).$

$$12x - 6 - 5x - 25 = 2x - 8 - 3x - 3.$$

$$12x - 5x - 2x + 3x = -8 - 3 + 6 + 25.$$

$$8x = 20.$$

$$x = 2\frac{1}{2}.$$

(f) $x = \frac{2}{3}$. Multiply both sides by 3, or cross-multiply.

$$x = 6.$$

(g) $x + \frac{x}{2} + \frac{x}{3} = 11$.

L.C.M. = 6.

$$6x + 3x + 2x = 66.$$

$$11x = 66.$$

$$x = 6.$$

$$(h) \frac{3(x-1)}{5} - \frac{2x-5}{2} = 1 - \frac{3(x-3)}{6}$$

L.C.M. = 30.

$$18(x-1) - 15(2x-5) = 30 - 15(x-3).$$

$$18x - 18 - 30x + 75 = 30 - 15x + 45.$$

$$18x - 30x + 15x = 30 + 45 + 18 - 75.$$

$$3x = 18.$$

$$x = 6.$$

Transposition of Formulae.

Express V and R in terms of the remaining quantities, given that $I = \frac{V}{R}$.

(i) $I = \frac{V}{R}$, so $\frac{V}{R} = I$. Multiply both sides by R.

$$\frac{V}{R} \times R = I \times R.$$

$$V = IR.$$

(ii) $V = IR$, so $IR = V$. Divide both sides by I.

$$\frac{IR}{I} = \frac{V}{I}.$$

$$R = \frac{V}{I}.$$

(i) $R = \frac{\rho l}{a}$; (ii) $\rho = \frac{Ra}{l}$; (iii) $l = \frac{Ra}{\rho}$; (iv) $Ra = \rho l$;

$$\text{so } a = \frac{\rho l}{R}.$$

Further examples :—

Find an expression for L in (a) and (c) and for X in (b).

$$(a) \lambda = 20\pi\sqrt{LC}.$$

$$\text{Answer.}-(a) L = \frac{\lambda^2}{400\pi^2 C}.$$

$$(b) I = \frac{E}{\sqrt{R^2 + X^2}}.$$

$$\text{Answer.}-(b) X = \sqrt{\left(\frac{E}{I}\right)^2 - R^2}.$$

$$(c) A = m \frac{\omega L}{\sqrt{r_a^2 + (\omega L)^2}}$$

$$\text{Answer.}-(c) L = \frac{Ar_a}{\omega \sqrt{m^2 - A^2}}$$

LOGARITHMS.

Definition.

The logarithm of a number to a given base is the index of the power to which the base must be raised to give the number.

No.	Base.	Power.	Index or Log.
8	2	2^3	3

$$\log_2 8 = \log_2 2^3 = 3.$$

$$\log_3 9 = \log_3 3^2 = 2.$$

$$\log_5 125 = \log_5 5^3 = 3.$$

$$\log_a a^x = x.$$

$$\log_9 3 = \log_9 9^{\frac{1}{2}} = \frac{1}{2}.$$

Consider the number 8 and take 2 as the base, then $8 = 2^3$. Then by definition $\log_2 8 = \log_2 2^3 = 3$.

Systems.

There are two systems in general use :—

(1) Common Logs.

(2) Naperian Logs.

In (1) the base is 10.

In (2) the base is 2.7183, represented by $e = \frac{1}{1} + \frac{1}{1} + \frac{1}{2} + \frac{1}{3} + \dots$

N.B. $[3 = 3 \times 2 \times 1, [5 = 5 \times 4 \times 3 \times 2 \times 1$, and so on.

No.	Power.	Log.	No.	Power.	Log.
1,000,000	10^6	6	0.1 or $\frac{1}{10}$	$\frac{1}{10^1}$ or 10^{-1}	- 1
100,000	10^5	5	0.01 or $\frac{1}{100}$	$\frac{1}{10^2}$ or 10^{-2}	- 2
10,000	10^4	4	0.001 or $\frac{1}{1,000}$	$\frac{1}{10^3}$ or 10^{-3}	- 3
1,000	10^3	3	0.0001 or $\frac{1}{10,000}$	$\frac{1}{10^4}$ or 10^{-4}	- 4
100	10^2	2	0.00001 or $\frac{1}{100,000}$	$\frac{1}{10^5}$ or 10^{-5}	- 5
10	10^1	1			
1	10	0			

Numbers which are not Exact Powers of 10.

Any number may be expressed as a power of 10. Its index will not be a whole number unless it is an exact power.

$$\text{Thus } 536 = 10^{2.7292}$$

$$\therefore \log_{10} 536 = 2.7292.$$

The reason that the log of 536 lies between 2 and 3 is because 536 lies between 100 and 1,000 (see table above).

Characteristic.

This is the whole number part of the log—obtained by inspection—and may be positive or negative.

Mantissa.

This is the decimal part of the log—obtained from the tables—and always positive.

Rules for Obtaining the Characteristic.

The characteristic of any number **greater than 1** is **positive**, and is one **less** than the number of **whole number** figures, *i.e.*, figures to the **left** of the decimal point.

E.g., Characteristic of log 324 = 2.
 Characteristic of log 12.62 = 1.
 Characteristic of log 8000 = 3.
 Characteristic of log 1.032 = 0.

The characteristic of any number **less than 1** is **negative** and is one **more** than the number of noughts between the decimal point and the **first** figure of the number which is not a nought.

E.g., Characteristic of log 0.0034 = $\bar{3}$
 Characteristic of log 0.0001006 = $\bar{4}$
 Characteristic of log 0.8073 = $\bar{1}$

N.B. A negative characteristic is written as, say, $\bar{3}$, and not -3 , because the mantissa is always positive: Thus $\bar{3}.7292$ means $-3 + 0.7292$, whereas -3.7292 would mean $-3 - 0.7292$.

Multiplication by Logs.

Add the logs of the individual numbers and take the antilog.

Example.

Find by logs the value of 326.4×0.2716 .

No.	Log.
326.4	2.5137
0.2716	$\bar{1}.4339$
88.63	1.9476

Answer.—88.63.

Division by Logs.

Subtract the log of the denominator from the log of the numerator and take the antilog.

Example.

Find by logs the value of $54.81 \div 0.0752$.

No.	Log.
54.81	1.7389
0.0752	$\bar{2}.8762$
729.0	2.8627

Answer.—729.0.

Powers.

Multiply the log of the number by the **index** of the power and take the antilog.

Example.

Find by logs the value of $(8.72)^3$.

No.	Log.	Log.
$(8.72)^3$	0.9405×3	1.8810
76.03	—	1.8810

Ans.—76.03.

Roots.

Divide the log of the number whose root is required by the root index.

Example.

Find by logs the value of $\sqrt[4]{0.02596}$.

No.	Log.	Log.
$\sqrt[4]{0.02596}$	$2.4143(-4 + 2.4143) \div 4$	$\bar{1}.60357$
0.4015	—	$\bar{1}.6036$

Ans.—0.4015.

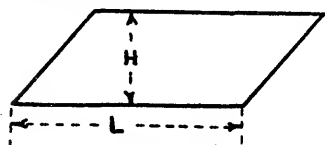
MEASURATION.**Parallelogram.**

FIG. 1.

$$\text{Area} = L \times H$$

Note :—H is the **perpendicular height**.

Triangle.

(1) Given base and perpendicular height :—

(i)

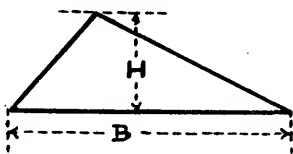


FIG. 2.

$$\text{Area} = \frac{1}{2} \text{ base} \times \text{height.}$$

$$= \frac{B \times H}{2}$$

(ii)

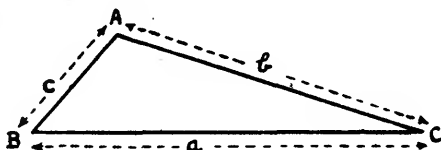


FIG. 3.

(ii) Given the lengths of the three sides.

s = half sum of sides,

$$\text{i.e., } s = \frac{a + b + c}{2}$$

Area =

$$\sqrt{s(s-a)(s-b)(s-c)}$$

Example.

Find the area of a triangle whose sides are 7, 11 and 14 feet long.

$$s = \frac{a + b + c}{2} = \frac{7 + 11 + 14}{2} = \frac{32}{2} = 16 \text{ feet.}$$

$$\text{Area} = \sqrt{16(16-7)(16-11)(16-14)} = \sqrt{16 \times 9 \times 5 \times 2} \text{ sq. ft.}$$

$$= 12\sqrt{10} \text{ sq. ft.} = 37.944 \text{ sq. ft.}$$

Right-Angled Triangles.

The theorem of Pythagoras states that in any right-angled triangle the square on the hypotenuse is equal to the sum of the squares on the other two sides—the hypotenuse being the side opposite the right angle.

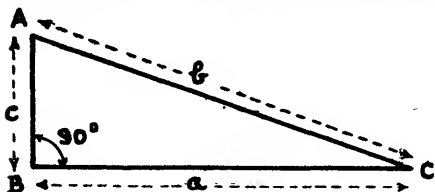


FIG. 4.

$$b^2 = a^2 + c^2 \text{ or } b = \sqrt{a^2 + c^2}.$$

$$a^2 = b^2 - c^2 \text{ or } a = \sqrt{b^2 - c^2}.$$

$$c^2 = b^2 - a^2 \text{ or } c = \sqrt{b^2 - a^2}.$$

Example.

A ladder AB rests against a wall with its foot 12 feet from the wall and its top 9 feet up the wall. Find the length of the ladder.

If the ladder is swung over to the opposite side of the road with its foot 6 feet from the wall, find the distance it reaches up the wall.

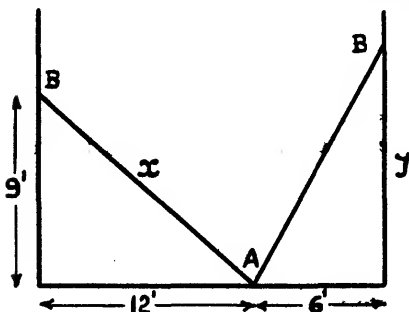


FIG. 5.

$$(i) \ x = \sqrt{12^2 + 9^2} = \sqrt{144 + 81} \\ = \sqrt{225} = 15 \text{ feet.}$$

$$(ii) \ y = \sqrt{15^2 - 6^2} = \sqrt{225 - 36} \\ = \sqrt{189} = 13.75 \text{ feet.}$$

Length of ladder = 15 feet.

Distance ladder reaches up wall = 13.75 feet.

The Circle.

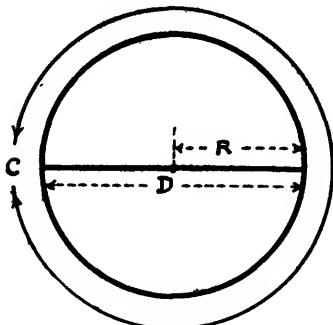


FIG. 6.

If the length of the circumference = C, and that of the diameter = D, it is found that in any circle the ratio of C to D is constant, and equal to 3.1416 (π) [pi].

Thus $\frac{C}{D}$ is a constant = π ,

$$\text{i.e., } C = \pi D = 2\pi R.$$

$$\text{Area of Circle} = \pi R^2 = \frac{\pi}{4} D^2.$$

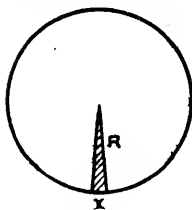


FIG. 7.

The small shaded portion of circle may be considered as a triangle of height R on a base x (regarded as a straight line).

Then area of circle—

$$= \frac{1}{2} R x + \frac{1}{2} R x_1 + \frac{1}{2} R x_2 + \text{etc.}$$

$$= \frac{1}{2} R (x + x_1 + x_2 + \text{etc.})$$

$$= \frac{1}{2} R C = \frac{1}{2} R 2\pi R = \pi R^2.$$

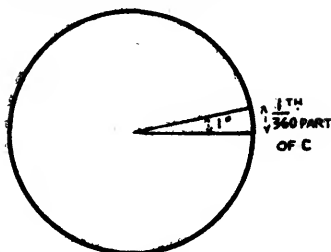
Measurement of Angles.

FIG. 8.

The two units in general use are the **degree** and the **radian**.

If the circumference of a circle is divided into 360 equal parts and lines drawn to the centre from the ends of any one part the two radii so drawn contain an angle of 1° between them.

N.B.—A **right angle** contains 90° .

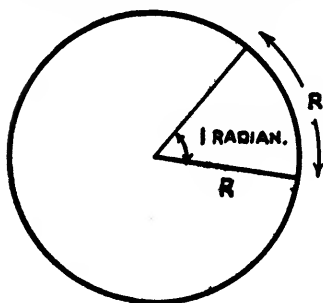
Radian (or Circular) Measure.

FIG. 9.

A **radian** is the angle subtended at the centre of a circle by an arc equal in length to the radius R .

The measure in radians of any angle subtended at the centre of the circle by an arc of the circumference—

$$= \frac{\text{length of arc}}{\text{length of radius}} = \frac{l}{R} = \theta \text{ radians}$$

$$\text{i.e., } \theta = \frac{l}{R}, R = \frac{l}{\theta}, l = R\theta.$$

N.B.—Radian measure is a **natural measure**, i.e., it is a ratio and so is independent of units.

Relation between Radians and Degrees.

An arc equal in length to—

R subtends 1 radian.

$2R$ subtends 2 radians.

$2\pi R$ subtends 2π radians.

But $2\pi R = C$ of circle and subtends 360° at the centre.

$$\therefore 2\pi \text{ radians} = 360^\circ.$$

$$\pi \text{ radians} = 180^\circ.$$

$$1 \text{ radian} = \frac{180^\circ}{\pi} = \frac{180^\circ}{3.1416} = 57.3^\circ.$$

Example.

(i) Convert 162° to radians.

$$\frac{180^\circ}{\pi} = 1 \text{ radian, so } 1^\circ = \frac{\pi}{180} \text{ radians.}$$

$$\begin{aligned} 162^\circ &= \frac{162\pi}{180} \text{ radians.} \\ &= 2.828 \text{ radians.} \end{aligned}$$

(ii) Convert 2.42 radians to degrees.

$$1 \text{ radian} = \frac{180^\circ}{\pi}.$$

$$\begin{aligned} 2.42 \text{ radian} &= \frac{2.42 \times 180^\circ}{\pi} \\ &= 138.7^\circ. \end{aligned}$$

Graphical Representations.

In many cases the relation between two quantities is most easily exhibited by plotting corresponding values of the two quantities as points on squared paper and joining up the points thus obtained by a smooth curve. The two lines of reference from which measurements are made are called the **axes** (plural of axis). It is common practice to draw the axes at right angles to each other. They are then called rectangular axes. Their point of intersection is called the **origin O**. The axes are labelled XOX' and YOY' as shown in Fig. 10.

Rule of Signs.

Measurements to the **right** of the YOY' axis are positive, those to the **left** of it negative.

Measurements **upwards** from the XOX' axis are positive, those **downwards** from it negative.

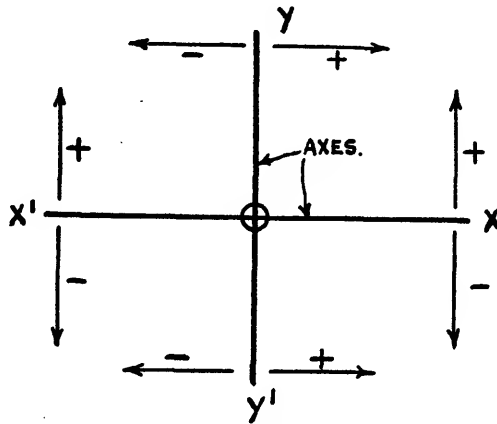


FIG. 10.

The **position of a point** is fixed by its perpendicular distances from the axes—called the **co-ordinates** of the point.

Abscissa.—The horizontal distance of a point from the YOY' axis. It is usually denoted by x and is **always given first**.

Ordinate.—The vertical distance of a point above or below the XOX' axis. It is usually denoted by y and is **always given second**.

Example.

(i) Plot the following points on graph paper to a scale of $\frac{1}{2}$ inch to 1 unit.

A (2, 1); B (-2, 3); C (-1, -4); D (2, -1); E (2.6, 3.2);
F (-3.4, 1.6); G (-2.2, 1.4); H (3.4, -3.4).

(ii) Plot the characteristic curve for the following valve, anode and filament voltages being kept constant at 60 volts and 3.6 volts respectively.

Use scales of 1 inch = 4 volts for V_g and 1 inch = 0.5 mA for I_a .

V_g (volts).	..	-12	-10	-8	-6	-4	-2	0	2	4	6	8	10
I_a (mA)	..	0	0.30	0.09	0.24	0.46	0.79	1.2	1.63	2.05	2.46	2.8	2.89

Straight Lines.

The general equation to a straight line is $y = mx + c$.

It is an equation of the first degree (no "roots" or "powers" other than the first) and all such equations give a straight line graph.

Meaning of "m" in the Equation.

"m" is a constant which indicates the **slope** or **gradient** of the straight line.

A line sloping thus \nearrow has positive gradient, *i.e.*, m is positive, while a line sloping thus \searrow has negative gradient, *i.e.*, m is negative.

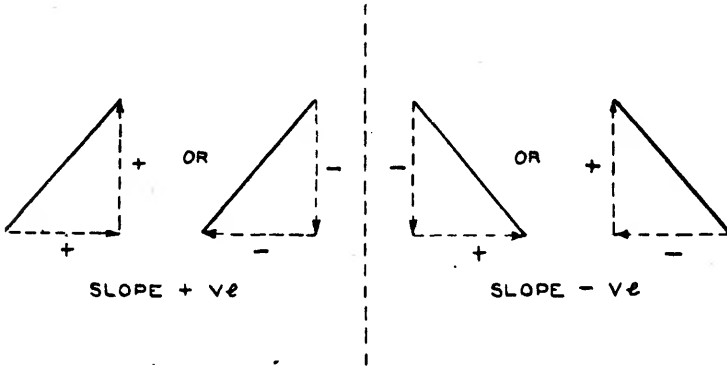


FIG. 11.

The gradient of a line parallel to XOX' is zero.

Meaning of "c" in the Equation.

"c" is a constant which indicates the point at which the straight line and the y axis intersect (or it is the value of y when $x = 0$). It is usually spoken of as **the intercept on the y axis**.

If the point of intersection is above the origin then c is +ve, if below the origin c is -ve.

If $c = 0$, the line must pass through the origin.

Thus, $y = \frac{1}{2}x$, $y = x$, $y = 2x$, $y = 3x$, $y = -\frac{1}{2}x$, $y = -x$, $y = -2x$, all pass through the origin but their slopes differ.

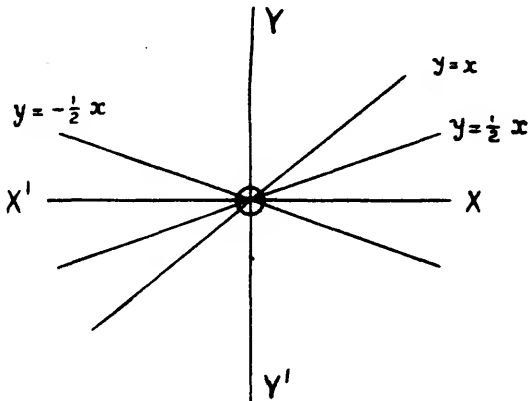


FIG. 12.

Example.

Draw the curve whose equation is $y = 2x - 1$.

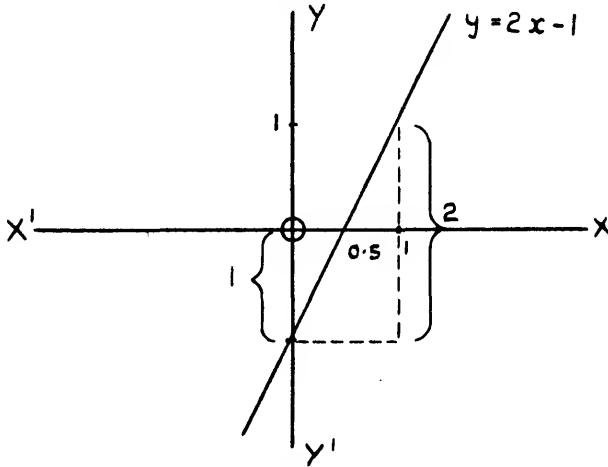


FIG. 13.

Intercept $c = -1$.

Slope $m = +\frac{2}{1} = +2$.

Equations of the Second Degree.**The Parabola.—**

General equation is $y = ax^2 + bx + c$.

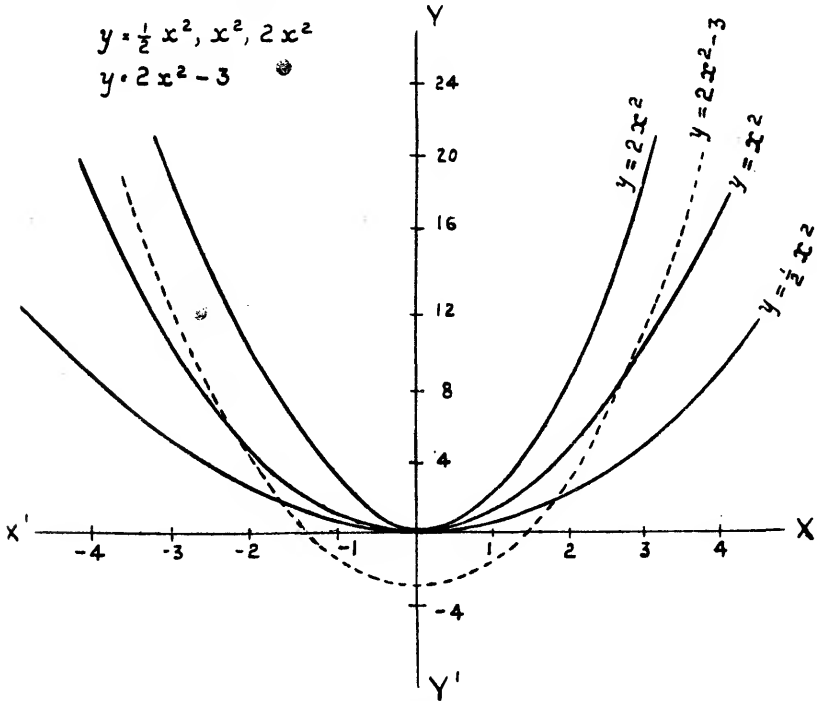


FIG. 14.

Method of Obtaining Values.

$$y = 2x^2 - 3x + 5.$$

x	-3	-2	-1	0	1	2	3
$2x^2$	18	8	2	0	2	8	18
$-3x$	9	6	3	0	-3	-6	-9
$+5$	5	5	5	5	5	5	5
y	32	19	10	5	4	7	14

TRIGONOMETRY.

Trigonometry is that branch of mathematics which deals with the measurement of the sides and angles of triangles.

The Trigonometrical Ratios are the relations between the sides of a right-angled triangle, taken two at a time.

Angle of Reference.—When one of the angles (other than the right angle) of a right-angled triangle is chosen and the trigonometrical ratios found for it, the angle chosen is called the angle of reference.

Hypotenuse, Adjacent Side and Opposite Side.—In any given right-angled triangle the hypotenuse is the side opposite the right angle; the adjacent side is the other side next the angle of reference, and the remaining side which is opposite the angle of reference is spoken of as the opposite side. The hypotenuse and the adjacent side form the bounding lines of the angle of reference.

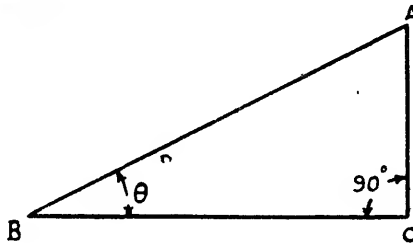
The Six Trigonometrical Ratios.

FIG. 15.

In the right-angled triangle ABC, θ = angle of reference, AB = hypotenuse, BC = adjacent side, AC = opposite side.

$$\text{sine } \theta = \frac{\text{opposite}}{\text{hypotenuse}} = \frac{AC}{AB}$$

written sin θ

$$\text{cosine } \theta = \frac{\text{adjacent}}{\text{hypotenuse}} = \frac{BC}{AB}$$

written cos θ

$$\text{tangent } \theta = \frac{\text{opposite}}{\text{adjacent}} = \frac{AC}{BC}$$

written tan θ

$$\text{cosecant } \theta = \frac{\text{hypotenuse}}{\text{opposite}} = \frac{AB}{AC} = \frac{1}{\sin \theta}$$

written cosec θ

$$\text{secant } \theta = \frac{\text{hypotenuse}}{\text{adjacent}} = \frac{AB}{BC} = \frac{1}{\cos \theta}$$

written sec θ

$$\text{cotangent } \theta = \frac{\text{adjacent}}{\text{opposite}} = \frac{BC}{AC} = \frac{1}{\tan \theta}$$

written cot θ

Example.

If $\sin \phi = \frac{12}{13}$, write down the values of all the other ratios.

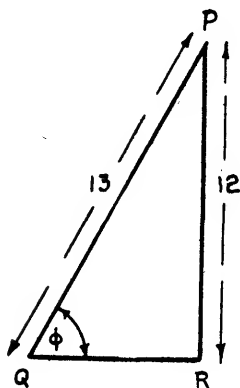


FIG. 16.

By the Theorem of Pythagoras—

$$PQ^2 = PR^2 + QR^2$$

$$\begin{aligned} QR &= \sqrt{PQ^2 - PR^2} \\ &= \sqrt{13^2 - 12^2} \\ &= \sqrt{169 - 144} \\ &= \sqrt{25} \\ &= 5. \end{aligned}$$

$$\therefore \cos \phi = \frac{5}{13}, \sec \phi = \frac{13}{5} = 2.6, \operatorname{cosec} \phi = \frac{13}{12} = 1.08.$$

$$\tan \phi = \frac{12}{5} = 2.4, \cot \phi = \frac{5}{12}$$

Use of Tables.

$$\sin 36^\circ 17' = 0.5918.$$

$$\cos 19^\circ 9' = 0.9446.$$

$$\cos 86^\circ 3' = 0.0689.$$

$$\tan 49^\circ 1' = 1.1511.$$

$$\sin^{-1} 0.1922 \text{ means the angle whose sine is } 0.1922 = 11^\circ 5'.$$

$$\cos^{-1} 0.9236 = 22^\circ 33'.$$

$$\tan^{-1} 0.1937 = 10^\circ 58'.$$

$$\tan^{-1} 2.9246 = 71^\circ 7'.$$

$$\log \sin 72^\circ 47' = 1.9801.$$

$$\log \cos 14^\circ 21' = 1.9862.$$

$$\log \tan 33^\circ 58' = 1.8285.$$

Example.

If $\theta = 36^\circ 51'$, show that $\sin 2\theta = 2 \sin \theta \cos \theta$.

$$2\theta = 73^\circ 42'$$

$$\sin 2\theta = \sin 73^\circ 42'$$

$$= 0.9598.$$

No.	Log.
2	0.3010
$\sin 36^\circ 51'$	1.7779
$\cos 36^\circ 51'$	1.9032
0.9596	1.9821

$$\text{L.H.S.} = 0.9598.$$

$$\text{R.H.S.} = 0.9596.$$

$$\therefore \sin 2\theta = 2 \sin \theta \cos \theta.$$

SOLUTION OF RIGHT-ANGLED TRIANGLES.

In order to solve a right-angled triangle it is necessary to know either :—

(a) 1 side and 1 angle other than the right angle, or

(b) 2 sides.

From this information, it is possible to find the remaining sides and angles of the triangle.

Example (a).

In the right-angled triangle ABC, $\hat{B} = 90^\circ$, $AB = 18.5$ cms., $AC = 27$ cms. Find BC, and the two angles A and C.

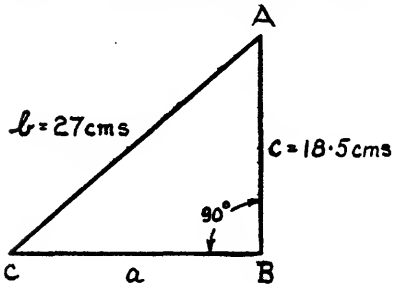


FIG. 17.

$$\frac{a}{b} = \cos C.$$

$$\begin{aligned} a &= b \cos C. \\ &= 27 \cos 43^\circ 15'. \\ &= 19.66 \text{ cms.} \end{aligned}$$

$$\begin{aligned} \sin C &= \frac{c}{b} = \frac{18.5}{27} \\ &= 0.6852 \end{aligned}$$

$$\therefore \hat{C} = 43^\circ 15'$$

$$\hat{A} = 90^\circ - 43^\circ 15' = 46^\circ 45'.$$

Answer.— $\hat{A} = 46^\circ 45'$; $\hat{C} = 43^\circ 15'$; $BC = 19.66$ cms.

Example (b).

Solve a right-angled triangle ABC given that $\hat{C} = 90^\circ$, $\hat{B} = 54^\circ$, $a = 2.7$ ft.

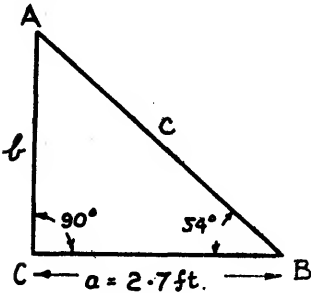


FIG. 18.

$$\begin{aligned} \frac{b}{2.7} &= \tan \hat{B} = \tan 54^\circ \\ \therefore b &= 2.7 \tan 54^\circ \\ &= 3.716 \text{ ft.} \end{aligned}$$

$$\frac{c}{2.7} = \sec 54^\circ$$

$$\frac{c}{2.7} = \frac{1}{\cos 54^\circ}$$

$$c \times \cos 54^\circ = 2.7$$

$$\begin{aligned} c &= \frac{2.7}{\cos 54^\circ} \\ &= 4.593 \text{ ft.} \end{aligned}$$

$$\begin{aligned} \text{Angle A} &= 90^\circ - \hat{B} \\ &= 90^\circ - 54^\circ \\ &= 36^\circ \end{aligned}$$

No.	Log.
2.7	0.4314
$\tan 54^\circ$	0.1387
3.716	0.5701

No.	Log.
2.7	0.4314
$\cos 54^\circ$	1.7692
4.593	0.6622

Answer.— $\hat{A} = 36^\circ$; $b = 3.716$ ft.; $c = 4.593$ ft.

(A 313/1198)Q

2 x 2

Example (c).

At a distance of 120 feet from the foot of a tower the angle of elevation of the top of the tower was found to be $58^\circ 12'$. Find the height of the tower.

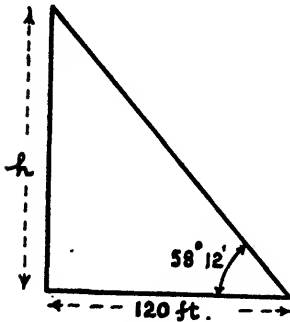


FIG. 19.

Let h = height of tower in feet.

$$\frac{h}{120} = \tan 58^\circ 12'.$$

$$h = 120 \tan 58^\circ 12' \text{ feet.} \\ = 193.6 \text{ feet.}$$

No.	Log.
120	2.0792
$\tan 58^\circ 12'$	0.2076
193.6	2.2868

Answer.—Height of tower = 193.6 feet.

Ratios of 0° , 30° , 45° , 60° , 90° .

0° (argued from a small angle).

$$\sin \theta = \frac{BC}{AB}; \text{ when } \theta \text{ becomes } 0^\circ, BC = \text{zero};$$

$$\text{so } \sin 0^\circ = \frac{0}{AB} = 0.$$

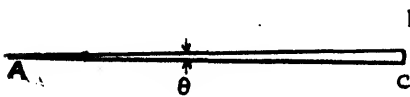


FIG. 20.

$$\cos \theta = \frac{AC}{AB}; \text{ when } \theta \text{ becomes } 0^\circ, \\ AC = AB; \text{ so } \cos 0^\circ = 1.$$

$$\tan 0^\circ = \frac{\sin 0^\circ}{\cos 0^\circ} = \frac{0}{1} = 0.$$

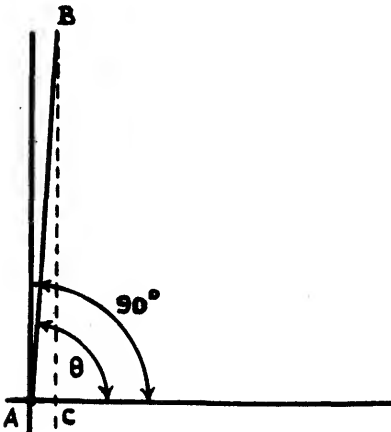


FIG. 21.

90° (by similar argument for an angle of nearly 90°).

$$\sin \theta = \frac{BC}{AB}; \sin 90^\circ = 1$$

$$\cos \theta = \frac{AC}{AB}; \cos 90^\circ = 0$$

$$\tan 90^\circ = \frac{\sin 90^\circ}{\cos 90^\circ} = \frac{1}{0} = \infty \text{ (infinity)}$$

30° and 60° (from half an equilateral triangle). $AB = AD = BD$.

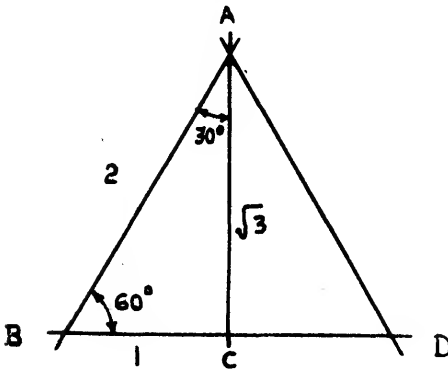


FIG. 22.

Since $AB = AD$, and AC bisects the angle BAD , AC also bisects BD at right angles.

$\therefore BC = CD = \frac{1}{2} BD$.

Calling BC 1 unit of length, $AB (= BD) = 2$ units.

$$AC = \sqrt{AB^2 - BC^2} = \sqrt{4 - 1} = \sqrt{3} \text{ units.}$$

$$\sin 60^\circ = \frac{AC}{AB} = \cos 30^\circ = \frac{\sqrt{3}}{2}.$$

$$\cos 60^\circ = \frac{BC}{AB} = \sin 30^\circ = \frac{1}{2}.$$

$$\tan 60^\circ = \sqrt{3}; \tan 30^\circ = \frac{1}{\sqrt{3}}.$$

[Note.— $\sqrt{3} = 1.732$.]

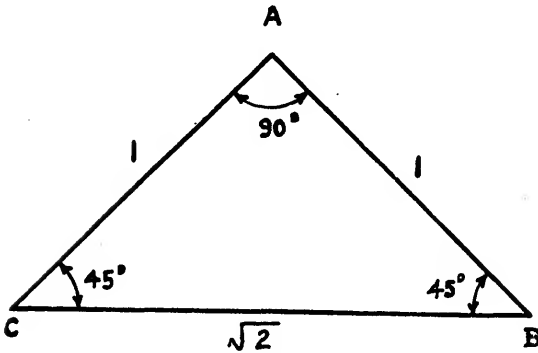


FIG. 23.

45° (from a right-angled isosceles triangle).

$AB = AC = 1$ unit of length.

$$BC = \sqrt{AB^2 + AC^2} = \sqrt{1+1} = \sqrt{2} = 1.414.$$

$$\sin 45^\circ = \cos 45^\circ = \frac{1}{\sqrt{2}}, \text{ and so } \tan 45^\circ = 1.$$

Results Tabulated for Reference.

Angle	0°	30°	45°	60°	90°
Sin	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1
Cos	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0
Tan	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	∞

Ratios of Angles greater than 90° .

For any angle, a line $OQ = OP$ (the circle having unit radius) rotates in an anti-clockwise direction from OP as its zero or starting position.

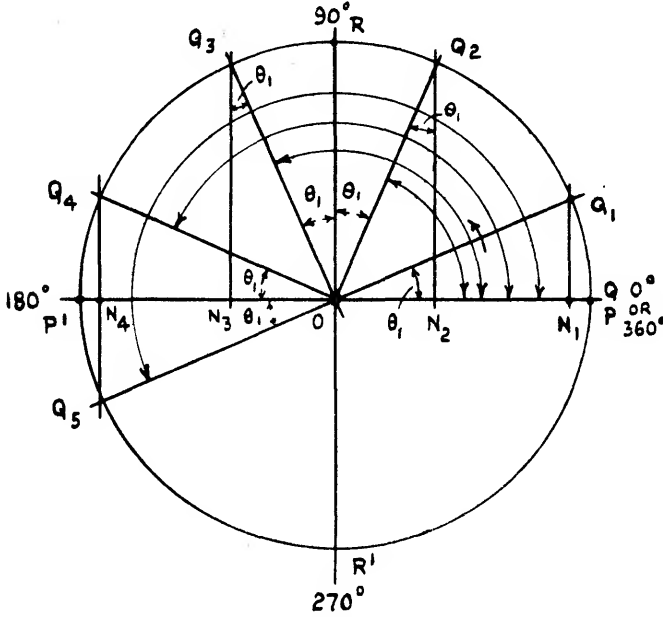


FIG. 24.

The angle under consideration is the amount of turning OQ has made from its starting position. The ratios of the angle are obtained from the vertical (sin) and horizontal (cos) components of OQ due regard being given to sign.

e.g., $\sin \theta_1 = Q_1N_1$ and $\cos \theta_1 = ON_1$ (OQ_1 being unity).

Sign of the Components.

The rotating line OQ is **always** assumed to be **positive**.

Q_1N_1 (the vertical component) is +ve if **above** POP' , -ve if **below**.

ON_1 (the horizontal component) is +ve if to the **right** of ROR' , -ve if to the **left**.

Ratios for $(90^\circ \pm \theta)$ and for $(180^\circ \pm \theta)$.

If $Q_2OR = \theta_1$, then $POQ_2 = (90^\circ - \theta_1)$. Now the triangles OQ_1N_1 , OQ_2N_2 , OQ_3N_3 , OQ_4N_4 and OQ_5N_5 are **all equal**.

So $\sin POQ_2 = \sin (90^\circ - \theta_1) = Q_2N_2 = ON_1 = \cos \theta_1$ and $\cos POQ_2 = \cos (90^\circ - \theta_1) = ON_2 = Q_1N_1 = \sin \theta_1$, whence $\tan (90^\circ - \theta_1) = \cot \theta$.

If θ_1 is an acute angle, both it and $(90^\circ - \theta_1)$ must be in the first quadrant POR , and all ratios of both angles must be +ve by convention. This result could have been obtained from the right-angled triangle (Fig. 25).

$$\hat{A} + \hat{C} = 90^\circ, \text{ so } \hat{A} = 90^\circ - \hat{C},$$

$$\sin \theta_1 = \frac{AB}{AC} = \cos (90^\circ - \theta_1) \text{ and so on.}$$

Similarly, $\sin POQ_3 = \sin (90^\circ + \theta_1) = Q_3N_3 = ON_1 = \cos \theta_1$, but $\cos POQ_3 = \cos (90^\circ + \theta_1) = ON_3$ and ON_3 is to the **left** of ROR' and is **negative**.

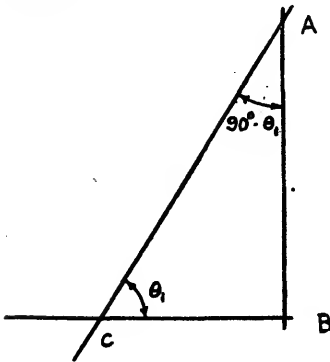


FIG. 25.

So $\cos (90^\circ + \theta_1) = ON_3 = -Q_1N_1 = -\sin \theta_1$.

It follows that $\tan (90^\circ + \theta_1) = -\cot \theta_1$.

The above are sometimes useful, but, as a rule, for angles in the second quadrant ROP' , we do not use $(90^\circ + \theta_1)$, but $(180^\circ - \theta_1)$ as there is then no change in the ratio but only in **sign** (for some of the ratios).

Thus—

(Fig. 1), $\sin POQ_4 = \sin (180^\circ - \theta_1) = Q_4N_4 = QN_1 = \sin \theta_1$,

while $\cos POQ_4 = \cos (180^\circ - \theta_1) = ON_4 = -ON_1 = -\cos \theta_1$

and $\tan (180^\circ - \theta_1) = -\tan \theta_1$,

e.g.,

$$\sin 123^\circ = \sin (180^\circ - 123^\circ) = \sin 57^\circ = 0.8387$$

$$\cos 123^\circ = -\cos (180^\circ - 123^\circ) = -\cos 57^\circ = -0.5446$$

$$\tan 123^\circ = -\tan (180^\circ - 123^\circ) = -\tan 57^\circ = -1.5399.$$

Similarly for angles in the third quadrant, $P'OR'$, we use $(180^\circ + \theta_1)$ and it should be clear from the figure that both \sin and \cos in this quadrant are **negative** making the tangent **positive**, e.g., $\sin 265^\circ = -\sin (180^\circ + 85^\circ) = -\sin 85^\circ = -0.9962$.

For angles in the fourth quadrant, $R'OP$, we use $(360^\circ - \theta_1)$ and the \sin is there **negative**, the \cos ine **positive**, and hence the tangent **negative**.

Graphs of Trigonometrical Functions.—From the previous section, it is clear that as the generating line of the angle θ passes through the first quadrant in the positive direction, the \sin of θ increases from 0 to 1, and, as the generating line passes through the second quadrant, $\sin \theta$ passes back through the same positive values from 1 to 0. In the third and fourth quadrants the numerical values of $\sin \theta$ are the same, but the signs are negative.

Consider the equation $y = \sin \theta$. The values of y , for values of θ at regular intervals, may be obtained directly from the tables provided the angle is in the first quadrant. Values of y corresponding to values of θ between 90° and 360° may now be written down without further reference to the tables.

The values of y corresponding to values of θ in degrees from 0° to 360° , at intervals of 30° , are given in the following table:—

θ	0°	30°	60°	90°	120°	150°	180°	210°	240°	270°	300°	330°	360°
y	0	0.5	0.87	1.0	0.87	0.5	0	-0.5	-0.87	-1.0	-0.87	-0.5	0

In Fig. 26, the graph of $y = \sin \theta$ has been plotted from the values in the table.

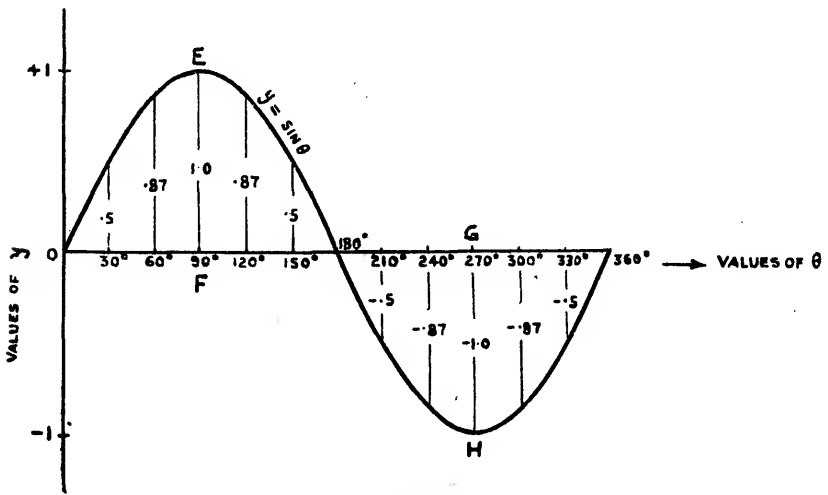


FIG. 26.

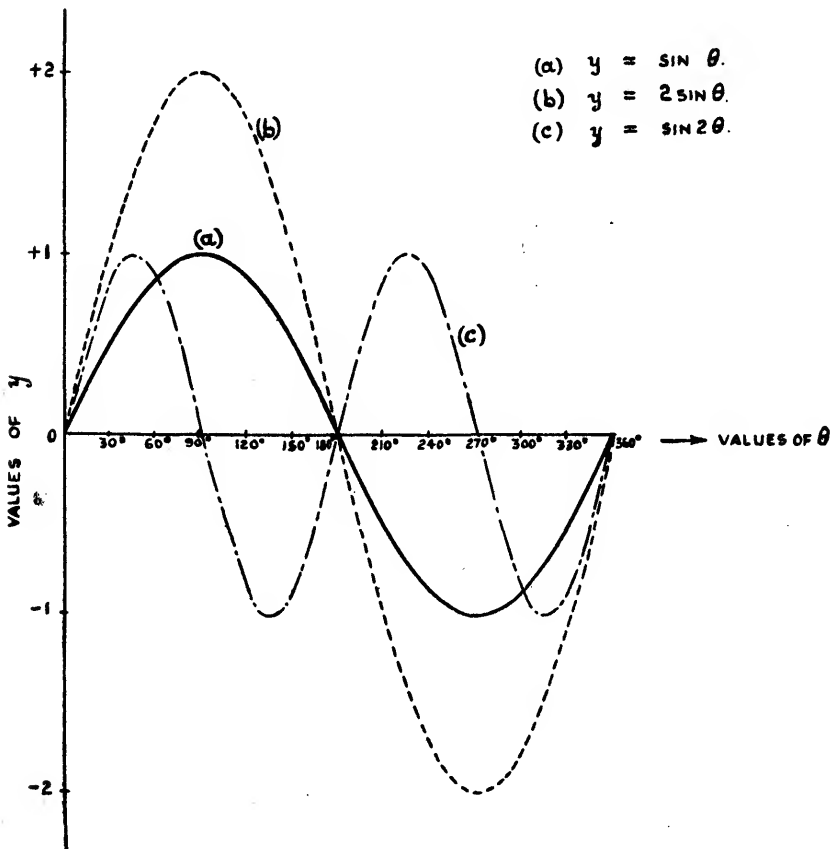


FIG. 27.

As the generating line continues through 360° , $\sin \theta$ begins again at 0, and passes through the same values in the same order as before. Thus the curve repeats itself indefinitely. One complete sequence of values is called a **cycle** of the curve.

The maximum distance (EF or GH) of the curve from the horizontal axis is unity in the case of $y = \sin \theta$. This maximum distance is called the **amplitude** of the sine curve.

When $y = A \sin \theta$ the values found for $\sin \theta$ must be multiplied by the coefficient A in order to obtain the corresponding values for y . In other words, each ordinate is increased in the ratio of A to 1. The maximum ordinate of $y = A \sin \theta$ is, therefore, A .

The graph of $y = 2 \sin \theta$ has been plotted on the same axes as $y = \sin \theta$ in Fig. 27, Curve (b). It has a maximum ordinate of 2.

If the graph of $y = \sin 2\theta$ is plotted, the maximum ordinate will be the same as for $y = \sin \theta$, but the value of $\sin 60^\circ$, for example, will be plotted above 30° and so on. As a result the curve will be completed between 0° and 180° instead of between 0° and 360° , and will then be repeated as shown by Fig. 27, Curve (c). Two complete cycles of $y = \sin 2\theta$ occur between the limits of one complete cycle of $y = \sin \theta$. For the curve of $y = \sin n\theta$, n cycles are completed between the same limits as for one cycle of $y = \sin \theta$.

Next consider the equation $y = \sin (\theta + 15^\circ)$. Before using the tables to find the sine of the quantity in brackets, it is necessary to add 15° to each chosen value of θ . A table of values of y for 30° intervals of θ follows.

θ	0°	30°	60°	90°	120°	150°	180°
$(\theta + 15^\circ)$	15°	45°	75°	105°	135°	165°	195°
$y = \sin (\theta + 15^\circ)$	0.26	0.71	0.97	0.97	0.71	0.26	-0.26

θ	210°	240°	270°	300°	330°	360°
$(\theta + 15^\circ)$	225°	255°	285°	315°	345°	375°
$y = \sin (\theta + 15^\circ)$	-0.71	-0.97	-0.97	-0.71	-0.26	+0.26

Fig. 28 shows these values of y plotted against the corresponding values of θ . The graph of $y = \sin \theta$ is also plotted on the same axes for reference. Each curve has the same maximum ordinate, but the curve $y = \sin (\theta + 15^\circ)$ has been displaced bodily to the left through a distance which corresponds to an angle of 15° .

15° is called the **angular phase** of $\sin (\theta + 15^\circ)$ with respect to $\sin \theta$.

Similarly, $y = \sin (\theta - 15^\circ)$ is the equation of a sine curve which is displaced 15° to the right of the curve $y = \sin \theta$.

The equation $y = A \sin (n\theta + \phi)$ therefore represents a sine curve whose maximum ordinate is A ; which has n complete cycles in a distance corresponding to one complete cycle of $y = \sin \theta$, and which is displaced to the left of $y = \sin \theta$ by a distance corresponding to the angle ϕ .

In the paragraph on ratios of angles greater than 90° it was shown that $\cos \theta = \sin (\theta + 90^\circ)$. The graph of $y = \sin (\theta + 90^\circ)$ is a sine curve displaced to the left through 90° ; but $y = \sin (\theta + 90^\circ)$ may be written as $y = \cos \theta$. Hence the latter equation is represented by the dotted curve in Fig. 29.

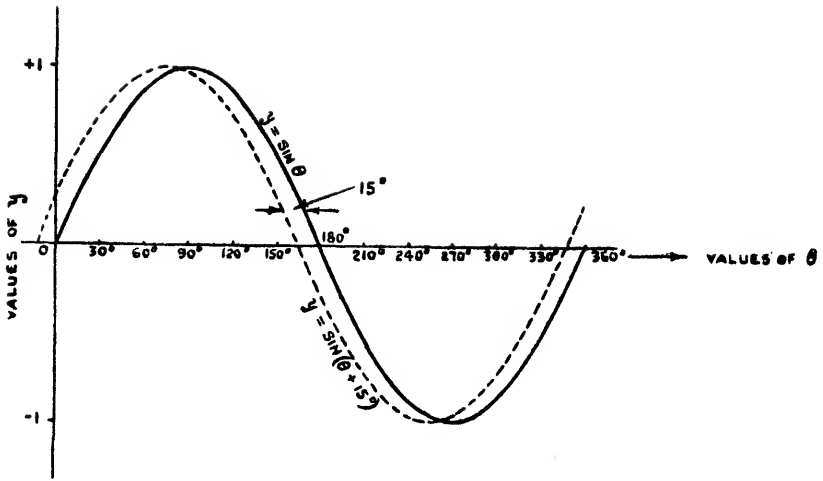


FIG. 28.

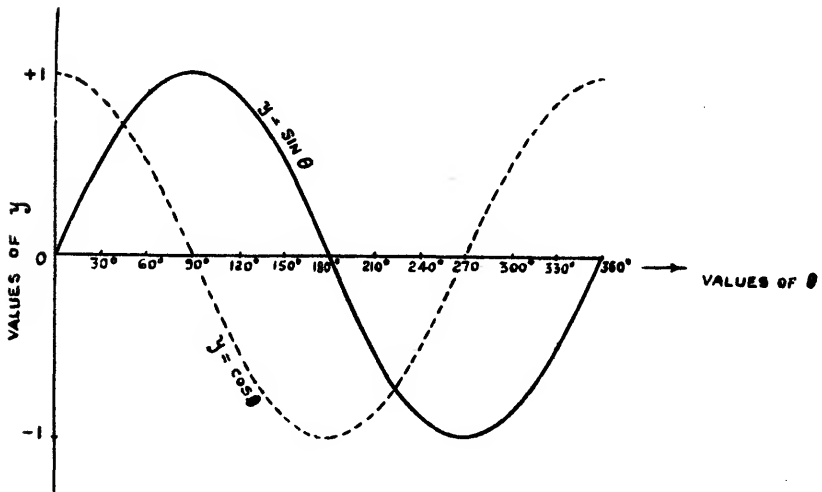


FIG. 29.

The curve of $y = \cos \theta$ can also be plotted in the same way as was that of $y = \sin \theta$, by taking values of $\cos \theta$ from the tables for values of θ at regular intervals.

Trigonometrical Functions of Time.—In the study of Wireless Telegraphy many of the quantities dealt with, *e.g.*, alternating currents and voltages may be considered to vary in time after the manner in which y varies with the angle θ in the functions considered above. In other words, the value of, say, an alternating current (i) at any instant of time t , can be expressed by the equation—

$$i = I \sin \omega t$$

where ωt corresponds to the angle θ above.

As ωt is an angle, ω is of the nature of an angular velocity (Appendix C), and, in fact, when the current is being taken from an alternator, it is the angular velocity of rotation of the armature in a two-pole machine, and a simple multiple of it in a multipolar machine. ω is measured in radians per second.

\mathcal{J} is the maximum value or amplitude of the alternating current.

The current i completes a cycle of values while the angle ωt or θ increases from 0° to 360° , or from 0 to 2π radians, i.e., the time (T) taken to complete one cycle is given by $\omega T = 2\pi$

$$\text{or } T = \frac{2\pi}{\omega} \text{ seconds.}$$

T is called the **periodic time** or **period** of the alternating current.

Another important idea in the study of alternating quantities is the **frequency (f)** which is defined as the number of cycles that take place in one second. The connection between frequency and period is a matter of simple proportion.

1 cycle takes place in T seconds ;
f cycles take place in 1 second ;

$$\therefore f = \frac{1}{T} \text{ cycles per second.}$$

$$\text{Since } T = \frac{2\pi}{\omega}, f = \frac{\omega}{2\pi}, \text{ or } \omega = 2\pi f,$$

which gives the relation between the frequency of the alternating quantity, and its mode of representation as

$$i = \mathcal{J} \sin \omega t.$$

Differences of phase between alternating quantities can be represented as before by phase angles. For instance,

$$v = \mathcal{V} \sin (\omega t + \phi_1)$$

represents an alternating voltage which differs in phase by an angle ϕ_1 from the current i .

This angular difference ϕ_1 corresponds to a difference in time phase, say, t_1 . In other words, values in the voltage cycle (e.g., the maximum value), occur t_1 seconds earlier than the corresponding values in the current cycle.

The connection between angular and time phase difference is easily found from the fact that the angle is directly proportional to the time, ($\theta = \omega t$),

$$\therefore \phi_1 = \omega t_1 \text{ and } t_1 = \frac{\phi_1}{\omega},$$

or, in terms of the period,

$$t_1 = \frac{\phi_1}{2\pi} T.$$

When ϕ_1 is **positive**, the voltage has a **leading phase** on the current.

When ϕ_1 is **negative**, the voltage has a **lagging phase** on the current.

VECTORS.

Scalar quantities have **magnitude only**, e.g., mass and time.

Vector quantities have both **magnitude and direction**, e.g., force and current.

They may be represented by straight lines, the length of the straight line representing to some convenient scale the magnitude of the vector quantity, and the direction of the straight line representing the **direction** of the vector quantity (generally with reference to some other direction or other vector quantity).

To describe a vector completely, we must know :—

1. The point of application of the quantity being represented.
2. Its magnitude.
3. Its direction with respect to a given datum line.
4. Its sense (e.g., a vertical force may be acting either upwards or downwards).

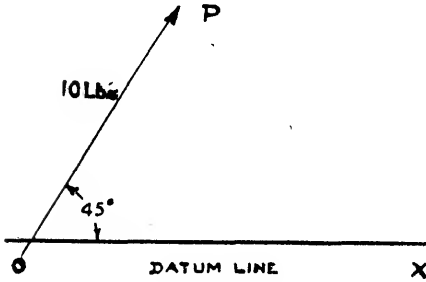


FIG. 30.

OP is a **vector** representing a force of 10 Lb.

O is the **point of application** of the force.

The length of OP represents the **magnitude** of the force.

The angle XOP which OP makes with the datum line OX gives the **direction** of the force.

The arrow head on OP gives the **sense** of the force.

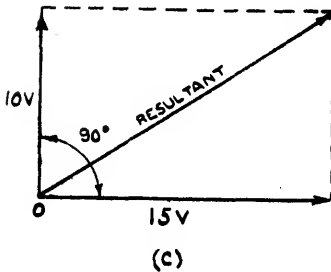
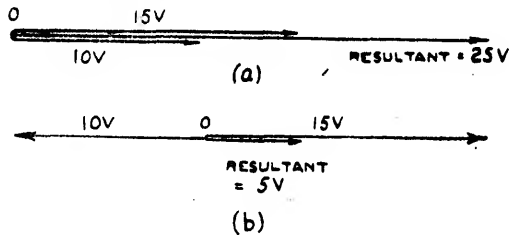
Addition of Vectors.

Case (1). Fig. 31 (a)—Vectors in the **same** direction are added arithmetically.

Vector sum = $10v + 15v = 25v$ in the same direction as each of the vectors being added.

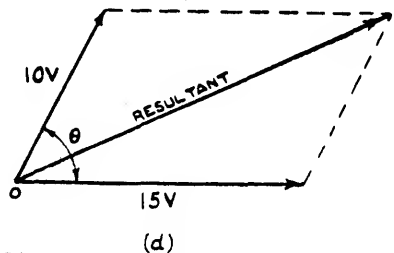
Case (2). Fig. 31 (b)—Vectors in the same direction but of **opposite sense** (180° out of phase).

Vector sum = $15v - 10v = 5v$ of the same sense as the larger vector. The vector sum here is the **algebraic sum**, i.e., due regard must be had to sign.



(c)

Case (3).



(d)

Case (4).

FIG. 31.

Case (3) and Case (4) are dealt with by the Parallelogram Law.

The Parallelogram of Vectors.

If two vectors with the same point of application are represented by the two adjacent sides of a parallelogram then their resultant is completely represented by that **diagonal** of the parallelogram which passes through the point of application.

[Note.—A rectangle is a parallelogram and so is a square.]

Example.

Two alternating currents of 30 amps. and 40 amps. (maximum) have a phase difference of 90° . Find the magnitude and phase angle of their resultant.

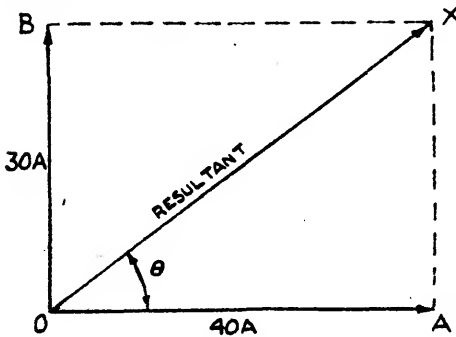


FIG. 32.

$$\begin{aligned}\text{Resultant} &= OX = \sqrt{OA^2 + OB^2} \\ &= \sqrt{1600 + 900} \\ &= \sqrt{2500} = 50 \text{ amps.}\end{aligned}$$

$$\begin{aligned}\text{Phase angle } \theta &= \tan^{-1} \frac{AX}{OA} \\ &= \tan^{-1} 0.75, \text{ whence } \theta = 36^\circ 52' .\end{aligned}$$

The above means that two currents of 30 A and 40 A, 90° out of phase and flowing simultaneously, are **in every respect equivalent** to a single current of 50 A which leads by $36^\circ 52'$ on the 40 A current.

Resolution of Vectors.

This is the previous idea reversed. A single vector may be replaced by two vectors, generally taken at right angles—the original vector being the resultant of the two vectors at right angles.

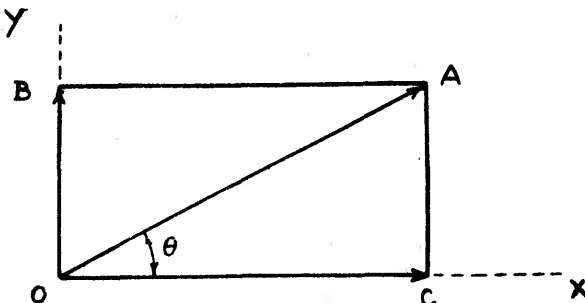


FIG. 33.

OA is the given vector, its point of application being O.

It is required to **resolve** OA into two components in the directions OX and OY.

From A drop perpendiculars (AC and AB) to OX and OY.

Then OC is the **horizontal** component of OA, and OB is the **vertical** component of OA.

If the angle COA be θ , then :—

$$\frac{OC}{OA} = \cos \theta, \text{ so } OC = OA \cos \theta,$$

$$\text{and } \frac{AC}{OA} = \frac{OB}{OA} = \sin \theta, \text{ so } OB = OA \sin \theta.$$

i.e., **Horizontal component = Vector $\times \cos \theta$.**

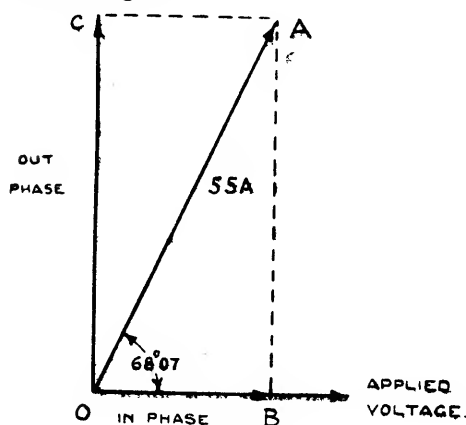
Vertical component = Vector $\times \sin \theta$.

Examples.

(1) An alternating current of 55A leads the applied voltage by $68^\circ 07'$. Draw vectors to illustrate this.

Find (a) the in-phase component; (b) the out-phase component of the current.

Vector Diagram.



The **positive** direction of angular phase is **anti-clockwise** so if the applied voltage vector be drawn as shown (as datum line) OA will be the current vector.

FIG. 34.

$$(a) \text{ In-phase component of current} = OB = 55 \cos 68^\circ 07' \text{ A.} \\ = \mathbf{20.5A.}$$

$$(b) \text{ Out-phase component of current} = OC = 55 \sin 68^\circ 07' \text{ A.} \\ = \mathbf{51.04A.}$$

(2) An alternating current of 20A leads the applied voltage by $61^\circ 07'$, another current of $5.25A$ leads 90° and a third current of $36.2A$ lags $58^\circ 43'$. Find the total in-phase and out-phase components of the resultant current and its magnitude and phase angle.

$$(a) \text{ Out-phase (20A)} = AB \\ = 20 \sin 61^\circ 07' \text{ A.} \\ = \mathbf{17.512A.}$$

$$\text{In-phase (20A)} = OB \\ = 20 \cos 61^\circ 07' \text{ A.} \\ = \mathbf{9.66A.}$$

$$(b) \text{ Out-phase (5.25A)} = \mathbf{5.25A.}$$

$$(c) \text{ Out-phase (36.2A)} = -CD. \\ = -36.2 \sin 58^\circ 43' \text{ A.} \\ = \mathbf{-30.94A.}$$

$$\text{In-phase (36.2A)} = OD. \\ = 36.2 \cos 58^\circ 43' \text{ A.} \\ = \mathbf{18.79A.}$$

Total out-phase component = $17.512A + 5.25A - 30.94A = -8.18A$
(current lags).

Total in phase component = $9.66A + 18.79A = 28.45A$.

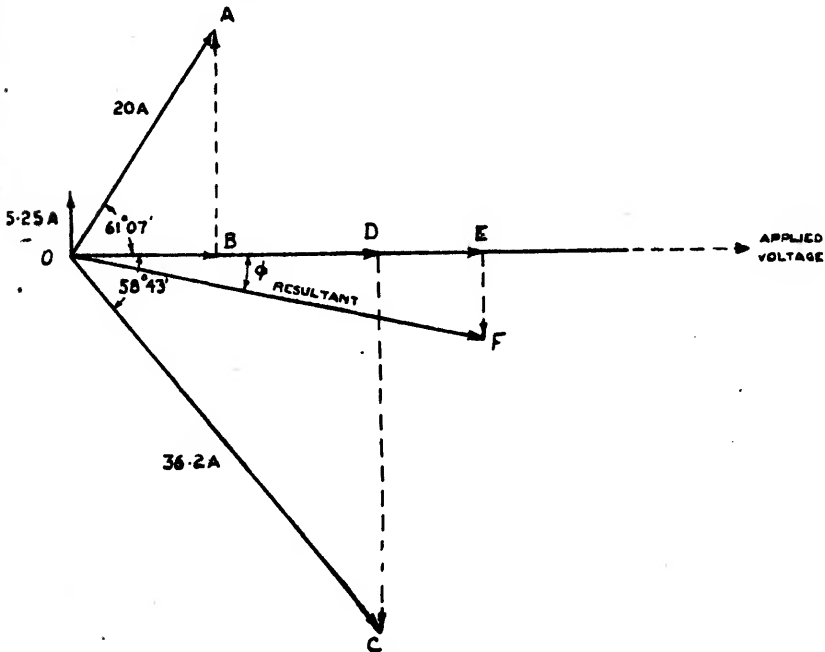


FIG. 35.

These two components are shown on the figure as EF and OE. One half only of the parallelogram has been drawn in each case.

Resultant current (OF) = $\sqrt{28.45^2 + 8.18^2} = 29.61A$.

Phase angle (of lag) = $\phi = \tan^{-1} \frac{8.18}{28.45} = 16^\circ 03'$.

Rotating Vectors.

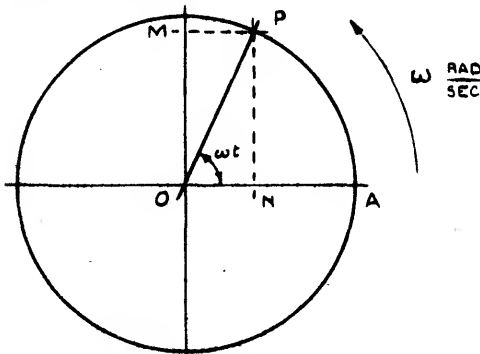


FIG. 36.

OP is a vector capable of rotating from the zero position OA in an anticlockwise direction with an angular velocity of ω radians per second.

Suppose after a time " t " seconds OP takes up the position shown in the sketch.

Then the angle AOP traced out by OP after " t " secs. will equal ωt radians.

If PM and PN are drawn from P perpendicular to the axes it will be seen that as OP rotates (from the zero position OA) M will move upwards and downwards, *i.e.*, OM will vary from zero to maximum + ve, back to zero, then to maximum - ve and back again to zero.

OP is known as a **rotating vector**.

OM is known as an **alternating vector**.

Note that, for every position of P, $OM = PN = OP \sin \omega t$, and $ON = OP \cos \omega t$.

RATES OF CHANGE.

In giving a systematic description of natural processes, it is often found that it is just as important to know how some quantity is changing in value as it is to know its actual value at any particular moment. We should expect this to be the case since change is the great law of nature. Nothing stands still. Even the "everlasting hills" alter their contours by slow degrees under the disintegrating influence of wind and rain.

It is therefore necessary to develop a method of studying and classifying rates of change, and the branch of mathematics which deals with this is called the "Differential Calculus." This is rather a discouraging name for what is in essence a very simple idea; for "calculus" simply means a method of calculating, and the processes of arithmetic might equally well be called the Arithmetical Calculus. The title "Differential" arose from the nature of the calculation found to be necessary when the study of rates of change was developed, so that the subject might equally well be termed "the process of calculating rates of change." It was in fact called "fluxions," which conveys the same idea more directly, by Sir Isaac Newton, who first formulated the processes.

The idea of "rate" is perhaps most simply grasped from the consideration of a moving body. A train is travelling at 60 miles per hour. If it maintains this speed for one hour it will have gone 60 miles. We say that the rate at which the train is covering the ground is 60 miles per hour. It will be seen that the idea involved in "rate" is the **ratio** of the change in one quantity (a change of 60 miles in the train's position on the earth's surface), to the change in another quantity (a change of one hour in time).

All rates are of this nature, although the quantities whose change is being considered may be of many different kinds.

In the example above, the train is supposed to be travelling at 60 miles per hour at every instant of the hour, *i.e.*, in a quarter of an hour it has gone 15 miles, in 20 minutes 20 miles, and so on, but this would not normally be the case in practice. The train would slow down at uphill gradients and possibly stop for a few minutes at stations en route. If it still covered 60 miles in an hour, it would require to travel at a faster rate than 60 miles per hour during some part of the hour to make up for the stoppages, &c. During the hour in question, its **average** rate of travel would be 60 miles per hour, but this would only be its actual rate at certain selected moments, and the rate at other moments would be widely different. It would, for instance, be zero while the train was stopped. We may sum this up by saying that in the first case the instantaneous rate is **uniform**, *i.e.*, the same at all instants; in the second case the instantaneous rate is varying at different instants.

Average rates are of importance in many enquiries. Birth and death rates, for example, are of this nature. To say that the death rate in a certain country is 365 people per year does not mean that one person dies every day. On some days nobody might die, on others half a dozen people, but **on the average** one person dies every day throughout the year. It is obvious, however, that the average rate gives no indication of the actual rate at any particular period of the year, except in the special case where the instantaneous rate is uniform and therefore equal to the average rate.

Graphical representations of the relation of one quantity to another are very convenient for illustrating rates of change. Fig. 13, for example, shows the relation of a quantity labelled " y " to one labelled " x ." The graph is a straight line.

When x is 0, y is -1 and when x is 0.5, y is 0, *i.e.*, y rises vertically by one unit while x moves to the right by 0.5 unit. The ratio of the change in y to the change in x , which is the rate at which y is changing with respect to x , is $\frac{1}{0.5} = 2$. To preserve the idea of rate we may call it a rate of two vertical units upwards per horizontal unit to the right.

When $x = 0.5$, $y = 0$ and when $x = 1$, $y = 1$. The rate is again $\frac{1 - 0}{1 - 0.5} = 2$ vertical units upwards per horizontal unit to the right, and this would also be found to be the rate between any two corresponding values of y and x . In other words, y is changing with respect to x at a uniform rate.

It is always the case that when one quantity is changing uniformly with respect to another, the relation between them can be represented graphically by a straight line. For example, the straight parts of valve mutual characteristics are employed for amplification because the change of anode current with grid voltage is then uniform, and so amplification without distortion is obtained.

The straight line in Fig. 13 was plotted from the relation between x and y given by $y = 2x - 1$ and it was then shown that 2 was the slope or gradient of the line. But it has just been seen that 2 is the rate of change of y with regard to x . Hence the slope of the straight line which is the graph of the relation between the two quantities is the measure of the uniform rate of change of one of them with regard to the other. This is a perfectly general rule and applies to any quantity varying uniformly with respect to another. It can be seen, for example, from Fig. 12, that the amount by which y changes for a certain change in x is twice as great for the relation $y = x$ as it is for $y = \frac{1}{2}x$.

According to the graphical convention, lines sloping upwards to the right have a positive gradient, and lines sloping downward to the right have a negative gradient. Thus a positive slope corresponds to a rate of increase and a negative slope to a rate of decrease of y in the positive direction of x (to the right).

The rate of change of any quantity y with another quantity x is written in mathematical shorthand as $\frac{dy}{dx}$, and for our purposes this may be taken to be of the same nature as any other shorthand or code symbol requiring to be memorised and not necessarily bearing any obvious relation to the long-hand words (plain language) for which it stands. Every letter is pronounced as in the alphabet, *i.e.*, as spoken, $\frac{dy}{dx}$ is dee-wy by dee-eks, not die by decks. We may note, however, that it is in the form of a rate (one thing divided by another) and that the letter d is prefixed to each of the quantities being discussed; $\frac{y}{x}$ would be quite a different ratio for every point on the line representing $y = 2x - 1$, *e.g.*, when $x = 0$, $y = -1$, and $\frac{y}{x} = \frac{-1}{0} = -\infty$, when $x = \frac{1}{2}$, $y = 0$ and $\frac{y}{x} = 0$, and so on. There is nothing uniform about $\frac{y}{x}$ and it is no measure of the rate of change of y with x . Thus the " d " bears no resemblance to a numerical factor multiplying both y and x . The symbol $\frac{dy}{dx}$ must be taken as a whole.

The other point to notice about this code symbol is that y appears in the numerator and x in the denominator. This is because we are quoting the rate of change of y with respect to x . The rate of change of x with respect to y would be written $\frac{dx}{dy}$.

In the case of the train we might represent the distance it travels from its starting point by " s " and the time it takes to cover the distance " s " by " t ." The speed of the train, or the rate at which its distance from the starting point is altering with time, is then represented by the symbol $\frac{ds}{dt}$. For our train travelling at a uniform speed of 60 miles per hour, we can then write $\frac{ds}{dt} = 60$ miles per hour.

It has been pointed out that a rate of change may be either a rate of increase or a rate of decrease, and that positive and negative signs are used to distinguish these. Thus $+\frac{dy}{dx}$ (or simply $\frac{dy}{dx}$) stands for the rate of increase of y with x ; $-\frac{dy}{dx}$ stands for the rate of decrease of y with x . The relation $y = -2x - 1$ would be represented graphically by a line sloping downwards to the right. -2 is the slope or $\frac{dy}{dx}$ in this case. The negative sign indicates that the rate of increase is negative, i.e., it is a positive rate of decrease, and $-\frac{dy}{dx} = 2$, i.e., the rate of decrease of y is 2 units vertically downward for every unit of x to the right.

The extension of these ideas to cases where the rate is not uniform will now be briefly considered. The idea of average rate in such cases has already been explained and its inability to give any indication of instantaneous rate pointed out. We have a perfectly definite idea of what we mean by the instantaneous rate of our train which eventually covers 60 miles in an hour, although its speed may never be the same for two successive instants. The engine driver for example may proudly say afterwards that at one moment during the run he was touching 80 miles per hour. By this he does not mean that he actually covered 80 miles in an hour, but that if he had kept on for an hour at a uniform rate equal to the instantaneous rate at that exalted moment, then he would have covered 80 miles in that time. If he were a mathematician he might say that his $\frac{ds}{dt}$ was 80 miles per hour for a certain definite value of t , but not for any other value.

Suppose we apply this idea to the graphical representation of the variation of one quantity with another. We have already seen that if the variation is at a uniform rate, the graph is a straight line. If the rate is not uniform, the line will no longer be straight. It will be some kind of curved line. For a straight line, the slope or gradient was a measure of the rate, but what meaning can be attributed to the slope of a curve? We do, however, talk of slopes and gradients in connection with curves. Suppose we are climbing a hill. At no point could the profile of the hill be considered as a straight line, but we may still say the slope of the hill is steeper at some heights than others. Road books even quote a numerical value of the steepest gradient to be encountered on a hilly road. If the road book states that the gradient at some point on a road is 1 in 5, it does not mean that the road actually rises by a vertical height of one mile while you travel five miles along it. The meaning is that if the road continued to rise at the same rate as it actually is rising at a particular point, then it would rise one mile in five. The gradient

may be quite different, say 1 in 10, at points 100 yards away in either direction, but in the neighbourhood of this point the height of the road is increasing at the particular rate of one mile in five. It might, for example, rise one inch in five inches. Now a road which increased uniformly in height by one mile in five miles would have a straight line profile over these five miles, and this profile would have the same slope as the actual profile of the hill over five inches in the neighbourhood of the point where the gradient is 1 in 5. The ideal road would touch the actual road at this point but would rise far above it at other points, since its slope would be so much greater (Fig. 37).

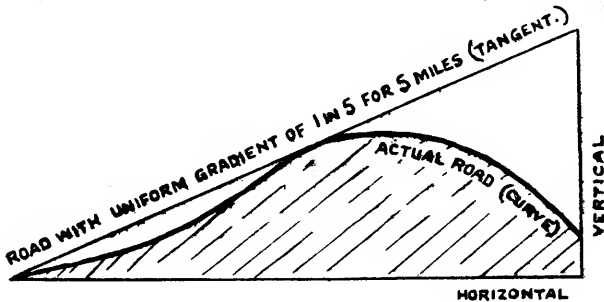


FIG. 37.

The ideal and actual road profiles bear the same relation to each other as a curve and a straight line drawn to touch the curve at some particular point. Such a line is called a **tangent** to the curve at the point. By the slope or gradient of a curve at a point, we mean the slope or gradient of the tangent to the curve at that point.

There is one important practical distinction. In the case of roads, a gradient of 1 in 5 means a rise of 1 unit for 5 units travelled along the road, but in the mathematical case it would correspond to a rise of 1 unit while traversing 5 units along a horizontal axis; which would be equivalent to more than 5 units (actually $\sqrt{5^2 + 1^2} = \sqrt{26} = 5.1$ units) along a uniformly rising road with this gradient. This distinction is only for convenience, and the two different conventions in surveying and mathematics merely alter the actual number quoted as the gradient.

As an electrical illustration, we may take Faraday's Law for finding the magnitude of the E.M.F. induced in a circuit when the current is changing. The law states that the E.M.F. is equal to the rate of decrease of the current, (with respect to time), multiplied by the self-inductance (L) of the circuit. At some moment which suits our convenience we "start the watches." This moment is our zero moment and we reckon our time from it. After " t " (any number) seconds have elapsed we shall suppose that the value of the current is represented by " i " amps. Since the current is changing, this only refers to its instantaneous value at the actual moment " t ." The rate of increase of the current at this moment is represented in the above notation by $\frac{di}{dt}$, and its rate of decrease is $-\frac{di}{dt}$. Hence, calling " e " the induced

E.M.F., we may represent the law symbolically by $e = -\frac{di}{dt} \times L = -L \frac{di}{dt}$.

This is as far as we can go unless we are told how the current is changing. In one particular case of great importance we do know this, viz., the case when the current is alternating. Its variation with time is then represented by $i = J \sin \omega t$, where J is its amplitude and $\omega = 2\pi \times \text{frequency}$, as explained in the section on Trigonometry. From this expression we can go on to

derive the value of $\frac{di}{dt}$ by using the graphical treatment outlined in the last paragraph, *i.e.*, deriving the rate of change from the slope of the graph representing the variation of current with time.

The first problem is to find the rate of change of a quantity (i) which can be represented as the sine of another quantity (ωt). A curve representing this type of variation for quantities y and θ has already been shown in Fig. 29, $y = \sin \theta$. The rate of change of y for any value of θ is given by the slope of the curve (or the tangent to it) for that particular value of θ .

At $\theta = 0$, the curve is rising most steeply and so the slope $\frac{dy}{d\theta}$ has its greatest value there. This value is positive since the slope is upward to the right. Its actual value is unity and it is represented by the point +1 on the y axis. As θ increases, the slope becomes less steep and when its value is plotted on the same axes it follows the dotted curve. When $\theta = 90^\circ$, the curve $y = \sin \theta$ is momentarily horizontal. Its slope is zero. Hence the dotted curve meets the θ axis at this point. Between $\theta = 90^\circ$ and $\theta = 270^\circ$, $y = \sin \theta$ steadily decreases. Its slope is downwards to the right, *i.e.*, negative. Hence the dotted curve representing the slope is negative over this region. The downwards slope is steepest when $\theta = 180^\circ$. The maximum negative value of $\frac{dy}{d\theta}$ therefore occurs at this point.

At $\theta = 270^\circ$, the curve $y = \sin \theta$ is again horizontal and $\frac{dy}{d\theta} = 0$. Thereafter the slope is upwards to the right, *i.e.*, positive, and rises to its maximum value at 360° . It will be seen that the dotted curve representing the rate of change of the sine curve is itself a cosine curve or, as it may also be considered, a sine curve leading in phase on the original sine curve by 90° , *i.e.*, $\frac{dy}{d\theta} = \cos \theta$, or $\frac{dy}{d\theta} = \sin(\theta + 90^\circ)$, is the expression representing the rate of change of $y = \sin \theta$.

If the curve $y = 2 \sin \theta$ in Fig. 27 is compared with $y = \sin \theta$, it will be seen that the only difference is that the slope has been doubled at every point and so the rate of change of $y = 2 \sin \theta$ is twice the rate of change of $y = \sin \theta$. It is therefore $\frac{dy}{d\theta} = 2 \cos \theta$.

Again, if the curve $y = \sin 2\theta$ in Fig. 27 be compared with $y = \sin \theta$, the effect graphically is to give two cycles of $\sin 2\theta$ for every one of $\sin \theta$ and so the slope of $y = \sin 2\theta$ is everywhere twice as great as that of $y = \sin \theta$. The curve representing the slope plotted against θ is a cosine curve as before and performs one cycle during one cycle of $\sin 2\theta$. It is therefore a $\cos 2\theta$ curve and since its slope is twice the slope of the $\sin \theta$ curve it may be represented as $\frac{dy}{d\theta} = 2 \cos 2\theta$.

Now consider a curve $y = 2 \sin 2\theta$. Its amplitude is twice that of $y = \sin 2\theta$, which doubles the slope, and the number of cycles over a given range of values of θ is twice as great as for the curve $y = 2 \sin \theta$ which again doubles its slope relatively. Its actual slope is therefore $2 \times 2 = 4$ times as great as that of $y = \sin \theta$. Similarly, the slope of the curve $y = 4 \sin 3\theta$ is 4×3 times as great as that of the curve $y = \sin \theta$ and so $\frac{dy}{d\theta} = 4 \times 3 \cos 3\theta$.

From these examples we can deduce a simple rule for finding the rate of change of any sine function of θ ; namely, to substitute cosine for sine and multiply the amplitude by the coefficient of θ , *e.g.*, if $y = 5 \sin 7\theta$, then $\frac{dy}{d\theta} = 7 \times 5 \cos 7\theta = 35 \cos 7\theta$.

This rule may now be applied to $i = J \sin \omega t$. Here t corresponds to θ and its coefficient is ω . Hence $\frac{di}{dt} = \omega \times J \cos \omega t = \omega J \cos \omega t$.

Substituting this in Faraday's Law, $e = -L \frac{di}{dt}$, gives

$$\begin{aligned} e &= -L \times \omega J \cos \omega t, \\ &= -\omega L J \cos \omega t. \end{aligned}$$

$\omega L J$ is the amplitude of this expression. In other words, the amplitude of the induced E.M.F. is $\omega L J$. To determine its phase with respect to the current i which induces it, we have from Trigonometry that

$$-\cos \omega t = \sin (\omega t - 90^\circ),$$

and so the induced E.M.F. may be written as

$$e = \omega L J \sin (\omega t - 90^\circ).$$

Comparing this with the current $i = J \sin \omega t$, it is seen that the induced E.M.F. lags 90° on the current.

APPENDIX C.

MECHANICS.

In mechanics we examine (a) the conditions under which a body may remain at rest under the action of a system of forces, and (b) the changes of position which a body may experience under such a system.

Motion.

We can only determine whether a body has changed its position when we can compare its position with that of some other body. This is generally expressed by saying that motion is *relative to some fixed reference body*, and can be readily recognised by anyone who has ever travelled by train. The landscape appears to be in motion and the train at rest, although to anyone standing by the railway line, of course, the change of position will appear to be undergone by the train. We commonly take the earth as our body of reference, but astronomers often use the sun or the fixed stars.

For a body to change its position, a certain time must elapse; and so the study of motion involves the measurement of distance (or length) and time.

Two **units of length** are in common use. In Britain, the unit is the **foot**, which is one-third of the distance at 62° Fahrenheit between two marked points on a bronze bar kept in the Houses of Parliament at Westminster.

The unit usually employed in scientific measurements is the **centimetre**, which is one-hundredth of the International Metre, the distance at the melting point of ice between two marked points on a bar of platinum kept at Sevres near Paris.

The **unit of time** is the **second**, which is derived from the times taken for the earth to rotate on its axis and to complete its orbit round the sun.

It will be seen that these units are quite arbitrary, being settled, so to speak, by Act of Parliament. Once fixed, however, they serve as a basis from which other units may be derived.

Velocity.

The rate at which a body is changing its position (with respect to the reference body) is called its velocity, and so velocity will be measured in feet per second or cms. per second. The units of velocity are **1 cm./sec.** and **1 ft./sec.** on our two systems.

Before a velocity is completely described, it must be specified both in amount and direction. Velocities of 10 ft./sec. North and 10 ft./sec. South-West will land their possessors in very different places. In other words, velocity is a vector quantity.

Acceleration.

The rate at which a body is changing its velocity is called its acceleration. A common example of accelerated motion is that of bodies falling freely to the earth's surface. Such bodies are uniformly accelerated, *i.e.*, their velocity increases by the same number of ft./sec. or cms./sec. during every second of their fall (actually 32 ft./sec. approximately).

Acceleration is thus measured in **ft. per sec. per sec.** and **cms. per sec. per sec.** For brevity these are usually written as **ft./sec.²** and **cms./sec.²**.

The two units of acceleration are 1 ft./sec.² and 1 cm./sec.². Acceleration is a vector quantity. Although a projectile fired from a gun may never have a velocity in a vertical direction, it eventually returns

to the earth's surface because (owing to the earth's gravitational attraction) it has an acceleration directed vertically downwards.

It follows that whenever the direction of a body's velocity is altered, even though its amount is unchanged, the body has an acceleration.

Dynamics.

Dynamics deals with the motion of bodies under a system of forces.

Everybody knows what is meant familiarly by force when it refers to the muscular exertion involved in pushing or pulling a body. It is this idea which is generalised in mechanics. Apart from our own muscular sensations, the commonest evidence we have of force being applied is that the motion of the body to which we apply the force is altered. If the body is at rest, force will commonly set it in motion, if it is in motion, its velocity will be changed. Thus one criterion of force is change of velocity or acceleration.

Our muscular sensations also inform us that if we apply roughly the same force to two different bodies the results, as far as change of velocity is concerned, may be widely different; evidently acceleration alone is not sufficient to estimate force. Colloquially we account for this by saying that one of the bodies is heavier than the other, but a little consideration shows that it is not the weight of a body that determines its motion in such a case. The weight of a body is simply the earth's gravitational attraction on the body. The earth pulls the body vertically downwards. If the body is set on a horizontal plane surface like a table top, for instance, it cannot move vertically downwards as long as the table top is strong enough to support its weight. The weight of the body is balanced by the upward pressure of the table top. But it will still be found that the forces required to set different bodies in motion with the same velocity are widely different. Evidently, then, it is some inherent property of the body, and not its weight, which is a mutual property of the body and the earth, that for a given force determines what velocity the body will acquire. This property is called the **mass** or **inertia** of the body.

Mass.

This property of a body only comes into evidence when we attempt to alter its motion. If we consider, for instance, a railway truck at rest on the rails, considerably greater muscular exertion is required to start it in motion than is necessary to keep it moving at a steady speed once it has acquired that speed.

This is because at a steady speed we need only exert sufficient force to overcome the friction between the rails and the wheels of the truck, whereas while it is getting up this speed, its inertia or tendency to resist a change in its motion is operative and determines the force which must be exerted to reach this speed in a given time.

It should now be evident that the mass or inertia of a body is quite a different thing from its weight. The weight of a body depends on the position of the body on the earth's surface and would vanish, for instance, at the centre of the earth where the gravitational attraction is zero. The mass of the body is unaltered as long as the body is unaltered and would be the same at the centre of the earth as it is at the surface.

The next question that presents itself is how to measure the mass of a body. To do this we have first to settle what we mean when we say two masses are equal. This is exactly the same problem in essence as confronts us in the measurement of length. We have to settle first what we mean by equal lengths, then we choose a standard length (the foot or the centimetre) and express any length as a number of lengths, each equal to our standard. Fractions of the standard are developed on the same principle. The meaning of equal lengths seem so obvious that we do not analyse the process mentally, but it should be realised that the method adopted for measuring mass is based on exactly the same principle as the method of measuring length or time, which we have already considered.

As mass obtrudes itself on our notice whenever we try to change the motion of a body, it is reasonable to adopt this as a criterion of the amount of mass a body possesses. Two masses are said to be equal when the same force applied to each in turn, over the same period of time, produces in both masses the same velocity. One body will thus have twice the mass of another if under those conditions it only acquires half the velocity and so on. (It will be seen below that we can make sure that we are applying the same force from statical considerations, and so this definition of equal masses does not imply that we have already settled how we are to measure forces.)

We have now only to choose a unit mass and our equipment for measuring mass will be complete.

Two standards units of mass are in use :—

- (1) The British Imperial **pound** (usually called the pound) which is the mass of a lump of platinum kept at Westminster.
- (2) The International Kilogram, which is the mass of a cylinder of platinum-iridium kept at Sevres. The **gram**, which is the mass of one-thousandth part of this cylinder, is usually taken as the unit.

It will be observed that our units of mass are just as arbitrary as our units of length and time. No more arbitrary units are, however, required. The units of all other mechanical quantities may be derived from these three fundamental ones. The two units of length and of mass give rise to two systems of units in mechanics, which are called the foot-lb.-sec. or British system and the cm.-gm.-sec. (C.G.S.) system. The latter is the one commonly employed in scientific work.

Force.

We need not go far in the study of natural phenomena before we realise that the velocity of a body may be altered by other means than a direct muscular effort. If the body is near the earth's surface, for instance, it acquires an acceleration vertically downwards; if it is given an electric charge, it may undergo various erratic types of motion in the neighbourhood of other electric charges. The idea of force was generalised by Sir Isaac Newton to include all such cases of "action at a distance," as well as direct pressures and tensions.

From the remarks above leading up to the idea of mass it will be seen that the two mechanical quantities which are involved when a force is applied to a body are (1) the mass of the body, and (2) the acceleration produced. The greater the mass the less the acceleration under a given force; and if different forces are applied to the same body (the same mass) proportionately different accelerations will be produced.

Hence, when Newton was faced with the problem of deciding how his "generalised" forces were to be measured, he made use of these two ideas and stated that the force acting on a body was proportional to the product of its mass and its acceleration. For instance, if the same force acts on two bodies and the mass of the first is three times the mass of the second, then the acceleration of the first will be one-third of the acceleration of the second; and if two forces acting on the same body produce accelerations in the ratio of 5 to 1, say, then the first force is five times the second force.

This rule for measuring force, which is called Newton's Second Law of Motion, may be written algebraically as

$$F \propto ma, \text{ or } F = Kma$$

where F is the force acting, m is the mass of the body, a is the acceleration, and K is the constant of proportion which gives the ratio of F to ma .

From this rule we can derive whatever units of force we choose by giving different values to the constant K . The method should be carefully observed as it is widely employed in deriving other units also, e.g., absolute electrical units of various quantities.

The units of mass (gm. or lb.) and acceleration (1 ft./sec.² or cm./sec.²) have already been chosen. Suppose now we were to take $K = 4$, say, and consider the action of a force on a mass of 1 gm. If the force were 1 unit, we could then derive the acceleration from $F = Kma$. Substituting, $1 = 4 \times 1 \times a$, $\therefore a = 1/4$ cm./sec.², i.e., the unit of force on this system would produce an acceleration of $1/4$ cm./sec.² when acting on 1 gm. mass. Similarly, if we took $K = 13$, we should derive another unit of force such that when this unit acted on 1 gm. mass, it would produce an acceleration of $1/13$ cm./sec.².

Obviously, the simplest value to choose for K is unity. Substituting in $F = Kma$, this gives

$$1 = 1 \times 1 \times a, \text{ or } a = 1 \text{ cm./sec.}^2,$$

i.e., on this system, the unit of force acting on 1 gm. mass produces an acceleration of 1 cm./sec.².

This simplest possible value of K , viz., $K = 1$ is chosen to define what are called the "absolute" units of force. They are called "absolute" because they depend only on the units of length, mass, and time. There will thus be two units of force :—

- (1) On the C.G.S. system. This unit is called the **dyne** and is the force which, acting on a mass of 1 gm., produces an acceleration of 1 cm./sec.² ($1 \text{ dyne} = \frac{1 \text{ gm. cm.}}{\text{sec.}^2}$).
- (2) On the British system. The unit is the force which acting on a mass of 1 lb. produces an acceleration of 1 ft./sec.². It is called the **poundal** ($1 \text{ poundal} = \frac{1 \text{ lb. ft.}}{\text{sec.}^2}$).

Example.

A body of mass 200 gms. has an acceleration of 2 cm./sec.². Find the force acting in dynes.

$$F = ma = 200 \text{ gms.} \times 2 \text{ cm./sec.}^2 \\ = 400 \text{ gm. cm./sec.}^2 = 400 \text{ dynes.}$$

The absolute units of force, although they are the simplest, are not the only units of force used. Long before they were thought of, the idea of force was in common vogue, particularly in connection with the weight of a body. "The weight of a pound mass" for instance, was a familiar criterion of force, whose value conveyed something to anyone who heard the expression, and it is still this weight which is commonly referred to as the "pound," although the "pound-weight" is the accurate expression for it.

The "pound-weight" is the force with which the earth attracts a mass of a pound. It therefore varies from place to place on the earth's surface. For standardising purposes, it is taken as the weight of a pound mass at Greenwich.

We can find out how the pound-weight compares with the poundal by measuring the acceleration of a pound mass when it is dropped from a height at Greenwich. If air resistance is allowed for, the only force acting on the pound mass is its weight. The acceleration is found to be 32.16 ft./sec.².

If we consider the equation $F = ma$, which gives the force in poundals when the acceleration is in ft./sec.² and the mass in lbs., it is obvious that the force required to give an acceleration of 32.16 ft./sec.² to a mass of 1 lb. is 32.16 poundals ($F = 1 \text{ lb.} \times 32.16 \text{ ft./sec.}^2$).

Hence 1 pound-weight (at Greenwich) = 32.16 poundals, or the poundal is a force of approximately the same value as the weight of half an ounce.

The pound-weight is called a "gravitational" unit of force because of its derivation.

The gravitational unit of force on the C.G.S. system is the "gram-weight" which as a standard of force is taken as the weight of a gram at Paris. It is determined in the same way as the pound-weight and is found to equal 981 dynes, i.e., a dyne is nearly the same as the weight of a milligram.

Example.

A truck weighing 200 Lbs. is moved from rest with an acceleration of 2 ft./sec.² until it acquires a velocity of 10 ft./sec. against a constant frictional resistance of 40 Lbs. Find the force required.

(Note.—To distinguish pound-weight from pound-mass, Lb. (capital "L") is used for the force and lb. (small "l") for the mass.)

(a) While the body is being accelerated, the force in poundals is found from $F = ma$.

The truck weighs 200 Lbs. and so its mass is 200 lbs.,

$$\therefore F = 200 \text{ lbs.} \times 2 \text{ ft./sec.}^2 = 400 \text{ poundals.}$$

If we take 1 Lb. = 32 poundals as an approximation,

$$F = \frac{400}{32} \text{ Lbs.} = 12\frac{1}{2} \text{ Lbs.}$$

The force required to overcome friction is 40 Lbs., and so the total force applied to the truck is $40 + 12\frac{1}{2} = 52\frac{1}{2}$ Lbs.

(b) As soon as a velocity of 10 ft./sec. is acquired, the only force necessary is that overcoming friction, i.e., 40 Lbs.

This illustrates the effect of the inertia or mass of the truck while its velocity is altering, and should be compared with the effect of inductance (or "electrical inertia") when the current in a circuit is altering.

Momentum.

Momentum is the name given to the quantity obtained by multiplying the mass of a body by its velocity, e.g., the momentum of the truck in the above example, when moving at 10 ft./sec., is $200 \text{ lb.} \times 10 \text{ ft./sec.} = 2,000 \text{ lb. ft./sec.}$

The "way" on a boat is its momentum. It gives a measure of how long any force will have to operate before the boat is brought to rest.

We are often not so interested in how long a force will have to act to produce a required state of motion as we are in how far or over what distance the force is required to operate. This gives rise to the mechanical ideas of work and energy.

Work.

In the mechanical sense, work is only done provided motion takes place. The point at which the force is applied must alter its position before mechanical work is done. Ordinarily we might consider from our muscular exertions that we had been working rather hard while tugging for instance at a large body but unless we succeeded in moving it, no work could be credited to us in this technical sense. Another instance is that of a table supported by its legs. The legs do no mechanical work and in fact, prevent it being done. If the legs were removed the table top would crash to the ground and work would be done by the mutual attraction between the table top and the earth.

Mechanical work is thus done whenever a force applied to a body moves it through a certain distance, and the measure of the work done is taken as the product of the force and the distance through which it has moved its point of application. Care must be taken in applying this rule. Consider a sailing boat driven along by the wind. The force acting on the sails is not entirely engaged in sending the boat along, unless the wind is directly astern. In any other direction of the boat's head, part of the wind force is occupied in producing leeway. Similarly in the case of a barge being towed along a canal by a horse on the towpath, only part of the pull of the rope moves the barge in the line of the canal. The effect of the total pull would in time bring the barge to the bank if the leeway were not neutralised. These examples indicate what is the effective force when calculating the useful work done on the boat or barge. It is the component (according to the parallelogram law for vectors) of the total force which acts in the line of motion of the body.

In Fig. 1 the component of the rope's pull F , in the barge's direction of motion, is $F \cos \theta$ and the work done on the barge as it moves through a distance " d " in this direction is $F \cos \theta \times d$.

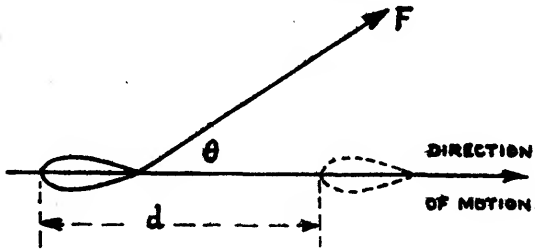


FIG. 1.

The unit of work is the work done when a unit of force acting on a body moves its point of application through unit length. When the number of different units of these two quantities is recalled, it will be seen that a large number of units of work might be derived. The two common ones are:—

- (1) the **ft.Lb.**, which is the work done when a force of 1 pound-weight acts through a distance of 1 foot;
- (2) the **erg**, which is the work done when a force of 1 dyne acts through a distance of 1 cm.

The **erg** is the C.G.S. absolute unit of work. It is inconveniently small in practice, and, particularly in electrical work, a multiple of the **erg** called the **joule** is normally used.

$$1 \text{ joule} = 10^7 \text{ (ten million) ergs.}$$

Energy.

The ordinary meaning of energy conveys in a general way its exact scientific meaning. It is defined as "capacity for doing work" and the amount of energy a body possesses is taken as being exactly the same as the amount of work it can do. The measurement of this work has just been discussed above and so the energy of a body can be measured. If we return to the truck example, a force of $12\frac{1}{2}$ Lbs. was exerted over and above the forces overcoming friction while the truck was being accelerated. During this time the truck was, of course, moving a certain distance over the rails and it could be shown that the actual distance covered was 25 feet. The work done on the truck in altering its motion was therefore $25 \times 12\frac{1}{2}$ ft. Lbs. = $312\frac{1}{2}$ ft. Lbs., and this work is exhibited as energy of motion of the truck, *i.e.*, the truck by virtue of its mass of 200 lbs. in motion with a velocity of 10 ft./sec. is capable of doing $312\frac{1}{2}$ ft. Lbs. of work before it comes to rest again. Energy which a body possesses due to its motion is called "Kinetic Energy."

A body may also possess energy due to its position with respect to the earth's surface. A wound-up clock weight drives the mechanism of the clock in its slow descent to its lowest position. The water in the weir drives round the mill-wheel as it finds its way to a lower level. Energy of this nature is called potential energy, and is due to gravitation in the above examples. It may also be due to elastic deformation of a body. The archer bends the bow and the arrow flies to its target as the bow returns to its unstrained position. The potential energy of the deformed bow, which was derived from work done by the archer in the first place, is converted to kinetic energy of the arrow.

The importance of energy as a scientific idea is due to its ready conversion from one form to another, thus giving a connecting link between various

branches of scientific study. The examples in the last paragraph illustrate conversion of gravitational or elastic potential energy into kinetic energy, *i.e.*, into another form of mechanical energy, but it may also be utilised in other ways. In one famous experiment, the energy from a falling weight was used to turn a paddle-wheel in a quantity of water with the result that the water became heated. Heat is a form of energy. The temperature of a body is a measure of the kinetic energy of its molecules. The experimental results showed that the energy lost by the falling weight was exactly equivalent to the energy gained as heat by the water, as measured by its rise in temperature. In other words, although the energy had been transformed from one kind of energy to another, its total amount was unaltered by the transformation.

It has since been shown that energy may exist in many other forms. A piece of coal possesses energy. When it burns, heat energy is developed. Energy of this kind is called chemical energy. Other forms are light, sound, and electrical energy.

All the processes of nature are accompanied by a transformation of energy from one form to another, and it is always found that such transformations leave the energy unchanged in amount. This result, which is one of the most general conclusions in science, is called the "Principle of Conservation of Energy." It asserts that :—

Energy cannot be created or destroyed. The total amount of energy in the Universe is constant. It is merely converted from one form into another.

When we speak of energy being "produced" we mean to imply that we have obtained it in one form from another form; and when we say that energy is "wasted" or "lost" we do not imply that it is destroyed, but that it has escaped from us because it was impracticable or inconvenient for us to utilise it.

The source of most of the energy that we "produce" and utilise is coal. Coal is a store-house of energy derived from the heat energy of the sun in ages past.

We liberate this energy from coal in furnaces and utilise a fraction of it to heat water in boilers, *i.e.*, transfer the heat energy to the water and produce steam.

By the medium of steam at high pressure we create motion in engines. We have converted the heat energy (or some of it) into mechanical energy.

An engine drives a dynamo, and electrical energy results. Mechanical energy has been converted into electrical energy.

The latter is utilised to burn electric lamps resulting in light and heat again; to run electric motors (mechanical energy); to radiate electromagnetic waves through the æther, ultimately producing sound; and so on.

At every stage we "lose" energy in overcoming friction in various forms. But it is not actually lost; it is converted mostly into heat which is given to the earth or atmosphere, and there it serves to assist the multitudinous processes of nature.

Power.

Power is defined as the rate of doing work or the rate at which energy is being transformed. On the C.G.S. system the unit is the erg per second. The practical unit is the joule per second, which has received the name of the Watt.

$$1 \text{ Watt} = 1 \text{ Joule per sec.} = 10^7 \text{ Ergs per sec.}$$

Another unit, which is used in British engineering, is the horse-power, so called because it was derived by Watt from experiments on the rate of working of the dray-horses of Barclay and Perkins Brewery.

$$1 \text{ Horse power (H.P.)} = 550 \text{ ft. Lbs. per sec.}$$

From this it can be easily shown that

$$1 \text{ H.P.} = 746 \text{ watts approximately.}$$

Static.

It has been seen that when a body is acted on by a system of forces, it will normally be accelerated. If this does not happen, the system of forces must balance, *i.e.*, there is no nett or "**resultant**" force acting on the body. The usual practical case of this is when the body is at rest and remains so. It is then said to be in "**equilibrium**" under the action of the forces.

If there are only two forces acting on a body which is in equilibrium, then it is obvious that the forces must be equal in magnitude and exactly opposite in direction. This gives the criterion for equality of forces which was made use of above in deriving the definition of equal masses. For instance, two spring balances might be hooked together. If the balances are then pulled apart, their readings will indicate equal forces.

Composition of Forces.

Forces are vectors—their direction as well as their magnitude must be specified before they can be completely described. Two forces acting in different directions through the same point are therefore combined according to the parallelogram law.

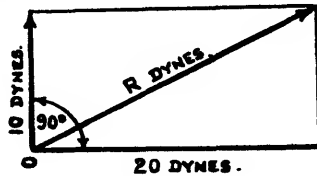


FIG. 2.

Two forces of 10 dynes and 20 dynes (represented in Fig. 2 by vectors) act at an angle of 90° at a point O. Their resultant (*i.e.*, a force which will produce the same effect as the two together) is found by completing the parallelogram or rectangle about them, and drawing the diagonal through O. Then, the diagonal (R) will represent their resultant in magnitude and direction.

In this case, $R^2 = 10^2 + 20^2 = 500$, and $R = \sqrt{500} = 22.36$ dynes.

Resolution of Forces.

Conversely, if it is desired to find the components of a force in any two particular directions, these directions may be taken for the sides of a parallelogram of which the force is the diagonal. Usually the parallelogram is taken as a rectangle. This gives the components of the force in any required direction, and at right angles to this chosen direction.

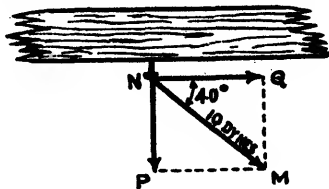


FIG. 3.

A force of 10 dynes (NM) pulls on a nail in a beam at an angle of 40° to the horizontal. It tends partly to pull the nail out and partly to bend it to the side. It is required to find the vertical and the horizontal force.

Complete a rectangle about MN.

Then NQ and NP represent the two required forces.

They are the two "components" of the original force (NM), and we have "resolved" the force NM into two components.

$$NQ = NM \times \cos 40^\circ = 10 \times .766 = 7.66 \text{ dynes.}$$

$$NP = NM \times \sin 40^\circ = 10 \times .643 = 6.43 \text{ dynes.}$$

An application of this principle occurs in considering the calculation of power in an A.C. circuit.

Moments.

The principle on which a lever operates is well known. A force applied at a considerable distance from the fulcrum (point of support), can be used to exert a much greater force at a point nearer the fulcrum. By this means a man may raise a heavier weight than he could ever hope to lift directly. The same principle is exemplified in jacking up the wheel of a car.

These examples illustrate the principle that the effect of a force when employed to produce a turning movement about a fixed axis or fulcrum depends on its distance from the axis.

The product of a force and its perpendicular distance from an axis is called the **moment** of the force about the axis and is the measure of the force's ability to produce turning movement. It is also called a **torque**. This name brings out its twisting or turning effect, as it is derived from the Latin word meaning "to turn."

Another illustration of a torque or moment is obtained in the electric motor. The interaction of the magnetic field of the poles with that due to the armature current produces mechanical forces on the armature conductors. These forces are applied at a distance of the armature radius from the axis and so tend to rotate the armature.

The motion which results in a case of this kind when there is no motion of the moving body as a whole, but merely a rotation about a fixed axis is called **angular motion**.

Angular Motion.

The useful ideas for describing angular motion are parallel to those already developed for linear motion. The moving body turns through an angle instead of covering a distance. The corresponding thing to linear velocity is thus the rate in time at which the angle is turned through. This is called **angular velocity**. Angles are always measured in radians and so angular velocity is measured in radians per second, *e.g.*, if a point on a rotating body turns through six radians in three seconds, its angular velocity is two radians per second.

Relation between angular and linear velocity of a point on a rotating body.

In Fig. 4, the axis of the rotating body is through O and perpendicular to the plane of the paper. As it rotates about this axis, every point on the body describes a circle which has a point on the axis as its centre. The circle described by a typical point A in the plane through O perpendicular to the axis is shown in the figure. The body is rotating with uniform velocity. Two velocities can thus be ascribed to A :—

- (1) a **linear velocity** of constant magnitude, say V ft./sec., the rate at which A travels round the circle ;
- (2) a constant **angular velocity**, say ω (omega) radians/sec., the rate at which the line OA is turning about O.

The distance of A from O, *i.e.*, the radius of the circle is taken to be R feet. Suppose that in t seconds A moves from A to B, going through an arc AB, and turning through an angle $AOB = \theta$:—

- (1) A goes V ft. in one second and so goes Vt ft. in t seconds.

$$\therefore AB = Vt.$$

- (2) A turns through ω radians in one second and so turns through ωt radians in t seconds.

$$\therefore \theta = \omega t.$$

But from the definition of a radian :—

$$AB = R \times \theta.$$

$$\therefore Vt = R \times \omega t.$$

$$\text{or } V = R \omega.$$

This may also be written in the forms $\omega = \frac{V}{R}$ and $R = \frac{V}{\omega}$.

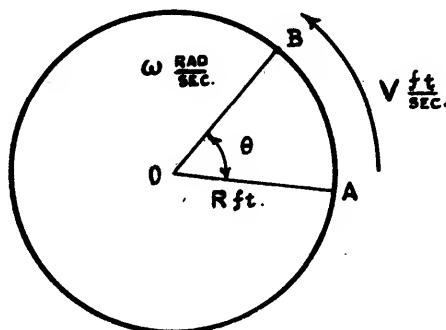


FIG. 4.

Example.

A spoked flywheel has a diameter of 5 ft. and a point on the rim has a linear velocity of $10 \frac{\text{ft.}}{\text{sec.}}$ Find :—

- (i). The angular velocity in $\frac{\text{rad.}}{\text{sec.}}$.
- (ii). The angle swept out by a spoke in $\frac{1}{4}$ sec.
- (iii). The angular velocity in r.p.m.

$$(i) \quad \omega = \frac{V}{R} = \frac{10\text{-ft.}}{2.5 \text{ ft./sec.}} = 4 \frac{\text{rad.}}{\text{sec.}}$$

$$(ii) \quad \theta = \omega t = \frac{4 \text{ rad.} \times 1 \text{ sec.}}{\text{sec.} \times 4} = 1 \text{ radian.}$$

$$(iii) \quad 4 \frac{\text{rad.}}{\text{sec.}} = \frac{240 \text{ rad.}}{\text{min.}} = \frac{240 \text{ rev.}}{2\pi \text{ min.}} = 38.19 \text{ r.p.m.}$$

It should be observed that, although the linear velocity of A above is constant in magnitude, it is continually altering in direction and so A actually has a linear acceleration. Its angular velocity is, however, constant and it has no angular acceleration about O.

Angular Acceleration.

The angular acceleration of a body about an axis is the rate at which its angular velocity about the axis is altering, *e.g.*, if the body's angular velocity is 5 radians per sec. at a certain instant and 7 radians per sec., a second later, its angular acceleration during that second is 2 radians per sec. per sec.

It has been seen that if a body has a linear acceleration there is a force acting on it. If a body has an angular acceleration it is also under the action of external forces. The appropriate property of the force is that which measures its turning ability, *i.e.*, its moment or torque. A torque acting on a body capable of rotation produces angular acceleration of the body.

A good example of this is the electric motor. The torque on the armature already referred to increases the speed of rotation until the back E.M.F. is so large that the resultant torque on the armature is zero. The motor then runs at constant angular velocity.

APPENDIX D

TABLE I.

The following conversion table is based on the formulæ :—

$$\lambda = \frac{3 \times 10^8}{f}, \text{ and } LC = \left(\frac{2 \times 10^4}{2\pi f} \right)^2, \text{ where}$$

f = frequency in kilocycles per second.

λ = wave-length in metres.

LC = oscillation constant, in microhenries and jars.

To obtain the LC value in microhenries and microfarads, the values given in the LC column should be divided by 900.

It should be noted that wave-length and frequency are reciprocal; *i.e.*, 50 metres correspond to 6,000 kcs., and 50 kcs. to 6,000 metres.

Frequency (f) in kilocycles per second.	Wave- length (λ) in metres.	LC value (mic.-jars).	Frequency (f) in kilocycles per second.	Wave- length (λ) in metres.	LC value (mic.-jars).
300,000	1	0.0002533	9,677	31	0.2434
150,000	2	0.001013	9,375	32	0.2594
100,000	3	0.002280	9,091	33	0.2758
75,000	4	0.004053	8,823	34	0.2928
60,000	5	0.006333	8,571	35	0.3103
50,000	6	0.009119	8,333	36	0.3283
42,855	7	0.01241	8,108	37	0.3468
37,500	8	0.01621	7,895	38	0.3658
33,333	9	0.02052	7,692	39	0.3853
30,000	10	0.02533	7,500	40	0.4053
27,273	11	0.03065	7,317	41	0.4258
25,000	12	0.03647	7,143	42	0.4469
23,077	13	0.04281	6,977	43	0.4684
21,429	14	0.04965	6,818	44	0.4904
20,000	15	0.05699	6,667	45	0.5129
18,750	16	0.06485	6,522	46	0.5360
17,647	17	0.07320	6,383	47	0.5595
16,666	18	0.08207	6,250	48	0.5836
15,789	19	0.09144	6,122	49	0.6082
15,000	20	0.1013	6,000	50	0.6333
14,285	21	0.1117	5,454	55	0.7662
13,636	22	0.1226	5,000	60	0.9119
13,043	23	0.1340	4,615	65	1.070
12,500	24	0.1459	4,286	70	1.241
12,000	25	0.1583	4,000	75	1.425
11,538	26	0.1712	3,750	80	1.625
11,111	27	0.1847	3,529	85	1.830
10,714	28	0.1986	3,333	90	2.052
10,345	29	0.2130	3,158	95	2.286
10,000	30	0.2280	3,000	100	2.533

TABLE I—*continued.*

Frequency (<i>f</i>) in kilocycles per second.	Wave- length (λ) in metres.	LC value (mic.-jars).	Frequency (<i>f</i>) in kilocycles per second.	Wave- length (λ) in metres.	LC value (mic.-jars).
2,857	105	2.793	508.5	590	88.17
2,727	110	3.065	500.0	600	91.19
2,609	115	3.350	461.5	650	107.0
2,500	120	3.648	428.6	700	124.1
2,308	130	4.281	400.0	750	142.5
2,143	140	4.965	375.0	800	162.1
2,000	150	5.699	352.9	850	183.0
1,875	160	6.485	333.3	900	205.2
1,765	170	7.320	315.8	950	228.6
1,667	180	8.207	300.0	1,000	253.3
1,579	190	9.144	285.7	1,050	279.3
1,500	200	10.13	272.7	1,100	306.5
1,429	210	11.17	260.9	1,150	335.0
1,364	220	12.26	250.0	1,200	364.8
1,304	230	13.40	241.9	1,240	389.5
1,250	240	14.59	240.0	1,250	395.8
1,200	250	15.83	232.6	1,290	421.5
1,154	260	17.12	230.8	1,300	428.1
1,111	270	18.47	222.2	1,350	461.6
1,071	280	19.86	214.3	1,400	496.5
1,034	290	21.30	212.8	1,410	503.6
1,000	300	22.80	206.9	1,450	532.6
967.7	310	24.34	202.7	1,480	554.8
937.5	320	25.94	200.0	1,500	569.9
909.1	330	27.58	193.5	1,550	608.6
882.3	340	29.28	187.5	1,600	648.5
857.1	350	31.03	181.8	1,650	689.6
833.3	360	32.83	176.5	1,700	732.0
810.8	370	34.68	172.4	1,740	766.9
789.5	380	36.58	171.4	1,750	775.7
769.2	390	38.53	166.7	1,800	820.7
750.0	400	40.53	162.2	1,850	866.9
731.7	410	42.58	157.9	1,900	914.4
714.3	420	44.68	153.8	1,950	963.2
697.7	430	46.84	151.5	1,980	993.0
681.8	440	49.04	150.0	2,000	1,013
666.7	450	51.29	142.9	2,100	1,117
652.2	460	53.60	136.4	2,200	1,226
638.3	470	55.95	130.4	2,300	1,340
625.0	480	58.36	125.0	2,400	1,459
612.2	490	60.82	120.0	2,500	1,583
600.0	500	63.33	115.4	2,600	1,712
588.2	510	65.88	111.1	2,700	1,847
576.9	520	68.49	107.1	2,800	1,986
566.0	530	71.15	103.4	2,900	2,130
555.6	540	73.86	100.0	3,000	2,280
545.4	550	76.62	96.77	3,100	2,434
535.7	560	79.44	93.75	3,200	2,594
526.8	570	82.30	90.91	3,300	2,758
517.1	580	85.21	88.24	3,400	2,928

TABLE I—*continued.*

Frequency (<i>f</i>) in kilocycles per second.	Wave- length (λ) in metres.	LC value (mic.-jars).	Frequency (<i>f</i>) in kilocycles per second	Wave- length (λ) in metres.	LC value (mic.-jars).
85.71	3,500	3,103	34.29	8,750	19,393
80.00	3,750	3,562	33.33	9,000	20,517
78.95	3,800	3,658	31.58	9,500	22,861
76.92	3,900	3,853	30.00	10,000	25,330
75.00	4,000	4,053	27.27	11,000	30,650
71.34	4,200	4,468	25.00	12,000	36,476
70.59	4,250	4,575	24.19	12,400	38,948
66.67	4,500	5,129	23.08	13,000	42,808
63.83	4,700	5,595	21.43	14,000	49,647
63.16	4,750	5,715	20.00	15,000	56,993
60.00	5,000	6,333	18.75	16,000	64,846
54.54	5,500	7,662	17.65	17,000	73,204
50.00	6,000	9,119	16.67	18,000	82,070
46.15	6,500	10,702	15.79	19,000	91,442
42.86	7,000	12,412	15.00	20,000	101,321
40.00	7,500	14,248	12.00	25,000	158,314
37.50	8,000	16,211	10.00	30,000	227,973
35.29	8,500	18,301			

TABLE II.

SPECIFIC INDUCTIVE CAPACITIES.

Air (at 0° C. and 1 atmosphere), measured at zero frequency 1.00586	Sulphur 3.6-4.3
Glass, flint 7-10	Silica, fused 3.5-3.6
Glass, plate 5-7	Quartz 4.5
Ebonite 2.7-2.9	Vulcanite 2.5
Mica 5.7-7	Porcelain 4.4-6.8
India-rubber 2.12-2.34	Distilled water 81
Shellac 3.3-7	Paraffin wax 2.0-2.3
		Service insulating oil 2.217
		Petroleum and turpentine	2.2-2.3

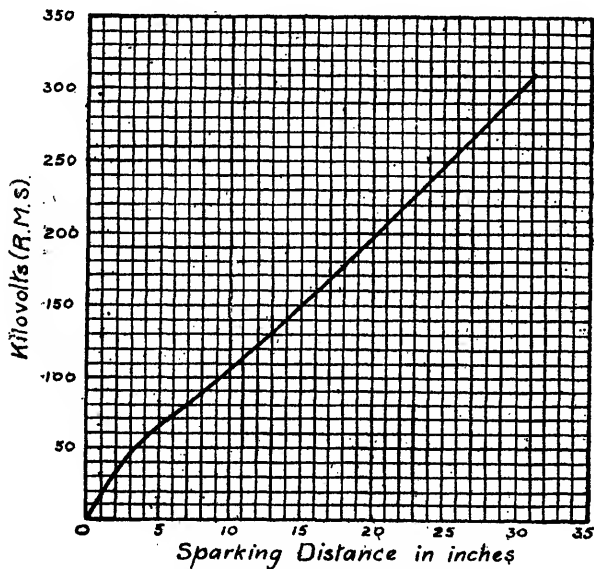
The S.I.C. of water at 8,000 kc/s. is 3.32; and at 25,000 kc/s. is 2.79..

TABLE III.

DIELECTRIC STRENGTHS.

Material	Volts per mm.	Material.	Volts per mm.
Celluloid 14,000	Paraffin 11,500
Ebonite 30,000	Porcelain 9,000-16,000
Empire cloth 10,000	Presspahn 4,000-10,000
Fuller board 16,000	Resin 11,000
Glass, ordinary 8,000	Wax 11,500
Micanite plate 40,000		

Curve of Sparking Voltages between sharp points in Air.



Note 1.—The dielectric strength per unit length falls as the thickness increases. 9,000 v/mm. is that for porcelain 15 mm. thick and 16,000 for 0.5 mm. thickness.

Note 2.—The effect of inserting stronger insulation of higher S.I.C. may result in a breakdown of the combination, *e.g.*, if the insulation of a 2 cm. gap between two plates is increased by the insertion of a 0.2 cm. sheet of insulation of S.I.C. ; 4, and much higher dielectric strength, say, 10kV/mm., and the applied voltage is 60 kV., the combination would break down while the air alone would stand.

TABLE IV.
Conversion Tables.

To reduce—		Multiply by	
Length ..	Kilometres to miles	0.62	
	Kilometres to nautical miles	0.54	
	Metres to yards	1.1 (1.0936)	
	Metres to feet	3.28	
	Centimetres to inches	0.394	
	Millimetres to mils	40 (39.4)	
	Miles to kilometres	1.61	
	Nautical miles (6,080 feet) to metres ..	1.853	
	Yards to metres	0.914	
	Feet to metres	0.305	
	Inches to centimetres	2.54	
	Miles to millimetres	1.609 × 10 ⁶	
	Square metres to square yards	1.2	
	Square metres to square feet	11 (10.76)	
Area ..	Square centimetres to square inches ..	0.155	
	Square millimetres to square inches ..	0.00155	
	Square yards to square metres	0.84	
	Square feet to square metres	0.093	
	Square inches to square centimetres ..	6.45	
	Square inches to square millimetres ..	645	
Volume ..	Cubic metres to cubic yards	1.31	
	Cubic metres to cubic feet	35.31	
	Cubic cms. to cubic inches	0.061	
	Cubic yards to cubic metres	0.76	
	Cubic feet to cubic metres	0.0283	
	Cubic inches to cubic cms.	16.4	
Capacity ..	Litres to cubic feet	0.035	
	Litres to gallons	0.22	
	Cubic metres to gallons	220	
	Gallons to cubic feet	0.16	
	Gallons to cubic inches	277	
	Gallons to cubic centimetres	4,541	
	Gallons to litres	4.54	
	Cubic feet to litres	28.3	
Mass ..	Cubic inches to cubic cm.	16.4	
	Kilograms to pounds	2.2	
	Kilograms to ounces	35 (35.3)	
	Grams to ounces	0.035	
	Grams to grains	15.4	
	Milligrams to grains	0.0154	
	Cwt. to kilograms	50 (50.8)	
	Pounds to kilograms	0.45	
	Pounds to grams	454 (453.6)	
	Ounces to grams	28.35	
	Grains to grams	0.065	
	Grains to milligrams	65 (64.8)	
	Pounds avoird. to grains	7,000	

TABLE IV—*continued.***Conversion Tables—*continued.***

To reduce—		Multiply by
Water ..	Litres of water to lbs.	2.2
	Pounds of water to litres	0.454
	Gallons of water to lbs.	10
	Pounds of water to gallons	0.1
	Cubic feet of water to lbs.	62.41 at 0°C
	Cubic feet of salt water to lbs.	64.3
	Pounds of water to cubic feet	0.016
	Cubic feet of water to ounces	997
	Cubic inches of water to ounces	0.58
	Feet per minute to miles per hour	0.0113
Velocity ..	Feet per minute to centimetres per second	0.508
	Miles per hour to feet per minute	88
	Kilometres per hour to centimetres per second	27.8
	Metres per second to feet per minute	197
	Miles per hour to centimetres per second	44.7
	Knots to cm. per second	51.5
Work and Energy	Joules to ergs	10 ⁷
	Joules to foot-Lb.	0.737
	Foot-Lb. to joules	1.356
	Calories (gm.) to joules	4.186
	Calories to foot-Lb.	3.08
Power ..	Kilowatts to joules per second	1,000
	Kilowatts to foot-Lb. per second	737.6
	Kilowatts to horse-power	1.34
	H.P. to watts	746
	Foot-Lb. per second to watts	1.356

TABLE V.
TRIGONOMETRICAL RATIOS, &c.

Angle.	Radians.	Sine.	Tangent.	Co-tangent.	Cosine.		
0°	0	0	0	∞	1	1.5708	90°
1	.0175	.0175	.0175	57.2900	.9998	1.5533	89
2	.0349	.0349	.0349	28.6363	.9994	1.5359	88
3	.0524	.0523	.0524	19.0811	.9986	1.5184	87
4	.0698	.0698	.0699	14.3007	.9976	1.5010	86
5	.0873	.0872	.0875	11.4301	.9962	1.4835	85
6	.1047	.1045	.1051	9.5144	.9945	1.4661	84
7	.1222	.1219	.1228	8.1443	.9925	1.4486	83
8	.1396	.1392	.1405	7.1154	.9903	1.4312	82
9	.1571	.1564	.1584	6.3138	.9877	1.4137	81
10	.1745	.1736	.1763	5.6713	.9848	1.3963	80
11	.1920	.1908	.1944	5.1446	.9816	1.3788	79
12	.2094	.2079	.2126	4.7046	.9781	1.3614	78
13	.2269	.2250	.2309	4.3315	.9744	1.3439	77
14	.2443	.2419	.2493	4.0108	.9703	1.3265	76
15	.2618	.2588	.2679	3.7321	.9659	1.3090	75
16	.2793	.2756	.2867	3.4874	.9613	1.2915	74
17	.2967	.2924	.3057	3.2709	.9563	1.2741	73
18	.3142	.3090	.3249	3.0777	.9511	1.2566	72
19	.3316	.3256	.3443	2.9042	.9455	1.2392	71
20	.3491	.3420	.3640	2.7475	.9397	1.2217	70
21	.3665	.3584	.3839	2.6051	.9336	1.2043	69
22	.3840	.3746	.4040	2.4751	.9272	1.1868	68
23	.4014	.3907	.4245	2.3559	.9205	1.1694	67
24	.4189	.4067	.4452	2.2460	.9135	1.1519	66
25	.4363	.4226	.4663	2.1445	.9063	1.1345	65
26	.4538	.4384	.4877	2.0503	.8988	1.1170	64
27	.4712	.4540	.5095	1.9626	.8910	1.0996	63
28	.4887	.4695	.5317	1.8807	.8829	1.0821	62
29	.5061	.4848	.5543	1.8040	.8746	1.0647	61
30	.5236	.5000	.5774	1.7321	.8660	1.0472	60
31	.5411	.5150	.6009	1.6643	.8572	1.0297	59
32	.5585	.5299	.6249	1.6003	.8480	1.0123	58
33	.5760	.5446	.6494	1.5399	.8387	.9948	57
34	.5934	.5592	.6745	1.4826	.8290	.9774	56
35	.6109	.5736	.7002	1.4281	.8192	.9599	55
36	.6283	.5878	.7265	1.3764	.8090	.9425	54
37	.6458	.6018	.7536	1.3270	.7986	.9250	53
38	.6632	.6157	.7813	1.2799	.7880	.9076	52
39	.6807	.6293	.8098	1.2349	.7771	.8901	51
40	.6981	.6428	.8391	1.1918	.7660	.8727	50
41	.7156	.6561	.8693	1.1504	.7547	.8552	49
42	.7330	.6691	.9004	1.1106	.7431	.8378	48
43	.7505	.6820	.9325	1.0724	.7314	.8203	47
44	.7679	.6947	.9657	1.0355	.7193	.8029	46
45	.7854	.7071	1.0000	1.0000	.7071	.7854	45
		Cosine.	Co-tangent.	Tangent.	Sine.	Radians.	Angle.

TABLE VI.
MATHEMATICAL TABLES.
LOGARITHMS.

	0	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9
10	0000	0043	0086	0128	0170	0212	0253	0294	0334	0374	4	9	13	17	21	26	30	34	38
11	0414	0453	0492	0531	0569	0607	0645	0682	0719	0755	4	8	12	15	19	23	27	31	35
12	0792	0828	0864	0899	0934	0969	1004	1038	1072	1106	3	7	11	14	18	21	25	28	32
13	1139	1173	1206	1239	1271	1303	1335	1367	1399	1430	3	7	10	13	16	20	23	26	30
14	1461	1492	1523	1553	1584	1614	1644	1673	1703	1732	3	6	9	12	15	18	21	24	28
15	1761	1790	1818	1847	1875	1903	1931	1959	1987	2014	3	6	9	11	14	17	20	23	26
16	2041	2068	2095	2122	2148	2175	2201	2227	2253	2279	3	5	8	11	14	16	19	22	25
17	2304	2330	2355	2380	2405	2430	2455	2480	2504	2529	3	5	8	10	13	15	18	20	23
18	2553	2577	2601	2625	2648	2672	2695	2718	2742	2765	2	5	7	9	12	14	16	19	21
19	2788	2810	2833	2856	2878	2900	2923	2945	2967	2989	2	4	7	9	11	13	16	18	20
20	3010	3032	3054	3075	3096	3118	3139	3160	3181	3201	2	4	6	8	11	13	15	17	19
21	3222	3243	3263	3284	3304	3324	3345	3365	3385	3404	2	4	6	8	10	12	14	16	18
22	3424	3444	3464	3483	3502	3522	3541	3560	3579	3598	2	4	6	8	10	12	14	15	17
23	3617	3636	3655	3674	3692	3711	3729	3747	3766	3784	2	4	6	7	9	11	13	15	17
24	3802	3820	3838	3856	3874	3892	3909	3927	3945	3962	2	4	5	7	9	11	12	14	16
25	3979	3997	4014	4031	4048	4065	4082	4099	4116	4133	2	3	5	7	9	10	12	14	15
26	4150	4166	4183	4200	4216	4232	4249	4265	4281	4298	2	3	5	7	8	10	11	13	15
27	4314	4330	4346	4362	4378	4393	4409	4425	4440	4456	2	3	5	6	8	9	11	13	14
28	4472	4487	4502	4518	4533	4548	4564	4579	4594	4609	2	3	5	6	8	9	11	12	14
29	4624	4639	4654	4669	4683	4698	4713	4728	4742	4757	1	3	4	6	7	8	10	12	13
30	4771	4786	4800	4814	4829	4843	4857	4871	4886	4900	1	3	4	6	7	9	10	11	13
31	4914	4928	4942	4955	4969	4983	4997	5011	5024	5038	1	3	4	6	7	8	10	11	12
32	5051	5065	5079	5092	5105	5119	5132	5145	5159	5172	1	3	4	5	7	8	9	11	12
33	5185	5198	5211	5224	5237	5250	5263	5276	5289	5302	1	3	4	5	6	8	9	10	12
34	5315	5328	5340	5353	5366	5378	5391	5403	5416	5428	1	3	4	5	6	8	9	10	11
35	5441	5453	5465	5478	5490	5502	5514	5527	5539	5551	1	2	4	5	6	7	9	10	11
36	5563	5575	5587	5599	5611	5623	5635	5647	5658	5670	1	2	4	5	6	7	8	10	11
37	5682	5694	5705	5717	5729	5740	5752	5763	5775	5786	1	2	3	5	6	7	8	9	10
38	5798	5809	5821	5832	5843	5855	5866	5877	5888	5899	1	2	3	5	6	7	8	9	10
39	5911	5922	5933	5944	5955	5966	5977	5988	5999	6010	1	2	3	4	5	7	8	9	10
40	6021	6031	6042	6053	6064	6075	6085	6096	6107	6117	1	2	3	4	5	6	8	9	10
41	6128	6138	6149	6160	6170	6180	6191	6201	6212	6222	1	2	3	4	5	6	7	8	9
42	6232	6243	6253	6263	6274	6284	6294	6304	6314	6325	1	2	3	4	5	6	7	8	9
43	6335	6345	6355	6365	6375	6385	6395	6405	6415	6425	1	2	3	4	5	6	7	8	9
44	6435	6444	6454	6464	6474	6484	6493	6503	6513	6522	1	2	3	4	5	6	7	8	9
45	6532	6542	6551	6561	6571	6580	6590	6599	6609	6618	1	2	3	4	5	6	7	8	9
46	6628	6637	6646	6656	6665	6675	6684	6693	6702	6712	1	2	3	4	5	6	7	8	9
47	6721	6730	6739	6749	6758	6767	6776	6785	6794	6803	1	2	3	4	5	5	6	7	8
48	6812	6821	6830	6839	6848	6857	6866	6875	6884	6893	1	2	3	4	4	5	6	7	8
49	6902	6911	6920	6928	6937	6946	6955	6964	6972	6981	1	2	3	4	4	5	6	7	8
50	6990	6998	7007	7016	7024	7033	7042	7050	7059	7067	1	2	3	3	4	5	6	7	8

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MATHEMATICAL TABLES.

LOGARITHMS.

	0	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9
81	7076	7084	7093	7101	7110	7118	7128	7135	7143	7152	1	2	3	3	4	5	8	7	8
82	7160	7168	7177	7185	7193	7202	7210	7218	7226	7235	1	2	2	3	4	5	8	7	7
83	7243	7251	7259	7267	7275	7284	7292	7300	7308	7316	1	2	2	3	4	5	6	6	7
84	7324	7332	7340	7348	7358	7364	7372	7380	7388	7396	1	2	2	3	4	5	6	6	7
85	7404	7412	7419	7427	7435	7443	7451	7459	7466	7474	1	2	2	3	4	5	5	8	7
86	7482	7490	7497	7505	7513	7520	7528	7536	7543	7551	1	2	2	3	4	5	5	8	7
87	7559	7566	7574	7582	7589	7597	7604	7612	7619	7627	1	2	2	3	4	5	5	8	7
88	7634	7642	7649	7657	7664	7672	7679	7686	7694	7701	1	1	2	3	4	4	5	6	7
89	7709	7716	7723	7731	7738	7745	7752	7760	7767	7774	1	1	2	3	4	4	5	6	7
90	7782	7789	7796	7803	7810	7818	7825	7832	7839	7848	1	1	2	3	4	4	5	8	6
91	7853	7860	7868	7875	7882	7889	7896	7903	7910	7917	1	1	2	3	4	4	5	6	6
92	7924	7931	7938	7945	7952	7959	7966	7973	7980	7987	1	1	2	3	3	4	5	8	8
93	7993	8000	8007	8014	8021	8028	8035	8041	8048	8055	1	1	2	3	3	4	5	5	6
94	8062	8069	8075	8082	8089	8098	8102	8109	8116	8122	1	1	2	3	3	4	5	5	6
95	8129	8136	8142	8149	8156	8162	8169	8176	8182	8189	1	1	2	3	3	4	5	5	6
96	8195	8202	8209	8215	8222	8228	8235	8241	8248	8254	1	1	2	3	3	4	5	5	6
97	8261	8267	8274	8280	8287	8293	8299	8308	8312	8319	1	1	2	3	3	4	5	5	6
98	8325	8331	8338	8344	8351	8357	8363	8370	8378	8382	1	1	2	3	3	4	4	5	6
99	8388	8395	8401	8407	8414	8420	8428	8432	8439	8445	1	1	2	2	3	4	4	5	6
70	8451	8457	8463	8470	8478	8482	8488	8494	8500	8506	1	1	2	2	3	4	4	5	6
71	8513	8519	8525	8531	8537	8543	8549	8555	8561	8567	1	1	2	2	3	4	4	5	5
72	8573	8579	8585	8591	8597	8603	8609	8615	8621	8627	1	1	2	2	3	4	4	5	5
73	8633	8639	8645	8651	8657	8663	8669	8675	8681	8686	1	1	2	2	3	4	4	5	5
74	8692	8698	8704	8710	8716	8722	8727	8733	8739	8745	1	1	2	2	3	4	4	5	5
75	8751	8756	8762	8768	8774	8779	8785	8791	8797	8802	1	1	2	2	3	3	4	5	5
76	8808	8814	8820	8825	8831	8837	8842	8848	8854	8859	1	1	2	2	3	3	4	5	5
77	8865	8871	8878	8882	8887	8893	8899	8904	8910	8915	1	1	2	2	3	3	4	4	5
78	8921	8927	8932	8938	8943	8949	8954	8960	8965	8971	1	1	2	2	3	3	4	4	5
79	8976	8982	8987	8993	8998	9004	9009	9015	9020	9025	1	1	2	2	3	3	4	4	5
80	9031	9036	9042	9047	9053	9058	9063	9069	9074	9079	1	1	2	2	3	3	4	4	5
81	9085	9090	9096	9101	9106	9112	9117	9122	9128	9133	1	1	2	2	3	3	4	4	5
82	9138	9143	9149	9154	9159	9165	9170	9175	9180	9186	1	1	2	2	3	3	4	4	5
83	9191	9196	9201	9208	9212	9217	9222	9227	9232	9238	1	1	2	2	3	3	4	4	5
84	9243	9248	9253	9258	9263	9269	9274	9279	9284	9289	1	1	2	2	3	3	4	4	5
85	9294	9299	9304	9309	9315	9320	9325	9330	9335	9340	1	1	2	2	3	3	4	4	5
86	9345	9350	9355	9360	9365	9370	9375	9380	9385	9390	1	1	2	2	3	3	4	4	5
87	9395	9400	9405	9410	9415	9420	9425	9430	9435	9440	0	1	1	2	2	3	3	4	4
88	9445	9450	9455	9460	9465	9469	9474	9479	9484	9489	0	1	1	2	2	3	3	4	4
89	9494	9499	9504	9509	9513	9518	9523	9528	9533	9538	0	1	1	2	2	3	3	4	4
90	9542	9547	9552	9557	9562	9566	9571	9576	9581	9586	0	1	1	2	2	3	3	4	4
91	9590	9595	9600	9605	9609	9614	9619	9624	9628	9633	0	1	1	2	2	3	3	4	4
92	9638	9643	9647	9652	9657	9661	9666	9671	9675	9680	0	1	1	2	2	3	3	4	4
93	9685	9689	9694	9699	9703	9708	9713	9717	9722	9727	0	1	1	2	2	3	3	4	4
94	9731	9736	9741	9745	9750	9754	9759	9763	9768	9773	0	1	1	2	2	3	3	4	4
95	9777	9782	9786	9791	9795	9800	9805	9809	9814	9818	0	1	1	2	2	3	3	4	4
96	9823	9827	9832	9838	9841	9845	9850	9854	9859	9863	0	1	1	2	2	3	3	4	4
97	9868	9872	9877	9881	9886	9890	9894	9899	9903	9908	0	1	1	2	2	3	3	4	4
98	9912	9917	9921	9926	9930	9934	9939	9943	9948	9952	0	1	1	2	2	3	3	4	4
99	9956	9961	9965	9969	9974	9978	9983	9987	9991	9996	0	1	1	2	2	3	3	3	4

MATHEMATICAL TABLES.

ANTILOGARITHMS.

	0	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9
00	1000	1002	1005	1007	1009	1012	1014	1016	1019	1021	0	0	1	1	1	1	2	2	2
01	1023	1026	1028	1030	1033	1035	1038	1040	1042	1045	0	0	1	1	1	1	2	2	2
02	1047	1050	1052	1054	1057	1059	1062	1064	1067	1069	0	0	1	1	1	1	2	2	2
03	1072	1074	1076	1079	1081	1084	1086	1089	1091	1094	0	0	1	1	1	1	2	2	2
04	1096	1099	1102	1104	1107	1109	1112	1114	1117	1119	0	1	1	1	1	1	2	2	2
05	1122	1125	1127	1130	1132	1135	1138	1140	1143	1146	0	1	1	1	1	1	2	2	2
06	1148	1151	1153	1156	1159	1161	1164	1167	1169	1172	0	1	1	1	1	1	2	2	2
07	1175	1178	1180	1183	1186	1189	1191	1194	1197	1199	0	1	1	1	1	1	2	2	2
08	1202	1205	1208	1211	1213	1216	1219	1222	1225	1227	0	1	1	1	1	1	2	2	2
09	1230	1233	1236	1239	1242	1245	1247	1250	1253	1256	0	1	1	1	1	1	2	2	2
10	1259	1262	1265	1268	1271	1274	1276	1279	1282	1285	0	1	1	1	1	1	2	2	2
11	1288	1291	1294	1297	1300	1303	1306	1309	1312	1315	0	1	1	1	1	1	2	2	2
12	1318	1321	1324	1327	1330	1333	1337	1340	1343	1346	0	1	1	1	1	1	2	2	2
13	1349	1352	1355	1358	1361	1365	1368	1371	1374	1377	0	1	1	1	1	1	2	2	2
14	1380	1384	1387	1390	1393	1396	1400	1403	1406	1409	0	1	1	1	1	1	2	2	2
15	1413	1416	1419	1422	1426	1429	1432	1435	1439	1442	0	1	1	1	1	1	2	2	2
16	1445	1449	1452	1455	1459	1462	1466	1469	1472	1476	0	1	1	1	1	1	2	2	2
17	1479	1483	1486	1489	1493	1496	1500	1503	1507	1510	0	1	1	1	1	1	2	2	2
18	1514	1517	1521	1524	1528	1531	1535	1538	1542	1545	0	1	1	1	1	1	2	2	2
19	1549	1552	1556	1560	1563	1567	1570	1574	1578	1581	0	1	1	1	1	1	2	2	2
20	1585	1589	1592	1596	1600	1603	1607	1611	1614	1618	0	1	1	1	1	1	2	2	2
21	1622	1626	1629	1633	1637	1641	1644	1648	1652	1656	0	1	1	1	1	1	2	2	2
22	1660	1663	1667	1671	1675	1679	1683	1687	1690	1694	0	1	1	1	1	1	2	2	2
23	1698	1702	1706	1710	1714	1718	1722	1726	1730	1734	0	1	1	1	1	1	2	2	2
24	1738	1742	1746	1750	1754	1758	1762	1766	1770	1774	0	1	1	1	1	1	2	2	2
25	1778	1782	1786	1791	1795	1799	1803	1807	1811	1816	0	1	1	1	1	1	2	2	2
26	1820	1824	1828	1832	1837	1841	1845	1849	1854	1858	0	1	1	1	1	1	2	2	2
27	1862	1866	1871	1875	1879	1884	1888	1892	1897	1901	0	1	1	1	1	1	2	2	2
28	1905	1910	1914	1919	1923	1928	1932	1936	1941	1945	0	1	1	1	1	1	2	2	2
29	1950	1954	1959	1963	1968	1972	1977	1982	1986	1991	0	1	1	1	1	1	2	2	2
30	1995	2000	2004	2009	2014	2018	2023	2028	2032	2037	0	1	1	1	1	1	2	2	2
31	2042	2046	2051	2056	2061	2065	2070	2075	2080	2084	0	1	1	1	1	1	2	2	2
32	2089	2094	2099	2104	2109	2113	2118	2123	2128	2133	0	1	1	1	1	1	2	2	2
33	2138	2143	2148	2153	2158	2163	2168	2173	2178	2183	0	1	1	1	1	1	2	2	2
34	2188	2193	2198	2203	2208	2213	2218	2223	2228	2234	1	1	1	1	1	1	2	2	2
35	2239	2244	2249	2254	2259	2265	2270	2275	2280	2286	1	1	1	1	1	1	2	2	2
36	2291	2296	2301	2307	2312	2317	2323	2328	2333	2339	1	1	1	1	1	1	2	2	2
37	2344	2350	2355	2360	2366	2371	2377	2382	2388	2393	1	1	1	1	1	1	2	2	2
38	2399	2404	2410	2415	2421	2427	2432	2438	2443	2449	1	1	1	1	1	1	2	2	2
39	2455	2460	2466	2472	2477	2483	2489	2495	2500	2506	1	1	1	1	1	1	2	2	2
40	2512	2518	2523	2529	2535	2541	2547	2553	2559	2564	1	1	1	1	1	1	2	2	2
41	2570	2576	2582	2588	2594	2600	2606	2612	2618	2624	1	1	1	1	1	1	2	2	2
42	2630	2636	2642	2648	2655	2661	2667	2673	2679	2685	1	1	1	1	1	1	2	2	2
43	2692	2698	2704	2710	2716	2723	2729	2735	2742	2748	1	1	1	1	1	1	2	2	2
44	2754	2761	2767	2773	2780	2786	2793	2799	2805	2812	1	1	1	1	1	1	2	2	2
45	2818	2825	2831	2838	2844	2851	2858	2864	2871	2877	1	1	1	1	1	1	2	2	2
46	2884	2891	2897	2904	2911	2917	2924	2931	2938	2944	1	1	1	1	1	1	2	2	2
47	2951	2958	2965	2972	2979	2985	2992	2999	3006	3013	1	1	1	1	1	1	2	2	2
48	3020	3027	3034	3041	3048	3055	3062	3069	3076	3083	1	1	1	1	1	1	2	2	2
49	3090	3097	3105	3112	3119	3126	3133	3141	3148	3155	1	1	1	1	1	1	2	2	2

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ANTILOGARITHMS.

	0	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9
50	3162	3170	3177	3184	3192	3199	3206	3214	3221	3228	1	1	2	3	4	4	5	6	7
51	3236	3243	3251	3258	3266	3273	3281	3289	3296	3304	1	2	2	3	4	5	5	6	7
52	3311	3319	3327	3334	3342	3350	3357	3365	3373	3381	1	2	2	3	4	5	5	6	7
53	3388	3396	3404	3412	3420	3428	3436	3443	3451	3459	1	2	2	3	4	5	5	6	7
54	3467	3475	3483	3491	3499	3508	3516	3524	3532	3540	1	2	2	3	4	5	5	6	7
55	3548	3556	3556	3573	3581	3589	3597	3606	3614	3622	1	2	2	3	4	5	5	6	7
56	3631	3639	3648	3656	3664	3673	3681	3690	3698	3707	1	2	3	3	4	5	5	6	7
57	3715	3724	3733	3741	3750	3758	3767	3776	3784	3793	1	2	3	3	4	5	5	6	7
58	3802	3811	3819	3828	3837	3846	3855	3864	3873	3882	1	2	3	4	4	5	5	6	7
59	3890	3899	3908	3917	3926	3936	3945	3954	3963	3972	1	2	3	4	5	5	5	6	7
60	3981	3990	3999	4009	4018	4027	4036	4046	4055	4064	1	2	3	4	5	6	6	7	8
61	4074	4083	4093	4102	4111	4121	4130	4140	4150	4159	1	2	3	4	5	6	7	8	8
62	4169	4178	4188	4196	4207	4217	4227	4236	4246	4256	1	2	3	4	5	6	7	8	9
63	4266	4276	4285	4295	4305	4315	4325	4335	4345	4355	1	2	3	4	5	6	7	8	9
64	4365	4375	4385	4395	4406	4416	4426	4436	4446	4457	1	2	3	4	5	6	7	8	9
65	4467	4477	4487	4498	4508	4519	4529	4539	4550	4560	1	2	3	4	5	6	7	8	9
66	4571	4581	4592	4603	4613	4624	4634	4645	4656	4667	1	2	3	4	5	6	7	8	10
67	4677	4688	4699	4710	4721	4732	4742	4753	4764	4775	1	2	3	4	5	7	8	9	10
68	4786	4797	4808	4819	4831	4842	4853	4864	4875	4887	1	2	3	4	6	7	8	9	10
69	4898	4909	4920	4932	4943	4955	4966	4977	4989	5000	1	2	3	5	6	7	8	9	10
70	5012	5023	5035	5047	5058	5070	5082	5093	5105	5117	1	2	4	5	6	7	8	9	11
71	5129	5140	5152	5164	5176	5188	5200	5212	5224	5236	1	2	4	5	6	7	8	10	11
72	5248	5260	5272	5284	5297	5309	5321	5333	5346	5358	1	2	4	5	6	7	9	10	11
73	5370	5383	5395	5408	5420	5433	5445	5458	5470	5483	1	2	4	5	6	8	9	10	11
74	5495	5508	5521	5534	5546	5559	5572	5585	5598	5610	1	3	4	5	6	8	9	10	12
75	5623	5636	5649	5662	5675	5689	5702	5715	5728	5741	1	3	4	5	7	8	9	10	12
76	5754	5768	5781	5794	5808	5821	5834	5848	5861	5875	1	3	4	5	7	8	9	11	12
77	5888	5902	5916	5929	5943	5957	5970	5984	5998	6012	1	3	4	5	7	8	10	11	12
78	6026	6039	6053	6067	6081	6095	6109	6124	6138	6152	1	3	4	6	7	8	10	11	16
79	6166	6180	6194	6209	6223	6237	6252	6266	6281	6295	1	3	4	6	7	9	10	11	13
80	6310	6324	6339	6353	6368	6383	6397	6412	6427	6442	1	3	4	6	7	9	10	12	13
81	6457	6471	6486	6501	6516	6531	6546	6561	6577	6592	2	3	5	6	8	9	11	12	14
82	6607	6622	6637	6653	6668	6683	6699	6714	6730	6745	2	3	5	6	8	9	11	12	14
83	6761	6776	6792	6808	6823	6839	6855	6871	6887	6902	2	3	5	6	8	9	11	13	14
84	6918	6934	6950	6966	6982	6998	7015	7031	7047	7063	2	3	5	6	8	10	11	13	15
85	7079	7096	7112	7129	7145	7161	7178	7194	7211	7228	2	3	5	7	8	10	12	13	15
86	7244	7261	7278	7295	7311	7328	7345	7362	7379	7396	2	3	5	7	8	10	12	14	16
87	7413	7430	7447	7464	7482	7499	7516	7534	7551	7568	2	3	5	7	9	10	12	14	16
88	7586	7603	7621	7638	7656	7674	7691	7709	7727	7745	2	4	5	7	9	11	12	14	16
89	7762	7780	7798	7816	7834	7852	7870	7889	7907	7925	2	4	5	7	9	11	13	14	16
90	7943	7962	7980	7998	8017	8035	8054	8072	8091	8110	2	4	6	7	9	11	13	15	17
91	8128	8147	8166	8185	8204	8222	8241	8260	8279	8299	2	4	6	8	9	11	13	15	17
92	8318	8337	8356	8375	8395	8414	8433	8453	8472	8492	2	4	6	8	10	12	14	15	17
93	8511	8531	8551	8570	8590	8610	8630	8650	8670	8690	2	4	6	8	10	12	14	16	18
94	8710	8730	8750	8770	8790	8810	8831	8851	8872	8892	2	4	6	8	10	12	14	16	18
95	8913	8933	8954	8974	8995	9016	9036	9057	9078	9099	2	4	6	8	10	12	15	17	19
96	9120	9141	9162	9183	9204	9226	9247	9268	9290	9311	2	4	6	8	11	13	15	17	19
97	9333	9354	9376	9397	9419	9441	9462	9484	9506	9528	2	4	7	9	11	13	15	17	20
98	9550	9572	9594	9616	9638	9661	9683	9705	9727	9750	2	4	7	9	11	13	15	17	20
99	9772	9795	9817	9840	9863	9886	9908	9931	9954	9977	2	5	7	9	11	14	16	18	20

APPENDIX E.

RESUSCITATION FROM APPARENT DEATH BY ELECTRIC SHOCK.

The urgent necessity for prompt and persistent efforts at resuscitation of victims of accidental shocks by electricity is very well emphasised by the successful results in the instances recorded. In order that the task may not be undertaken in a half-hearted manner, it must be appreciated that accidental shocks seldom result in death unless the victim is left unaided too long, or efforts at resuscitation are stopped too early.

In the majority of instances the shock is only sufficient to suspend animation temporarily, owing to the momentary and imperfect contact of the conductors, and also on account of the resistance of the body submitted to the influence of the current. It must be appreciated also that the body under the conditions of accidental shocks seldom receives the full force of the current in the circuit, but only a shunt current, which may represent a very insignificant part of the whole.

When an accident occurs, the following rules should be promptly executed with care and deliberation pending the arrival of a doctor :—

- (1) Remove the body at once from the circuit by breaking contact with the conductors. This may be accomplished by using a dry stick of wood, which is a non-conductor, to roll the body over to one side, or to brush aside a wire if that is conveying the current. When a stick is not at hand, any dry piece of clothing may be utilised to protect the hand in seizing the body of the victim, unless rubber gloves are available. If the body is in contact with the earth, the coat tails of the victim, or any loose or detached piece of clothing may be seized with impunity to draw him away from the conductor. When this has been accomplished, observe Rule 2. The object to be attained is to make the subject breathe, and if this can be accomplished and continued, he can be saved.
- (2) Lay the man on the ground, face downwards. Turn his head on one side. No time should be lost by removing or loosening clothes. Begin artificial respiration at once. Tell one of the bystanders to prepare some sort of pad like a folded coat and slip it in under the patient's body just above his waist; but do not wait for this. You will probably have performed several movements of respiration before the pad is ready and have thus gained all-valuable time. Kneel by the patient's side or across his body facing his head. Spread your hands out flat on his back at his lowest ribs. Press gradually and slowly for about three seconds by leaning forward on to your hands. Use no violence. Relax the pressure by falling back into your original upright kneeling position for two seconds. The process of artificial respiration consists in repeating this swaying motion backwards and forwards about 12 to 15 times a minute.
- (3) The dashing of cold water into the face will sometimes produce a gasp and start breathing, which should then be continued as directed above. If this is not successful the spine should be rubbed vigorously with a piece of ice. Alternate applications of heat and cold over the region of the heart will accomplish the same object in some instances. It is both useless and unwise to attempt to administer stimulants to the victim in the usual manner by pouring them down his throat.

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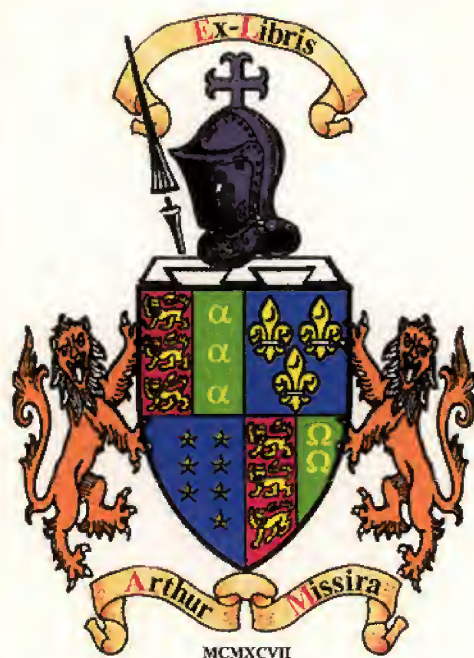
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1931

H.M. STATIONERY OFFICE